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Calculations for Crack Growth Rate in Whole Process Realized with the Single Stress-Strain-Parameter Method for Elastic-Plastic Materials Contained Crack

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Abstract
In consideration of the short crack and the long crack behaviors there are distinctly different, to use the theoretical approach, to adopt the simple stress-parameter, or the strain-parameter-method, to establish some new calculation models in whole crack propagation process for elastic-plastic steels, which are the crack growth driving forces, the crack growth rate equations for different stages, the crack growth rate-linking-equation in whole process; For the transitional crack size and the crack growth rate at transitional point from short crack to long crack growth process respectively to put forward different expressions, also to provide the concrete and detailed calculation the steps and the methods; With respect to some key materials parameters for new discovering and there are functional relations, respectively to give the new calculable formulas, the new definitions, the new physical meanings and geometrical meanings for them. Thereby to make linking and communication between for the modern fracture mechanics and the traditional material mechanics; to realize calculations for the crack growth rate in whole process based on conventional material constants.

1. Introduction

In consideration of the micro crack and the macro crack behaviors are obviously different under different loading conditions, In view of complexity of elastic-plastic material properties contained crack, so to research the problems of the driving forces under so many factors and conditions, to establish the crack propagation rate models in the whole process, which are all very complicated problems.

As everyone knows for the traditional material mechanics, that is a calculable subject, and it has done valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the crack growth rate problems for some structures when it is pre-existing flaws and concentrated stress under repeated loading. In that it has no to contain such calculable parameters as crack variable a or as the damage variable D in its calculating models. But in the fracture mechanics and the damage mechanics, due to there are these variables, so they can all calculate above problems. Nowadays latter these disciplines are all subjects mainly depended on tests. So that, for above elastic-plastic materials and structures of contained defects, if want to solve the crack growth rate calculations for the whole propagation process from the micro crack to macro crack, that are more difficult, to pay the manpower and money for experiments are more huge.

