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Fuzzy Set and Fuzzy Rough Set Concepts in Some Decision Making

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Abstract: Rapid growth in technology and its accessibility by general public produce voluminous, heterogeneous and unstructured data resulted in the emergence of new concepts. Different computational tools such as rough-set theory, fuzzy-set theory and fuzzy-rough-set that are often applied to analyze such kind of data are the focus of this chapter. Real-life data is often vague, so fuzzy logic and rough-set theory are applied to handle uncertainty and maintain consistency in the data sets. The aim of the fuzzy-rough-based method is to generate optimum variation in the range of membership functions of linguistic variables. In this paper we have discussed the definition of Fuzzy set and Fuzzy rough set. Then we have used fuzzy set and fuzzy rough set concept in some decision making of a real life problem of uncertainty. Here we have taken fuzzy set in universe X where all concepts are described. A lower and upper approximation of acoefficient of belongingness of an object x belongs to X to a fuzzy decision concept have been described. This can be done by means of a family of fuzzy concept defined on the universal set X.

Keywords: *Crisp set* Fuzzy Sets, Fuzzy Rough Sets, Deterministic Rough Set, Approximation, Decision Making

1. Introduction

The concept of rough set was originally proposed by Z. pawlak [12] as a mathematical approach to handle imprecision and uncertainty in data analysis. The theory of rough set deals with the approximation of an arbitrary subset of a universe by lower and upper approximations. As per F. Li and Y. Yin [7] this set has been used as an useful tool for approximation indecision situation. The main idea of rough approximation isthat of finding a lower and an upper bounds for a set A. Mrozek and R. slowinski [9] have described some advantage of this theory in real life situation. D. Dubois and Hprade [3-5] have explained some decision on real life problem by combination of fuzzy set and rough set several times. In section –2 we have described some basic concept and notation of rough set and fuzzy rough set. In section -3 we have explained our approach to the approximation of fuzzy decision sets which is based on degree of inclusion of fuzzy sets and in last we have given some numerical examples. On the basis of the classical variable precision rough sets model, Shenand Wand [10] defined the variable precision rough sets model over two universes. Then, the classical variable precision rough sets model was generalized to two universes. They studied the fuzzy rough set on two different universes based on a fuzzy compatible relation. They generalized the fuzzy rough set model on two different universes proposed by Sunand Maa [11] and presented the bipolar fuzzy rough set model on two different universes.

2. Fuzzy Sets

A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership (characteristic) function which assigns to each object a grade of membership ranging between zero and one. The notions of inclusion, union, intersection, complement, relation, convexity, etc., are extended to such sets, and various properties of these notions in the context of fuzzy sets are established Now we give the definition of a fuzzy set by Zadeh and C. Y. Wang [2]. Let U be a set called universe. A fuzzy set X in U is a membership function $\mu_x(x)$, which to every element x€U associated a real number from the interval (0 1), and $\mu_x(x)$ is the grade of membership of x in X. the union and intersection of fuzzy sets X and Y are defind as follows $\mu_{X\cup Y}(x) \equiv \max(\mu_x(x), \mu_y(x))$, $\mu_{X\cap Y}(x) = \min(\mu_X(x), \mu_Y(x))$ for every x ∈ U. The complement –X of a fuzzy set X is defined by the membership function $\mu_{-X}(x) = 1 - \mu_X(x)$, for every x∈ X,

3. Rough Sets

Rough sets are introduced by Z-Pawlak which belong to the family of concepts concerning the modeling and representation of knowledge. Let U denote the set of universe and let R be an equivalence relation on U called Indiscernibility relation. Equivalence classes of the relation R are called elementary sets in A. Any union of elementary set is called a composed set in A. The family of composed sets in A is denoted com (A). The pair A=(U, R) will be called an approximation space. Let X$\subseteq U$ be a subset of U and let D(X) and G(X) respectively as follows

$$D(X)=\{ x\in U;\ [x]_R \subset X \}$$

$$G(X)=\{x\in U;[x]_R \cap X \neq O\} B(X)=G(X) - D(X),$$

Where $[x]_R$ represents the equivalence class of x∈ U and B(X) is called boundary of X in A.G(X) is called best upper approximation of X in A and D(X) is called best lower approximation of X in A. As per J. Hu, T. Li, C. Luo, H. Fujita and S. Li, [8] a rough set X^r corresponding to the classical but may be, imprecisely described subset X$\subset U$. We shall mean the ordered pair composed of D(X) and G(X). Then we write X^r ={D(X), G(X)} and assume that the impreciseness of the description of X arises from the lack of complete information rather than fuzziness. Using D(X) and G(X) one can introduce two types of belonging to X in ∈ A. If x∈ $D(X)$ and x∈ $G(X)$ respectively then we can say that x surely belongs to X Moreover, it is possible to define various kinds of approximation inclusion and equalities of sets.

