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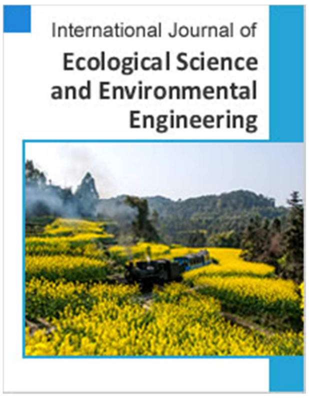


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# Moist Potential Vorticity as Diagnostic Tool of Rainfall Events in Tanzania

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## Abstract
This article, presents a first attempt to use Moist Potential Vorticity (MPV) in the diagnosis of rainfall events over different regions in Tanzania. The main purpose of the article is to compute and compare qualitatively the patterns of MPV derived from the gradient of the moist air entropy potential temperature ($\theta_s$) and heavy rainfall events that occurred over different regions in Tanzania on 20th December, 2011. Moreover, the article aims at assessing the relative contributions of horizontal and vertical components of MPV on detecting the location received heavy rainfall events. Wind speed, temperature, atmospheric pressure and relative humidity from the numerical output generated by the Weather Research and Forecasting (WRF) model were used to compute $\theta_s$ and MPV. Results indicated that MPV at 700 hPa exhibit positive values, over/near the areas received heavy rainfall due to local instability associated with possible continent/ocean contrast and continent/Lakes contrast. When compared with the observed rainfall, MPV provides fairly accurate tracking of location received rainfall, suggesting its potential use as a dynamic tracer of heavy rainfall events in Tanzania. Finally, it is found that in contrast to the mid-latitude and extra tropical regions where the vertical component of the MPV is larger than horizontal parts, in Tanzania which is located close to the Equator the horizontal parts are larger than the vertical part in the upper troposphere and the vertical part is larger than the horizontal parts in the lower troposphere.

## 1. Introduction

[1], defined potential vorticity (PV) as

$$PV(\theta) = \rho^{-1}\zeta_a \cdot \nabla(\theta), \tag{1}$$

where, $\rho$ is the density of air, $\zeta_a$ is the absolute vorticity and $\nabla(\theta)$ is the three dimension gradient of potential temperature. This definition emerged from fundamental concepts on circulation and vorticity laid in the work of pioneers like [2-6].

The PV thinking has many applications in meteorology, oceanography and aerodynamics [7]. It can be used to understand the dynamics and thermal conditions of atmospheric flow to the lower limit to the fineness of the structures that may occur, all the way down to the length scales on which molecular diffusion acts [7]. Many important synoptic scale processes can be understood within the framework of PV [7]. Recently, [8] mentioned that "PV is important for understanding balanced flow and thence a vast range of basic dynamical processes, such as Rossby-wave propagation and breaking and its many consequences, including, in the Earth's atmosphere, global-scale teleconnections, anti-frictional phenomena such as jet stream self-sharpening, and the genesis of cyclones, anticyclones and storm tracks, answering the child's age-old question of where the wind comes from".

The conservative and inevitability properties of PV form the bases of understanding many important atmospheric flow processes. The conservative nature applies for atmospheric flows in absence of friction and diabatic heating [9]. However, [10], generalized that even under diabatic heating, frictional and other forces, PV cannot be transport across any isentropic surface, it cannot be created or destroyed within a layer bounded by two isentropic surfaces but it can be advected as a conserved scalar quantity. Under inevitability principle, it is mentioned in [7] that "PV distribution and suitable temperature distribution at the lower atmosphere boundary is sufficient to deduce, diagnostically, all other dynamical field such as winds, temperature, geopotential heights, static stabilities and vertical velocities, under a suitable balance condition".

