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Influence of Connection Flexibility on the Dynamic Capacity of Semi-rigid Frames

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Abstract: The dynamic response of semi-rigid frames is studied by using a computer program-MATLAB. The connection flexibility is modeled by rigidity index ρ, and advanced deflection-rotation equation is used to build the stiffness matrix of semi-rigid element and frames. Having the same geometry and cross-section semi-rigid frames with different indexes, are examined. The reducing coefficient and lateral rigidity values, representing the real behavior of these frames, are determined. To represent the real behavior, all deformations of a frame are accounted for a dynamic analysis with EI-centro wave. Response characteristics of those multistory frames are compared with reference to their model attributes. The study indicates that connection flexibility tends to increase vibration periods, while it causes vibration frequencies decrease.

Keywords: Semi-rigid, Rigidity Index, Deformation-Rotation Equation, Dynamic Response

1. Introduction

In the present days, the constructions become more and more ambitions and their structures more complex, at the same time, with the raise in the demand for increased safety measures. This is the most challenging to civil engineers, so the fact is as that, the increase in the load bearing capacity is usually associated with the decrease in ductility, the structures becoming sensitive to the seismic action.

Defining behaviors of frames under dynamic loads exactly takes an important place in earthquake engineering; the engineering must placed an emphasis on simulating the nonlinear response of a structural system to seismic action, to know the real behavior of a structure is provided by determining geometrical, damping, mass and connection model well. In fact, structures having flexible connections in which connections flexibility becomes important are called semi-rigid frames—the beam-to-column joints are neither pinned nor rigid.

The results of the damage produced by strong earthquakes, such as Northridge 1994, Japan 1996, Kobe Chi-Chi, Taiwan 1998, has shown that the hypothesis of rigid (fixed) or pinned modes is not correct and the real mechanical properties of beam-to-column connections must be taken into account while an accurate analysis is required.

2. Geometrical Nonlinear Finite Element Model for the Member with Semi-rigid Connection

Firstly, the relationship bending moment-relative must be chosen to represent true properties of semi-rigid connection. Relationship $M-\theta$ is repress test roughly the polynomial model by Lu [1]

$$R_k=\frac{dM}{d\theta}=(R_{ki}-R_p)\left(\frac{15000}{1+\frac{(R_{ki}-R_p)\theta^{\frac{1}{m}}}{M_u}}\theta^{\frac{1}{m}-1}+\frac{1}{1+\frac{(R_{ki}-R_p)\theta}{mM_u}}\right)+R_p \tag{1}$$

Where R_k is the rigidity of connection, R_{ki} is the original rigidity of joint, R_p is the connection rigidity while the plastic deformation just occurs, m is the shape coefficient of curve. M_u is the ultimate moment capacity of connection.

So that, the rigidity index ρ_i can be as

$$\rho_i = \frac{\theta'}{\theta} = \frac{1}{1 + \frac{(C - \frac{S^2}{C})\eta EI}{R_{ki} l}} \tag{2}$$

Secondly, the rotation-displacement equation which consider the influence of axial force are used to analysis the effect of connection stiffness on the capacity of frame. In past, the researcher not consider the influence of axial force more, but the inflection become more important with the complex and more size in buildings.

Rotation at two ends and axial displacement of a semi-rigid frame element is as in Figure 1:

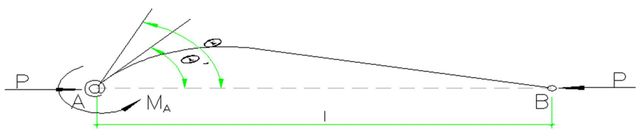


Figure 1. *Displacements at one end of a semi-rigid frame element.*

In Figure 1, the θ is total rotation at the end, θ' is the rotation occurred without spring at the end of element, the axial force in element is no considered, so the can be written by using

$$M_A = \frac{EI}{l}[S^*\theta_A + C^*\theta_B]; \; M_B = \frac{EI}{l}[C^*\theta_A + D^*\theta_B] \tag{3}$$

Where

$$S^* = \frac{12\rho_A}{4 - \rho_A\rho_B}, \; C^* = \frac{6\rho_A\rho_B}{4 - \rho_A\rho_B}, \; D^* = \frac{12\rho_B}{4 - \rho_A\rho_B} \tag{4}$$

$\rho_i (i = A, B)$ take as in eq. (2).

