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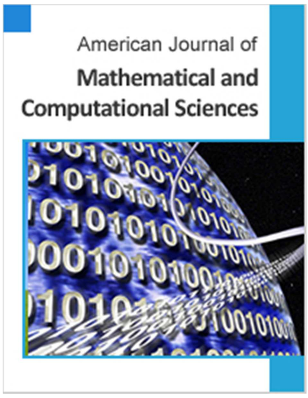

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Dimension of Topologies Associated with Information Systems

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Abstract
Many phenomena in quantum mechanics and theories of modern dynamics systems are represented in topological spaces with high dimensions and modern information systems use topological space as mathematical model which express it**.** As far as linking between dimensions of topological spaces and accuracy of approximation are not taught before and mentioned only in Pawlak space. The topology associated with Pawlak approximation space has a zero dimension, the generalized space generated by general binary relation is not generally of zero dimension. The purpose of this paper is to compute dimensions of topologies associated with information systems which resulted from general relations and connection between the dimension of topologies and approximation accuracy of uncertain concepts. Examples for topologies resulted from various subsets of features are given. We use the concept of topology, boundary, basis, the upper and lower approximation for calculation approximation accuracy. Also, we construct topologies from data table (information systems) by using general relations and compute dimensions of these topologies for its importance in finding connection between accuracy of approximation and dimensions.

1. Introduction

Abstract topological spaces play an important role in applicable fields such as artificial intelligence, digital topology and photo processing in additional to several other trends.

So far as the special studies to calculate the dimensions of abstract topological spaces which exist by theoretical method and do not indicate finite and infinite applicable examples. So the purpose of this paper is calculation dimensions of topologies associated with information systems where information systems were used extensively in the areas of data retrieval and decision making in many aspects of life.

Topology is generally considered to be one of the three linchpins of modern abstract mathematics (along with analysis and algebra). In the early history of topology, results were primarily motivated by investigations of real-world problems. Rough Set Theory (RST) is one of the newest mathematical tools to deal with the imperfect knowledge.

2. Basic Concepts

2.1. Information Systems [1, 5, 4, 7, 8, 9, 10, 11, 12, 13]

Topology is one of the important sciences concerned with the problem of ambiguities in the information, since the topological view of the boundary region is the clearest approach to implement the area of uncertainty in knowledge, which was first formulated in 1893 by GottlobFrege. Lately, the use of topology in many applications has been expanded, for example in structural analysis, in chemistry, physics and biology. Rough Set Theory (RST) is a new mathematical tool based on topology, it had a structure depended on a topological space.

A data set is represented in two forms: as information tables and as decision tables, In information tables all variables are called attributes while in decision tables one of the variables is called a decision. For both tables are called information system (IS) . Formally the triple $IS = (U, A, \{val_a\}_{a\in A})$, where U is the set of all cases, A is the set of all attributes and $\{val_a\}_{a\in A}$ is a value of the attribute a, each attribute $a \in A$ is a function, which is defined as below:

$a: U \to val_a$such that $a(x) \in val_a\ \forall\ x \subseteq U$

$IS$is called a total if and only if $a(x) \neq \emptyset$ for all $a \in A$, and for all $x \in U$.

2.2. Topological Space Generated by a Family of General Relations

In this section, several definitions of lower, upper approximation and accuracy.

Definition 2.2.1. [3, 7, 11]

Let U be a non empty finite universe and R is an arbitrary relation on U, then (U, R) is a generalized approximation space, frequently topology resulted from this relation called topological approximation space denoted by TAS (sometimes called Yao space).

In Yao space (U, R), for $x \in U$

The after set of x with respect to R on U is denoted by $(x)R = \{y \in U: (x, y) \in R\}$, and the for set of x with respect to R on U is denoted by $R(x) = \{y \in U: (y, x) \in R\}$.

The upper and lower approximations of a subset X of U are defined respectively as follows

$$\overline{apr}(X) = \{x \in U: R(x) \cap X \neq \emptyset\}$$

$$\underline{apr}(X) = \{x \in U: R(x) \subseteq X\}$$

The accuracy of the rough-set representation of the set X can be given (Pawlak 1991) by the following:

$$\text{Accuracy}(X) = \frac{|\underline{apr}(X)|}{|\overline{apr}(X)|}$$

Where Accuracy(X) is greater than or equal zero and less than or equal 1.

If Accuracy(X) $= 1$, then X is called definable set.

Definition 2.2.2. [2, 8]

We consider the family of all after sets (for sets) as a sub-base of a topological structure on U.

If$\gamma = \{R_\alpha: \alpha \in \Delta\}$ be a family of general relations on U, then a topological structure on U can be generated in the following approaches :

I: - Consider that $sb_\alpha = \{(x)R_\alpha: x \in U\}(sb_\alpha = \{R_\alpha(x): x \in U\})$ as the family of all after sets induced by R_α, then let $sb = \{\bigcup_{\alpha\in\Delta} sb_\alpha\}$ be a sub-base of a topological structure τ_I on U, then $\beta = \{\bigcap_{i=1}^{n} u_i: \{u_i\}_{i=1}^{n} \subseteq sb\} \cup \{U\}$ is a base of a topological structure τ_I on U.

II: - Let $sb_\alpha = \{(x)R_\alpha: x \in U\}, \beta_\alpha = \{\bigcap_{i=1}^{n} v_i: \{v_i\}_{i=1}^{n} \subseteq sb\} \cup \{U\}$ is a base of τ_α on U.Now, consider $\beta = \bigcap_{\alpha\in\Delta} \beta_\alpha$ is a base of $\tau_{II} = \bigcap_{\alpha\in\Delta} \tau_\alpha$ on U.

III:- Suppose that R$=\bigcup_{\alpha\in\Delta} R_\alpha$, $sb = \{(x)\ : x \in U\}$ is a sub-base of a topological structure τ_{III} on U, then $\beta = \{\bigcap_{i=1}^{n} u_i: \{u_i\}_{i=1}^{n} \subseteq sb\} \cup \{U\}$ is a base ofa topological structure τ_{III} on U.

