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A Non-Isothermal Reacting MHD Flow over a Stretching Sheet Through a Saturated Porous Medium

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Abstract: Non-Newtonian fluid flow in porous media has variety of usefulness and applications in various fields of human endeavours. This work is aimed at studying non-isothermal reacting MHD flow with a view of investigating the detailed effects of various physical parameters of a stretching sheet through a saturated porous medium. In this work, variable thermal conductivity on a radiative MHD boundary layer flow of an incompressible, viscous, and electrically fluid over a non-isothermal stretching sheet through a saturated porous medium is considered. It is assumed that the fluid has chemical and react satisfying Arrhenius law. The governing partial differential equations were transformed into ordinary differential equations in terms of suitable similarity variable. Galerkin weighted residual method is employed to solve the resulting non-linear equation. The results showed the effects of variable thermal conductivity parameter, radiation parameter, Frank-Kamenetskii parameter, magnetic parameter, Prandtl number and Schmidt number on the system of flow. The effects of various physical parameters on the flow system were reported graphically. It is concluded from the analysis of the problem that thermal radiation parameter, thermal conductivity parameter and the Prandtl number greatly affect the mass flow and the energy transfer phenomena in the system.

Keywords: Thermal Conductivity, Weighted Residual Method, Arrhenius Reaction, Radiative MHD Boundary Layer, Electric Fluid

1. Introduction

The study of non-Newtonian fluid flow in the presence of Arrhenius reaction has attracted the attention of scientist and engineers in the recent times. It has given insight in generating electric power, the reservoir engineering, thermal recovery processes, flow of moisture through porous industrial materials, oil recovery processes, food processing, geothermal reservoir, polymer solution, and enhanced oil recovery to mention but just a few applications.

In fact, good working knowledge of convection through porous media helps in designing the pertinent equipment, and in understanding the phenomena correctly. Hence, applications of such flow have served as the central stimuli in the development of field of porous media flows. This effect is of particular significance in natural convection in various devices that are subjected to large variation of gravitational force or that operate at high rotational speeds. Gravity is naturally present in the reservoirs and is used as the main driving force to effect the oil movement. [1]

Gravity flow of non-Newtonian fluid through porous medium is involved in some important engineering practices, such as enhanced oil recovery by thermal methods, polymer solutions, geothermal power, geothermal reservoirs, food processing, and emulsions of oil, among others. Many processes in thermal engineering areas occur at high temperature and radiative heat transfer becomes very important for the design of pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are example of such engineering areas.

The magnetohydrodynamics of an electrically conducting fluid is a phenomenon that is encountered in our day to activities such as geophysics, astrophysics, engineering applications, industrial areas, to mention but just a few. Hydromagnetic free convection flow through a saturated porous medium have a great significance effect for it tremendous applications in the fields of steller, planetary magnetospheres, and aeronautics. Engineers employ magnetohydrodynamics principles in the design of heat exchangers, pumps in space vehicle propulsion, thermal protection, and in creating novel power generating systems. However, the study of hydromagnetic flow through porous medium and heat transfer phenomena have become more important in today's industrial processes. In many metallurgical processes which involve the cooling of many continuous strips or filaments by drawing them through an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and the final product of desired characteristics can be achieved. The importantance of hydromagnetic is applicable to metallurgy which lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field. [2]

Consequently, the significant effect of temperature is seen from radiation in space technology, solar power technology, space vehicle re-entry, nuclear engineering which are of great importance in electrical engineering. Many processes in industrial areas occur at high temperature and the knowledge of radiation heat transfer in the system can perhaps lead to a desired product with a desired characteristic. A few representative fields of interest in which combined heat and mass transfer plays an important role are designing of chemical processing equipment, formation and dispersion of fog, groves of fruit trees, crop damage due to freezing, environmental pollution, distribution of temperature and moisture over an agricultural fields. [3]

Studies in fluid mechanics involves the study of fluids either at rest (fluids static) or at motion (fluids dynamics and kinematics) and the subsequent effects of the fluid upon the boundaries which may be either at solid surfaces or interfaces with other fluids. Fluids unlike solid, lack ability to offer sustained resistance to a deforming force. Thus, a fluid is a substance which deforms continuously under the action of shearing forces, irrespective of the magnitude. Deformation is caused by shearing forces, forces that act tangentially to the surfaces to which they are applied. [4] In heat transfer, thermal radiation is a complex phenomenon which is to be account for because of its spectral and directional dependence in addition to the difficulty of determining accurate physical property values of the medium. Consequently, the addition of radiation term in the energy equation makes the equation to be more highly non-linear.

Erickson et al. [5] examined heat and mass transfer in the laminar boundary layer flow of a moving flat surface with constant surface velocity and temperature focusing on the effects of suction/injection.