While the axial force in element is considered, the advanced deflection-rotation is as:

$$S^* = \frac{\rho_A (C^2 - S^2)(C - \frac{S^2}{C})}{(C^2 + \frac{\rho_A\rho_B S^4}{C^2} - S^2 - S^2\rho_A\rho_B)} = \frac{\rho_A C(C - \frac{S^2}{C})}{C - \frac{\rho_A\rho_B S^2}{C}}$$

$$C^* = \frac{S\rho_A\rho_B (C - \frac{S^2}{C})^2}{(C^2 + \frac{\rho_A\rho_B S^4}{C^2} - S^2 - S^2\rho_A\rho_B)} = \frac{\rho_A\rho_B S(C - \frac{S^2}{C})}{C - \frac{\rho_A\rho_B S^2}{C}} \tag{5}$$

$$D^* = \frac{\rho_B (C^2 - S^2)(C - \frac{S^2}{C})}{(C^2 + \frac{\rho_A\rho_B S^4}{C^2} - S^2 - S^2\rho_A\rho_B)} = \frac{\rho_B C(C - \frac{S^2}{C})}{C - \frac{\rho_A\rho_B S^2}{C}}$$

Where C and S must be

While element is compressed, $P < 0$:

$$C = \frac{\alpha l(\sin\alpha l - \alpha l\cos\alpha l)}{2(1 - \cos\alpha l) - \alpha l\sin\alpha l}, \; S = \frac{\alpha l(\alpha l - \sin\alpha l)}{2(1 - \cos\alpha l) - \alpha l\sin\alpha l} \tag{6}$$

Where $\alpha^2 = \frac{-P}{EI}$;

(1) while element is in tension, $P > 0$:

$$C = \frac{\alpha l(\alpha l\cosh\alpha l - \sinh\alpha l)}{2(1 - \cosh\alpha l) + \alpha l\sin\alpha l}, \; S = \frac{\alpha l(\sinh\alpha l - \alpha l)}{2(1 - \cosh\alpha l) + \alpha l\sinh\alpha l} \tag{7}$$

Where $\alpha^2 = \frac{P}{EI}$;

(2) While C=4, S=2, the eq. (5) transform to eq. (4)

So that, the following can be get

While $\rho_A = 0$, $\rho_B = 0$: $M_A = 0, M_B = 0$;

while $\rho_A = 0$, $\rho_B \neq 0$: $M_A = 0, M_B = \eta\frac{EI}{l}(C - \frac{S^2}{C})\rho_B\theta_B = R_{kB}(1 - \rho_B)\theta_B$;

(3) while $\rho_A = 1$, $\rho_B = 1$: $M_A = \eta\frac{EI}{l}(C\theta_A + S\theta_B)$, $M_B = \eta\frac{EI}{l}(S\theta_A + C\theta_B)$, it is typical traditional deflection-rotation equation.

So that, the eq. (3) can describe the process of member capacity with the varying of end restrict condition, which the pinned, fixed and semi-rigid connection types were combined in the equation。

3. Semi-rigid Frame Model and Stiffness Matrix

$$\left[K^s\right]=\begin{bmatrix} k_{11}^s & k_{12}^s & k_{13}^s & k_{14}^s & k_{15}^s & k_{16}^s \\ k_{21}^s & k_{22}^s & k_{23}^s & k_{24}^s & k_{25}^s & k_{26}^s \\ k_{31}^s & k_{32}^s & k_{33}^s & k_{34}^s & k_{35}^s & k_{36}^s \\ k_{41}^s & k_{42}^s & k_{43}^s & k_{44}^s & k_{45}^s & k_{46}^s \\ k_{51}^s & k_{52}^s & k_{53}^s & k_{54}^s & k_{55}^s & k_{56}^s \\ k_{61}^s & k_{62}^s & k_{63}^s & k_{64}^s & k_{65}^s & k_{66}^s \end{bmatrix}$$ where: $k_{11}^s=\dfrac{EA}{l}$, $k_{12}^s=k_{21}^s=k_{13}^s=k_{31}^s=0$, $k_{14}^s=k_{41}^s=-\dfrac{EA}{l}$,

$k_{15}^s=k_{51}^s=k_{16}^s=k_{61}^s=0$, $k_{22}^s=\dfrac{\eta EI}{l^3}\left[\left(S^*+2C^*+D^*\right)-(\alpha l)^2\right]$, $k_{23}^s=k_{32}^s=\dfrac{\eta EI}{l^2}\left(S^*+C^*\right)$, $k_{24}^s=k_{42}^s=0$,