4. Basic Concepts and Notations of Rough Sets and Fuzzy Rough Sets

Recently, D. Chen, Y Yang and Z Dong [6] have explain some excellent incremental methods for updating knowledge while the variation of the object set based on classical rough set were reported in some real life problem and a lot of noticeable incremental methods were studied for generalized rough set model. Let X = {Xl, X2..... *Xn*} be a set of objects. Denoted by ~(X) the set of all crisp subsets from X and by~(X) the set of all fuzzy subsets from X. Obviously,~(X) C ~(X). Any subset A C X will be called a concept in X and/*A will denote the membership function of A. Let C = {C1.....*Cm}, Ci C ~(X)* be a family of concepts in X which forms a partition of X and let Y be a crisp subset of X. Z. Pawlak [12-15] introduced the lower and the upper approximation of Y by means of concepts Ci C C as:

$$\frac{C \quad Y}{CiC \quad Y} = UG \tag{1}$$

$$\frac{c' \quad Y}{c \quad my\#O} = UG \tag{2}$$

The pair (C(Y), C(Y)) is called a rough set, or deterministic rough set. It is evident, that for all

xEX:#_c(r)(x) ~< ~r(x) ~< ~&r)(x) (3)

A set Y E ~(X) is definable in approximation space (X,C) if and only if C(Y) = C(Y). A set Y E J/(X) is roughly definable in approximation space (X, C) if C(Y) # C(Y). The quality of approximation is

Measured by the coefficient card

C(Y)~; c(Y) (4)

card X. A set of decision rules can be created to determine whether an object satisfying the description *Des(c/)* of the known concept C/. Also satisfies the description *Des(Y)* of the concept Y. The decision rules are

if C/ E C(Y) then *Des(c/) --+ Des(Y),*

if C/ E X - C(Y) then *Des(c~) -+ not Des(Y).*

Objects from X are usually described by a family of attributes S = {$1..... Sp}, where each *Si* induces a partition of X. Then the approximation space for an approximation of a concept Y E ~(X) is *(X, Cs* =As, *E s Si).* In order to simplify the decision situation, we try to find a minimal subset S*C S such that

7q.(r) = 7cs(r).

Dubois and Prade [3] consider a situation where concepts C1..... *Cm;* C/E if(X) form a weak fuzzy partition of X, and concept Y is a fuzzy set on X. Let us recall that C = {Cl..... *Cm}* is a weak fuzzy partition of X provided each C/is a normal fuzzy set (i.e. max x*pq(x)* = 1) and for all i inf max

pc,(x) > 0 (5)

while for all i, j,

i#j supmin{pq(x), pq(x)} < 1 (6)

Then the lower and the upper approximations of Y by means of C are defined as fuzzy sets *of X/C* with membership functions given by

pc_(y)(C/) = inf max{1 – pq(x), pr(x)} (7)

Pd(r)(C/) = sup min{pc,(X), pr(x)} (8)

Pc(r)(c/) is the degree of certain membership of *C*~in Y and PS(r)(C/) is the corresponding degree of possible membership. The pair (Pc(r), P~(r)) is called a fuzzy rough set. A fuzzy rough set provides information about the strength of the relationship between Y and C. Ourgoal is to find fuzzy sets Pc(r) E if(X) and PC(r) 6,~-(X) such that for all x E X

p_c(r)(X) ~<pr(x) ~ p 8(r)(x) (9)

which is analogous to (3) in the case of crisp sets. We also require that pc(r) and PS(r) can be simply derived from Pc,, C/E C, which enables one to obtain decision rules with clear interpretation. A method of approximation satisfying the above conditions is explained in the next section.