IV: - Consider that $sb_\alpha = \{(x)R_\alpha: x \in U\}, \beta_\alpha = \{\bigcap_{i=1}^{n} v_i: \{v_i\}_{i=1}^{n} \subseteq sb\} \cup \{U\}$ is a base of τ_α on U. Let $\beta = \{\bigcap_{\alpha\in\Delta} u_\alpha : u_\alpha \in \beta_\alpha\}$, is a base of a topological structure τ_{IV} on U.

Remark 2.2.1. if we have only one relation R on a space X, we can compute a topological space from this relation as follows :

(A) $(x)R = \{y \in X: (x, y) \in R\}$

(B) $sb = \{(x)R: x \in X\}$ is sub-base for a topological space, then

(C) $\beta = \{\bigcap_{i=1}^{n} u_i: \{u_i\}_{i=1}^{n} \subseteq sb\} \cup \{X\}$ is a base of a topological structure on X and hence we compose the topology.

2.3. Inductive Dimension of Topological Spaces

The concept of dimensions of spaces plays an important role in operations represent real problems by Mathematics.

In physics and mathematics, the dimension of a mathematical space (or object) is informally defined as the minimum number of coordinates needed to specify any point within it. Thus a line has a dimension of one because only one coordinate is needed to specify a point on it-for example, the point at 5 on a number line. A surface such as a plane or the surface of a cylinder or sphere has a dimension of two because two coordinates are needed to specify a point on it-for example, both a latitude and longitude are required to locate a point on the surface of a sphere is three-dimensional because three coordinates are needed to locate a point within these space.

The works if the representation problems in geometric spaces such as line, plane and space. But when you represent data in topological space which neighborhoods and open sets play primary role in analysis and conclusion. Most studies on dimensions were limited on abstract definitions and did not address the spaces resulting from the actual data.

In view of Euclidean topology, points have dimension zero and the curve is one dimensional, where the walls (the boundary) of the curve are discrete points which are a zero dimensional, and the surface is two dimensional, where its walls are lines, which have dimension one, the solid body is three dimensional where the walls of the room are surfaces which are two dimensional.

From the above discussion, we notice that all dimensions are dependent on the concept of boundary. We aim in this paper to present a method of computing the inductive dimension as a type of topological dimensions and its relation with accuracy [7, 11].

Definition 2.3.1. [6, 5]

1. Let (X,τ) be a topological space, Then (X,τ) has inductive dimension equal to (-1) if and only if $X=\emptyset$which is denoted by $indX=-1$.

2. Let$n$ be an integer larger than or equal zero, then$(X.\tau)$ has inductive dimension less than or equal n, if it has a base β such that for every $B\in\beta$, the boundary $Fr(B)$ has inductive dimension less than or equal $(n-1)$ and we write $indX\leq n$

3. Let (X,τ) be topological space. If X has inductive dimension less than or equal n, and if it is false that X has inductive dimension less than or equal $(n-1)$, then the inductive dimension of X is n, and denoted by $indX=n$.

4. If for every $n\in Z_+$ it is false that X has inductive dimension less than or equal n, then X is said to have an infinite inductive dimension, and we have $indX=\infty$.

Remark 2.3. 1 [6] To compute the inductive dimension of the topology of the space (X,τ), we take a member B of the base of τ and find the boundary FrB.

(1). If FrB is $\emptyset$ then the inductive dimension of FrB is -1

(2). If FrB is not $\emptyset$, we consider FrB as a subspace and compute the boundary of all members of the base of τ_{FrB}.

(3). Repeat the above steps until the boundary is $\emptyset$ for all members of the base.

(4). If n is the number of steps to arrive that all members$B$ of the base has an empty boundary, then the inductive dimension of the space is less than or equal (n-1), then $indX\leq n-1$.

Remark 2.3.2 [6] A topological space (X,τ) is inductive zero-dimension space if it has a base of clopen sets.

3. Motivation Examples

Example 3.1

Consider the information system containing the results of exams in 4 subjects performed for 4 students in $X=\{x_1,x_2,x_3,x_4\}$ the set of all cases, and $A=\{M,A,E,S\}$ the set of all attributes where M =Mathematics, A =Arabic, E=English and S= Science in table 1.

Table 1. *Information system containing the results of exams.*

U	M	A	E	S
x_1	90	97	91	96
x_2	88	85	75	80
x_3	70	88	79	85
x_4	80	88	94	96

Definition 3.1

If $IS=(U,A,\{val_a\}_{a\in A})$be information system, For each $B\subset A$ the relation $R_B\subset U\times U$ is defined $xR_By\leftrightarrow\frac{\sum_{i\in B}|i(x)-i(y)|}{|B|}<\lambda$, where $|B|$ is the cardinality of B and $\lambda$is a represented any number (determined by the specialist and its value lies between the minimum and the maximum in data table), $|i(x)-i(y)|$ represents the absolute value of the difference values which corresponding to case. We determine the inductive dimension for the constructed topology associated with each R_B.

Let $B=\{M\},|B|=1,xR_By\leftrightarrow(|i(x)-i(y)|)/1<\lambda$, we compose table 2 from table 1 by subtraction the values of the cases for attribute $B=\{M\}$.

Table 2. *Subtraction the values of the cases for attribute* $B=\{M\}$.

M	x_1	x_2	x_3	x_4
x_1	0	2	20	10
x_2	2	0	18	8
x_3	20	18	0	10
x_4	10	8	10	0

When $\lambda\leq 5$, We find the subset information system from table 2 as follows:

$XR_{\{M\}}Y=\{(x_1,x_1),(x_1,x_2),(x_2,x_1),(x_2,x_2),(x_3,x_3),(x_4,x_4)\}$,

then $x_1R_{\{M\}}=\{x_1,x_2\},x_2R_{\{M\}}=\{x_1,x_2\},x_3R_{\{M\}}=\{x_3\},x_4R_{\{M\}}=\{x_4\}$,

$(x)R_{\{M\}}=\{\{x_1,x_2\},\{x_3\},\{x_4\}\}$as in Yao method.

Then,

$SR_{\{M\}}=\{\{x_1,x_2\},\{x_3\},\{x_4\}\}$as in TAS method.

In our method TAS "Topological Approximation Space", we get:

$BR_{\{M\}}=\{\emptyset,\{x_1,x_2\},\{x_3\},\{x_4\}\}$, which from remark 2.2.1

$\tau_{\{M\}}=\overline{\tau_{\{M\}}}=\{\emptyset,X,\{x_1,x_2\},\{x_3\},\{x_4\},\{x_1,x_2,x_3\},\{x_1,x_2,x_4\},\{x_3,x_4\}\}$ is the complement of the topological space.

therefore $indX=0$ since each element in $BR_{\{M\}}$is clopen furthermore boundary is empty.

