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# Developing Mathematical Relationship Between Solar Reflectors and Solar Enhanced Waste Stabilisation Pond

**Onuchukwu Okwudili John Chrysostom, Agunwamba Jonah Chukwuemeka**

Department of Civil Engineering, University of Nigeria, Nsukka, Nigeria

## Email address
johncoolk13@gmail.com (O. O. J. Chrysostom)

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**Abstract:** Wastewater stabilization pond is a basin dug in the ground to treat waste water. It is simple to construct, operate and maintain. However, its limitation is that it requires large area of land to treat waste water efficiently. It had been observed by previous researchers that solar reflectors improve the efficiency of waste water stabilization pond. Hence, this study investigates the mathematical relationships between a reflector and a pond with purpose of improving designs of enhanced waste stabilization pond. To achieve this, a portable pyranometer device was made and was used to measure solar radiation at different points in the pond. The study considered the distribution of solar radiation in the pond with regard to the variation of the reflector's dimension. Also a mathematical relationship to describe the performance of the pond was also derived using regression analysis. The findings showed that the length and width of the reflected rays have a linear relationship with the length and width of the reflectors. The correlation factor for the variable length and width are 0.84 and 0.98 respectively. Also the distribution of solar radiation along the depth of the pond was observed to be curve like. The mathematical expression developed to describe the solar enhanced pond performance has a correlation factor of 0.87. It was verified with a correlation factor that is above 0.5 and slope in the range of 1.2 to 2.8.

**Keywords:** Waste Water Stabilization Pond, Waste Water, Pyranometer, Solar Radiation, Solar Reflectors

## 1. Introduction

Waste water stabilization pond (WSP) is a basin dug on the earth for removal of organic and pathogenic organisms [1]. It is more economical than conventional treatment plants [5]. It is simple to construct and cheap to maintain because it does not require any input of external energy [1]. However, one of its limitations is that it requires large area of land to function effectively [1].

Because of large area requirement of WSP, researchers has been working on this area of study to improve the efficiency of ponds using attached growth system [11, 9], step feeding [10], water hyacinth [7], hydraulic jump [2], hydraulic jump and solar reflectors [3], solar reflectors [4].

From the past studies reviewed, it is indisputable that solar reflector improves the efficiency of WSP. However, from their reviewed studies there has not been any mathematical relationship shown between the reflector and the pond. Therefore, the main objective of this research is to develop mathematical relationships between the reflector and pond.

## 2. Method

### 2.1. Experimental Set Up

Different sets of solar enhanced ponds with dimensions are shown in Table 1 and Figure 1. It has a sewage tank (1.2m×1.2m×0.6m) that received its influent from sewage that was diverted temporary to a ditch. Half inches diameter inlet pipes were fitted centrally in the experimental ponds. The outlet pipes were also fitted centrally in the experimental ponds. This setup was situated at the sewage plant in University of Nigeria, Nsukka.

### 2.2. Solar Radiation Meter Setup

A device for measuring solar radiation was set up to suit this study and calibrated with the device in the Department of

Geography University of Nigeria Nsukka. The data for calibration is presented in Appendix (Table A2). The circuit diagram and picture of the device are shown in Appendix (Figures A1 and A2 respectively). The device consists of sensor unit, display unit (16×2 liquid crystal display), control unit and power unit. The sensor unit consists of operational amplifier and BL – L3522PD sensor. The sensor detects electromagnetic spectrum in the range of 190nm-1100nm. The control unit is made up of PIC16F877A micro controller and all powered by a 9 volt DC battery.

### 2.3. Solar Reflector Setup

A set of three ponds were constructed with frames to carry reflector of varying length fixed at angle $90^0$ to horizontal plane. The aim of this setup is to investigate the distribution of solar radiation on the surface of the pond and to develop a mathematical relationship describing the performance of solar enhanced waste pond. The dimensions of the reflectors are shown in Table 1. The reflectors are made of a flat ceiling board wrapped with foil paper. The foil paper was to act as solar reflector with each of the three ponds having one reflector each at the outlet position (west facing)

### 2.4. Radiation Along the Length of the Pond

A rope is tied horizontal along the center of the tank from one end to the other end. The rope is calibrated to the nearest 5 cm mark, the points the radiation reading was taken. Radiation reading was taken from the two calibrated ponds. One of the ponds is fitted with a reflector at the effluent side of the pond and the other has no reflector (control test). The difference in reading was taken as the reflected solar radiation as a result of the reflector.

### 2.5. Radiation Along the Width of the Pond

The end side of the pond is calibrated to mark the points the radiation reading will be taken. Radiation reading is taken with reflector fitted at the effluent side of the pond and when there is no reflector. The difference in reading is recorded as solar radiation as a result of the reflector.

### 2.6. Radiation Along the Depth of the Pond

The device as shown in Figure A2 has a glass tube that is calibrated from the sensor to the upper part of the tube in cm. The radiation reading in the wastewater was taken by inserting the calibrated tube in water to correspond to the respective depth and the corresponding radiation reading in the device was recorded. Different sizes of reflectors were inserted at the end of the pond to study the effect of variation of sizes of the reflectors with the penetration of solar radiation in the pond.