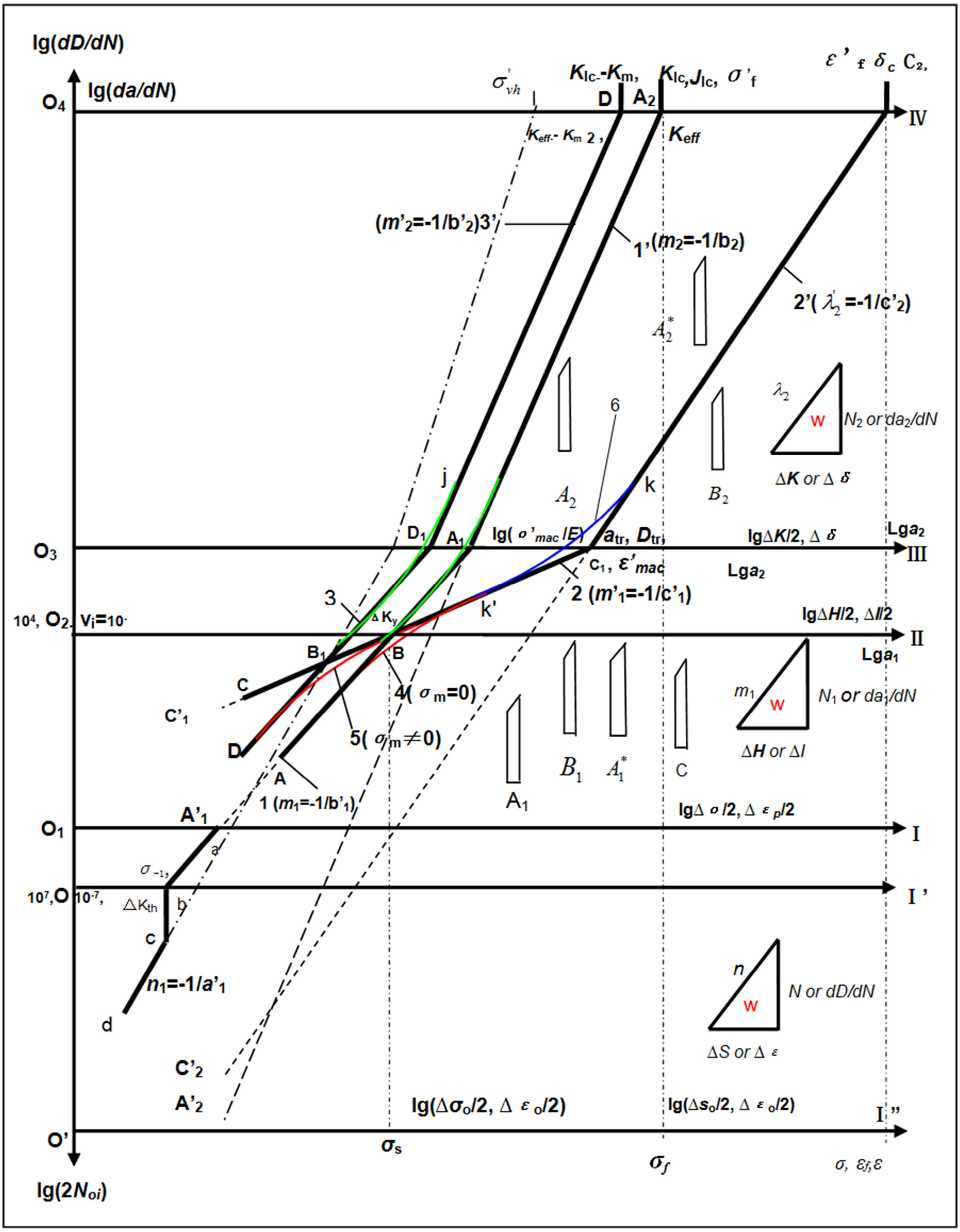

Figure 1. Comprehensive figure of material behaviors (Called calculating figure of material behaviors or Called bidirectional combined coordinate system and simplified schematic curves in the whole process) [1-3].

Author thinks, in the mechanics and the engineering fields, where are also to exist such a scientific law as similar to genetic elements and clone technology in life science. Author has used the theoretical approach as above the similar principles, proposes some calculation models [1-7], recently sequentially discovers some new scientific laws, adopts the simple stress-parameter or the strain-parameter-method, provides some new calculable models for the crack growth driving force in different stages and for the crack growth rate in whole process. Try to make the fracture mechanics, step by step become such calculable disciplines as the traditional material mechanics. That way, it may be having practical significances for decreasing experiments, to stint man powers and funds, for promoting engineering applying and developing to relevant disciplines.

2. Crack Growth Rate Calculations in Whole Process for Elastic-Plastic Steels of Containing Crack

For some elastic-plastic steels of pre-existed flaw, about its driving force, crack growth rate and life's calculation equations for short crack growth processes, for which some models have been proposed in reference[1-7]; And its driving force, crack growth rate and life's calculating problems for long crack growth processes, some models has been also provided related references.

Inside this paper, from short crack to long crack, it uses a called as "the single stress or strain parameters method" for the crack propagating rates puts up the whole process calculations, that are by means of the stress σ and the strain ε as "genetic element" [7-8], to establish various calculable models for the driving forces and the crack growth rates, thereby achieve the calculations of crack propagating rates in whole process under low cycle fatigue loading.

2.1. The Calculations for Short Crack Growth Process

Under the work stress is more than yield stress $\sigma > \sigma_s (= \sigma_y)$ or low fatigue condition, the short crack growth rate equation corresponded to positive direction curve CC_1 in attached fig.1, here to adopt the strain range $\Delta\varepsilon_p$ to express that is as following form

$$da_1 / dN_1 = B_1 \, (\Delta I)^{m'_1} \, (mm/cycle) \quad (1)$$

Here

$$I_1 = (\varepsilon_p)^{m_1'} \cdot a_1, (\%^{m_1'} \cdot mm) \quad (2)$$

$$\Delta I_1 = (\Delta\varepsilon_p)^{m_1'} \cdot a_1, (\%^{m_1'} \cdot mm) \quad (3)$$

Where the a_{01} is an initial micro crack size, The I_1 is defined as short crack growth strain factor, that is driving force of short crack growth under monotonous load; ΔI_1 is defined as strain factor range, that is driving force under fatigue load, their units are "$(\%)^{m_1'} \cdot mm$". ε'_f is a fatigue ductility factor, m'_1 is fatigue ductility exponent, $m'_1 = -1/c_1'$, c_1' just is also a fatigue ductility exponent under low cycle fatigue. The B_1 is defined as comprehensive material constants. Author thinks, its physical meaning of the B_1 is a concept of the power, it is a maximal increment value to give out energy in one cycle before failure. Its geometrical meaning of the B_1 is a maximal micro-trapezium area approximating to beeline (attached fig.1) that is a projection of corresponding to curve 2(CC_1) on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m'_1 of the formula (4). And the B_1 because there is functional relation with other parameters, so the B_1 is a calculable comprehensive material constants,