Similarity for attributes A,E,Set $\lambda\leq 5$, we find $indX=0$

When $\lambda\leq 10$, We find the subset information system from Table 2 as follows:

$$XR_{\{M\}}Y=\left\{\begin{matrix}(x_1,x_1),(x_1,x_2),(x_1,x_4),(x_2,x_1),(x_2,x_2),(x_2,x_4),\\(x_3,x_3),(x_3,x_4),(x_4,x_1)\\(x_4,x_2),(x_4,x_3),(x_4,x_4)\end{matrix}\right\},$$

$x_1R_{\{M\}}=\{x_1,x_2,x_4\},x_2R_{\{M\}}=\{x_1,x_2,x_4\},x_3R_{\{M\}}=\{x_4,x_3\}$, $x_4R_{\{M\}}=\{x_1,x_2,x_3,x_4\},(x)R_{\{M\}}=\{\{x_1,x_2,x_4\},\{x_4,x_3\},\{x_1,x_2,x_3,x_4\}\}$

$SR_{\{M\}}=\{\{x_1,x_2,x_4\},\{x_4,x_3\},\{x_1,x_2,x_3,x_4\}\}$

$BR_{\{M\}}=\{\{x_1,x_2,x_4\},\{x_4,x_3\},\{x_1,x_2,x_3,x_4\},\{x_4\}\}$

$\tau_{\{M\}}=\{\emptyset,X,\{x_1,x_2,x_4\},\{x_4,x_3\},\{x_4\}\}$

$\overline{\tau_{\{M\}}}=\{\emptyset,X,\{x_1,x_2\},\{x_3\},\}\{x_1,x_2,x_3\}$ where $\overline{\tau_{\{M\}}}$ be the complement of topological space for computing the inductive dimension follows remark 2.3.1. as follows:

$B_1=\{x_1,x_2,x_4\},B_2=\{x_4,x_3\},B_3=\{x_4\}$where $\{B_1,B_2,B_3\}$ are elements of a base for relation $BR_{\{M\}}$

$bdB_1=X\backslash\{x_1,x_2,x_4\}=\{x_3\}\neq\emptyset,bdB_2=\{x_1,x_2\}\neq\emptyset$, $bdB_3=\{x_1,x_2,x_3\}\neq\emptyset$

$\tau_{bdB_1}=\tau_{\{x_3\}}=\tau_{\{M\}}\cap\{x_3\}=\{\emptyset,\{x_3\}\},B_{\{x_3\}}=\{\emptyset,\{x_3\}\}$, $bd\{x_3\}=\emptyset$, $\tau_{bdB_2}=\tau_{\{x_1,x_2\}}=\{\emptyset,\{x_1,x_2\}\},B_{\{x_1,x_2\}}=\{\emptyset,\{x_1,x_2\}\},bd\{\{x_1,x_2\}\}=\emptyset$, $\tau_{bdB_3}=\tau_{\{x_1,x_2,x_3\}}=\{\emptyset,\{x_1,x_2\},\{x_1,x_2,x_3\}\},B_{\{x_1,x_2,x_3\}}=\{\{x_3\},\{x_1,x_2\},\{x_1,x_2,x_3\}\}$

$bd\{x_1,x_2\}=\{x_3\}\neq\emptyset,bd\{x_3\}=\{x_3\}\neq\emptyset,bd\{x_1,x_2,x_3\}=\{x_4\}\neq\emptyset$

3.$\tau_{bd\{x_1,x_2\}} = \tau_{\{x_3\}} = \{\emptyset, \{x_3\}\}, B_{\{x_3\}} = \{\emptyset, \{x_3\}\}$,
$bd\{x_1, x_2\} = \emptyset, \tau_{\{x_4\}} = \{\emptyset, \{x_4\}\}, B_{\{x_4\}} = \{\emptyset, \{x_4\}\}$
$indbd\{x_1, x_2, x_4\} = 0, indbd\{x_4, x_3\} = 0, indbd\{x_4\} = 2 - 1 = 1$
$indX = 3 - 1 = 2$

Table 3. *Subtraction the values of the cases for attribute $B = \{A\}$.*

A	x_1	x_2	x_3	x_4
x_1	0	12	9	9
x_2	12	0	3	3
x_3	9	3	0	0
x_4	9	3	0	0

When $\lambda \leq 5$, We find the subset information system from table 3 as follows:

Let $B = \{A\}, |B| = 1, XR_BY \leftrightarrow (|i(x) - i(y)|)/1 < \lambda$

$$XR_{\{A\}}Y = \left\{\begin{matrix}(x_1,x_1),(x_2,x_2),(x_2,x_3),(x_2,x_4),(x_3,x_2),\\(x_4,x_2),(x_4,x_4),(x_4,x_3),(x_3,x_3),\\(x_3,x_4)\end{matrix}\right\},$$

$x_1R_{\{A\}} = \{x_1\}, x_2R_{\{A\}} = \{x_2, x_3, x_4\}, x_3R_{\{A\}} = \{x_2, x_3, x_4\}$,
$x_4R_{\{A\}} = \{x_2, x_3, x_4\}, (x)R_{\{A\}} = \{\{x_1\}, \{x_2, x_3, x_4\}\}$ as in Yao method [7].

$SR_{\{A\}} = \{\{x_1\}, \{x_2, x_3, x_4\}\}$as in TAS method.