Salem [6] studied the effects of variable viscosity, viscous dissipation and chemical reaction on heat and mass transfer flow of MHD micropolar fluid along a permeable stretching sheet in a non-Darcian porous medium. The fundamental importance of convective flow in porous media has been established in the recent books by Nield and Bejan [7]. Sparrow and Cess [8] considered the effect of magnetic field on free convection heat transfer on isothermal vertical plate. Krishnendu et al. [9] considered similarity solution of mixed convective boundary layer slip flow over a vertical plate. Seddeck and Almushigeh [10] examined the effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. Anjalidevi and Kandasamy [11] considered the effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate. Hayat et al [12] they considered the effect of joule heating and thermal radiation in flow of third grade fluid over a radiative surface.

Whitaker [13] considered the effects of radiation on heat transfer in porous medium. Mohammed [14] examined the effects of chemical reaction on dissipative radiative MHD flow through a porous medium over a non-isothermal stretching sheet. [15-18] examined the effect of variable viscosity on convective heat transfer along an inclined plate embedded in porous medium with an inclined magnetic field. Motivated by the work of Mohammed, a reacting variable thermal conductivity and chemical reaction on MHD boundary layer flow over a non-isothermal stretching sheet through a porous medium is considered. [14]

2. Governing Equations

Following the work of Mohammed, the steady two-dimensional forced convection boundary-layer flow of viscous, incompressible, electrically conducting fluid in a saturated horizontal porous medium is considered. The radiation heat flux is approximated with the Rosseland approximation. It is assume that there exists a homogeneous chemical reaction between the fluid and species concentration. The governing equations are continuity, momentum and energy equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)-\frac{\mu_{ef}\,u}{\rho K}-\frac{\sigma B_0^2 u}{\rho}+g\beta\left(T-T_\infty\right)+g\beta^*\left(C-C_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+Q_c \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_r\left(C-C_\infty\right) \tag{4}$$

Together with the boundary conditions

$$\mathrm{u}=\mathrm{cx}, v=0, T=T_w\left(x\right)=dx^m+T_\infty, C=C_w\left(x\right)=dx^m+C_\infty, at\ \ y=0$$

$$u=0,\ T\to T_\infty, C\to C_\infty, at\ \ y\to\infty \tag{5}$$

where k -Thermal conductivity, $\rho-Density$, C_p -Specific heat at constant pressure,

μ -Dynamic viscosity, u,v are the dimensional velocity component in the horizontal and vertical directions, k_0 – The thermal conductivity of the fluid, β -coefficient of thermal expansion, β^* -coefficient of concentration expansion, C - species inside the boundary layer, σ -electric conductivity, B_0 -the uniform magnetic, D -molecular diffusivity of the species concentration, $c>0$, d are constants, C-species far away from the plate Kr^* -first-order homogeneous constant reaction rate, α – the wall temperature parameter, ψ is the Frank-Kamenetskii parameter, q_r -radiation heat flux, Gr - Grashof number, Gc -modified Grashof number, Kr -the chemical reaction parameter, Q_c is the chemical reaction term, $Q_c=QC_0K_0\left(T\right)$,

C_0 is the initial concentration of reactant species, Q is the heat energy and

$$K_0\left(T\right)=J\left(\frac{kT}{vh}\right)^m\exp\left(-\frac{E}{RT}\right) \tag{6}$$

where J is the rate constant, k is the Boltzmann's constant, L is the velocity gradient, v is the vibration frequency, h is the Plank's number, E is the activation energy, R is the universal gas constant and m is a numerical exponent. M - Magnetic parameter, K_2 -permeability, R_d -Radiation, Ec - Eckert number, Sc -Schmidt number, T -Temperature inside the boundary layer, T_1, T_2,.....T_∞ -Temperature far away from the plate, θ-Dimensionless temperature, and f -is the stream function. The Rosseland approximation is followed to describe the heat flux in the energy equation.

We seek variable we seek variable thermal conductivity of the form:

$$k\left(T\right)=k_0e^{-\gamma\theta} \tag{7}$$

Following Rosseland approximation for radiant energy in an optically thick fluid the radiant heat flux of the fluid is expressed as follows:

$$q_r=\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{8}$$

where

σ^* is the Stefan-Boltzman constant, k^* is the Rosseland mean assumption coefficient and T^4 is expressed as a linear function of temperature given as

$$T^4\cong 4T_\infty^3-3T_\infty^4 \tag{9}$$

The following similarity transformations were introduced

$$u=\frac{\partial\zeta}{\partial y}=xcf', v=-\frac{\partial\zeta}{\partial x}=\sqrt{cv}f, \theta=\frac{T-T_\infty}{T_w-T_\infty}, C=\frac{C-C_\infty}{C_w-C_\infty} \tag{10}$$