$k_{25}^s=k_{52}^s=\dfrac{\eta EI}{l^3}\left[-\left(S^*+2C^*+D^*\right)+(\alpha l)^2\right]$, $k_{26}^s=k_{62}^s=\dfrac{\eta EI}{l^2}\left(D^*+C^*\right)$, $k_{33}^s=\dfrac{\eta EI}{l}S^*$, $k_{34}^s=k_{43}^s=0$,

$k_{35}^s=k_{53}^s=\dfrac{-\eta EI}{l^2}\left(S^*+C^*\right)$, $k_{36}^s=k_{63}^s=\dfrac{\eta EI}{l}C^*$, $k_{44}^s=\dfrac{EA}{l}$, $k_{66}^s=\dfrac{\eta EI}{l}D^*$, $k_{45}^s=k_{54}^s=k_{46}^s=k_{64}^s=0$,

$k_{55}^s=\dfrac{\eta EI}{l^3}\left[\left(S^*+2C^*+D^*\right)-(\alpha l)^2\right]$, $k_{56}^s=k_{65}^s=\dfrac{-\eta EI}{l^2}\left(D^*+C^*\right)$

These equations are derived while there aren't the loading on the part between the end of element. while there were loading on the part between the end of member, the fixed end moment must be consider as in Figure 2, the fixed end moment $\bar{M}_{FA}$, $\bar{M}_{FB}$ can be gotten with the offsetting the rotation of member end.

In Figure 2 (a)~(d), the relationship can be found as followings:

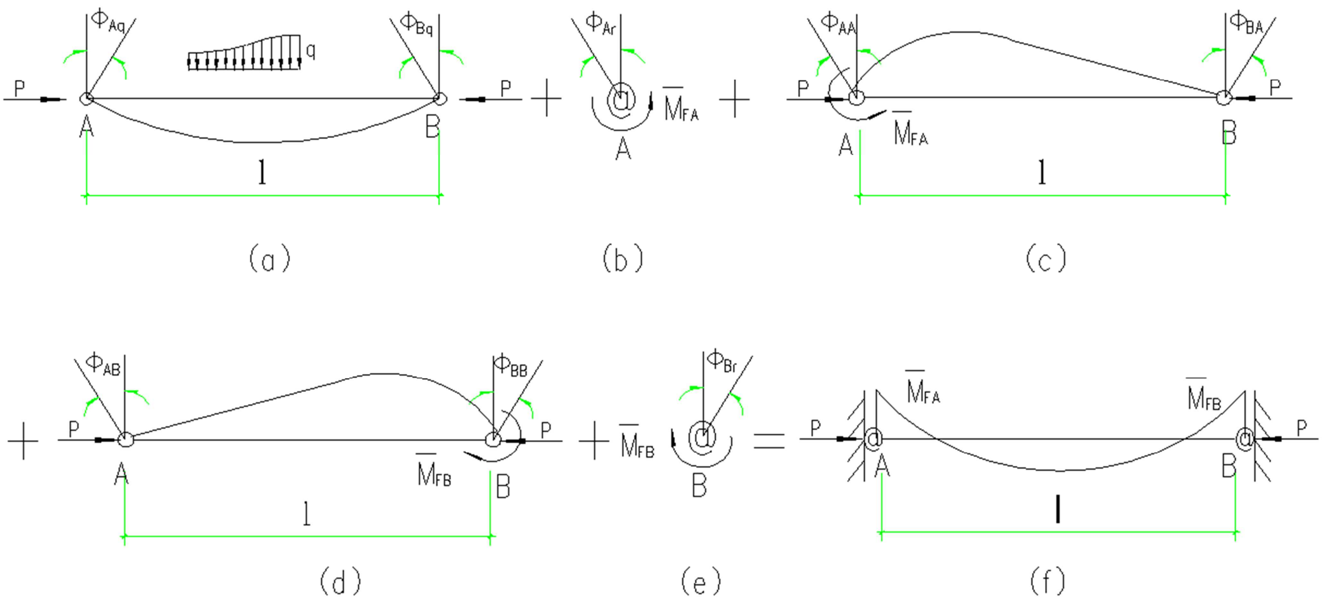


Figure 2. The fixed end moment.