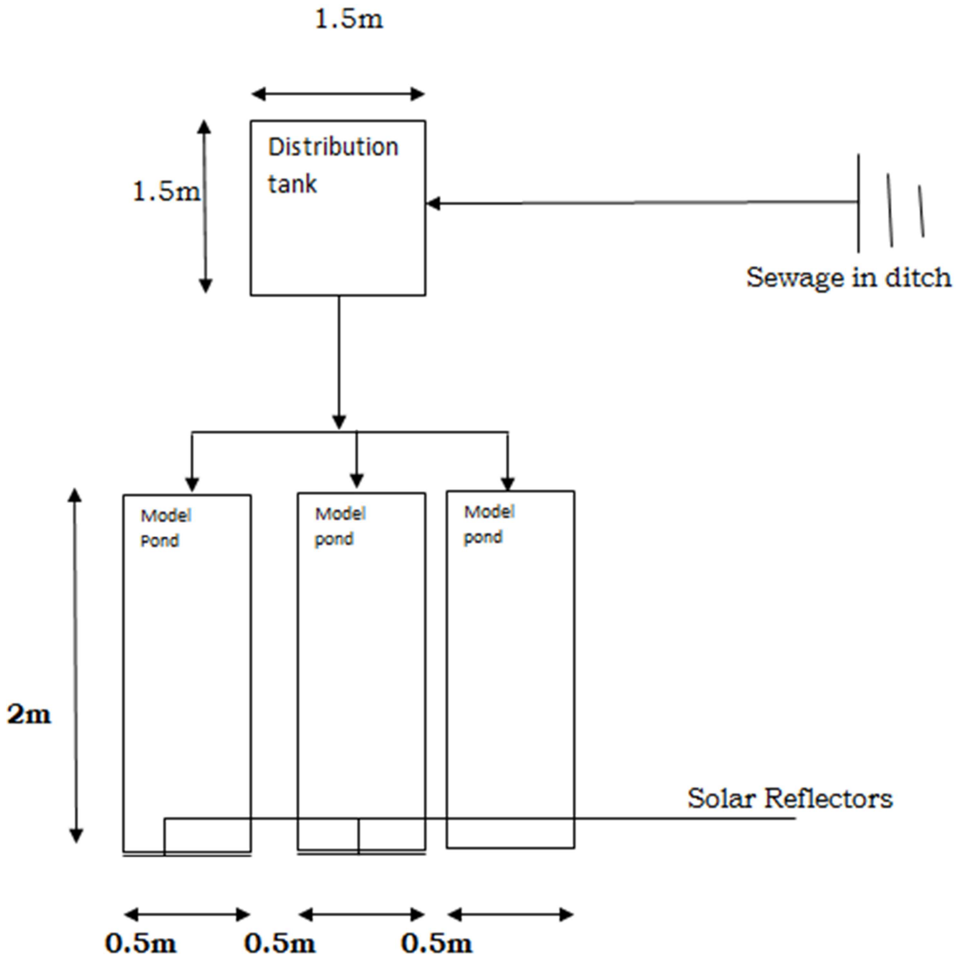


***Figure 1.*** *Schematic Diagram of Experimental Setup.*

***Table 1.*** *Detailed Description of Ponds and Its reflectors.*

| Experimental Pond | Size of Pond (m) | Size of Reflector (m) | Purpose |
|---|---|---|---|
| A | 2 × 0.5 × 0.3 | 0.4×0.1 (wxh) | To study the distribution of solar radiation |
| B | 2 × 0.5 × 0.3 | 0.3×0.1 (wxh) | To study the distribution of solar radiation |
| C | 2 × 0.5 × 0.3 | 0.2×0.1 (wxh) | To study the distribution of solar radiation |
| D | 2 × 0.5 × 0.3 | 0.1×0.1 (wxh) | To study the distribution of solar radiation |
| E | 2 × 0.5 × 0.3 | 0.1×0.2 (wxh) | To study the distribution of solar radiation |
| F | 2 × 0.5 × 0.3 | 0.1×0.3 (wxh) | To study the distribution of solar radiation |
| G | 2 × 0.5 × 0.3 | 0.1×0.4 (wxh) | To study the distribution of solar radiation |
| H | 2 × 0.5 × 0.3 | No Reflector | To study the distribution of solar radiation |
| I | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| J | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| K | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| L | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| M | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| N | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |
| O | 1 × 0.25 × 0.1 | 0.25× 0.1 (w×h) | To develop mathematical relationship between the pond and the reflector |

# 3. Result

## 3.1. Distribution of Radiation Along the Surface of the Solar Enhanced Pond

Figures 2 to 6 are graphs of reflected radiation as a result of varied height of reflectors versus distance along the length of the pond. The reflected radiation is the difference in radiation reading when there is reflector and no reflector. Also Figures 7-10 showed the reflected radiation along the width of the pond due to varied width of the reflectors.