Equations (1) -(10) yield

$$\frac{d}{d\eta}\left(f''\right)+ff''-\left(M-K_2\right)f'-\left(f'\right)^2+Gr\theta+Gc\varphi=0 \tag{11}$$

$$\theta'''\left(e^{-\gamma\theta}+\frac{4}{3}R_d\right)+\Pr f\theta'-\alpha\Pr f'\theta+Ec\Pr\left(f''\right)^2+\psi\left(1+\varepsilon\theta\right)^m e^{\frac{\theta}{(1+\varepsilon\theta)}}=0 \tag{12}$$

$$\varphi''+Scf\varphi-KrSc\varphi=0 \tag{13}$$

where

$$\Pr = \frac{\mu_0 \rho c_P}{k}, Ec = \frac{x^2 c^2}{c_P\left(T_w - T_\infty\right)}, Sc = \frac{\mu_0}{\rho} D, Kr = \frac{Kr^*}{c}, K_2 = \frac{\mu_{ef}}{cK\rho} \quad (14)$$

$$R_d = \frac{4\sigma^* T_\infty^3}{k^* K}, M = \frac{cB_0^2}{c\rho}, Gr = \frac{g\beta\left(T_w - T_\infty\right)}{c^2 x}, Gc = \frac{g\beta^*\left(C_w - C_\infty\right)}{c^2 x}, \psi = \frac{k^m h^2 T_0^{m-2}}{V^m h^m RK\mu_* V^2} \frac{EQC_0 A}{\left(RT_0^2 + T_0 E\right)} e^{-E/RT_0}$$

The transformed boundary conditions are as follows

$$f(0) = 0, f'(0) = 1, f(\infty) = 1, f'(\infty) = 0, \theta(0) = 1, \theta(\infty) = 0 \quad (15)$$

3. Result

Equations (9)-(11) subject to (13) are solve numerically using Galerkin-weighted residual method via f solve in Maple 18 Software as follows:

Let

$$f = \sum_{i=0}^{2} A_i e^{\left(-i/5\right)\eta}, \theta = \sum_{i=0}^{2} B_i e^{\left(-i/4\right)\eta}, \varphi = \sum_{i=0}^{2} C_i e^{\left(-i/4\right)\eta} \quad (16)$$

The results are presented in Figures 1-9 as follows:

