A Simple Approach to Extract Some Optical Properties of Uniaxial Crystals from the Raman Spectrum: Case of Lithium Iodate (LiIO₃)

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Citation

Abstract
In this work the improvement of the crystal rotation method established earlier for the quantitative analysis of the Raman spectra is presented. The integrated area is now a function of the damping parameter of the phonon modes, the phonon frequency and the maximum amplitude of the Lorentz profile. The cross sections are expressed as functions of the crystal rotation angle and the Raman polarizabilities through the anisotropy parameter. The integrated areas are fitted with a Lorentzian profile and evaluated. These areas are considered as experimental points for different crystal rotation angles to determine the angular dependence of the Raman scattering cross sections. A new nonlinear least square method (NLLSM) is used to derive the anisotropy parameters of the Raman polarizabilities and the cross sections for Transverse Optics (TO) modes. The derived quantities are later used to deduce the relative tensor of the Raman scattering cross sections. The phase differences of the Raman polarizabilities are deduced and therefore, the relative tensor of the Raman polarizabilities for the modes of the A symmetry. The angular dependence of the theoretical cross sections is also calculated for the modes of the E₁ and E₂ symmetries. The improvement of the crystal rotation method for the quantitative analysis of the Raman spectra is simple and leads to satisfactory results.

1. Introduction

Raman polarizabilities are important for the determination of electro-optic coefficients of materials. Despite proposals for absolute measurements by some authors [1], these coefficients have been given as approximations. Relative quantities obtained from a spectrometer are comparable to those obtained from other spectrometers so relative methods are currently preferred.
In the past decades, concerted efforts have been devoted to determine the Raman polarizabilities. Nippus [2], using a simplified Raman profile, established that the relative peak area is equivalent to the square of the relative Raman polarizabilities for LiNbO₃. This model was not suitable because the proportionality coefficient depends on the phonon frequency. Kaminow and Johnston [3] have determined the absolute Raman scattering efficiencies of LiNbO₃ and LiTaO₃, but the results were incomplete due to some missing polarizability values. Johnston [4] using a theoretical approach determined the Raman polarizabilities with a sign uncertainty. Otaguro et al. [5] using a relative method determined the second harmonic-generation coefficients and the linear electro-optic coefficient in LiIO₃ through oblique Raman measurements. Only one component of the Raman polarizability was extracted using this approach. More recently, Djiedeu et al. [6] have calculated the anisotropy factors and the phase differences of the Raman polarizabilities for uniaxial crystals using the crystal rotation method based on the fit parameter reduction method (FPRM) which consists of reducing the number of variables in the cross section before the fitting operation.

The purpose of this work is the determination of the relative tensor of the Raman scattering cross sections and the relative tensor of the Raman polarizabilities in Lithium Iodate (LiIO₃) single crystal. The Lithium Iodate has interesting optical properties [7] and ionic conductivity character [8]. Section 2 presents an improvement of the integrated area evaluation and the general expression of the Raman cross section. In section 3, the Raman polarizability tensors associated with the studied crystal point group, the cross sections are then expressed explicitly as functions of the crystal rotation angle and the anisotropy parameter. A new nonlinear least square method (NLLSM) associated with the FPRM is presented and used for the theoretical calculation of the cross sections. The residue is derived as a function of the anisotropy parameter which has a minimum for a unique value of the anisotropy parameter. The theoretical expressions for the relative tensor of the Raman scattering cross sections and the relative tensor of the Raman polarizabilities are then given. Section 4 presents the experimental and numerical results of the cross sections for some crystal rotation angles. The cross sections and the anisotropy parameters have been computed from the data using the Fortran 90 code and the relative values deduced. The phase differences of the Raman polarizabilities are also evaluated numerically. The generalization and the coherence of our method are then emphasized.

## 2. General Theory

### 2.1. Equivalence Between the Integrated Area and the Raman Scattering Cross Section

In general, the area $A$ under a peak is defined by the expression

$$A = \int f(x)dx$$

(1)

The differential cross section has been predicted to have a Lorentzian profile [9]. Thus, if $f(x)$ is the differential cross section, the integral will be the cross section that is the area under the peak. Nippus admitted this equivalence without an explicit demonstration [2]. We are going to determine the rough cross sections (cross section with arbitrary units) before deducing the relative cross sections.