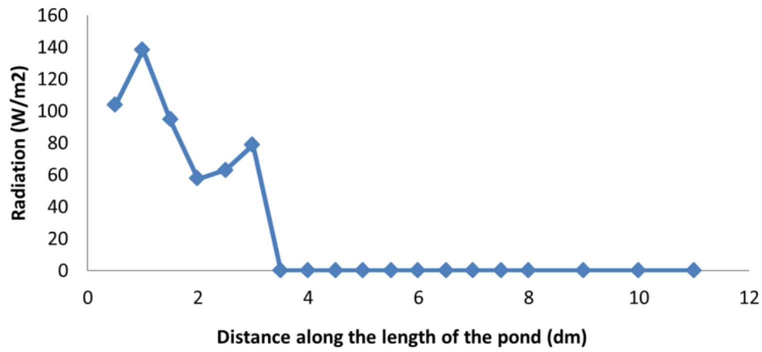


***Figure 2.*** *Solar Radiation along the Longitudinal Surface of the Pond with Reflector 0.1×0.1.*

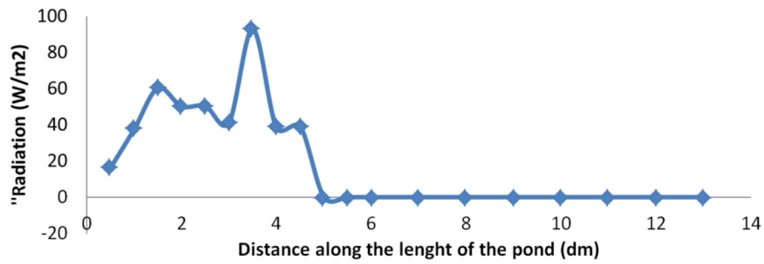


***Figure 3.*** *Solar Radiation along the Longitudinal Surface of the Pond with Reflector 0.2×0.1 (hxw).*

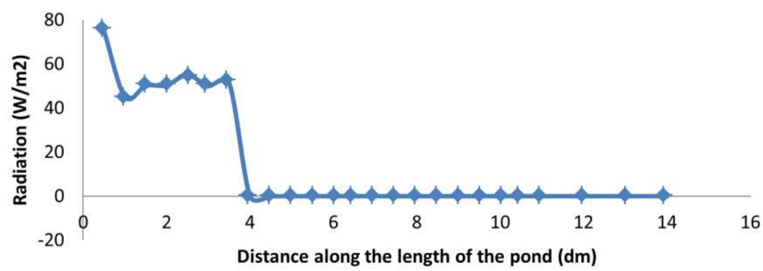


***Figure 4.*** *Solar Radiation along the Longitudinal Surface of the Pond with Reflector 0.3×0.1 (hxw).*

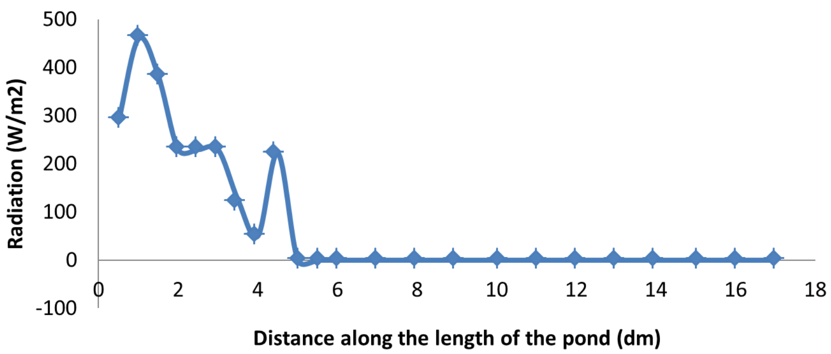


***Figure 5.*** *Solar Radiation along the Longitudinal Surface of the Pond with Reflector 0.4×0.1 (hxw).*

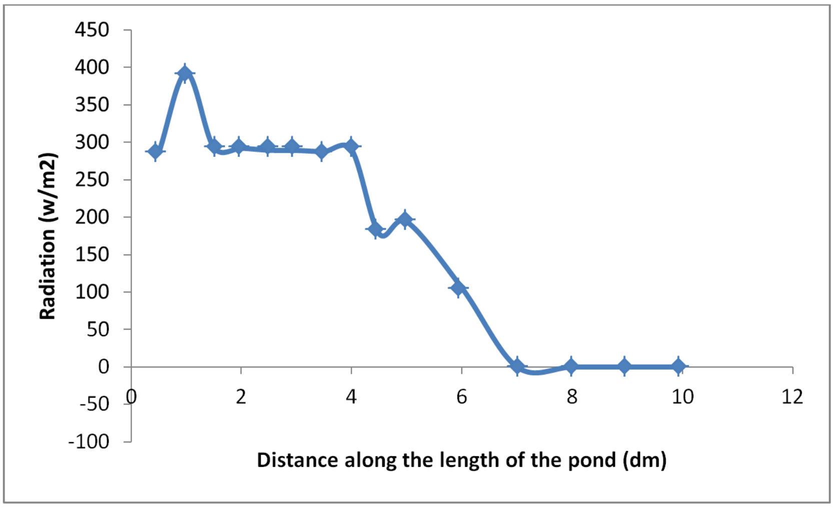


***Figure 6.*** *Solar Radiation along the Longitudinal Surface of the Pond with Reflector 0.6×0.1 (hxw).*

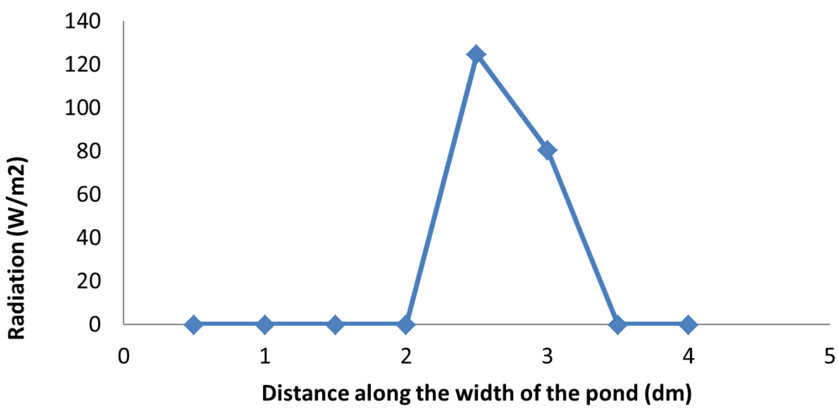