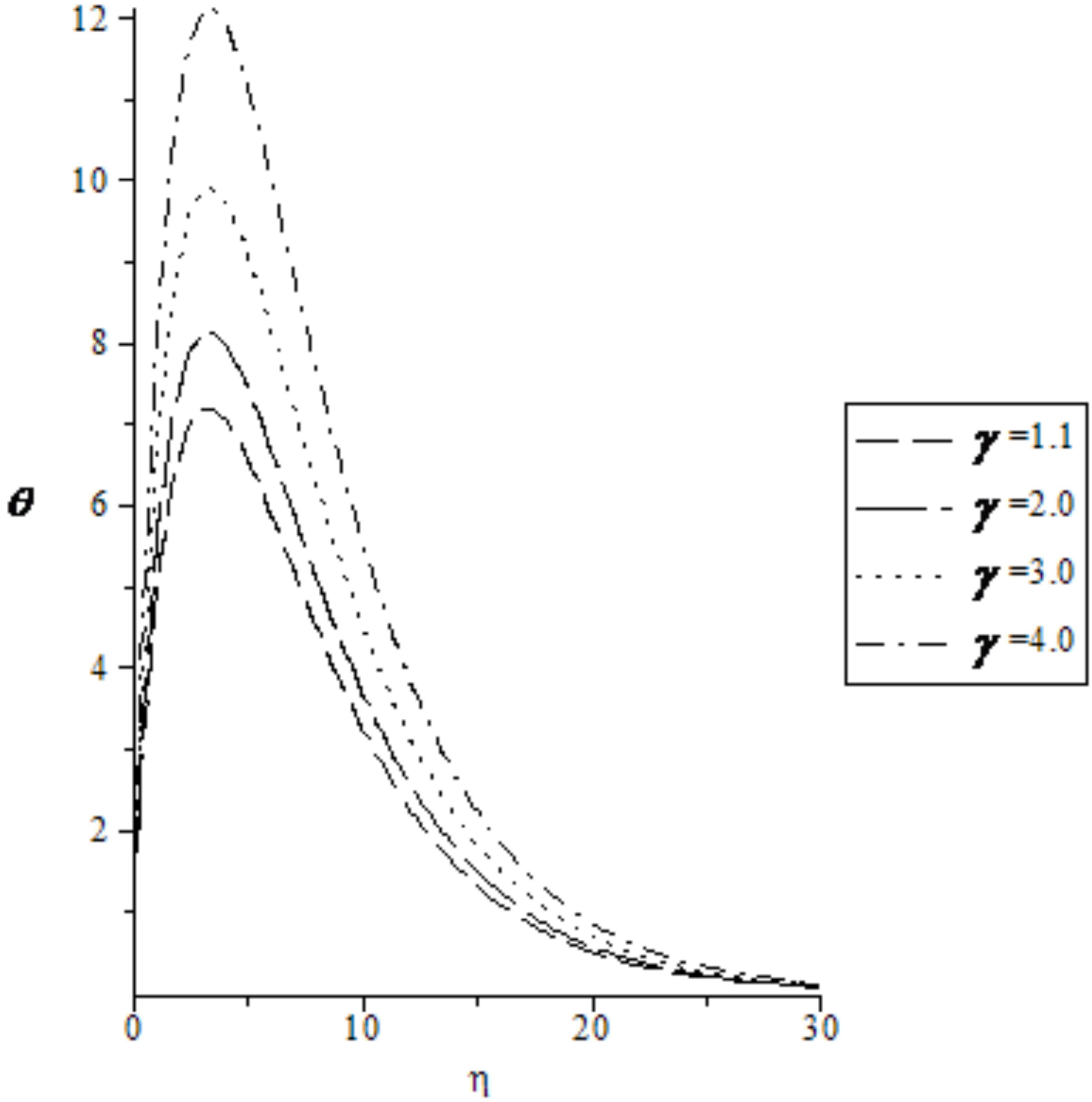

Figure 1. *Graph of the temperature function* θ *for various values of* $M = 0.5, \Pr = Ec = 1.0, Sc = \alpha = Kr = Gr = 1.0$

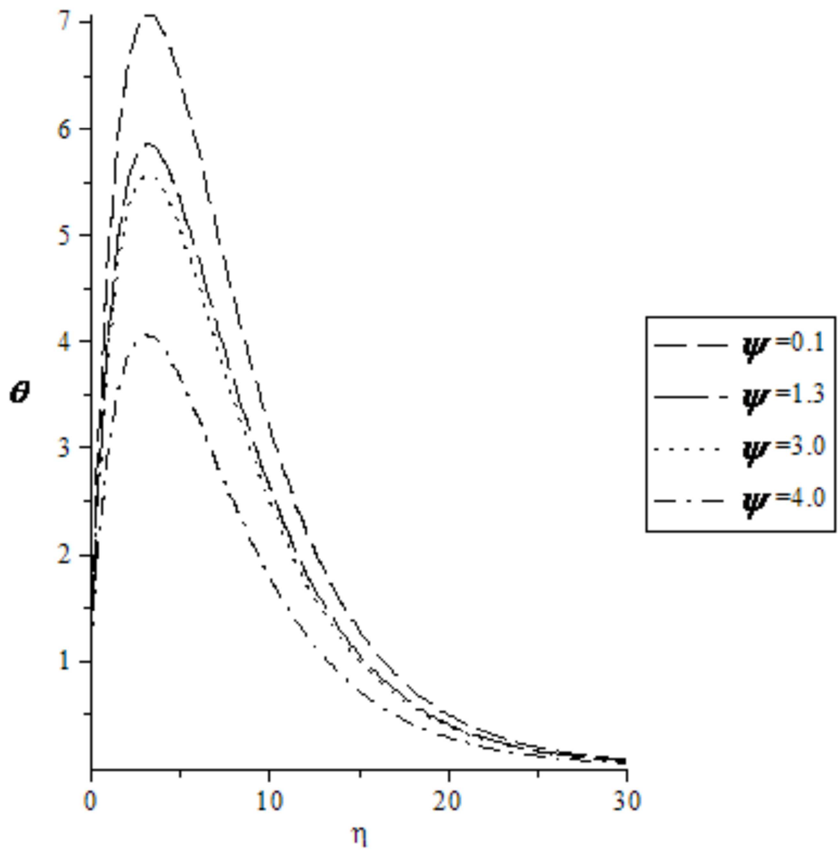

Figure 2. *Graph of the temperature function* θ *for various values of* $M=0.5, \Pr=Ec=1.0, Sc=\alpha=Kr=Gr=1.0$.