Here

$$B_1 = 2[2\varepsilon'_f]^{-m'_1} \times (v_{eff})^{-1}, (\%)^{m_1'} \times mm / cycle \quad (4)$$

$$\begin{aligned} v_{eff} &= \ln(a_{1fc} / a_0) / N_{1fc} - N_{01} \\ &= [\ln(a_{1fc}/a_0) - \ln a_1/a_{01})] / N_{1fc} - N_{01}, (mm/\text{cycle}) \end{aligned} \quad (5)$$

or

$$v_{eff} = [a_{1fc} \ln(1/1 - \psi)] / N_{1fc} - N_{01}, (mm/\text{cycle}) \quad (6)$$

The v_{eff} in eqn (4-6) is defined as an effective rate correction factor in first stage, its physical meaning is the effective damage rate to cause whole failure of specimen material in a cycle, its unit is $mm/cycle$. ψ is a reduction of area. a_0 is pre-micro-crack size which has no effect on fatigue damage under prior cycle loading [9]. a_{01} is an initial micro crack size, a_{fc} is a critical crack size before failure, N_{01} is initial life in first stage, $N_{01} = 0$; N_{1fc} is failure life, $N_{1fc} = 1$ .Such, its final expansion equation for (1) is as following form,

$$da_1 / dN_1 = 2[2\varepsilon'_f]^{-m'_1} \times (\Delta\varepsilon_p)^{m_1'} / v_{eff} (mm/cycle) \quad (7)$$

If the materials occur strain hardening, and want to via the stress σ to express it, due to plastic strain occur cyclic hysteresis loop effect, then the crack growth rate equation corresponded to positive direction curve CC_1 in Fig1should be

$$da_1 / dN_1 = A_1 \, (\Delta H_1 / 2)^{m_1}, (mm/cycle), (\sigma > \sigma_s) \quad (8)$$

Where

$$H_1 = \sigma \cdot a_1^{1/m_1} \quad (9)$$

$$\Delta H_1 = \Delta\sigma \cdot a_1^{1/m_1} \quad (10)$$

The H_1 is defined as the short crack stress factor, the $\Delta H_1/2$ is stress factor amplitude. Same, that H_1 is driving force of short crack growth under monotonous load, and the ΔH_1 is driving force of under fatigue loading. Its physical and geometrical meaning of the A_1 are similar to the B_1 .The A_1 is also calculable comprehensive material constant, for $\sigma_m = 0$, it is as below

$$A_1 = 2(2\sigma_f^{'})^{-m_1}(v_{eff})^{-1}, (\sigma_m = 0) \quad (11)$$

But if $\sigma_m \neq 0$, here for the eqn (8) to adopt the correctional method for mean stress by in reference [10] as follow

$$A_1 = 2[2\sigma_f^{'}(1-\sigma_m/\sigma_f)]^{-m_1}(v_{eff})^{-1}, (\sigma_m \neq 0) \quad (12)$$

Or

$$A_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1}, (\sigma_m \neq 0) \quad (13)$$

Where the $\sigma_f^{'}$ is the fatigue strength coefficient, K' is the cyclic strength coefficient. $m_1 = -1/b'_1$, m_1 and b'_1 are the fatigue strength exponent. $m_1 = -1/c'_1 \times n'$, $n' = b_1^{'}/c_1^{'}$, n' is a strain hardening exponent. So that, its final expansion equation for (8) is as below form,

$$da_1/dN_1 = 2(2\sigma_f^{'})^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), \quad (\sigma > \sigma_s, \sigma_m = 0) \quad (14)$$

$$da_1/dN_1 = 2[2\sigma_f^{'}(1-\sigma_m/\sigma_f)]^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), (\sigma > \sigma_s, \sigma_m \neq 0) \quad (15)$$

If to take formula (13) to replace A_1 into eqn. (8), its final crack rate expansion equation is as below forming

$$da_1/dN_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma_f)]^{1/c'}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), (\sigma > \sigma_s, \sigma_m \neq 0) \quad (16)$$

Here, when $\sigma >> \sigma_s$, influence of mean stress in eqn (15-16) can be ignored.