***Figure 7.*** *Solar Radiation along the Breadth Wise Surface of the Pond with Reflector 0.1×0.1.*

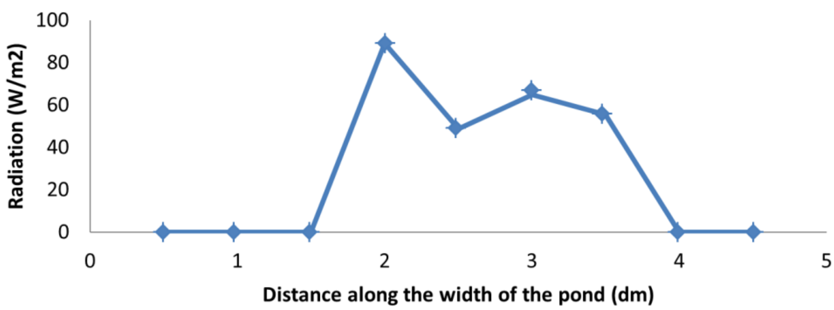


***Figure 8.*** *Solar Radiation along the Breadth Wise Surface of the Pond with Reflector 0.2×0.1 (WXH).*

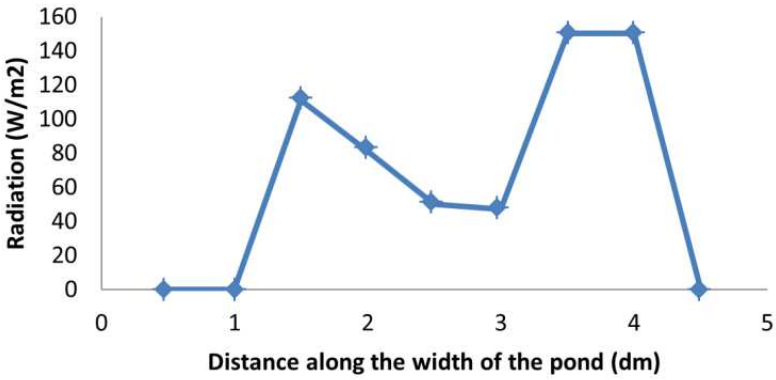


***Figure 9.*** *Solar Radiation along the Breadth Wise Surface of the Pond with Reflector 0.3×0.1 (wxh).*

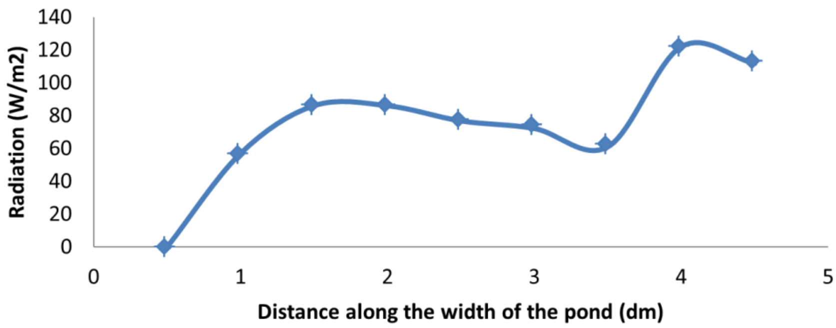


***Figure 10.*** *Solar Radiation along the Breadth Wise Surface of the Pond with Reflector 0.4×0.1 (wxh).*

## 3.2. Distribution of Solar Radiation Along the Depth of the Pond

The graphs of solar radiation against depth in wastewater are shown from Figures 11 to 15.