In spite of the fundamental usefulness of PV, already described above, it has not yet been used widely in operational weather forecasting community as a diagnostic variable of meteorological events. There are many possible reasons for this, including, a lack of awareness of the utility of PV in atmospheric studies and lack of diagnostic studies that can demonstrate that PV can provide useful information that cannot be obtained via more traditional means. The main purpose of this paper is to demonstrate the utility of PV in diagnosis of location specific rainfall events in Tanzania. Moreover the relative contributions of horizontal and vertical components of MPV in detection of location specific rainfall event are assessed. This study is of fundamental important to use PV fields in diagnosis of location specific rainfall in Tanzania which may be complicated to diagnose and predict due to complexity of dynamics in the tropics that are interdependent between convective, meso and large scale circulations. The complex topographical terrain of Tanzania even makes weather forecast and climate prediction to become difficult there [11]. To address the current needs of accurate location specific weather and climate information, some researchers example [12-13] have tried to explore the use of indigenous knowledge (IK) to provide location specific weather and climate forecast. IK is based on observation and monitoring behaviours of certain animals, birds, plants and insects (indicators) which may be linked with onset/cessation of rainfall season, and occurrence extreme weather event. However, the use of IK may not provide location specific weather and seasonal climate forecast, because the behaviour of the indicators used in IK may change and acquire different behaviours that may not represent the onset and cessation of rainfall or occurrence of any other weather event. Really, the use of IK indicators for provision location specific weather and climate forecast may be misleading and should not be advocated to be used by local farmers or any other decision makers. In this paper therefore a proper function of PV that describes the dynamics of dry and moist atmosphere is used in detection of location specific rainfall event.

The [1] PV formulation (Equation 1) is often used to study the thermodynamic properties of the atmosphere, while it is based only on dry-air potential temperature $\theta$. However, in moist atmosphere, [1] PV formulation is not conserved when latent heat release is taken into account [14-15]. In attempt to avoid the drawback of [1] formulation in moist atmosphere, [16] first defined a generalized formulation of PV by replacing dry-air potential temperature $\theta$ with equivalent potential temperature $\theta_e$ as

$$MPV(\theta_e) = \frac{1}{\rho}\zeta_a \cdot \nabla(\theta_e). \tag{2}$$

However, [17] have shown that still [16] formulation is conservative only in moist adiabatic and frictionless processes and its generalization to be used in dry atmosphere lead to annihilation of solenoidal term in PV tendency equation.

In fact the atmosphere has never been completely dry or saturated but it is non-uniformly saturated. Thus neither [1] or [16] can be used to study both dry and moist atmospheric flow and fulfil the demand to verify, at the same time, a moist and dry air conservative property and an invertibility principle. To overcome that drawback, [18] derived a Generalized Moist Potential Vorticity (GMPV) by replacing $\theta$ with a Generalized Potential Temperature (GPT) $\theta^*$, where GPT is written as

$$\theta^*(T, p, q) = \theta \; exp\left(\frac{Lq_s}{c_p T}\left(\frac{q}{qs}\right)^k\right), \tag{3}$$

where $q$ and $q_s$ is the specific humidity and saturated specific humidity respectively, $\left(\frac{q}{qs}\right)^k$ is a condensation probability function. It is noticeable that in the case of absolutely dry atmosphere where $q = 0$, equation 9 reduces to $\theta^*(T, p, q) = \theta$, while in completely saturated atmosphere where $q = q_s$ it reduces to $\theta^*(T, p, q) = \theta_e = \theta \; exp\left(\frac{Lq_s}{c_p T}\right)$. In more realistic atmosphere which is neither saturated nor un-saturated, the introduction of condensation probability function fixes the discontinuity of latent heat term due to the impact of water phase changes in the thermodynamic equation. Therefore a smooth transition from completely dry atmosphere and saturated atmosphere is achieved through the change of specific humidity from $q$ to $q_s$.

The GPT has been successfully tested in computation of MPV [15, 18-19], and found the solenoidal term does not cancel out in the MPV tendency equation in moist and dry atmosphere. However, [18] noted some limitations of applicability of the condensation density function in regions of no condensation or lower relative humidity conditions.