2.2. The Calculations for Long Crack Growth Process

Under $\sigma > \sigma_s$ condition, due to the material behavior comes into the long crack growth stage, the exponent in crack growth rate da_2/dN_2 equation also to show change from $m_1^{'}$ to λ_2; and due to it occurs cyclic hysteresis loop effect, its long crack growth rate model corresponded to positive direction curve C_1C_2 in figure 1 is as below form

$$da_2/dN_2 = B_2^{'}[y_2(a/b)\Delta\delta'_t/2]^{\lambda_2}, (mm/Cycle) \quad (17)$$

Where

$$\delta_t = 0.5\pi \times \sigma_s \times a_2(\sigma/\sigma_s + 1)/E, \quad (18)$$

$$\Delta\delta_t = 0.5\pi \times \sigma_s \times a_2(\Delta\sigma/2\sigma_s + 1)/E, \quad (19)$$

Where δ_t is a crack tip open displacement, $\Delta\delta_t$ is a crack tip open displacement range [12]. The $y_2(a/b)$ is correction factor[13] related to long crack form and structure size. Here should note the B_2 is also a calculable comprehensive material constant, for $\sigma_m = 0$, it is following form

$$B_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)a_{2eff}/E)]^{-\lambda_2} \times v_{pv}, (\sigma_m = 0) \quad (20)$$

for $\sigma_m \neq 0$,

$$B_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1-\sigma_m/\sigma'_f)a_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}, (\sigma_m \neq 0) \quad (21)$$

$$v_{pv} = \frac{(a_{2pv} - a_{02})}{N_{2eff} - N_{02}} \approx 3\times10^{-5} \sim 3\times10^{-4} = v^*(mm/Cycle) \quad (22)$$

Where the v_{pv} is defined to be the virtual rate, its physical meaning is an equivalent propagation rate contributed for the test specimen pre crack in the second stage, the unit is $mm/cycle$, its dimension is similar to the v^* -value in reference [14], but both units are different, where is the "$m/cycle$" . And the λ_2 is defined to be ductility exponent in long crack growth process, $\lambda_2 = -1/c_2^{'}$, $c_2^{'}$ is a fatigue ductility exponent under low cycle in second stage. So that, the conclusive expansion equations is derived from above mentioned eqn.(17) as follow

For $\sigma_m = 0$,

$$da_2/dN_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)a_{2eff}/E)\right]^{-\lambda_2} v_{pv} \times \left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s + 1)a_2}{E}\right]^{\lambda_2}, \quad (mm/cycle) \quad (23)$$

For $\sigma_m \neq 0$, it should be

$$da_2/dN_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1-\sigma_m/\sigma'_f)a_{2eff}/E)\right]^{-\lambda_2} v_{pv} \left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s + 1)a_2}{E}\right]^{\lambda_2}, \quad (mm/cycle) \quad (24)$$

Where, influence for mean stress usually can be ignored in the eqn (24). a_{2eff} is an effective crack size, it can be calculated from effective crack tip opening displacement δ_{2eff}

$$a_{2eff} = \frac{E \times \delta_{2eff}}{\pi\sigma_s(\sigma'_f / \sigma_s + 1)}, (mm) \quad (25)$$

And

$$\delta_{2eff} = (0.25 \sim 0.4)\delta_c, (mm) \quad (26)$$

Here the δ_c is critical crack tip open displacement.

2.3. Calculations for the Crack Growth Rate in Whole Process

Due to the short crack behaviors and the long crack ones there are distinctly differences, for availing to the crack rate calculation in whole process, author proposes a research result and calculating method: that is to be the calculating formula for the transition crack size a_{tr} at transition point from short crack to long crack growth process. It can be derived by both crack growth rate equations to make equal expression between two stages. The calculating model is as follow:

$$(da_1 / dN_1)_{a_{01} -> a_{tr}} <= da_{tr} / dN_{tr} =< (da_2 / dN_2)_{a_{tr} -> a_{eff}} \quad (27)$$

Here the equation (27) is defined as the crack growth rate-linking-equation in whole process, the da_{tr} / dN_{tr} in (27) is the crack growth rate at transition point.