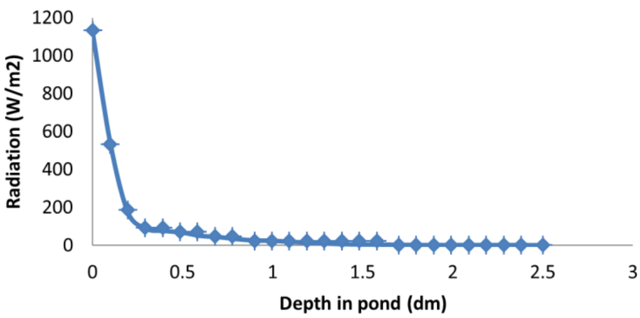


***Figure 11.*** *Solar Radiation along the Depth of the Pond with Reflector 0.4×0.1 (hxw).*

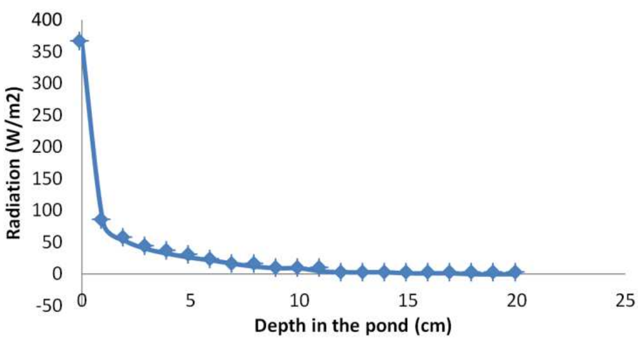


***Figure 12.*** *Solar Radiation along the Depth of the Pond with Reflector 0.1×0.1 (hxw).*

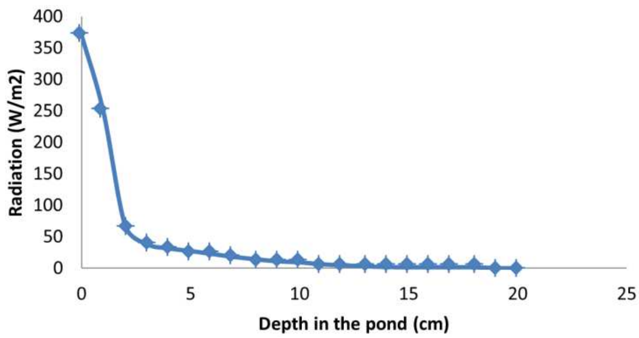


***Figure 13.*** *Solar Radiation along the Depth of the Pond with Reflector 0.2×0.1 (hxw).*

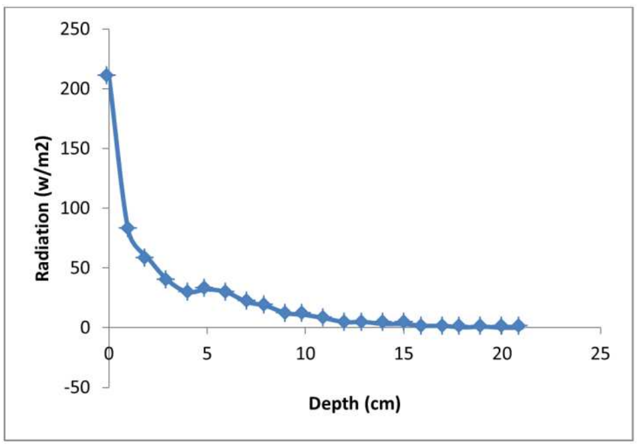


***Figure 14.*** *Solar Radiation along the Depth of the Pond with no Reflector.*

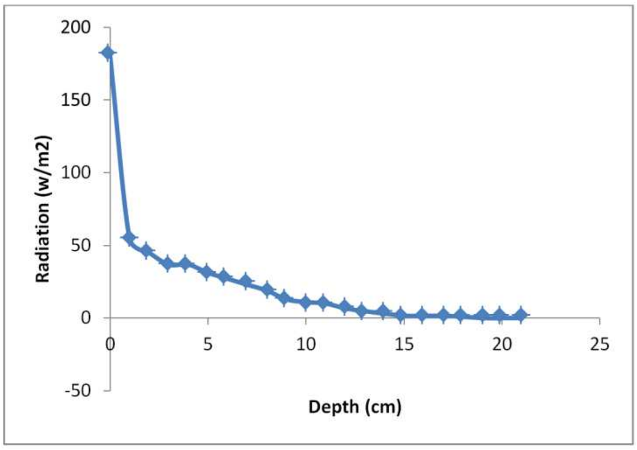


***Figure 15.*** *Solar Radiation along the Depth of the Pond with no Reflector.*

# 4. Discussion

## 4.1. Distribution of Solar Radiation on the Surface of the Wastewater

The length of reflected radiation and the corresponding height of the reflector were tabulated in Table 2. In the same way, the width of the reflected radiation and corresponding width of the reflector were tabulated in Table 3.

The relationship between the distance of reflected radiation and the height of the reflectors was obtained with regression analysis from Table 2 having correlation coefficient of (R) = 0.84 and it is shown in Equation 1

$$D = 0.426\ R_l + 0.334 \quad (1)$$

Where:

D is the length of reflected radiation along the length of pond

$R_l$ is the height of reflector

Using regression analysis method in the Table 3, the relationship between width of reflector and distance of reflected radiation along the width of the pond is shown in Equation 2 with a correlation coefficient of (R) = 0.985.

$$W = 0.85\ R_w + 0.075 \quad (2)$$

Where:

W is the distance of reflected radiation along the width of the pond

$R_w$ is the width of reflector

***Table 2.*** *Length of Reflected Radiation with Corresponding Length of Reflector.*

| S/no | Length of reflector | Length of reflected radiation |
|---|---|---|
| 1 | .10 | .35 |
| 2 | .20 | .50 |
| 3 | .30 | .40 |
| 4 | .40 | .50 |
| 5 | .60 | .60 |

***Table 3.*** *Width of Reflected Radiation with Corresponding Width of Reflector.*

| S/no | Width of reflector | Width of reflected radiation |
|---|---|---|
| 1 | .10 | 0.15 |
| 2 | .20 | 0.25 |
| 3 | .30 | 0.35 |
| 4 | .40 | 0.4 |

## 4.2. Distribution of Solar Radiation Along the Depth of the Wastewater

The shape of the graphs as shown from Figures 11 to 15 are curves indicating reduction in solar radiation with depth. A similar shape of graphs was observed by [6] when solar radiation was plotted against depth in ordinary water. It was demonstrated that light intensity decreases with concentration and depth according to the beer – Lambert law.