For it at the end of A

$$\Phi_{Aq}=\frac{-M_{FA}C+M_{FB}S}{\left(C^2-S^2\right)\frac{\eta EI}{l}},\quad \Phi_{Ar}=\frac{\bar{M}_{FA}}{R_{kA}},\quad \Phi_{AA}=\frac{C\bar{M}_{FA}}{\frac{\eta EI}{l}\left(C^2-S^2\right)},\quad \Phi_{AB}=\frac{S\bar{M}_{FB}}{\frac{\eta EI}{l}\left(S^2-C^2\right)} \tag{8}$$

$$\Phi_{Aq}+\Phi_{Ar}+\Phi_{AA}+\Phi_{AB}=0 \tag{9}$$

With relationship showing in the eq (1), and taking the eq (8) into the eq. (9) then the following can be gotten:

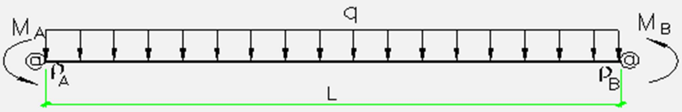
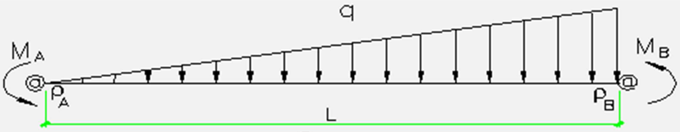
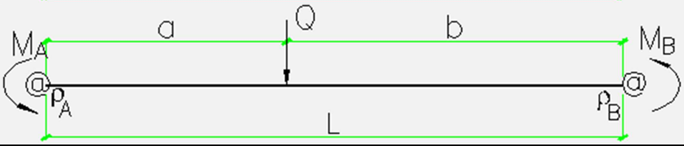
$$\bar{M}_{FA}=\frac{(C^2-S^2\rho_B)\rho_A M_{FA}-CS(1-\rho_B)\rho_A M_{FB}}{C^2-S^2\rho_A\rho_B} \tag{10}$$

And at the end B:

$$\bar{M}_{FB}=\frac{(C^2-S^2\rho_A)\rho_B M_{FB}-CS(1-\rho_A)\rho_B M_{FA}}{C^2-S^2\rho_A\rho_B} \tag{11}$$

Where, $\bar{M}_{FA}$, $\bar{M}_{FB}$ are the end moment which the end restrict condition is considered; M_{FA}, M_{FB} are the end moment without the connection.; ρ_i can be from the eq. (2).

Table 1. *End moment of element without axial force (L-length of element).*

Type of loading	End moment
	$\bar{M}_{FA}=\frac{qL^2}{12}[\frac{3\rho_A(2-\rho_B)}{4-\rho_A\rho_B}]$ $\bar{M}_{FB}=-\frac{qL^2}{12}[\frac{3\rho_B(2-\rho_A)}{4-\rho_A\rho_B}]$
	$\bar{M}_{FA}=\frac{qL^2}{30}[\frac{\rho_A(7-4\rho_B)}{4-\rho_A\rho_B}]$ $\bar{M}_{FB}=-\frac{qL^2}{60}[\frac{\rho_B(16-7\rho_A)}{4-\rho_A\rho_B}]$
	$\bar{M}_{FA}=\frac{\rho_A QL}{(4-\rho_A\rho_B)}\frac{a}{L}\frac{b}{L}[2(2-\frac{a}{L})-\rho_B(1+\frac{a}{L})]$ $\bar{M}_{FB}=\frac{\rho_B QL}{(4-\rho_A\rho_B)}\frac{a}{L}\frac{b}{L}[2(1+\frac{a}{L})-\rho_A(2-\frac{a}{L})]$

4. Mass matrix

The uniform mass matrix is used in the paper as:

$$[M]=\frac{\bar{m}L}{420D^2}\begin{bmatrix} 140D^2 & & & & & \\ 0 & 4f_1(P_1,P_2) & & \text{symmetry} & & \\ 0 & 2Lf_2(P_1,P_2) & 4L^2f_5(P_1,P_2) & & & \\ 70D^2 & 0 & 0 & 140D^2 & & \\ 0 & 2f_3(P_1,P_2) & Lf_4(P_1,P_2) & 0 & 4f_1(P_1,P_2) & \\ 0 & -Lf_4(P_1,P_2) & -L^2f_6(P_1,P_2) & 0 & -2Lf_2(P_1,P_2) & 4L^2f_5(P_1,P_2) \end{bmatrix} \tag{12}$$

The symbol in matrix can be found in Ref. [7].