For $\sigma_m \neq 0$, to select driving force equations (10) and (19), to select formula (13) and (21)for relative comprehensive material constant A_1 and B_2,with above related parameters are substituted into eqn (27), then to derive its expanded crack growth rate-linking-equation for eqn (27)corresponded to positive curve CC_1C_2 is as following form

$$\begin{aligned} \frac{da_1}{dN_1} &= \left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (v_f \times a_{tr})^{-1} \times (\Delta\sigma / 2)^{m_1} \times a\right\}_{a_{01} -> a_{tr}} <= \frac{da_{tr}}{dN_{tr}} =< \\ \frac{da_2}{dN_2} &= \left\{2\left[(\pi\sigma_s(\sigma'_f / \sigma_s + 1)a_{eff} / E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma / 2\sigma_s + 1)a}{E}\right]^{\lambda_2}\right\}_{a_{tr} -> a_{eff}}, (mm / cycle), (\sigma \neq 0) \end{aligned} \quad (28)$$

It should point that the calculations for the crack growth rate in whole process should be according to different stress level and loading condition, to select appropriate calculable equation. And here have to explain that its meaning of the eqns (27-28) is to make a linking for the crack growth rate between the first stage and the second stage, in which before the transition-point crack size, its crack growth rate should be calculated by the short crack growth rate equation; and after the transition-point crack size a_{tr} it should be calculated by the long crack growth rate equation. Note that it should not been added together by the crack growth rates for two stages. About calculation method, it can be calculated by means of computer doing computing by different crack size [17-18].

3. Calculating Example

3.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, fatigue limit $\sigma_{-1} = 267.2MPa$, reduction of area is $\psi = 0.51$, modulus of elasticity $E = 200000MPa$; Cyclic strength coefficient $K' = 1165MPa$, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma'_f = 947.1MPa$, fatigue strength exponent $b'_1 = -0.111$, $m_1 = 9.009$; Fatigue ductility coefficient $\varepsilon'_f = 0.464$, fatigue ductility exponent $c'_1 = -0.5395$, $m'_1 = 1.8536$. Threshold value $\Delta K_{th} = 8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7\text{MPa}\sqrt{\text{m}}$, critical damage stress intensity factor $K_{1c}(K_{2c})$. Its working stress $\sigma_{\max} = 450MPa$, $\sigma_{\min} = 0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b) = 1$. Other computing data are all included in table 1.

Table 1. *Computing data*

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv} **(mm/cycle)**	m_2	δ_c, mm	λ_2	$y_2(a/b)$	a_{th}, mm
92.7	28.23	8.6	2×10^{-4}	3.91	0.18	2.9	1.0	0.07

3.2. Required Calculation Data

Try by the simple stress-strain parameter calculating methods to calculate following different data and depicting their curves:

(1) To calculate crack size a_{tr} at the transitional point between two stages;

(2) To calculate the crack growth rate da_{tr} / dN_{tr} at transitional point;

(3) To calculate the short crack growth rate da_1/dN_1 in first stage from micro crack $a_{01} = 0.02mm$ growth to crack $a = 2mm$;

(4)To calculate the long crack growth rate da_2/dN_2 in second stage from $a_2 = 0.2mm$ to long crack effective size $a_{2\mathrm{eff}} = 5mm$;

(5)Calculating for crack growth rate da/dN in the whole process;

(6)To depict the curves of the crack growth rate da/dN in whole process.

3.3. Calculating Processes and Methods

3.3.1. Calculations for Relevant Parameters

The concrete calculation methods and processes are as follows,

1) Calculations for stress range and mean stress:

Stress range calculation: $\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450(MPa)$;

Mean stress calculation: $\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (450 - 0)/2 = 225MPa$.

2) Calculation for effective damage value a_{eff}

For the effective crack size a_{1eff} in first stage and the second stage, both can be calculated respectively, and can take smaller one of both. According to formulas (25), calculation for effective crack size a_{2eff} in second stage is as follow,

$$a_{2eff} = \frac{E \times \delta_{eff}}{\pi\sigma_s(\sigma_f/\sigma_s + 1)} = \frac{200000 \times 0.25 \times 0.18}{\pi 361(947.1/361 + 1)} = 2.1(mm),$$

Take $a_{2eff} = 2.0mm$, here for a_{1eff} in first stage to take same value by the second stage, $a_{1eff} = a_{2eff} = 2mm$.