Recently [20] derived a new MPV using a specific entropy formulation expressed in terms of moist-air entropy potential temperature (denoted as $\theta_s$ in his paper). This version of potential temperature is valid for a general mixing of dry air, water vapour and all possible condensed water species. It is mentioned in [20] that "$\theta_s$ verifies the same conservative properties as the moist entropy, even for varying dry air or total water content. The moist formulation for $\theta_s$ is equal to the dry formulation $\theta$ if dry air is considered and it verifies new properties valid for the moist air cases, both saturated or under-saturated ones''. Based on the literature, reviewed in this study, moist air entropy potential temperature $\theta_s$ is the only one which represents the moist-air entropy in all circumstance, including in non-adiabatic and very moist regions. For these reasons, only MPV computed from replacing $\theta$ with $\theta_s$ in equation 1 should be used to represent the moist-air PV in the moist lower troposphere. Thus in this paper, the recent development of entropic potential temperature defined by [20] used to compute MPV and test qualitatively: -(1) the patterns of the Moist Potential Vorticity (MPV) derived from moist air entropic potential temperature as compared to heavy rainfall event occurred over different regions in Tanzania on 20$^{th}$-22$^{nd}$ December, 2011 and 6-8$^{th}$ May 2015 (2) the relative contribution of vertical and horizontal components of the MPV in diagnosis of rainfall event.

## 2. Method

### 2.1. Model Description, Experimental Design and Data from the Model

The Weather Research and Forecasting (WRF) model version 3.3.1, jointly developed by the National Oceanic and Atmospheric Administration (NOAA) and National Centre for Atmospheric Research (NCAR) is used in this study. This model is a non-hydrostatic mesoscale NWP model, fully compressible and has terrain following sigma coordinates [21]. The WRF is chosen in this study because it has been used by different researchers at government institution and universities and is currently used by the Tanzania Meteorological Agency (TMA) modelling section for provision of NWP. It features multiple dynamical cores, a 3-dimensional variation and 4- dimensional (3DVAR and 4DVAR) data assimilation system, and a software architecture allowing for computational parallelism and system extensibility. WRF is suitable for a broad spectrum of applications across scales ranging from meters to thousands of kilometres [22].

The domain of WRF is set-up over East Africa domain (8°N to -18°S; 25°E to 52°E) on a horizontal resolution of 15 km, with 28 vertical levels (Figure 1). The model is set to simulate heavy rainfall events that occurred from 20-22, December, 2011 and 5-8, May 2015 over different regions in Tanzania. The WRF is initialized with the National Centre for Environmental Prediction (NCEP) global forecast system (GFS) dataset with a (0.5°x0.5°) resolution. The input data has a time interval of three hours. The simulations were initiated at 00h UTC and run for 48 hours ahead and output was archived for every 1 hour. Since the focus of this paper is to compute PV and compare its pattern with observed rainfall data, other simulated meteorological variables: wind speed (zonal, meridional and vertical components), pressure, temperature and relative humidity at different pressure levels were used.

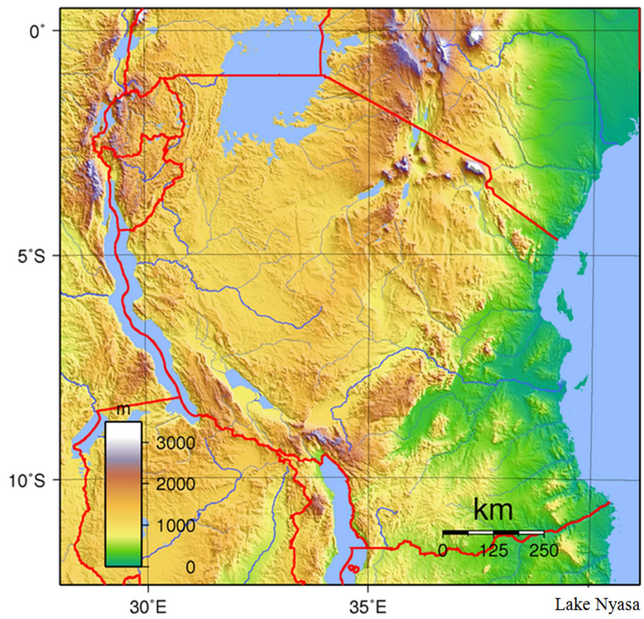


***Figure 1.*** *The topographical map in the domain where WRF was set in the study region.*

### 2.2. Observed Rainfall Data

Daily rainfall data on the 20 December, 2011 were collected from 25 synoptic weather stations managed by The Tanzania Meteorological Agency. The rainfall data were interpolated into the model grids and plotted using the GrADS graphical packages.