$BR_{\{A\}} = \{\emptyset, \{x_1\}, \{x_2, x_3, x_4\}\}$,
$\tau_{\{A\}} = \overline{\tau_{\{A\}}} = \{\emptyset, X, \{x_1\},, \{x_2, x_3, x_4\}\}$

Then
$ind\ X = 0$

When $\lambda \leq 10$, We find the subset information system from Table 3 as follows:

$$XR_{\{A\}}Y = \left\{\begin{matrix}(x_1,x_1),(x_1,x_3),(x_1,x_4),(x_2,x_2),(x_2,x_3),(x_2,x_4),(x_3,x_1)\\(x_3,x_2),(x_3,x_3),(x_3,x_4),(x_4,x_1),(x_4,x_2),(x_4,x_3),(x_4,x_4)\end{matrix}\right\},$$

$x_1R_{\{A\}} = \{x_1, x_3, x_4\}, x_2R_{\{A\}} = \{x_2, x_3, x_4\}, x_3R_{\{A\}} = \{x_1, x_2, x_4, x_3\}, x_4R_{\{A\}} = \{x_1, x_2, x_3, x_4\}, (x)R_{\{A\}} = \{\{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\}$
$(x)R_{\{A\}} = \{\{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\}$
$BR_{\{A\}} = \{\{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_3, x_4\}, X\}$
$\tau_{\{A\}} = \{\emptyset, X, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_3, x_4\}\}$

1.B_1=$\{x_1, x_3, x_4\}, B_2 = \{x_2, x_3, x_4\}, B_3 = \{x_3, x_4\}$
$bdB_1 = \{x_2\} \neq \emptyset, bdB_2 = \{x_1\} \neq \emptyset, bdB_3 = \{x_1, x_2\} \neq \emptyset$

2. $\tau_{bdB_1} = \{\emptyset, \{x_2\}\}$, $\tau_{bdB_2} = \{\emptyset, \{x_1\}\}$, $\tau_{bdB_3} = \{\emptyset, \{x_1\}, \{x_2\}, \{x_1, x_2\}\}$

$B_{bdB_1} = \{\emptyset, \{x_2\}\}$, $B_{bdB_2} = \{\emptyset, \{x_1\}\}$, $B_{bdB_3} = \{\{x_1\}, \{x_2\}, \{x_1, x_2\}\}$

After two steps each element of the base equals empty.
$ind\ X = 2 - 1 = 1$
$indbdB_1 = 0, indbdB_2 = 0, indbdB_3 = 0$

When $\lambda \leq 20$, We find the subset information system from Table 3 as follows:

$$XR_{\{A\}}Y = \left\{\begin{matrix}(x_1,x_1),(x_1,x_2),(x_1,x_3),(x_1,x_4),(x_2,x_1),(x_2,x_2),\\(x_2,x_3),(x_2,x_4),(x_3,x_1)\\(x_3,x_2),(x_3,x_3),(x_3,x_4),(x_4,x_1),(x_4,x_2),\\(x_4,x_3),(x_4,x_4)\end{matrix}\right\},$$

$x_1R_{\{A\}} = \{x_1, x_2, x_3, x_4\}, x_2R_{\{A\}} = \{x_1, x_2, x_3, x_4\}$,
$x_3R_{\{A\}} = \{x_1, x_2, x_4, x_3\}, x_4R_{\{A\}} = \{x_1, x_2, x_3, x_4\}$,

$(x)R_{\{A\}} = \{\{x_1, x_2, x_3, x_4\}\}$is classes of cases associated with attribute {A}

$SR_{\{A\}} = \{\{x_1, x_2, x_3, x_4\}\}$is the sub-basis associated with attribute {A}

$BR_{\{A\}} = \{\{x_1, x_2, x_3, x_4\}\}$ is the basis associated with attribute {A}

$\tau_{\{A\}} = \{\emptyset, X\}$is the topology associated with attribute {A}
$ind\ X = 0$

Table 4. *Subtraction the values of the cases for attribute $B = \{E\}$.*

E	x_1	x_2	x_3	x_4
x_1	0	16	12	3
x_2	16	0	4	19
x_3	12	4	0	15
x_4	3	19	15	0

Table 5. *Subtraction the values of the cases for attribute $B = \{S\}$.*

S	x_1	x_2	x_3	x_4
x_1	0	16	11	0
x_2	16	0	5	16
x_3	11	5	0	11
x_4	0	16	11	0

Example 3.2

We determine the inductive dimension for the constructed topology from attributes $B = \{M, A\}$in each λ in table 1. We construct table 5 by subtraction table of attribute M from table of attribute$A$ for matching cases then divide on 2 and similarity the attributes $\{M, E\}, \{A, E\}, \{A, S\}$ and $\{S, E\}$

Table 6. *Subtraction the corresponding values of attribute $\{M\}$ from attribute $\{A\}$ and divide on 2.*

M, A	x_1	x_2	x_3	x_4
x_1	0	7	14.5	9.5
x_2	7	0	10.5	5.5
x_3	14.5	10.5	0	5
x_4	9.5	5.5	5	0

When $\lambda \leq 5$, We find the subset information system from table 6 as follows:

$XR_{\{M,A\}}Y = \{(x_1, x_1), (x_2, x_2), (x_3, x_4), (x_4, x_4), (x_4, x_3), (x_3, x_3)\}$,
$x_1R_{\{M,A\}} = \{x_1\}, x_2R_{\{M,A\}} = \{x_2\}, x_3R_{\{M,A\}} = \{x_3, x_4\}, x_4R_{\{M,A\}} = \{x_3, x_4\}$,
$(x)R_{\{M,A\}} = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$
$SR_{\{M,A\}} = \{\{x_1\}, \{x_2\}, \{x_3, x_4\}\}$
$BR_{\{M,A\}} = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3, x_4\}\}$
$\tau_{\{M,A\}} = \overline{\tau_{\{M,A\}}} = \{\emptyset, X, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_3, x_4\}, \{x_1, x_2\}, \{x_1\}, \{x_2\}\}$
$indX = 0$

When $\lambda \leq 10$, We find the subset information system from Table 6 as follows:

$$XR_{\{M,A\}}Y = \left\{\begin{matrix}(x_1,x_1),(x_1,x_3),(x_1,x_4),(x_2,x_2),(x_2,x_3),(x_2,x_4),(x_3,x_1)\\(x_3,x_2),(x_3,x_3),(x_3,x_4),(x_4,x_1),(x_4,x_2),(x_4,x_3),(x_4,x_4)\end{matrix}\right\},$$

$x_1R_{\{M,A\}} = \{x_1, x_3, x_4\}, x_2R_{\{M,A\}} = \{x_2, x_3, x_4\}$,
$x_3R_{\{M,A\}} = \{x_1, x_2, x_4, x_3\}, x_4R_{\{M,A\}} = \{x_1, x_2, x_3, x_4\}$,
$(x)R_{\{M,A\}} = \{\{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\}$