5. Seismically Excital Analysis and Numerical Studies

In the present study, the seismically properties of semi-rigid frames and how connection flexibility influences them are investigated. The equation of motion for a semi-rigid frame in free vibration is given by

$$[M]\{\ddot{v}\}+[C]\{\dot{v}\}+[K]\{v\}=0 \tag{13}$$

where $\ddot{v}, \dot{v}, v$, respectively, acceleration and displacement of a structure.

The influence of connection flexibility on the frequency and period of a vibration will be studied by. eq. (13) and the motion equation for frame under the force is

$$[M]\{\ddot{v}\}+[C]\{\dot{v}\}+[K]\{v\}=p(t) \tag{14}$$

In the paper, story semi-rigid frames having different rigidness of semi-rigid connection and a rigid connection for the analysis are given in. All frames have the same physics property and material property to compare the influence of connection flexibility on dynamic characteristics under earthquake.

The lateral displacement was calculated for each frame.

The results of the conducted analysis are given for each model of vibration as below

Tale a 5-layer and 3-span framework as an example to do numerical analysis, for the effect of connection on the structural capacity. The structure is composed as in Figure 3

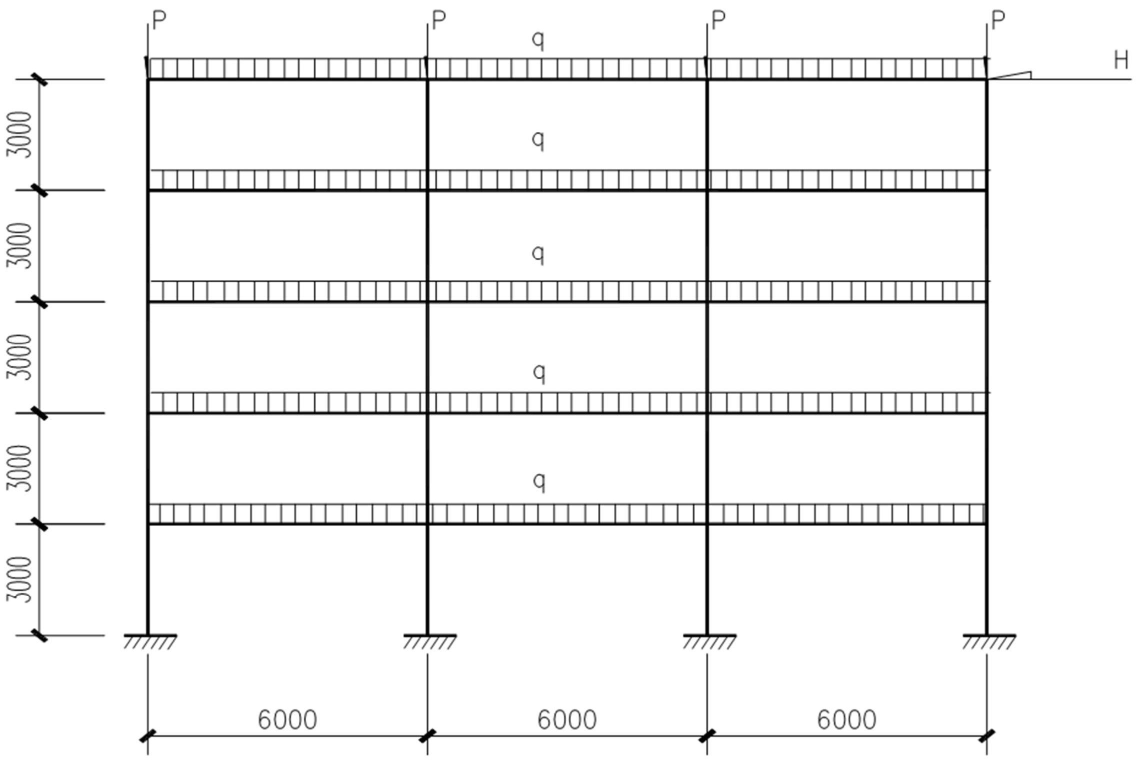


Figure 3. *Model of structural.*

In Figure 3, the side column is HM450x300, middle columns are HM500x300, the beams are all HN450x200, property of structural members is Q235, $P = 0.5P_y$, $P_y = A_{col} \times f_y$, where A_{col} --cross section of column, f_y --the yield strength of material..

The elastic reactions of top-layer with vary connection types are shown in Figure 5~8, they are with the spectrum of dynamic as shown in Figure 4.