The moist air entropy potential temperature $\theta_s$, is define in [23] as

$$\theta_s \equiv (\theta_s)_1 \left(\frac{T}{T_r}\right)^{\lambda q_t} \left(\frac{p}{p_r}\right)^{-\kappa\delta q_t} \left(\frac{r_r}{r_v}\right)^{\gamma q_t} \frac{(1+\eta r_v)^{\kappa(1+\delta q_t)}}{(1+\eta r_r)^{\kappa\delta q_t}}, \quad (4)$$

$$(\theta_s)_1 = \theta \; exp(\Lambda_r \, q_t) \; exp\left(-\frac{L_v q_l + L_s q_i}{c_{pd} T}\right), \quad (5)$$

where $\Lambda_r = \frac{(s_v^0 - s_d^0)}{c_{pd}} \approx 5.87$ is a key quantity. It is mentioned in [23], that $\Lambda_r$, depends on the standard entropies of water vapour and dry air ($s_v^0$) and ($s_d^0$). It is also mentioned that $(\theta_s)_1$ is a good approximation of $\theta_s$. For detailed derivation of $\theta_s$, please refer Marquet, [20, 23-24].

From equation (1), $\psi$ can be replaced by $\theta_s$, leading to a PV equation which can be written as

$$PV(\theta_s) = \frac{1}{\rho} \, \zeta_a \cdot \nabla(\theta_s). \quad (6)$$

In pressure coordinate under hydrostatic balance, (6) can be written as

$$PV(\theta_s) = -\,g\,\zeta_a \cdot \nabla(\theta_s). \tag{7}$$

There are two quantities to calculate in (7). The first quantity is the three dimensional absolute vorticity $\zeta_a$ which is written as:

$$\zeta_a = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial p}\right) i - \left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial p}\right) j + (f+\zeta)k. \tag{8}$$

According to a general class of vorticity theorems described in [10], the components of vorticity normal to fixed plane surface cannot be transported across the surface (there is strong vortex induced by high-speed of the rotating earth), therefore there is no advection of the vertical vorticity in the horizontal. Therefore the contribution of $\frac{\partial w}{\partial y}$, and $\frac{\partial w}{\partial x} \approx 0$. The components of absolute vorticity in (8) reduces to

$$\zeta_a = \left(-\frac{\partial v}{\partial p}\right) i + \left(\frac{\partial u}{\partial p}\right) j + (f+\zeta)k, \tag{9}$$

where $\left(-\frac{\partial v}{\partial p}\right)$, $\left(\frac{\partial u}{\partial p}\right)$ and $(f+\zeta)$ are components of absolute vorticity normal to isentropic surface.

The second quantity to calculate is the gradient of moist entropic potential temperature

$$\nabla(\theta_s) = \left(\frac{\partial \theta_s}{\partial x}\right) i + \left(\frac{\partial \theta_s}{\partial y}\right) j + \left(\frac{\partial \theta_s}{\partial p}\right) k. \tag{10}$$

Therefore, (8) can be written as

$$PV(\theta_s) = -g \cdot \left(\left(-\frac{\partial v}{\partial p} i + \frac{\partial u}{\partial p} j + (f+\zeta)k\right) . \left(\frac{\partial \theta_s}{\partial x} i + \frac{\partial \theta_s}{\partial y} j + \frac{\partial \theta_s}{\partial p} k\right)\right). \tag{11}$$

Equation (11) has three terms:

Term 3 = $g\ \frac{\partial v}{\partial p}\frac{\partial \theta_s}{\partial x}$, Term 2 = $-g\ \frac{\partial u}{\partial p}\frac{\partial \theta_s}{\partial y}$, and Term 1 = $-g\ (f+\zeta)\ \frac{\partial \theta_s}{\partial p}$.