$(x)R_{\{M,A\}} = \{\{x_1,x_3,x_4\},\{x_2,x_3,x_4\},\{x_1,x_2,x_3,x_4\}\}$
$BR_{\{M,A\}} = \{\{x_1,x_3,x_4\},\{x_2,x_3,x_4\},\{x_3,x_4\},X\}$
$\tau_{\{M,A\}} = \{\emptyset,X,\{x_1,x_3,x_4\},\{x_2,x_3,x_4\},\{x_3,x_4\}\}$

1.B_1=$\{x_1,x_3,x_4\}, B_2 = \{x_2,x_3,x_4\}, B_3 = \{x_3,x_4\}$
$bdB_1 = \{x_2\} \neq \emptyset, bdB_2 = \{x_1\} \neq \emptyset, bdB_3 = \{x_1,x_2\} \neq \emptyset$

2. $\tau_{bdB_1} = \{\emptyset,\{x_2\}\}$, $\tau_{bdB_2} = \{\emptyset,\{x_1\}\}$, $\tau_{bdB_3} = \{\emptyset,\{x_1\},\{x_2\},\{x_1,x_2\}\}$

$B_{bdB_1} = \{\emptyset,\{x_2\}\}$, $B_{bdB_2} = \{\emptyset,\{x_1\}\}$, $B_{bdB_3} = \{\{x_1\},\{x_2\},\{x_1,x_2\}\}$

After two steps each element of the base equals empty.

$ind\,X = 2 - 1 = 1$
$indbdB_1 = 0, indbdB_2 = 0, indbdB_3 = 0$

Table 7. *Subtraction the corresponding values of attribute $\{M\}$ from attribute $\{E\}$ and divide on 2.*

M,E	x_1	x_2	x_3	x_4
x_1	0	9	16	6.5
x_2	9	0	11	13.5
x_3	16	11	0	12.5
x_4	6.5	13.5	12.5	0

When $\lambda \leq 5$, We find the subset information system from table 7 as follows:

$$XR_{\{M,E\}}Y = \{(x_1,x_1),(x_2,x_2),(x_4,x_4),(x_3,x_3)\},$$

$$x_1R_{\{M,E\}} = \{x_1\}, x_2R_{\{M,E\}} = \{x_2\}, x_3R_{\{M,E\}} = \{x_3\}, x_4R_{\{M,E\}} = \{x_4\},$$

$$(x)R_{\{M,E\}} = \{\{x_1\},\{x_2\},\{x_3\},\{x_4\}\}$$

$$SR_{\{M,E\}} = \{\{x_1\},\{x_2\},\{x_3\},\{x_4\}\}$$

$$BR_{\{M,E\}} = \{\emptyset,\{x_1\},\{x_2\},\{x_3\},\{x_4\}\}$$

$$\tau_{\{M,E\}} = \overline{\tau_{\{M,E\}}} = \begin{Bmatrix} \emptyset,X,\{x_1,x_3,x_4\},\{x_2,x_3,x_4\},\{x_1,x_2,x_4\},\{x_1,x_2,x_3\},\{x_3,x_4\},\{x_1,x_2\},\{x_1\},\{x_2\},\{x_3\} \\ \{x_2,x_4\},\{x_1,x_3\},\{x_4\},\{x_2,x_3\},\{x_4,x_1\} \end{Bmatrix}$$

$$indX = 0$$

When $\lambda \leq 10$, We find the subset information system from Table 7 as follows

$$XR_{\{M,E\}}Y = \begin{Bmatrix} (x_1,x_1),(x_1,x_2),(x_1,x_3),(x_2,x_1),(x_2,x_2),(x_3,x_3), \\ (x_4,x_1),(x_4,x_4) \end{Bmatrix},$$

$$x_1R_{\{M,E\}} = \{x_1,x_2,x_3\}, x_2R_{\{M,E\}} = \{x_1,x_2\}, x_3R_{\{M,E\}} = \{x_3\},$$

$$x_4R_{\{M,E\}} = \{x_1,x_4\}, (x)R_{\{M,E\}} = \{\{x_1,x_2,x_3\},\{x_1,x_2\},\{x_3\},\{x_1,x_4\}\}$$

$$SR_{\{M,E\}} = \{\{x_1,x_2,x_3\},\{x_1,x_2\},\{x_3\},\{x_1,x_4\}\}$$

$$BR_{\{M,E\}} = \{\{x_1,x_2,x_3\},\{x_1,x_2\},\{x_3\},\{x_1,x_4\},\{x_1\},\emptyset\}$$

$$\tau_{\{M,E\}} = \{\emptyset,X,\{x_1,x_2,x_3\},\{x_1,x_2\},\{x_3\},\{x_1,x_4\},\{x_1\},\{x_1,x_3,x_4\},\{x_1,x_3\},\{x_1,x_2,x_4\}\}$$

$$\overline{\tau_{\{M,E\}}} = \{\emptyset,X,\{x_1,x_2,x_4\},\{x_2,x_4\},\{x_3\},\{x_2\},\{x_3,x_4\},\{x_4\},\{x_2,x_3,x_4\},\{x_2,x_3\}\}$$

1.$B_1 = \{x_1,x_2,x_3\}, B_2 = \{x_1,x_2\}, B_3 = \{x_3\}, B_4 = \{x_1,x_4\}, B_5 = \{x_1\}$

$bdB_1 = \{x_4\}, bdB_2 = \{x_3,x_4\}, bdB_3 = \emptyset, bdB_4 = \{x_2\}, bdB_5 = \{x_2,x_4\}$

2.$\tau_{\{x_4\}} = \{\emptyset,\{x_4\}\}$,$\tau_{\{x_3,x_4\}} = \{\emptyset,\{x_3,x_4\},\{x_3\},\{x_4\}\}$,$\tau_{\{x_2\}} = \{\emptyset,\{x_2\}\}$,
$\tau_{\{x_2,x_4\}} = \{\emptyset,\{x_2,x_4\},\{x_2\},\{x_4\}\}$
$indbdB_1 = 0, indbdB_2 = 0, indbdB_4 = 0, indbdB_5 = 0$

$ind\,X = 1$ since after two steps become boundary of each element in the basis equals empty.