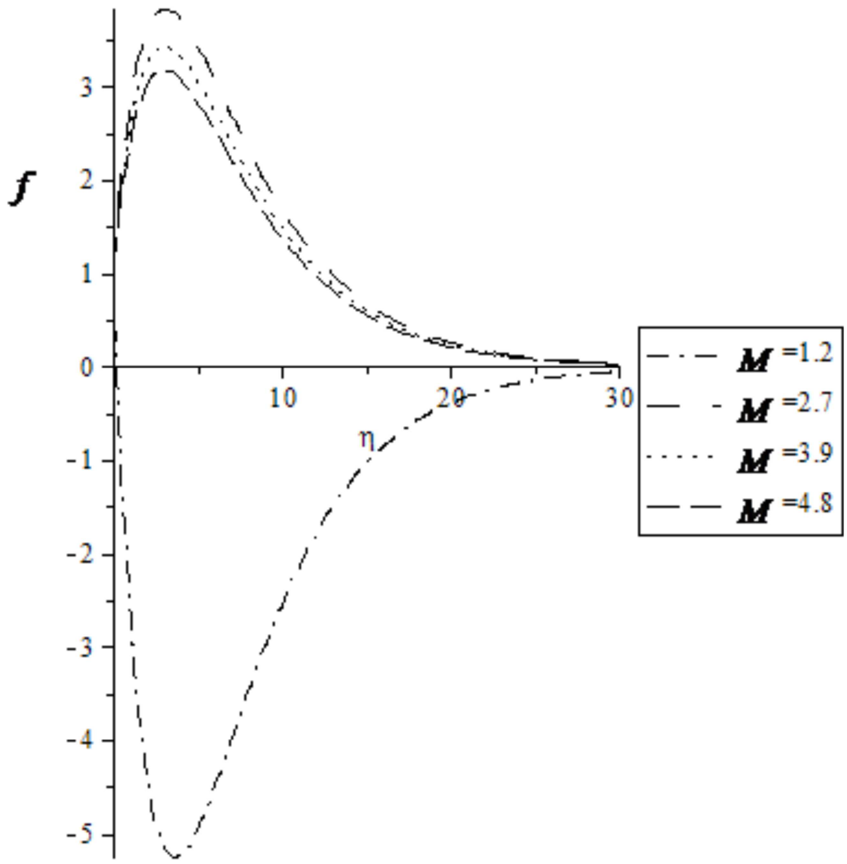

Figure 3. *Graph of the velocity function* f *for various values of* $M=0.5, \Pr=Ec=1.0, Sc=\alpha=Gc=Gr=1.0$.

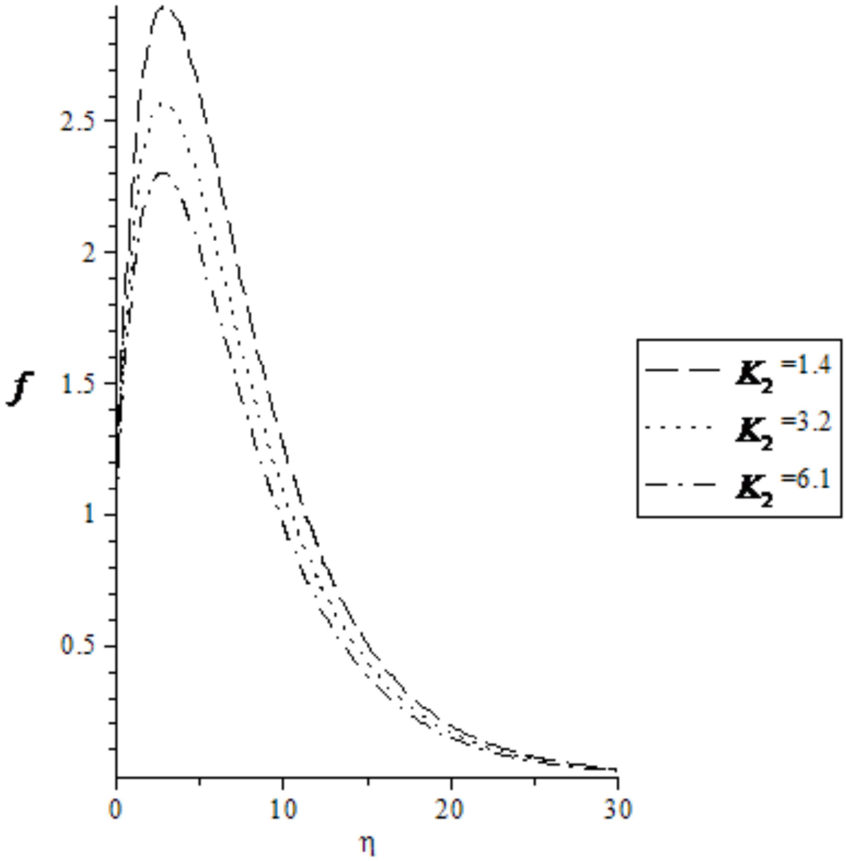

Figure 4. *Graph of the velocity function* f *for various values of* $M = 0.5, \Pr = Ec = 1.0, Sc = \alpha = Gc = Gr = 1.0$.