## 3. Results

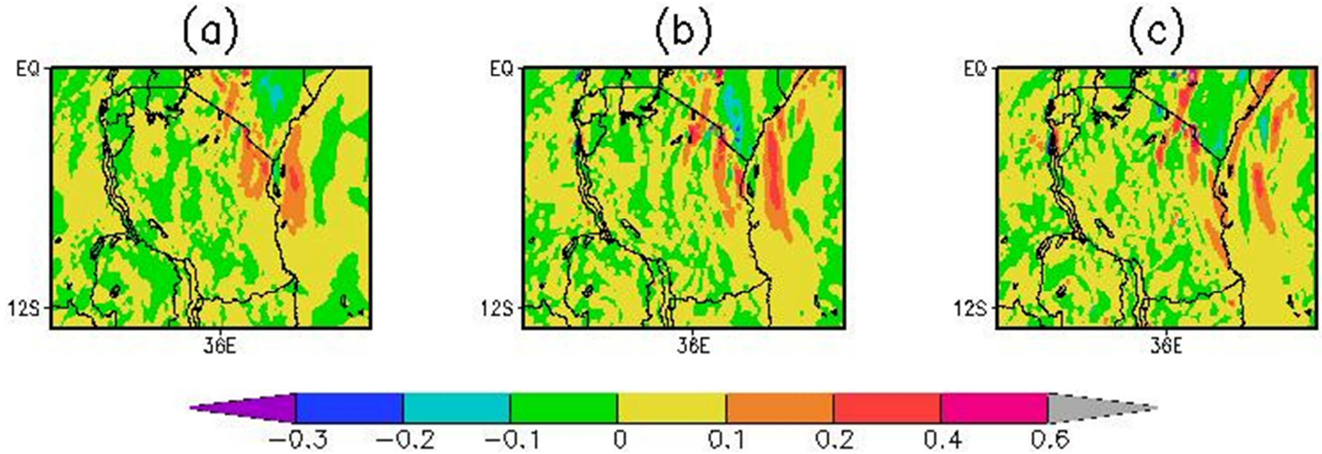


***Figure 2.*** *Zonal term of MPV (PV units) (a) from 00-06Z (b) 06-12Z(c) 12-18Z on the 20th December, 2011 at 700hPa.*