5. Fuzzy Rough Sets

For V, W C ~(X) we use:

#z u w(x) = max{#z(x), pw(x)} (10)

P v n w(X) = min{pz(x), pw(x)} (11)

VC W iffpv(x) <~pw(x) for all x EX (12)

supp V = {x E *X; pv(x)* > 0} (13)

card V = ~ *pv(x)* (14)

There are numerous examples of inclusion grades I between two fuzzy sets [1]. For example, for V, W E~-(X), the inclusion grade of V in W can be calculate das follows

Card(Wn V) *I(W, V)* - card V (15)

We propose the following formal definition of an inclusion function of fuzzy sets.

Definition 1. A function I: ~(X) × ~(X) ~ [0, 1] satisfying the conditions:
for all

V, W E o~(X): *I(W, V)* = liff VCW (16)

if VNW =~J then I(W,V)=l(V,W)=O (17)

is called an inclusion function of fuzzy sets. The value of I(W, V) is the degree of inclusion of fuzzy set Vin fuzzy set W or the degree of covering of fuzzy set V by fuzzy set W. We will use inclusion functions in definition of 2-approximable fuzzy sets

Definition 2. Let X be a finite set of objects. Let C *={Cl.....fro}* be a family of fuzzy sets from ~-(X) such that X C supp *Uc, ~c Ci* and let Y E ~(X). Let I be an inclusion function of fuzzy sets and 2 E (0, 1]. We say that Y is 2-approximable in approximation space *(X, C,1)*

if min{l(Y, N c i), I (U ci, YI} >~.\ cicc \c, cc / (18)

The coefficient

Tc= card_~c~cc(H C/- ('l, c,~c*Ci*) card X (19)

is called the tolerance of approximation.

It is obvious that ifI(Y, *~c, cc* C/) = 2L, we can say with degree of certainty 2L that/~ n ci (x) ~</~ r (x) for allx E X. Analogously, *If I(Uc, cc* c/, Y) = 2u, we can say with degree of certainty 2u that #~.(x)<.#uc,(x) for all x E X. If 2L = 2e = 1, we are sure that for all

x EX:/ln<(X) ~</~r(x) ~< #u <(x) (20)

The expression (20) will help to approximate/~r(x) only if/~uc,(x) < 1 and *#nc,(x) > O.*

Let Y E ~(X) be a 2-approximable set in approximation space *(X, C,I). Our* goal is to find A* C C and *B*CC such that I(Y, Ncic A. Ci)>>.2,* I(Uc, cB* *Ci, Y)>>,2 and card (UcicB. Ci - Nc, c,~.Ci)* is minimal. That is, we want to find the largest intersection of fuzzy sets from C included in the fuzzy set Y at least to degree 2, and the smallest union of fuzzy sets from C covering Y at least to degree 2. The pair (C~(Y), Ca(Y)) is called a modified fuzzy rough set. From the approximations of Y E ~(X) by the Modified fuzzy rough set, one can derive the following decision rule: With degree of certainty 2, object x E X belongs to fuzzy set Y with the coefficient of membership being at least as high as min c,c A*{#c,(x)}, but not higher than max c, c B* {#ci(x)}. The quality of approximation is given by the tolerance

Coefficient card((~(Y) - C~(Y)) (21)

V c~(r) = card X

Note: If there are more pairs of sets A*, B* satisfying the condition from Definition 3, then we can derive more than one approximation of Y E ~-(X) by modified fuzzy rough sets in approximation space *(X, C,I).*

Example 1. Suppose X = {xl..... x8}. Let each object x E X be described by the following attributes: condition attribute $1 with condition concepts (categories) *C1, C2,* C3; condition attribute $2 with condition concepts Ca, C5, C6 and decision attribute Y with decision concepts Y1, *Y2, Y3.* All concepts are crisp sub sets of X. First of all we find an approximation of *Yj E Y,* j = 1, 2, 3, in approximation space (X, S = S1 NS2) by deterministic rough sets. Then we approximate Yj in approximation space (X, C = {$1, $2}, I) by modified fuzzy rough sets. The inclusion function I is defined as follows:

For A, B C X:

card (A N B) *I(A,B)* – card B (22)

Then *C={&, S2}* ={CI = {2,3,8}, C2 = {1,5}, C3 ={4,6,7}, C4 = {4,6}, C5 = {3,5,7,8}, C6 = {1,2}}.