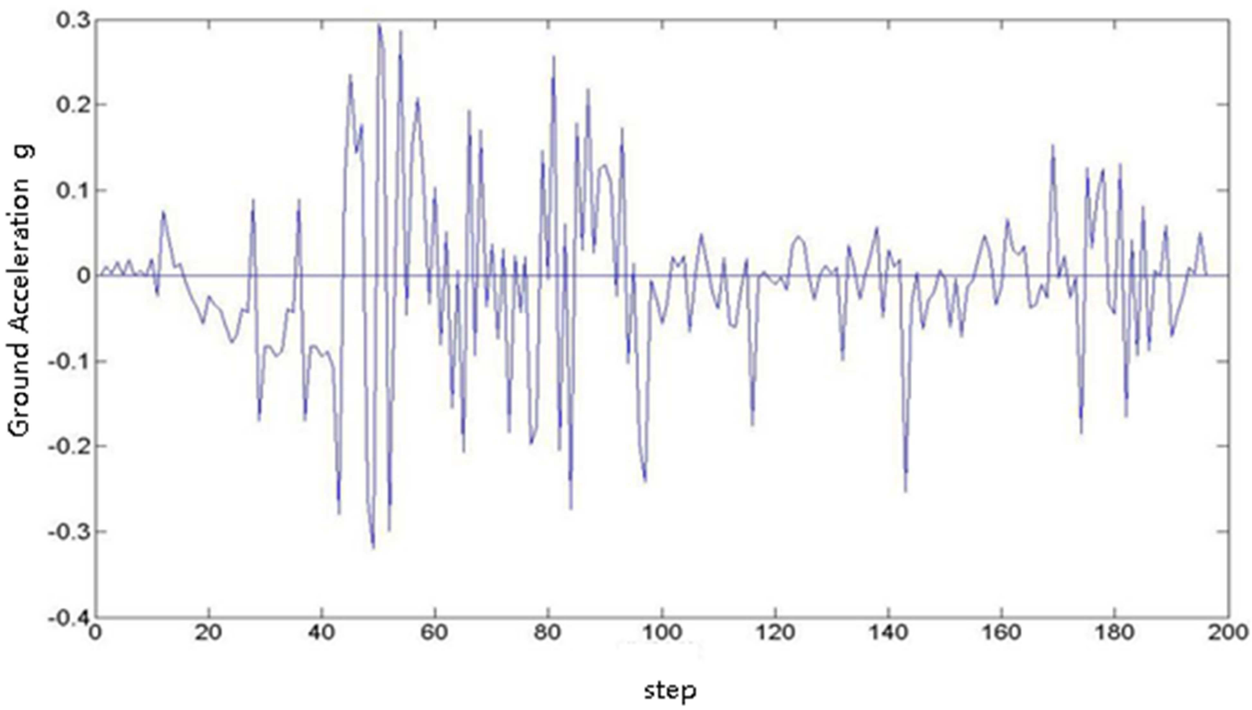


Figure 4. *Spectrum of dynamic.*

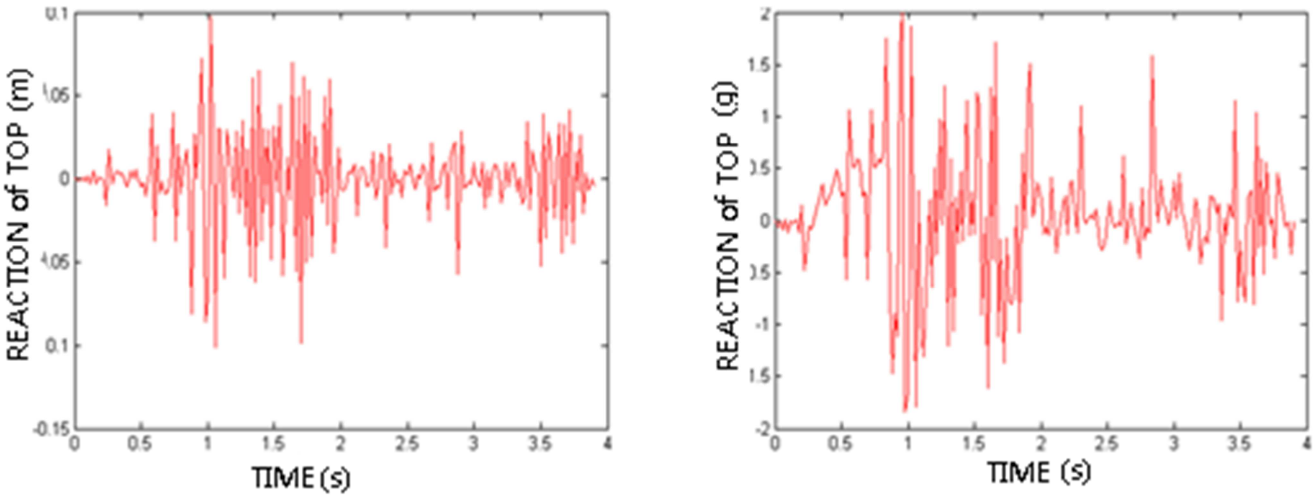


Figure 5. *Reaction with* $\rho = 1$.