When $\lambda \leq 20$, We find the subset information system from Table 7 as follows:

$$XR_{\{M,E\}}Y = \begin{Bmatrix} (x_1,x_1),(x_1,x_2),(x_1,x_3),(x_1,x_4),(x_2,x_1),(x_2,x_2),(x_2,x_3),(x_2,x_4),(x_3,x_1) \\ (x_3,x_2),(x_3,x_3),(x_3,x_4),(x_4,x_1),(x_4,x_2),(x_4,x_3),(x_4,x_4) \end{Bmatrix},$$

$$x_1R_{\{M,E\}} = \{x_1,x_2,x_3,x_4\}, x_2R_{\{M,E\}} = \{x_1,x_2,x_3,x_4\},$$

$x_3R_{\{M,E\}} = \{x_1,x_2,x_4,x_3\}, x_4R_{\{M,E\}} = \{x_1,x_2,x_3,x_4\},$

$(x)R_{\{M,E\}} = \{\{x_1,x_2,x_3,x_4\}\}$

$SR_{\{M,E\}} = \{\{x_1, x_2, x_3, x_4\}\}$

$BR_{\{M,E\}} = \{\{x_1, x_2, x_3, x_4\}\}$

$\tau_{\{M,E\}} = \{\emptyset, X\}$

$indX = 0$

Table 8. Subtraction the corresponding values of attribute $\{M\}$ from attribute $\{S\}$ and divide on 2.

M,S	x_1	x_2	x_3	x_4
x_1	0	9	15.5	5
x_2	9	0	11.5	12
x_3	15.5	11.5	0	10.5
x_4	5	12	10.5	0

Table 9. Subtraction the corresponding values of attribute $\{A\}$ from attribute $\{E\}$ and divide on 2.

A,E	x_1	x_2	x_3	x_4
x_1	0	2	1.5	3
x_2	2	0	0.5	8
x_3	1.5	0.5	0	7.5
x_4	3	8	7.5	0

Table 10. Subtraction the corresponding values of attribute $\{S\}$ from attribute $\{E\}$ and divide on 2.

S,E	x_1	x_2	x_3	x_4
x_1	0	0	0.5	1.5
x_2	0	0	0.5	1.5
x_3	0.5	0.5	0	2
x_4	1.5	1.5	2	0

Table 11. Subtraction the corresponding values of attribute $\{A\}$ from attribute $\{S\}$ and divide on 2.

A,S	x_1	x_2	x_3	x_4
x_1	0	2	1	4.5
x_2	2	0	1	6.5
x_3	1	1	0	5.5
x_4	4.5	6.5	5.5	0

Example 3.3

We construct table 12 by subtraction the corresponding values of cases of table of $\{M, A\}$ from table of S then divide on 3. We determine the inductive dimension of attribute $\{M, A, S\}$ at cases $\lambda \leq 5, \lambda \leq 10$ and $\lambda \leq 20$ accordingly to relation $xR_By \leftrightarrow \frac{\sum_{i\in B}|i(x)-i(y)|}{|B|} < \lambda$ and similarity the attributes $\{M, E, S\}, \{A, E, S\} and\ \{M, E, A\}$

Table 12. Subtraction the corresponding values of cases of table 3 of $\{M, A\}$ from attribute {S} then divide on 3.

M,A,S	x_1	x_2	x_3	x_4
x_1	0	10	13.333333	6.3333333
x_2	10	0	8.6666667	9
x_3	13.333333	8.6666667	0	7
x_4	6.3333333	9	7	0

When $\lambda \leq 5$, We find the subset information system from table 12 as follows:

$XR_{\{M,A,S\}}Y = \{(x_1, x_1), (x_2, x_2), (x_4, x_4), (x_3, x_3)\}$,

$x_1R_{\{M,A,S\}} = \{x_1\}, x_2R_{\{M,A,S\}} = \{x_2\}, x_3R_{\{M,A,S\}} = \{x_3\}, x_4R_{\{M,A,S\}} = \{x_4\}$,

$(x)R_{\{M,A,S\}} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$

$SR_{\{M,A,S\}} = \{\{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$

$BR_{\{M,A,S\}} = \{\emptyset, \{x_1\}, \{x_2\}, \{x_3\}, \{x_4\}\}$

$$\tau_{\{M,A,S\}} = \overline{\tau_{\{M,A,S\}}} = \left\{\begin{matrix}\emptyset, X, \{x_1\}, \{x_2\}, \{x_4\}, \{x_3\}, \{x_1, x_2\}, \{x_2, x_3\}, \{x_3, x_4\}, \{x_1, x_4\}, \{x_1, x_3\}, \{x_2, x_4\}, \\ \{x_1, x_3, x_4\}, \{x_1, x_2, x_3\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_4\}\end{matrix}\right\}$$

$indX = 0$

When $\lambda \leq 10$, We find the subset information system from Table 12 as follows:

$XR_{\{M,A,S\}}Y = \left\{\begin{matrix}(x_1, x_1), (x_1, x_2), (x_1, x_4), (x_2, x_1), (x_2, x_2), (x_2, x_4), \\ (x_3, x_3), (x_3, x_4), (x_4, x_1), (x_4, x_2)(x_4, x_3), (x_4, x_4)\end{matrix}\right\}$,

$x_1R_{\{M,A,S\}} = \{x_1, x_2, x_4\}, x_2R_{\{M,A,S\}} = \{x_1, x_2, x_4\}, x_3R_{\{M,A,S\}} = \{x_3, x_4\}$,

$x_4R_{\{M,A,S\}} = \{x_1, x_2, x_3, x_4\}, (x)R_{\{M,A,S\}} = \{\{x_1, x_2, x_4\}, \{x_3, x_4\}, X\}$

$SR_{\{M,A,S\}} = \{\{x_1, x_2, x_4\}, \{x_3, x_4\}, X\}$

$BR_{\{M,A,S\}} = \{\{x_1, x_2, x_4\}, \{x_3, x_4\}, X, \{x_4\}\}$

$\tau_{\{M,A,S\}} = \{\emptyset, X, \{x_1, x_2, x_4\}, \{x_3, x_4\}, \{x_4\}\}$

$\overline{\tau_{\{M,A,S\}}} = \{\emptyset, X, \{x_1, x_2, x_3\}, \{x_1, x_2\}, \{x_3\}\}$