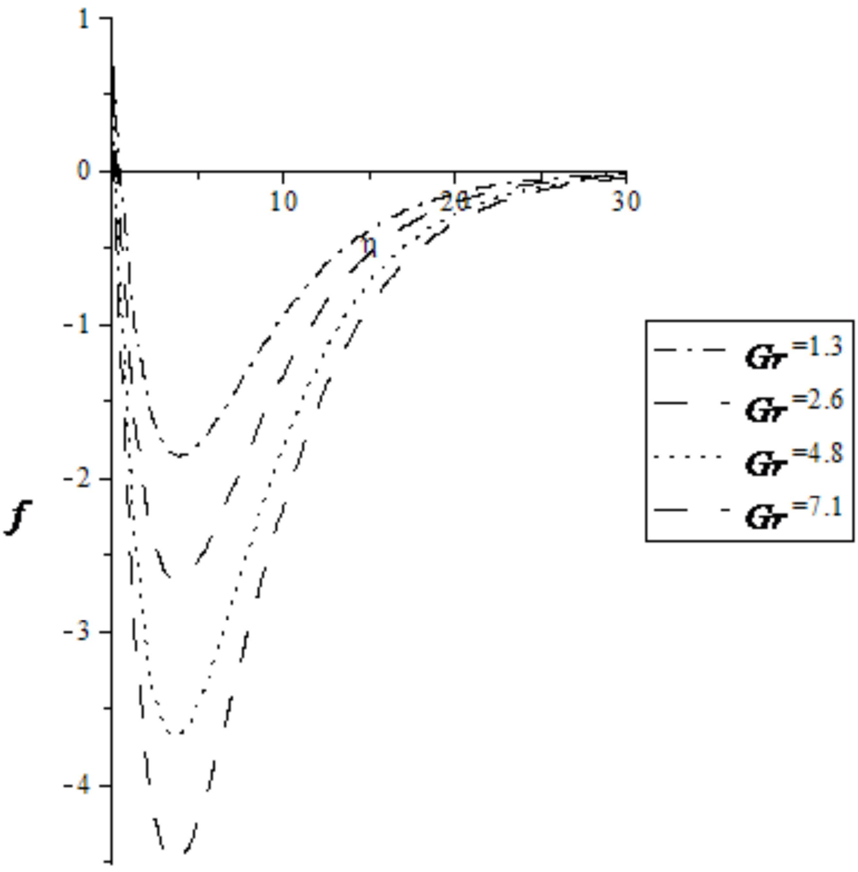

Figure 5. *Graph of the velocity function* f *for various values of* $M = 0.5, \Pr = Ec = 1.0, Sc = \alpha = Gc = Kr = 1.0$.

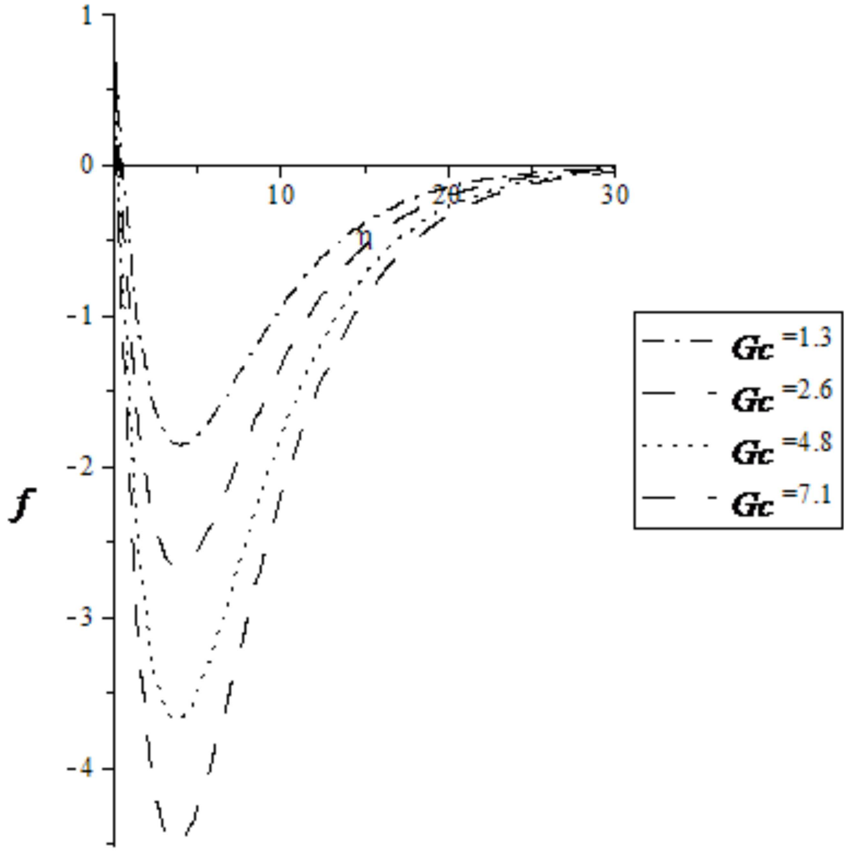

Figure 6. *Graph of the velocity function* f *for various values of* $M = 0.5, \Pr = Ec = 1.0, Sc = \alpha = Kr = Gr = 1.0$