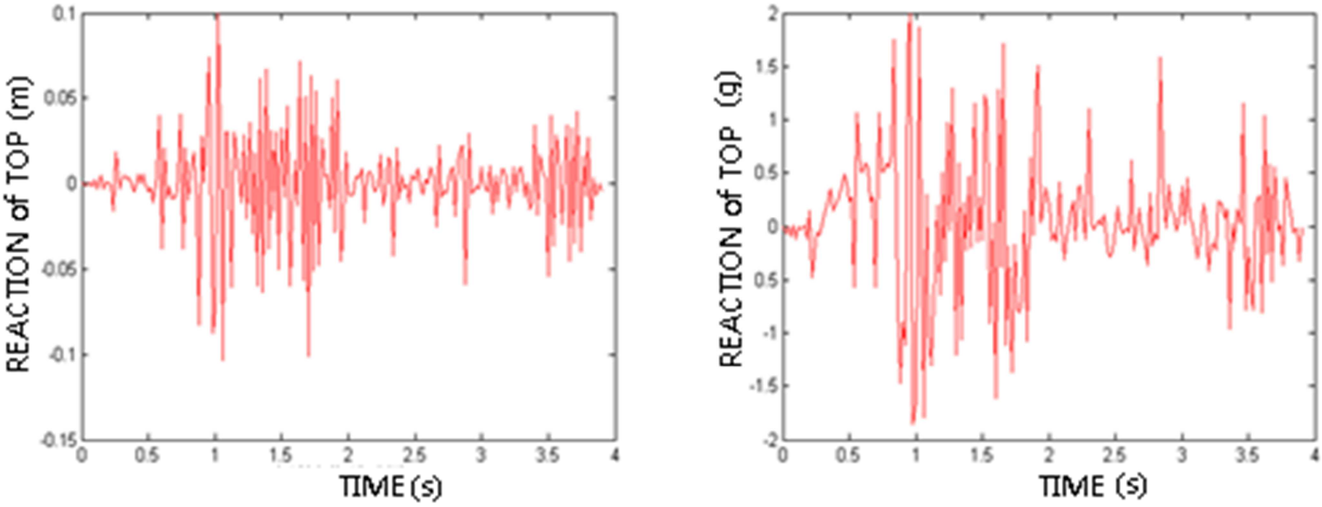


Figure 6. *Reaction with* $\rho = 0.8$.