1. $B_1 = \{x_1, x_2, x_4\}, B_2 = \{x_3, x_4\}, B_3 = \{x_4\}$

$bdB_1 = \{x_3\}, bdB_2 = \{x_1, x_2\}, bdB_3 = \{x_1, x_2, x_3\}$

2. $\tau_{bdB_1} = \{\emptyset, \{x_3\}\}, \tau_{bdB_2} = \{\emptyset, \{x_1, x_2\}\}$,

$\tau_{bdB_3} = \{\emptyset, \{x_3\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}\}$

$B_{\{x_3\}} = \{\emptyset, \{x_3\}\}$, $B_{\{x_1,x_2\}} = \{\emptyset, \{x_1, x_2\}\}, B_{\{x_1,x_2,x_3\}} = \{\{x_3\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}\}, bd_{\{x_3\}} = \emptyset, bd_{\{x_1,x_2\}} = \emptyset$

$indbdB_1 = 1 - 1 = 0, indbdB_2 = 0, ind\ bdB_3 = 0$

$ind\ X = 2 - 1 = 1$

Table 13. Subtraction the corresponding values of cases of $\{M, E\}$ from attribute {S} then divide on 3.

M,E,S	x_1	x_2	x_3	x_4
x_1	0	11.333333	14.333333	4.3333333
x_2	11.333333	0	9	14.333333
x_3	14.333333	9	0	12
x_4	4.3333333	14.333333	12	0

When $\lambda \leq 5$, We find the subset information system as follows:

$XR_{\{M,E,S\}}Y = \{(x_1, x_1), (x_1, x_4), (x_2, x_2), (x_4, x_1), (x_4, x_4), (x_3, x_3)\}$,

$x_1R_{\{M,E,S\}} = \{x_1, x_4\}, x_2R_{\{M,E,S\}} = \{x_2\}, x_3R_{\{M,E,S\}} = \{x_3\}, x_4R_{\{M,E,S\}} = \{x_1, x_4\}$,

$(x)R_{\{M,E,S\}} = \{\{x_1, x_4\}, \{x_2\}, \{x_3\}\}$

$SR_{\{M,E,S\}} = \{\{x_1, x_4\}, \{x_2\}, \{x_3\}\}$

$BR_{12} = \{\emptyset, \{x_1, x_4\}, \{x_2\}, \{x_3\}\}$

$\tau_{\{M,E,S\}} = \overline{\tau_{\{M,E,S\}}} = \left\{\begin{matrix}\emptyset, X, \{x_2\}, \{x_3\}, \{x_2, x_3\}, \{x_1, x_4\}, \\ \{x_1, x_3, x_4\}, , \{x_1, x_2, x_4\}\end{matrix}\right\}$

$ind\ X = 0$

When $\lambda \leq 10$, We find the subset information system from Table 13 as follows:

$$XR_{\{M,E,S\}}Y = \begin{Bmatrix} (x_1,x_1),(x_1,x_4),(x_2,x_2),(x_2,x_3), \\ (x_3,x_3),(x_4,x_1),(x_4,x_4),(x_3,x_2) \end{Bmatrix},$$

$$x_1R_{\{M,E,S\}} = \{x_1,x_4\}, x_2R_{12} = \{x_2,x_3\}, x_3R_{\{M,E,S\}} = \{x_2,x_3\}, x_4R_{\{M,E,S\}} = \{x_1,x_4\}, (x)R_{12} = \{\{x_1,x_4\},\{x_2,x_3\}\}$$

$$SR_{\{M,E,S\}} = \{\{x_1,x_4\},\{x_2,x_3\}\}$$

$$BR_{\{M,E,S\}} = \{\emptyset,\{x_1,x_4\},\{x_2,x_3\}\}$$

$$\tau_{\{M,E,S\}} = \{\{x_1,x_4\},\{x_2,x_3\},\emptyset,X\}$$

$$ind\ X = 0$$

at $\lambda \leq 20, ind\ X = 0$

Table 14. *Subtraction the corresponding values of cases of* $\{M,A\}$ *from attribute* $\{S\}$ *then divide on 3.*

M,A,S	x_1	x_2	x_3	x_4
x_1	0	10	13.333333	6.3333333
x_2	10	0	8.6666667	9
x_3	13.333333	8.6666667	0	7
x_4	6.3333333	9	7	0

Table 15. *Subtraction the corresponding values of cases of* $\{A,E\}$ *from attribute* $\{S\}$ *then divide on 3.*

A,E,S	x_1	x_2	x_3	x_4
x_1	0	14.666667	10.666667	4
x_2	14.666667	0	4	12.666667
x_3	10.666667	4	0	8.6666667
x_4	4	12.666667	8.6666667	0

We summarize the previous results of inductive dimension in the following table:

Table 16. *Summary of the previous results of inductive dimension.*

	$\lambda \leq 5$	$\lambda \leq 10$	$\lambda \leq 20$
M	$indX = 0$	$indX = 2$	$indX = 0$
A	$indX = 0$	$indX = 1$	$indX = 0$
E	$indX = 0$	$indX = 0$	$indX = 0$
S	$indX = 0$	$indX = 0$	$indX = 0$
M,A	$indX = 0$	$indX = 1$	$indX = 0$
M,E	$indX = 0$	$indX = 1$	$indX = 0$
M,S	$indX = 0$	$indX = 1$	$indX = 0$
E,A	$indX = 2$	$indX = 0$	$indX = 0$
E,S	$indX = 0$	$indX = 0$	$indX = 0$
A,S	$indX = 2$	$indX = 0$	$indX = 0$
A,E,S	$indX = 0$	$indX = 1$	$indX = 0$
M,A,E	$indX = 0$	$indX = 1$	$indX = 0$
M,E,S	$indX = 0$	$indX = 0$	$indX = 0$
M,A,S	$indX = 0$	$indX = 1$	$indX = 0$
A,E,S	$indX = 0$	$indX = 1$	$indX = 0$

Example 3.4

We find the lower and upper approximation for a subset $f = \{x_1,x_3\}$ frequently calculation accuracy for each previous cases.

At $\lambda \leq 5$, we determine the accuracy of a subset $f = \{x_1,x_3\}$ corresponding to the topologies for each attribute from table 1.

Table 17. *Summary of accuracy at* $\lambda \leq 5$.