From aforementioned figures, it will be observed that near the surface of the waste water, there is higher reduction of solar radiation than deeper part of the waste water. This is because most solar energy are absorbed at near the surface of the waste water which will led to increase in temperature that will aid in biological reaction. In fact more than 90% of solar radiation is transformed into heat and less than 10% into chemical energy [8]. The remaining 10% that is converted to chemical energy represent the amount that is absorbed by algae for photosynthesis activities. This in return reduces the solar energy penetration in the pond.

However, actual transmission of solar radiation will vary from pond to pond depending on the water type. In an 8m deep facultative pond in Murcia, Spain, light penetration was limited to the top 0.5m when algal and bacterial populations were blooming. However, when algivorous zooplankton were blooming, a deeper penetration of down to 2m was reported [12].

It was also observed that the variation of sizes of reflector (Figures 11-13) has no effect on the penetration of the solar radiation in the pond. Penetration of solar radiation in the pond seems to depend on water type and incident solar radiation in the pond.

The distribution of solar radiation in the pond along the depth of the pond is describe by equation 3 with a correlation coefficient of ($R^2$) = 0.99

$$R_i = R_o(1 - 1.1614h^{0.0844}) \quad (3)$$

Where: $R_o$ is the radiation at the surface of the pond water

$R_i$ is the radiation corresponding with the depth of pond water

h is the depth of the pond water

from non linear regression analysis with data from Figures 11 to 15.

Different values of atmospheric radiation and corresponding surface radiation were computed and tabulated in Table A1. The average reduction of atmospheric solar radiation on the surface ($R_o/R_a$) of waste water is 0.622 as shown in Table A1.

$$R_i = R_a 0.622(1 - 1.1614h^{0.0844}) \quad (4)$$

$R_a$ is the atmospheric radiations

### 4.3. Mathematical Relationship Between the Area of Reflector and the Efficiency of the Solar Enhanced Waste Water Stabilization Pond

***Table 4.*** *The Values of the Ratio of the Area of Reflector and Pond with the Corresponding Increase in Efficiency of Solar Enhance Waste Stabilization Pond.*

| Area of reflector ($m^2$) | Area of Reflector/Area of Pond | Height (m) | Sample | Total Coliform (MPN/100ml) | Efficiency of Total Coliform | Difference in Efficiency |
|---|---|---|---|---|---|---|
| | | | Outlet with ref | 150 | | |
| 0.15×0.25 | 0.15 | .1 | Outlet with no ref | 1100 | 0.938<br>0.542 | 0.396 |
| | | | Inlet | 2400 | | |
| | | | Outlet with ref | 460 | | |
| 0.07×0.25 | 0.07 | .1 | Outlet with no ref | 1100 | 0.808<br>0.542 | 0.266 |
| | | | Inlet | 2400 | | |
| | | | Outlet with ref | 210 | | |
| 0.2×0.25 | 0.2 | .1 | Outlet with no ref | 1100 | 0.913<br>0.542 | 0.371 |
| | | | Inlet | 2400 | | |
| | | | Outlet with ref | 210 | | |
| 0.4×0.25 | 0.4 | .1 | Outlet with no ref | 1100 | 0.988<br>0.542 | 0.371 |
| | | | Inlet | 2400 | | |
| | | | Outlet with ref | 150 | | |
| 0.5×0.25 | 0.5 | .1 | Outlet with no ref | 1100 | 0.938<br>0.542 | 0.395 |
| | | | inlet | 2400 | | |
| | | | Outlet with ref | 28 | | |
| 0.6×0.25 | 0.6 | .1 | Outlet with no ref | 1100 | 0.988<br>0.542 | 0.446 |
| | | | inlet | 2400 | | |
| | | | Outlet with ref | 28 | 0.988 | |
| 0.7×0.25 | 0.7 | .1 | Outlet with no ref | 1100 | 0.542 | 0.446 |
| | | | inlet | 2400 | | |
| | | | Outlet with ref | 28 | | |
| 0.8×0.25 | 0.8 | .1 | Outlet with no ref | 1100 | 0.988<br>0.542 | 0.446 |
| | | | inlet | 2400 | | |
| | | | Outlet with ref | 21 | | |
| 1.0×0.25 | 1.0 | .1 | Outlet with no ref | 1100 | 0.991<br>0.542 | 0.449 |
| | | | inlet | 2400 | | |
| | | | Outlet with ref | 120 | | |
| 1.0×0.3 | 1 | .2 | Outlet with no ref | 240 | .891<br>.782 | 0.11 |
| | | | Inlet | 1100 | | |
| | | | Outlet with ref | 210 | | |
| 1.0×0.3 | 1 | .2 | Outlet with no ref | 460 | .81<br>.58 | 0.23 |
| | | | Inlet | 1100 | | |
| | | | Outlet with ref | 240 | | |
| 1.0×0.3 | 1 | .2 | Outlet with no ref | 460 | .78<br>.58 | 0.20 |
| | | | Inlet | 1100 | | |
| | | | Outlet with ref | 240 | | |
| 1.0×0.3 | 1 | .2 | Outlet with no ref | 460 | .78<br>.58 | 0.20 |
| | | | inlet | 1100 | | |