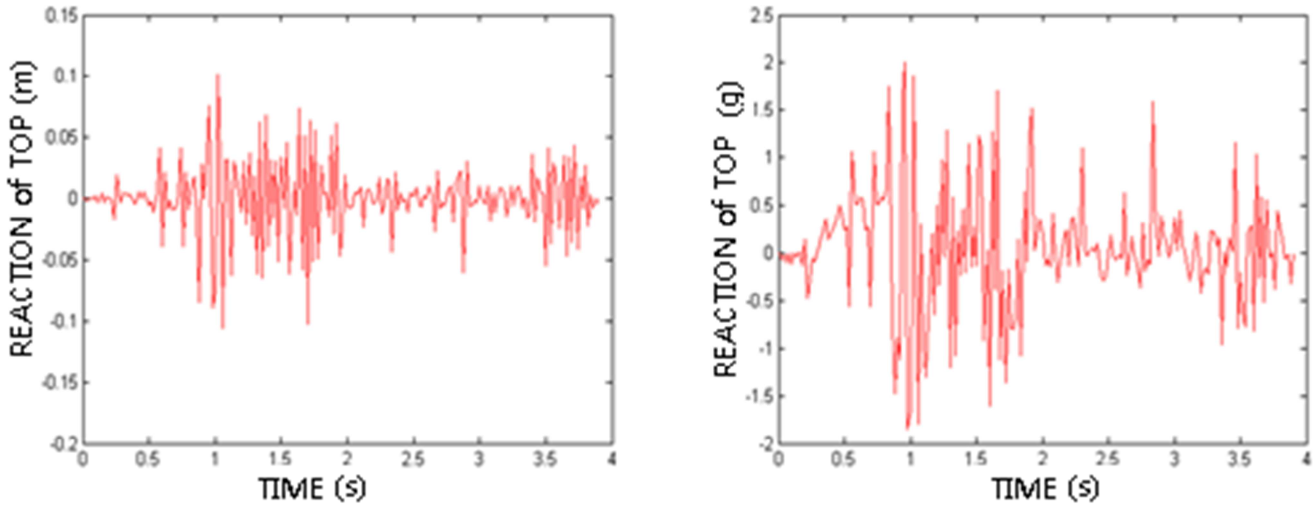
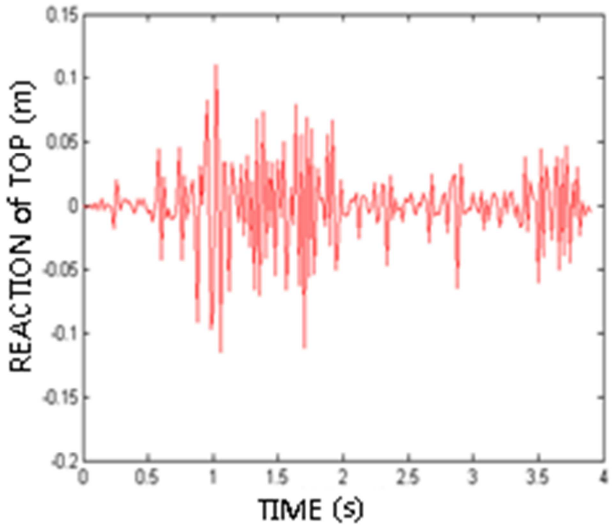


Figure 7. *Reaction with* $\rho = 0.6$.



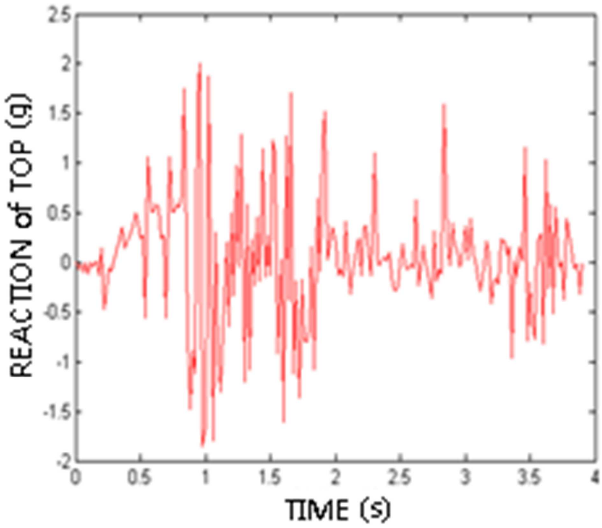


Figure 8. *Reaction with* $\rho = 0$.

Table 2. *The effect of connection on the capacity of structural.*

Coeff. of connection rigidness	The max. displacement of top (m)	The max. hor. Acc. of top (m/s^2)
$\rho = 1$	0.0975	1.995
$\rho = 0.9$	0.100	1.996
$\rho = 0.8$	0.100	1.9998
$\rho = 0.7$	0.100	2.0085
$\rho = 0.6$	0.1025	2.0002
$\rho = 0.5$	0.1025	2.0004
$\rho = 0.4$	0.105	2.0006
$\rho = 0.3$	0.105	2.0008
$\rho = 0.2$	0.1075	2.0010
$\rho = 0.1$	0.1100	2.0011
$\rho = 0.0$	0.1125	2.0075

It can be found that the connection has great effect on the characteristic of structural – the displacement increased, and natural period of vibration more longer with the decrease of connection rigidness

6. Conclusions

(1). The semi-rigid frames were modeled by rotational rigidness. The stiffness matrix was obtained by using rigidity index (ρ) at the ends of a semi-rigid frame element. A computer program was written to obtain the stiffness matrix with semi-rigid rigidity index (ρ). Dynamic analysis with EI centro was performed for different types of connection. The effects of connection flexibility were investigated.

(2). In the semi-rigid frame, the reducing coefficients for frames with lower rotational coefficients are lower than the reducing coefficients for frames with higher rotational.

(3). The dynamic behavior of a semi-rigid frame is great different from the dynamic behavior of a rigid connected frame. The study indicates that connection flexibility tends to increase periods especially in lower modes, while it tends to decrease the frequency.

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