The attributes defined on X create the following partitions of X:

S~ ={C1 = {2,3,8}, C2 = {1,5}, C3 = {4,6,7}},
$2 = {C4 = {4,6}, C5 = {3,5,7,8}, C6 = {1,2}},
S = S1 (-)$2 = {C1,5 = {3, 8}, C1,6 = {2},
C2,5 z {5}, C2,6 = {1},
C3, 4 ~-- {4, 6}, C3.5 = {7}},
Y = {I/1 = {1,5,6,7}, Y2 = {2,4,8}, I/3 = {3}}.
Approximation of Y m.
1. Deterministic rough sets:
S(Yl) = C2,5 I..j C2,6 [-J C3,5 = {1, 5, 7},

S(Y~) = C2,5 U C2,6 U C3,s tO C3,4 = {1,4,5,6,7}.

2. Modified fuzzy rough sets, 2 = 1:

C2(Y1) = C 2 = {1,5},

C;~(Y~) = C2 U C3 = {1,4,5,6,7}.

Therefore for each x E X, *#c2(x) <<.pr, (x)* <~ max{Pc2(x),/~c3(x)}.

In this case C~(Y1) C S(Y1) and C~(Y1) = S(Y1).For the deterministic rough set approximation we used combinations of concepts C2, C3, Ca, C5 and C6; while for modified fuzzy rough set approximation only combinations of concepts *Cz* and C3

Approximation of I/2.

1. Deterministic rough sets:

s(r2) = C1,6 • {2},

S(Y2) = C1,5 to C1,6 UC3,4 z {2,3,4,6,8}.

2. Modified fuzzy rough sets, 2 = 1:

c;.(Y2) = c~ n c6 = {2},

C~(Y2) = Cl to Ca = {2,3,4,6,8}, or C~(Y2) =C1 U C3 = {2,4,6,7,8}.

Therefore for each x E X,

min { pc, (x), Pc6 (x)}<~ *Pr2 (x)* <~ max{pc, (x),/gc4 (x)}

Or

min{ #c, (x), ktc6 (x) }<~ *#r2(x)* <~ max{pc, (x), Pc3 (x)}.

In this case C~(Y2) = S(Y2) and Ca(Y2) A ~q(Y2) ¢ 0. The approximation by deterministic rough sets requires combinations of concepts C1, C3, C4, C5 and C6; while the approximation by modified fuzzy rough sets requires combinations of Cl, C3, C4 and C6.

Approximation of Y3.

1. Deterministic rough sets:

S(Ys) = O,

S(Y3) = C1,5 = {3,8}.

2. Modified fuzzy rough sets, 2 = 1:

c~,(r3) = O,

C;~(Y3) = Cl = {2,3,8}.

In this case C;~(Y3) = S(Y3) and S~(Y3) C C(Y1).

The approximation by deterministic rough sets requires combinations of concepts C1 and C5; while the approximation by modified fuzzy rough set uses only the concept C~. The lower approximation of Y3 by deterministic rough sets, and the lower approximation of Y3 by Modified fuzzy rough sets for 2 -- 1, is empty set. If we consider 2 = ½, then C;~(Y3) = C1. Because *(J;~(Y3)* =C1, we can expect, with degree of certainty 2 = ½, that #y;(x) = #c, (x)

6. Conclusion

We have introduced a new method of approximation of a fuzzy decision concept Y E ~(X) by fuzzy condition concepts *Ca..... C, n; Ci C ~(X).* This method can be used in a very general decision situation, because the only requirement is that X c supp U *ci.* However, from the computational point of view, it requires a large space to store data. Therefore, it is recommended only in cases where the number of concepts is small. If {c1...., *Cm}* forms a weak fuzzy partition of X, we can also use approximation by fuzzy rough sets [3]. Approximation of Y by modified fuzzy rough sets provides the lower and the upper bounds of the coefficient of membership #r(X) for each x € X. The quality of approximation is measured by the tolerance coefficient. (Fuzzy rough sets give information only about the strength of a relationship between the condition and decision fuzzy sets.) Modified fuzzy rough sets enable one to derive decision rules with simple interpretation. This method also provides some partial decision rules where the degree of approximation of #r(x) is 2 E (0, 1). The approximation of Y depends on the choice of inclusion function 1; therefore, I must be specified in approximation space *(X, C, I).* Modified fuzzy rough sets can also be used in the approximation of concepts described by crisp sets.

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