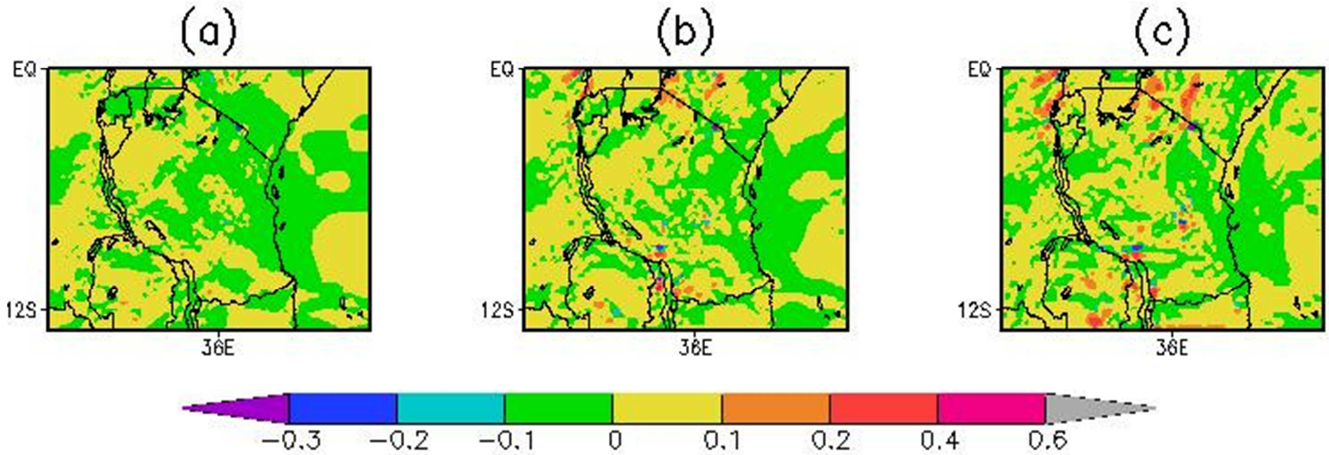


***Figure 3.*** *Meridional term of MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 700hPa.*

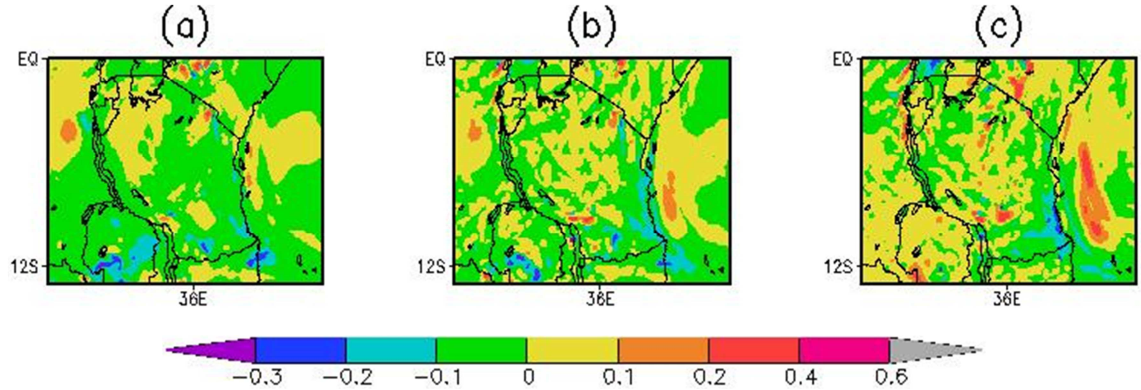


***Figure 4.*** *Vertical term of MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 700hPa.*

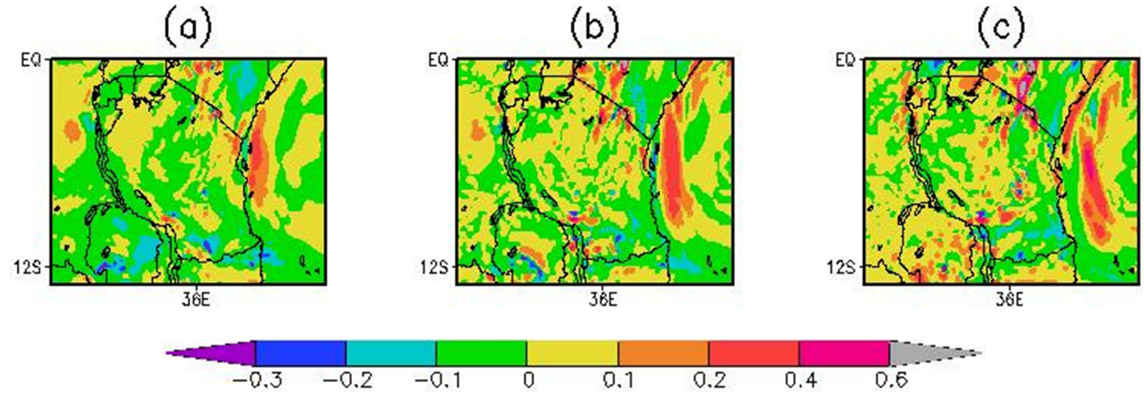


***Figure 5.*** *Total MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 700hPa.*

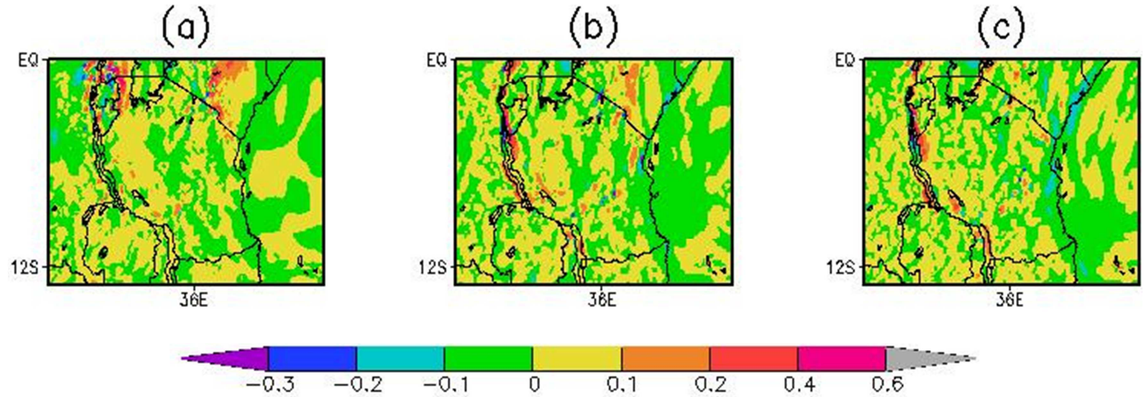


***Figure 6.*** *Zonal term of MPV (PV units) (a) from 00-06Z (b) 06-12Z(c) 12-18Z on the 20th December, 2011 at 850hPa.*