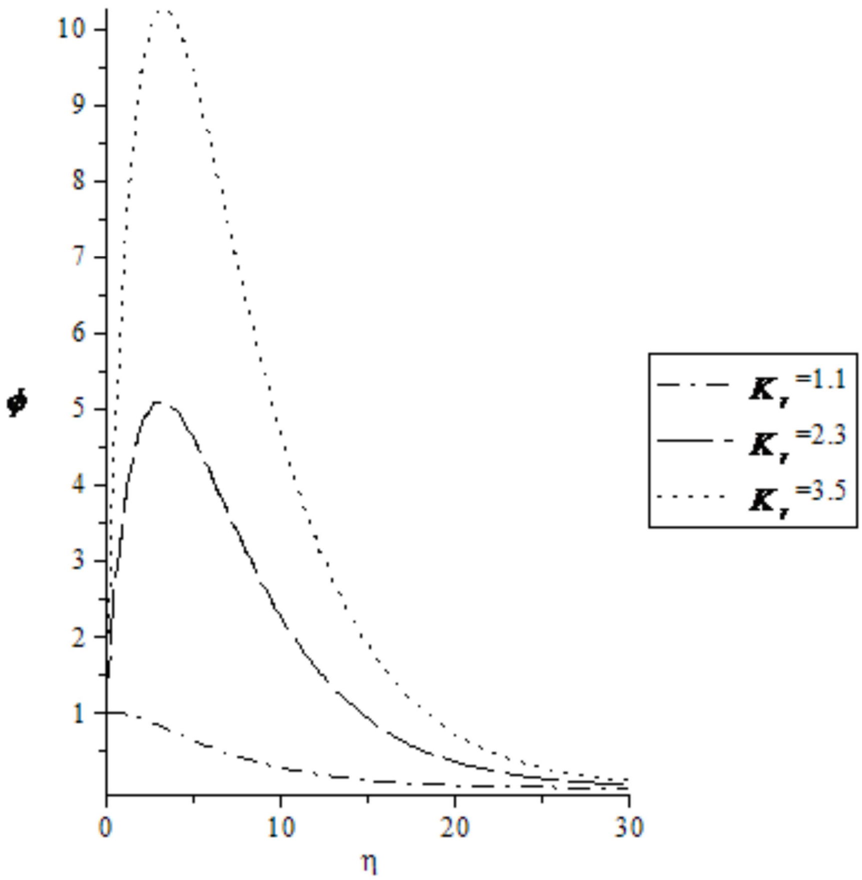

Figure 7. *Graph of the species function* φ *for various values of* $M = 0.5, \Pr = Ec = 1.0, Sc = \alpha = Kr = Gr = 1.0$

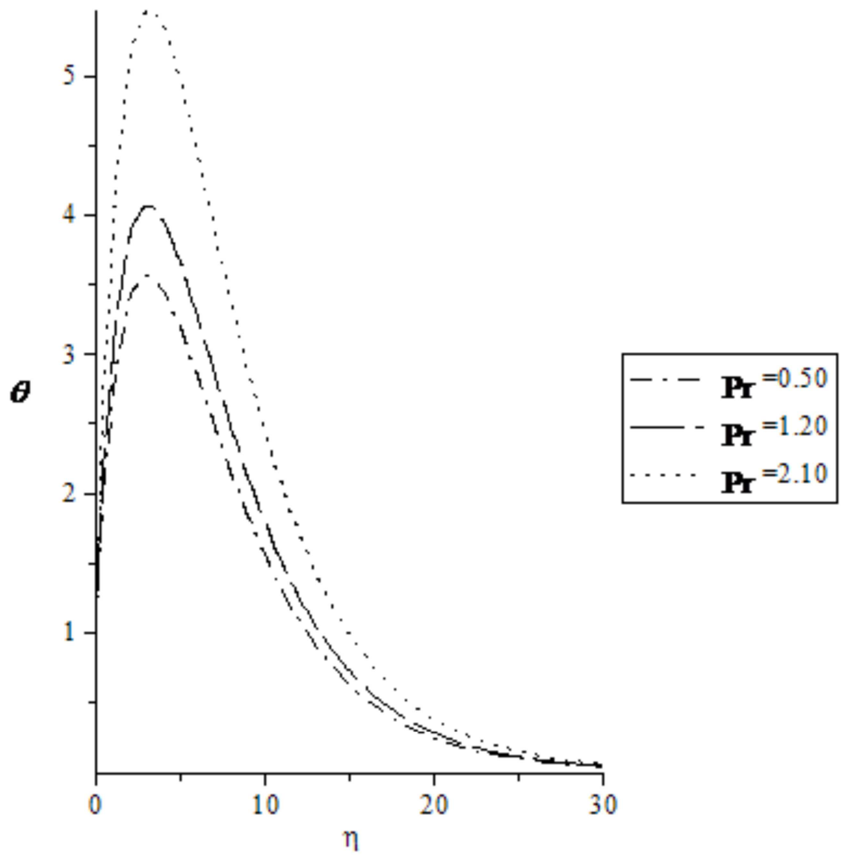

Figure 8. *Graph of the temperature function* θ *for various values of* $M=0.5, \mathrm{Pr}=Ec=1.0, Sc=\alpha=Kr=Gr=1.0$.