Applying non linear regression analysis from data in Table 4, Equation 5 was derived with a correlation coefficient of (R) = 0.923

$$E = 0.023\, h^{-1.302} (A_r/A_p)^{0.167} \quad (5)$$

$$E = \left(1 - \frac{C_{er}}{C_{ir}}\right) - \left(1 - \frac{C_{en}}{C_{in}}\right)$$

By substitution with mixed flow equation

$$\left(1 - \frac{C_{er}}{C_{ir}}\right) = E + 1 - \frac{1}{(kt+1)^n}$$

$$\frac{C_{er}}{C_{ir}} = \frac{1}{(kt+1)^n} - 0.023h^{-1.302}\left(\frac{A_r}{A_p}\right)^{0.167} \quad (6)$$

### 4.4. Verification of the Model

Figures 16-18 shows the verification of the Equation 6. The measured values of total coliform from model ponds were plotted against the bacteria degradation of corresponding theoretical model. The figures showed that the slopes were in range of 2.6 to 2.9 and the intercepts varied from 0.09 to 0.25. The correlations are all above 0.5.

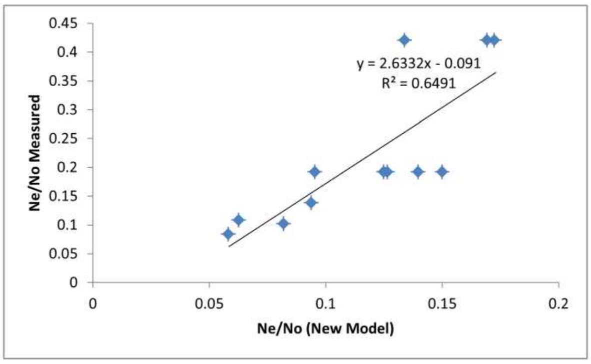


***Figure 16.*** *Theoretical and Practical Bacteria Degradation for Model Verification.*

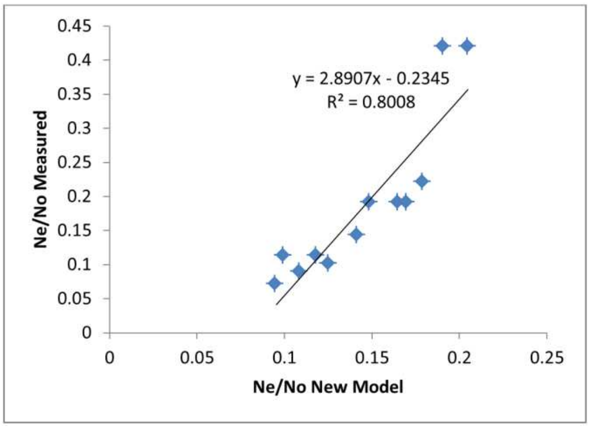


***Figure 17.*** *Theoretical and Practical Bacteria Degradation for Model Verification.*

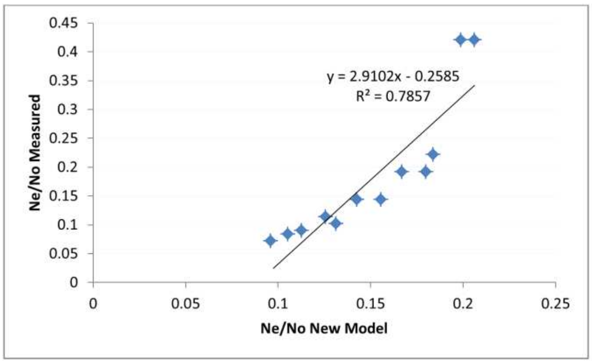


***Figure 18.*** *Theoretical and Practical Bacteria Degradation for Model Verification.*

## 5. Conclusion

Waste water stabilization pond is a simple way of treating waste water effectively but its limitation is that it requires large area of land for effective performance. The aim of this research is to improve the efficiency of waste water stabilization pond without increasing the area of the pond by installing solar reflectors at the effluent side of the pond. To achieve this, the study developed mathematical relationship between the reflector and the pond using regression analysis. The findings suggested a relationship between the dimensions of the reflector and the dimensions of the reflected radiation and decrease in solar radiation with depth in wastewater. And also, a model that describes the performance of solar enhanced ponds was developed and verified. If the findings of this research study are implemented in the design and construction of waste stabilization pond, it will reduce the cost of construction of waste stabilization ponds

## 6. Recommendation

Based on the findings of this research, the following recommendations would be useful in improving the efficiency of waste stabilization ponds in sewage treatment.

1. To enhance the efficiency of WSP, reflector made of flat foil paper should be installed at the effluent side of the pond.
2. The dimension of the reflector to be installed in the WSP should be determined by the expected qualities of effluent of the WSP
3. The depth of the pond should be reduced towards the effluent side of the WSP so as to maximize solar radiation penetration in the pond.
4. The bottom of the pond near the reflector should be lined with concrete or plastic material to prevent the growth of grasses.