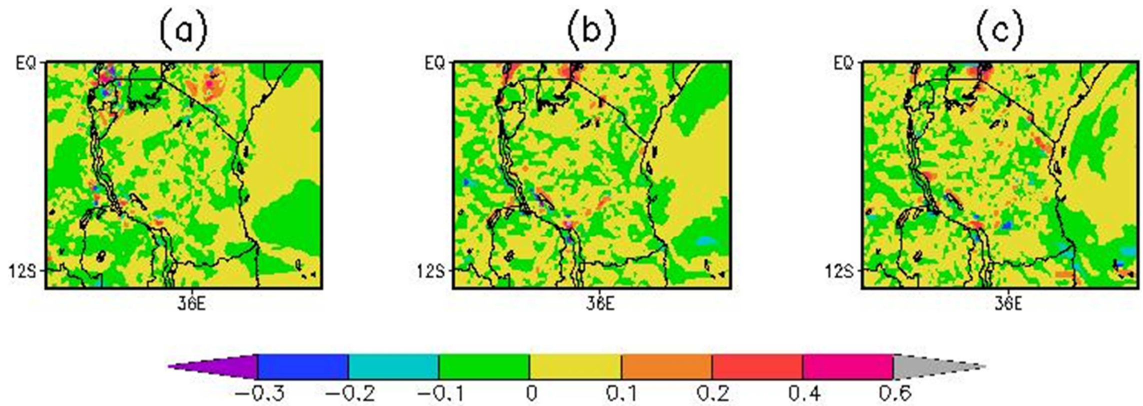


***Figure 7.*** *Meridional term of MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 850hPa.*

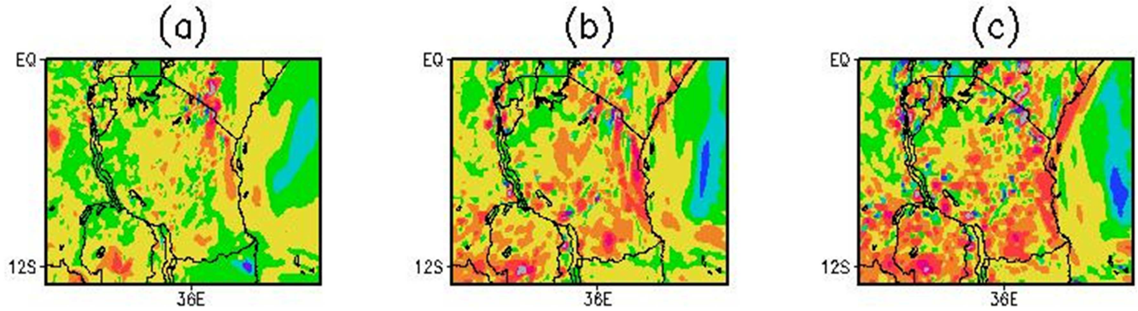


***Figure 8.*** *Vertical term of MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 850hPa.*

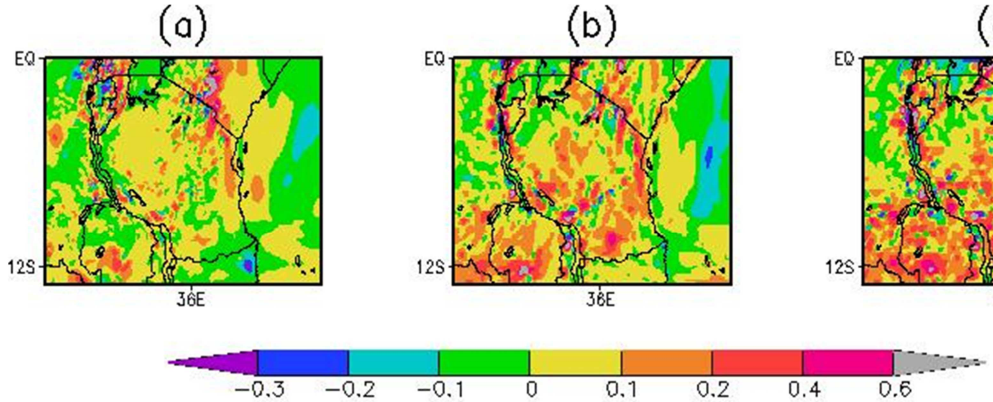


***Figure 9.*** *Total MPV (PV units) (a) from 00-06Z (b) 06-12Z (c) 12-18Z on the 20th December, 2011 at 850hPa.*

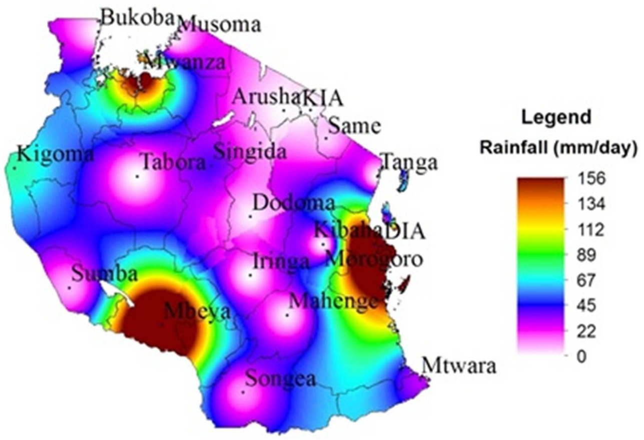


***Figure 10.*** *Distribution of observed rainfall over different regions in Tanzania on 20-12-2011.*

## 4. Discussion

In this paper, the MPV was computed using the recent new moist entropy potential temperature [23]. This temperature is linked to the second law of thermodynamics. It can be used to describe the thermodynamics of moist air which become active within the lower troposphere and this especially in the tropics and in equatorial regions. The zonal, meridional, vertical terms and the total MPV maps at 700hPa are presented in (Figure 2-5). The interesting features in these figures, especially figure 2 and 5 are the positive patterns of MPV over the northern coast of Tanzania and off the coast of Zanzibar close to Dar es Salaam where heavy rainfall was observed (Figure 10). The zonal, meridional, vertical terms and the total MPV maps at 850hPa are presented in (Figure 6-9). The vertical term and the total MPV maps indicate high positive values over coastal regions, especially over Dar es Salaam where heavy rainfall was observed. High positive values of MPV are observed in the southwestern highland. This may be the sources of heavy rainfall observed over those areas. It is important to note here that the vertical term of MPV close to the surface (925 and 850hPa) contribute much on the total MPV maps. While at 700hPa the zonal term contributed much on the total MPV map. Therefore based on the presented case study the dominant contributor on the total MPV is the zonal term. The meridional term shows little contribution on the total MPV.