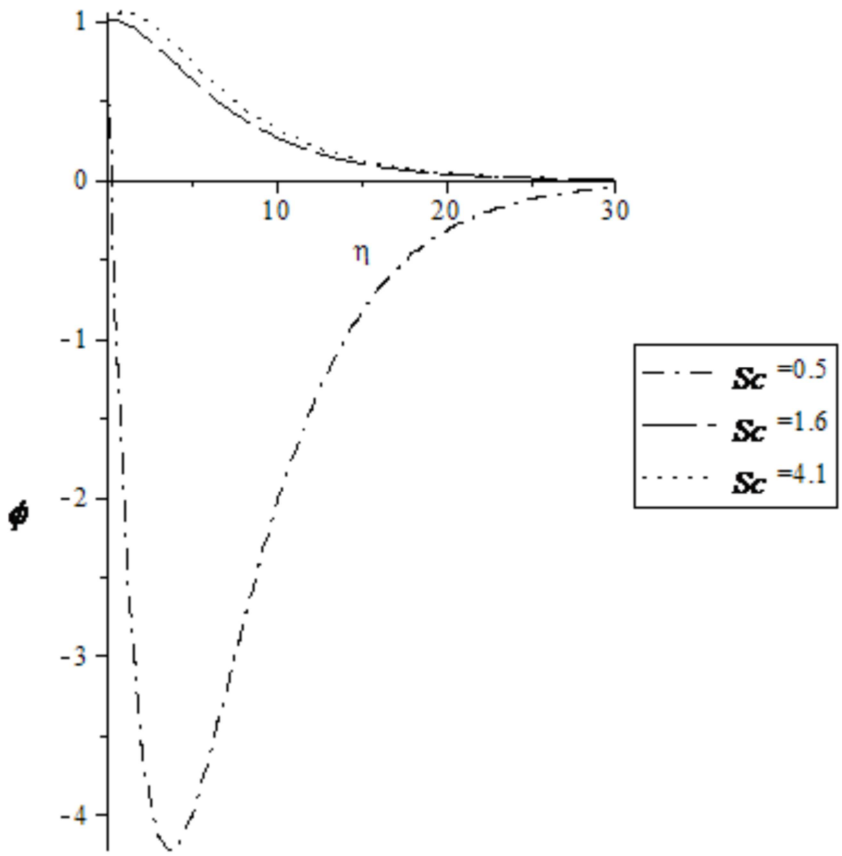

Figure 9. *Graph of the species function* φ *for various values of* $M=0.5, \mathrm{Pr}=Ec=1.0, R_d=\alpha=Kr=Gr=1.0$.

4. Discussion

From Figures 1-9 the results show that the velocity profile, concentration profile and temperature profile decreases with increase in each of variable thermal conductivity parameter, $\Pr = 0.71; Kr; Gc, Gr, K_2, Kr, Ec, \alpha, Sc$ parameters and Frank-Kamenetskii ψ parameter

It is seen from Figures 1 and 8 that as variable thermal conductivity parameter and Prandtl number increases, the viscosity of the oil in the reservoir decreases and more oil is recovered in an enhanced oil recovery processes. Prandtl number being the ratio of momentum diffusion to thermal diffusion that is, increase in Prandtl number means that the thermal diffusion decreases and the thermal boundary layer (being the temperature difference between the surface and the moving fluid) becomes thinner. High Prandtl number leads to low thermal diffusion consequently low Prandtl number leads to an increase in the thermal conductivity. Low Prandtl number plays a significant role in the fluid that is, heat conduction is very effective which implies that thermal diffusion is dominant in the flow system.

5. Conclusion

It is observed that velocity and temperature profiles decreases as Magnetic parameter and permeability parameter increases. It is noted that the concentration profile decreases as chemical reaction parameter increases. It is noticed from figure 1 that the temperature profile decreases as variable thermal conductivity parameter increases. It is observed from figure 2 that as Frank-Kamenetskii ψ parameter increases the temperature profile decreases. It is also shown from figures 1-2 the effects of various parameters on the temperature profile. It is noted that the temperature gradually decreases from a maximum value near the plate surface to zero far away from the plate satisfying the free stream conditions.

It is observed that minimum point exist in figures 3 and 4. For engineering purpose, the results of this problem are of great interest in oil recovery processes. Figures 3-6 depicts the effects of various physical parameters on the fluid velocity profiles. It is noted that for all the pertinent parameters, the velocity is maximum at the moving plate surface but decreases gradually to zero at the free stream far away from the plate surface thus satisfying the boundary conditions. It is also observed that as the reacting parameter increases it enhances the temperature which enhances quick recovery of oil from the reservoir.

It is observed from figure 8 that as Prandtl number increases, the viscosity of the fluid increases and thereby decreasing the temperature. Physically, it implies that for smaller values of Prandtl number the heat spread out quickly to the heated surface more rapidly compared to the momentum (velocity). It can be concluded that the increase physical parameters i.e. Schmidt number, Prandtl number, permeability parameter, and chemical reaction parameter leads to a corresponding decrease in the viscosity of the fluid. These will be of great interest to the field engineers in various processes of oil recovery.

It is shown that, increasing the values of permeability parameter lead to a decrease in the momentum boundary layer thickness and an increase in the thermal boundary layer thickness. From the application point of view, it is obvious that the cooling effect on the convectively heated plate surface is enhanced with increasing values and while an increase in Eckert number decreases the cooling effect.

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