3) According to formulas (6), to calculate correction coefficient v_{eff} in first stage:

$$v_{eff} = a_{eff}\ln[1/(1-\psi)] = 2 \times \ln[1/(1-0.51)] = 1.43,(mm/\text{cycle}).$$

4) By eqn (22), to select virtual rate v_{pv} in second stage, here take:

$$v_{pv} = \frac{a_{2eff} - a_{02}}{N_{2f} - N_{02}} \approx 2.0 \times 10^{-4}(mm/Cycle),\ (N_{2f} = 1,\ N_{02} = 0).$$

Here by means of two kinds of methods to calculate respectively as below:

3.3.2. The Calculating Process, Steps and Methods

(1) To calculate crack size a_{tr} at the transitional point between two stages

1) By the crack-rate-link formulas (27-28), for short crack growth rate calculating in first stage to select equation (8) and (16):

At first, calculation for comprehensive material constant A_1 by eqn (13)

$$\begin{aligned} A_1 &= 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'} \times (D_{ef} \times v_f)^{-1} \\ &= 2 \times 1165^{-9.01} \times [2 \times 0.464(1 - 225/947.1)]^{1/-0.5395}(2 \times 0.713)^{-1} \\ &= 6.28 \times 10^{-28}, (MPa\sqrt[m_1]{mm})^{-m_1} \times mm/cycle \end{aligned}$$

Then, to simplify calculations as follow form,

$$\begin{aligned} da_1/dN_1 &= A_1 \times (\Delta\sigma/2)^{m_1} \times a_1 = 3.193 \times 10^{-28} \times (450/2)^{9.01} \times a_1 \\ &= 6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times a_1 = 9.8 \times 10^{-7} \times a_1 \end{aligned}$$

2) To select equation (24), calculating for long crack growth rate in second stage:

Calculation for comprehensive material constant B_2 by eqn (21)

$$\begin{aligned} B_2 &= 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1 - \sigma_m/\sigma'_f)a_{eff}/E)\right]^{-\lambda_2} \times v_{pv} \\ &= 2[2(3.1416 \times 361(947.1/361 + 1)(1 - 225/947.1) \times 2/200000)]^{-2.9} \\ &\quad \times 2 \times 10^{-4} = 9.1988,\ (mm)^{-\lambda_2} \times mm/Cycle \end{aligned}$$

Then to simplify calculation equation as follow form,

$$\begin{aligned} da_2/dN_2 &= B_2\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)a_2}{E}\right]^{\lambda_2} \\ &= 9.1988 \times \left[\frac{0.5\pi 361(450/(2 \times 361) + 1)a_2}{E}\right]^{2.9} \\ &= 9.1988 \times 1.6698 \times 10^{-7} a_2^{2.9} \\ &= 1.5384 \times 10^{-6} a_2^{2.9}(mm/cycle) \end{aligned}$$

3) Calculation for crack size a_{tr} at transitional point:

According to the equations (27) and (28), to do calculation for crack size a_{tr} at the transitional point; here, to take brief crack-rate-linking-calculating-formulas as follow form,

$$6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times a_{tr} = 9.1988 \times 1.6698 \times 10^{-7} \times a_{tr}^{2.9}$$

$$a_{tr} = (0.638)^{\frac{1}{1.9}} = (0.638)^{0.5263} = 0.789(mm)$$

So to obtain the transitional point crack size $a_{tr} = 0.789(mm)$.

(2) To calculate the crack rate at transitional point a_{tr}

$$\begin{aligned} da_1/dN_1 &= da_{tr}/dN_{tr} = 9.8 \times 10^{-7} a_1 \\ &= 9.8 \times 10^{-7} \times 0.789 = 7.74 \times 10^{-7}(mm/cycle) \end{aligned}$$

$$\begin{aligned} da_2/dN_2 &= da_{tr}/dN_{tr} = 1.5384 \times 10^{-6} a_{tr}^{2.9} \\ &= 1.5384 \times 10^{-6} \times (0.79)^{2.9} = 7.74 \times 10^{-7}(mm/cycle) \end{aligned}$$

Here it can be seen, the crack-rate at the transition point $(a_{tr} = 0.789mm)$ is same.