## 5. Conclusion

This study is the attempt to diagnose rainfall event with the MPV in Tanzania. It is clear from the presented results that the positive values of MPV collocate with the area received rainfall events. In contrast to the mid and extra tropical regions, where the vertical term of MPV contribute much on the total MPV, in tropics the zonal term contribute much when compared to the vertical term in the upper the troposphere. The vertical term dominate over the horizontal terms in the lower troposphere. The distribution of positive values of MPV from vertical term and total MPV at 850hPa level collocates with the distribution of rainfall. The high positive values of zonal term of the MPV at 700hPa are found to be closer to the region received heavy rainfall. This suggests that the use of MPV to diagnose rainfall events require special examination of all terms especially at different pressure levels from the lower troposphere. The maximum values of MPV are associated with strong convective process that may influence or trigger the formation heavy rainfall. It is recommended that more case studies need to be done to validate the usefulness of MPV in operational weather forecasting. Moreover, contrary to the extra tropics the vertical term of MPV in the tropics does not explain the expected convective activities in throughout the troposphere. This is partly due to the small values of the Coriolis parameter and the small variation in the vertical temperature due to strong vertical mixing exists in the tropics. It is recommended that a modified version of MPV the moist potential vorticity vector (MPVV) defined by [25] better represent the convective activities and location received heavy rainfall events in the tropics. This is due to the fact that rainfall in the tropics in mainly convective in nature driven by the buoyancy and shear forces.

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