The Raman intensity defined by Eq. (2) also has the inverse of a solid angle. $\Omega$: solid angle, $\omega$: the phonon frequency, $\Gamma$: damping parameter of the phonon modes, $n(\omega)$: population factor. Dimensional analysis of Eq. (2) leads to $[A_0]=[\Omega]^{-1}sr^{-1}$. Then $A_0$ has the inverse unit of a solid angle. The Raman intensity defined by Eq. (2) also has the inverse unit of a solid angle given that the remaining part of the Lorentzian profile is dimensionless.

The integrated areas obtained earlier [6] from Eq. (2) were independent of the damping parameter because of the approximation that was done, but the damping parameter was used to normalize those areas. Therefore, to improve the evaluation of these integrated areas, an approximation is made here only for the population factor, which doesn’t change much from $\omega_0$. Using a change of variable, the integration of Eq. (2) in the interval $[0; +\infty]$, leads to:

$$
\frac{d^2I}{d\omega d\Omega} = A_0 \frac{\Gamma \omega^2}{(\omega^2 - \omega_0^2)^2 + \Gamma^2 \omega^2} (n(\omega) + 1)
$$

(2)

$A_0$ is the amplitude of the Lorentzian profile and is now a dimensionless quantity and not a pure constant. $I$ is the ratio of the scattered and the incident light intensities. It is a dimensionless quantity. $\Omega$: solid angle, $\omega$: the phonon frequency, $\Gamma$: damping parameter of the phonon modes, $n(\omega)$: population factor. Dimensional analysis of Eq. (2) leads to $[A_0]=[\Omega]^{-1}sr^{-1}$. Then $A_0$ has the inverse unit of a solid angle. The Raman intensity defined by Eq. (2) also has the inverse unit of a solid angle given that the remaining part of the Lorentzian profile is dimensionless.

The phase differences of the Raman polarizabilities are then given. Section 4 presents the experimental and numerical results of the cross sections for some crystal rotation angles. The cross sections and the anisotropy parameters have been computed from the data using the Fortran 90 code and the relative values deduced. The phase differences of the Raman polarizabilities are also evaluated numerically. The generalization and the coherence of our method are then emphasized.
explicitly defines the Raman scattering cross section which equals the integrated area. The Raman scattering cross section now depends on the three main parameters of the Lorentz profile \((S_{\text{max}}, \Gamma, \omega_0)\). For small values of the damping parameter, we retrieve the result published recently [6]. The only difference is that the variables used here are dimensional quantities.

Dimensional analysis of Eq. (3) shows that \(S\) has the unit of a cross section.

\[
[S] = [S_{\text{max}}][\Gamma] = [A_s][\Gamma] = sr^{-1}cm^{-1}
\]

Instead of \(sr^{-1}cm^{-1}\), arbitrary unit (a.u) will be used because \(S_{\text{max}}\) is generally given with arbitrary units.

### 2.2. Raman cross Section as Function of the Crystal Rotation Angle

The Raman scattering cross section of the optical mode is defined by [10,11,12]:

\[
S_{\text{opt}} = \frac{dI}{I_{\text{opt}}d\Omega} = \sigma_{k, \text{TO}}^2 = \sigma_{k, \text{TO}}^2 \left| \vec{P}_{\text{e}} \right|^2
\]

Where \(\sigma_{k, \text{TO}}\) is the Raman polarizability,

\[
\sigma_{k, \text{TO}} = \frac{\hbar (\omega_k - \omega_0)^4 (n_e + 1)}{32\pi^2 c^4 \omega_k}
\]

\(A(z) = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix}, E_i(x) = \begin{pmatrix} 0 & c \\ 0 & 0 \end{pmatrix}, E_i(y) = \begin{pmatrix} 0 & 0 & -d \\ 0 & 0 & c \\ d & 0 \end{pmatrix}
\]

\(E_z = \begin{pmatrix} e & f & 0 \\ f & -e & 0 \\ 0 & 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} f & -e & 0 \\ -e & f & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

The letters in the parentheses (x,y,z) indicate the electric polarizations associated with the modes and the basic functions of the infra-red actives modes. The modes of the \(E_2\) irreducible representations are Raman active and infra-red inactive. The theory of the crystal rotation [6] allows us to express the Raman scattering cross section as a function of the crystal rotation angle.