(3) Calculations for the crack growth rates da/dN in whole process

According to eqn (28), Calculation for the da/dN from micro-crack $a_{01} = 0.02mm$ to transitional point $a_{tr} = 0.789mm$, again to long-crack $a_{eff} = 5mm$ is as follow:

1) To select eqn (28) as below

$$\frac{da_1}{dN_1} = \left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (v_f \times a_{tr})^{-1} \times (\Delta\sigma/2)^{m_1} \times a\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}} =<$$

$$\frac{da_2}{dN_2} = \left\{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)a_{eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)a}{E}\right]^{\lambda_2}\right\}_{a_{tr}->a_{eff}}, (mm/cycle), (\sigma \neq 0)$$

2) To put into relevant data above mentioned

$$\frac{da_1}{dN_1} = \left\{3.193\times10^{-28} \times (450/2)^{9.01} \times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}}$$

$$=< \frac{da_2}{dN_2} = \left\{9.1988 \times \left[\frac{0.5\pi 361(450/(2\times361)+1)a_2}{E}\right]^{2.9}\right\}_{a_{tr}->a_{eff}},$$

$(mm/cycle), (\sigma \neq 0)$

3) To calculate and simplify,

$$\frac{da_1}{dN_1} = \left\{6.28\times10^{-28} \times 1.56\times10^{21} \times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}}$$

$$=< \frac{da_2}{dN_2} = \left\{9.1988 \times 1.6698\times10^{-7} a_2^{2.9}\right\}_{a_{tr}->a_{eff}},$$

$(mm/cycle), (\sigma \neq 0)$

4) For above formulas, it can derive more simplified crack-rate-linking-equation in whole process corresponded to different crack size as follow form

$$\frac{da_1}{dN_1} = \left\{9.8\times10^{-7} \times a_1\right\}_{a_{01}->a_{tr}}$$

$$<= \frac{da_{tr}}{dN_{tr}} =< \frac{da_2}{dN_2} = \left\{1.5384\times10^{-6} a_2^{2.9}\right\}_{a_{tr}->a_{eff}},$$

$(mm/cycle), (\sigma \neq 0)$

5) According to above the simplified rate-linking-equation, by means of a computer, to do the crack growth rate computing in whole process from micro crack $a_{01} = 0.02mm$ to transitional point size $a_{tr} = 0.789mm$, again to long crack size $a_{eff} = 5mm$. The crack growth rate data corresponded to different crack sizes is all included in table 2~4.

(4) To depict the crack growth rate curves in the whole process.

By the data in tables 2-4, the crack growth rate curves for two stages and whole process are depicted respectively in figure 2 and 3.

Table 2. *Data of crack growth rate in whole process*

Data point of number	**1**	**2**	**3**	**4**	**5**
Crack size (mm)	0.02	0.04	0.1	0.2	0.4
The first stage	1.96×10^{-8}	3.92×10^{-8}	9.8×10^{-8}	1.96×10^{-7}	3.92×10^{-7}
The second stage	Invalid section			1.446×10^{-8}	1.079×10^{-7}

Table 3. *Data of crack growth rate in whole process*

Data point of number	**5**	**6**	**7**	**Transition point**	**8**
Crack size (mm)	0.5	0.6	0.7	0.789	1.133
The first stage	4.95×10^{-7}	5.88×10^{-7}	6.869×10^{-7}	7.732×10^{-7}	1.11×10^{-6}
The second stage	2.06×10^{-7}	3.497×10^{-7}	5.468×10^{-7}	7.732×10^{-7}	2.21×10^{-6}

Table 4. *Data of crack growth rate in whole process*

Data point of number	**9**	**10**	**11**	**12**	**13**
Crack size (mm)	1.5	2.0	3.0	4	5
The first stage	1.47×10^{-6}	1.96×10^{-6}	Invalid section		
The second stage	4.986×10^{-6}	1.148×10^{-5}	3.72×10^{-5}	8.57×10^{-5}	1.64×10^{-4}