	(interior)$\underline{f}$	**(closure)**$\overline{f}$	**Accuracy**$=\frac{\lvert\underline{f}\rvert}{\lvert\overline{f}\rvert}$
M	$\{x_3\}$	$\{x_1,x_2,x_3\}$	1/3
A	$\{x_1\}$	X	1/4
E	$\emptyset$	X	0
S	$\emptyset$	X	0
M,A	$\{x_1\}$	$\{x_1,x_3,x_4\}$	1/3
M,E	$\{x_1,x_3\}$	$\{x_1,x_3\}$	1
M,S	$\{x_3\}$	$\{x_1,x_3,x_4\}$	1/3
M,A,E	$\{x_1,x_3\}$	$\{x_1,x_3\}$	1
M,E,S	$\{x_3\}$	$\{x_1,x_3,x_4\}$	1/3
M,A,S	$\{x_1,x_3\}$	$\{x_1,x_3\}$	1
A,E,S	$\emptyset$	X	0
E,S	$\emptyset$	X	0
S,A	$\emptyset$	X	0
A,E	$\{x_1\}$	X	1/4

At $\lambda \leq 10$, we determine the accuracy of a subset $f = \{x_1,x_3\}$ corresponding to the topologies for each attribute from table 1.

Table 18. *Summary of accuracy at* $\lambda \leq 10$.

	(interior)$\underline{f}$	**(closure)**$\overline{f}$	**Accuracy**$=\frac{\lvert\underline{f}\rvert}{\lvert\overline{f}\rvert}$
M	$\emptyset$	$\{x_1,x_2,x_3\}$	0
A	$\emptyset$	X	0
E	$\emptyset$	X	0
S	$\emptyset$	X	0
M,A	$\emptyset$	$\{x_1,x_2,x_3\}$	0
M,E	$\{x_1,x_3\}$	X	1/2
M,S	$\{x_1,x_3\}$	X	1/2
M,A,E	$\emptyset$	$\{x_1,x_3\}$	0
M,E,S	$\emptyset$	X	0
M,A,S	$\emptyset$	$\{x_1,x_3\}$	0
A,E,S	$\{x_3\}$	X	1/4
E,S	$\emptyset$	X	0
S,A	$\emptyset$	X	0
E,A	$\emptyset$	X	0

At $\lambda \leq 20$

Table 19. *Summary of accuracy at* $\lambda \leq 20$.

	(interior)$\underline{f}$	**(closure)**$\overline{f}$	**Accuracy**$=\frac{\lvert\underline{f}\rvert}{\lvert\overline{f}\rvert}$
M	$\emptyset$	X	0
A	$\emptyset$	X	0
E	$\emptyset$	X	0
S	$\emptyset$	X	0
M,A	$\emptyset$	X	0
M,E	$\emptyset$	X	0
M,S	$\emptyset$	X	0
M,A,E	$\emptyset$	X	0
M,E,S	$\emptyset$	X	0
M,A,S	$\emptyset$	X	0
A,E,S	$\emptyset$	X	0
E,S	$\emptyset$	X	0
S,A	$\emptyset$	X	0
A,E	$\emptyset$	X	0

We summarize the accuracy of the previous cases in the following table

Table 20. *Summary of the previous results of accuracy.*

	$\lambda \leq 5$	$\lambda \leq 10$	$\lambda \leq 20$
M	1\3	0	0
A	1\4	0	0
E	0	0	0
S	0	0	0
M, A	1\3	0	0
M, E	1	1\2	0
M, S	1\3	1\2	0
M, A, E	1	0	0
M, E, S	1\3	0	0
M, A, S	1	0	0
A, E, S	0	1\4	0
A, E	1\4	0	0
E, S	0	0	0
A, S	0	0	0

We summarize the previous results in the following table:

Table 21. *Summary of the previous results of inductive dimension and accuracy.*

	$\lambda \leq 5$	$\lambda \leq 10$	$\lambda \leq 20$	$\lambda \leq 5$	$\lambda \leq 10$	$\lambda \leq 20$
M	$indX = 0$	$indX = 2.$	$indX = 0$	Acc=$1/3$	Acc=0	Acc=0
A	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=$1/4$	Acc=0	Acc=0
E	$indX = 0.$	$indX = 0.$	$indX = 0$	Acc=0	Acc=0	Acc=0
S	$indX = 0.$	$indX = 0.$	$indX = 0$	Acc=0	Acc=0	Acc=0
M, A	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=$1/3$	Acc=0	Acc=0
M, E	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=1	Acc=1\2	Acc=0
M, S	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=$1/3$	Acc=1\2	Acc=0
M, A, E	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=1	Acc=0	Acc=0
M, E, S	$indX = 0.$	$indX = 0.$	$indX = 0$	Acc=$1/3$	Acc=0	Acc=0
M, A, S	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=1	Acc=0	Acc=0
A, E, S	$indX = 0.$	$indX = 1.$	$indX = 0$	Acc=0	Acc=1\4	Acc=0
A, E	$indX = 2.$	$indX = 0.$	$indX = 0$	Acc=1\4	Acc=0	Acc=0
E, S	$indX = 0.$	$indX = 0.$	$indX = 0$	Acc=0	Acc=0	Acc=0
S, A	$indX = 2.$	$indX = 0.$	$indX = 0$	Acc=0	Acc=0	Acc=0

Lemma 3.1. If $indX > 0$, then degree of accuracy approximation for elements of the base is less than 1.

Proof In the case $indX = 0$ (i.e. there is base of clopen and frequently accuracy approximation is 1), but in the case $indX > 0$ (i.e. there exists at the least element from the base its boundary is not empty) and frequently the cardinality of interior less than the cardinality of closure) implies accuracy less than 1.

Remark 3.1

When value of λ takes the minimum and the maximum in data table for each attribute, we obtain discrete and indiscrete topology and frequently $indX = 0$

4. Conclusion

The approach presented in this work for computing dimension of topologies associated with information systems open the way for choosing suitable topologies for information systems that give the highly accurate approximations and this help in all fields of rough set applications such as vagueness and imperfect knowledge. When value of λ takes the minimum and the maximum in data table for each attribute, we obtain discrete and indiscrete topology and frequently $indX = 0$. The accuracy of any subset f from X depends on itself.

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