## Acknowledgements

I thank Maduka Nsude for his contributions to this work.

## Appendix

***Table A1.*** *Reduction of Solar Radiation on the Surface of Waste Water.*

| s/no | Air radiation | Surface radiation | Ratio | % reduction |
|---|---|---|---|---|
| 1 | 370.611 | 246.03 | 1.51 | |
| 2 | 1128 | 534 | 2.11 | |
| 3 | 525 | 366 | 1.43 | |
| 4 | 323.29 | 180.73 | 1.79 | |
| 5 | 268.9 | 85.64 | 3.1 | |
| 6 | 324 | 210.64 | 1.54 | |
| 7 | 352.03 | 219.5 | 1.6 | |
| 8 | 283.07 | 159.64 | 1.77 | 53.7% |

***Table A2.*** *Determination of Factor Used for Calibration of Portable Pyranometer on 27th October 2015.*

| Time | Radiation reading In pyranometer in Geography dept.(w/m$^2$) | Volt reading in Portable pyranometer (v) | Ratio | Average ratio (factor) |
|---|---|---|---|---|
| 8:30am | 213 | 0.62 | 343.54 | |
| 10am | 464.7 | 2.2 | 211.23 | |
| 12noon | 390.9 | 1.42 | 275.28 | |
| 2pm | 320.6 | 0.73 | 439.18 | |
| 3:30pm | 112 | 0.33 | 339.39 | 322 |

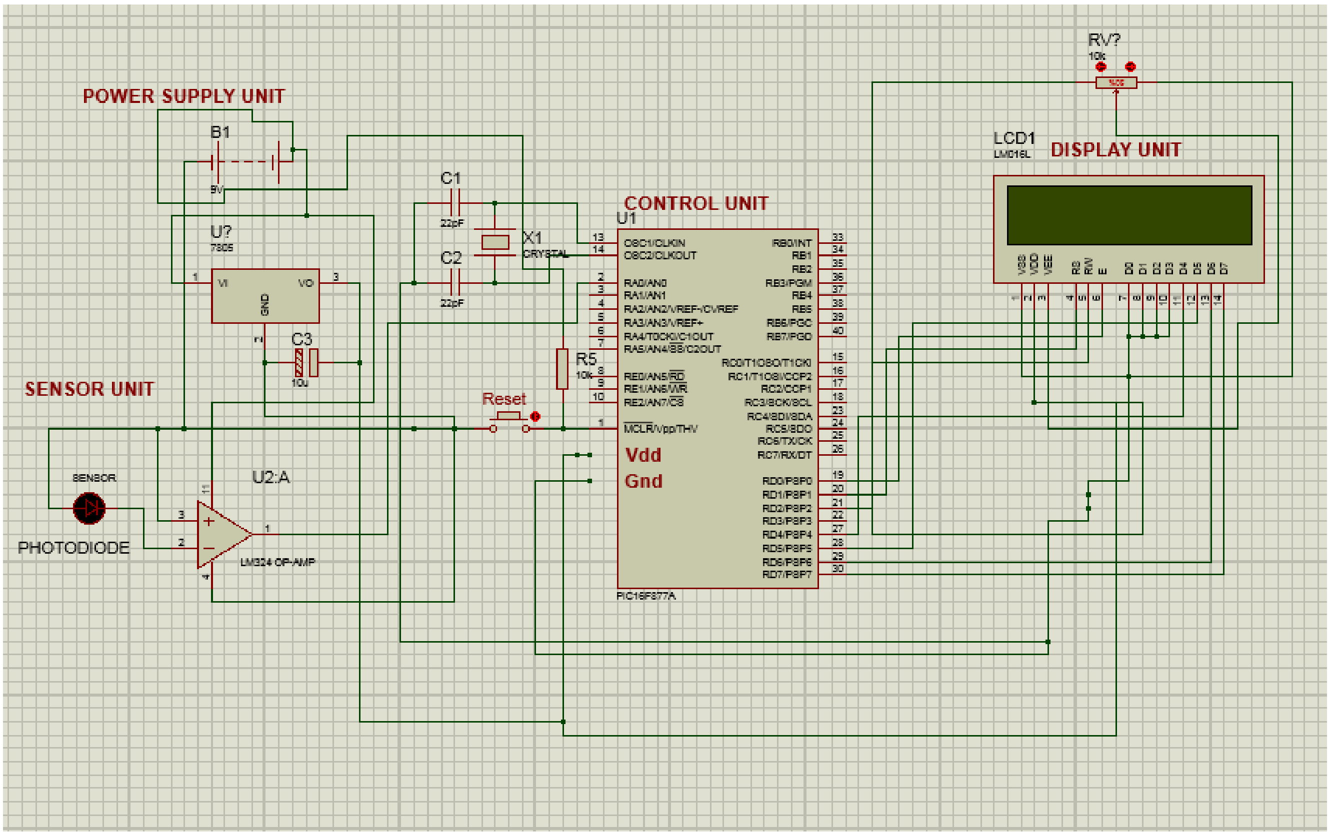


***Figure A1.*** *Circuit Diagram for the Portable Pyranometer.*

***Figure A2.*** *Picture of the Portable Pyranometer.*

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