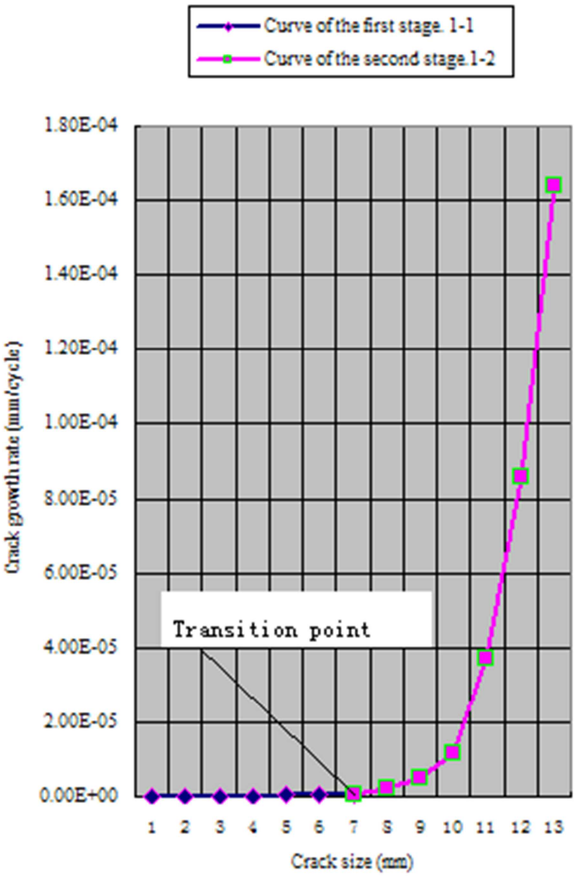

Figure 2. Crack growth rate curves in whole process (in decimal coordinate system)

(a) 1-1---Curve in first stage depicted by single-parameter calculating data;
(b)1-2---Curve in second stage depicted by single-parameter calculating data;
(c) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789*mm*).

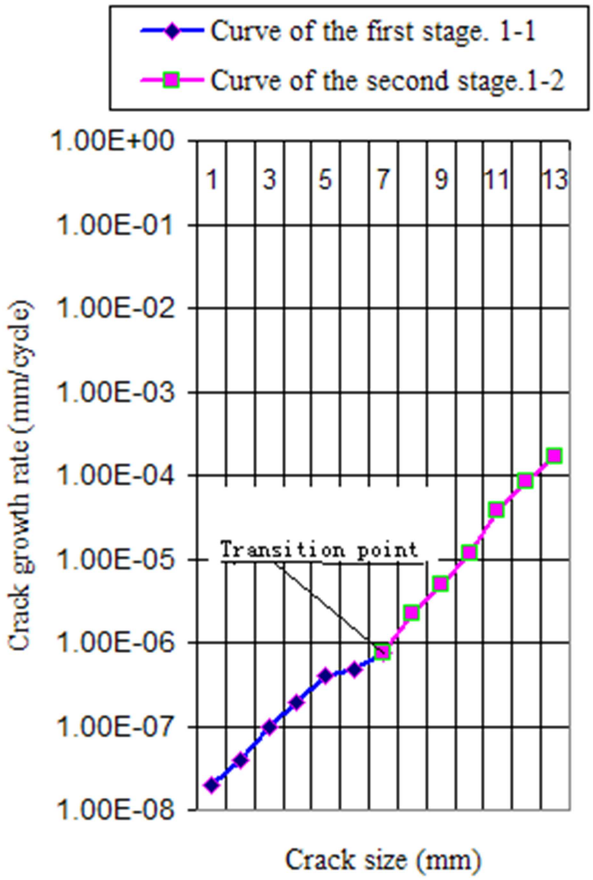

Figure 3. Crack growth rate curves in whole process (in logarithmic coordinate system)

(a) 1-1---Curve in first stage depicted by single-parameter calculating data;
(b)1-2---Curve in second stage depicted by single-parameter calculating data;
(c) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789*mm*).

4. Discussions and Conclusions

(1) About new cognition for some key material constants: For some new material constants A_1 and B_2 about the crack growth rate equations in the fracture mechanics, in practice there are functional relations with other parameters, they are all calculable parameters by means of the relational expressions(12-13), (20-21) etc. Therefore for this kind of key parameters can be defined as comprehensive materials constants.

(2) About the theory basis of the whole process rate model: Although the short crack behavior and the long crack behavior there are obvious differences, from the short crack growth to the long crack growth process it must exist a same crack size at the transition point, and the crack propagation rate at this point must be equal. According to this reasoning, with the help of the same location at transition point as the linking point, therefore to establish the crack- growth rate-linking-equation between the first and the second stage in whole process, this is just the theory basis of crack rate equation as whole process.

(3) About cognitions to the physical and geometrical meanings for key parameters: The parameters A_1 in the first stage and the B_2 in the second stage, their physical meanings are all a concept of the power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

(4) About the calculating methods for the crack growth rate-linking-equation in whole process: Calculation for the crack growth rate before the transition size a_{tr}, it should be calculated by the short crack growth rate equation; after the transition size a_{tr} it should be calculated by the long crack growth rate equation.

(5) Total conclusion: Based on the traditional material mechanics is a calculable subject, in consideration of the

conventional constants there are "the hereditary characters", In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions (1)~(4), then make the fracture mechanics disciplines become calculable subjects, that will be to exist the possibility.

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