### 3. Crystal of \(C_6^6\) Point Group: Case of the Lithium Iodate Crystal

The Lithium iodate crystal belongs to the \(C_6^6\) point group and the Raman polarizability tensors of the active modes are given by [14]:

\[
\begin{align*}
A_1 & = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & b \end{pmatrix} \\
A_2 & = \begin{pmatrix} e & f & 0 \\ f & -e & 0 \\ 0 & 0 & 0 \end{pmatrix} \\
A_3 & = \begin{pmatrix} f & -e & 0 \\ -e & f & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{align*}
\]

The anisotropy parameter \(\beta\) is given by:

\[
\cos \beta = \frac{|a|}{\sqrt{|a|^2 + |b|^2}}, \quad \sin \beta = \frac{|b|}{\sqrt{|a|^2 + |b|^2}}
\]

\(P_k\) is the Raman tensor. Instead of \(P_k\) we can also use:

\[
P_k (\theta) = MP_k M^{-1}
\]

\(P_k (\theta)\) is the Raman tensor due to the rotation of the crystal and \(M\) is its rotation matrix because a \(\theta\) rotation of the crystal corresponds to a \(-\theta\) rotation of the polarization unit vectors [6]. \(\alpha\) and \(\beta\) are the initial polar directions of the polarization unit vectors.

A relation similar to Eq. (8) was derived by Koningstein [13].

Using Eq.(6) and Eq. (8) we obtained:

\[
S_{\text{opt}} = \sigma_{k, \text{TO}}^2 \left| \vec{P}_{\text{e}} \right|^2
\]

Eq. (3) is used to evaluate the Raman Scattering intensity from the experimental data and Eq. (9) is used to fit these points because they depend on the crystal rotation angle.

### 3.1. Rotation around the X Axis: Case of A Modes

We recall the Raman cross sections obtained recently [6] from Eq. (9) and Eq. (10) for A modes using the method of the crystal rotation. We can write these equations as the following forms:

\[
S_{xy}^{(\theta)} = S_{y}^{(\theta)} = (1/2) S_{\alpha \beta} (1 - \sin(2\beta) \cos \phi_\alpha) \sin^2(2\theta)
\]

\[
S_{yx}^{(\theta)} = S_{\alpha \beta} \left( \cos^2 \beta \cos^4 \theta + \sin^2 \beta \sin^4 \theta + (1/4) \sin(2\beta) \cos \phi_\alpha \sin^2(2\theta) \right)
\]

\[
S_{yz}^{(\theta)} = S_{\alpha \beta} \left( \cos^2 \beta \sin^4 \theta + \sin^2 \beta \cos^4 \theta + (1/4) \sin(2\beta) \cos \phi_\alpha \sin^2(2\theta) \right)
\]

Where the anisotropy parameter \(\beta\) is given by:

\[
\cos \beta = \frac{|a|}{\sqrt{|a|^2 + |b|^2}}, \quad \sin \beta = \frac{|b|}{\sqrt{|a|^2 + |b|^2}}
\]

\[r = (\sigma_{\text{TO}})^{1/2} \left| |a| + i|b| \right| = |r| e^{i\beta}\]
\[ S_s = r^2 = S_{ab} e^{2i\beta} \]  
\[ S_{ab} = |\rho|^2 = S_s + S_a(1 + \tan^2 \beta) \]  
\[ S_a = S_{ab} \cos^2 \beta \]  
\[ S_b = S_{ab} \sin^2 \beta \]

The magnitude of a complex number and its phase are independent variables. Hence, \(|\rho|\) and \(\beta\) defined in Eq. (15) form a set of two independent variables. According to Eq. (16), \(S_{ab}\) is also an independent variable.

The relative quantity for the different modes have been deduced by dividing the results obtained from the previous calculations by some chosen quantities.

### 3.3. Relative Evaluation of the Raman Polarizabilities and the Raman Scattering Cross Sections

The relative tensor of the Raman polarizabilities (RP) is then given by:

\[ RP(A(z)) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{|B|}{|A|} e^{i\varphi_{ba}} \end{pmatrix} \]  
\[ \frac{|\rho|}{|a|} = \tan \beta \]

The relative tensor of the Raman scattering cross sections (RCS) is then given by:

\[ RCS(A(z)) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & S_b/S_a \end{pmatrix} \]  
\[ \frac{S_b}{S_a} = \tan^2 \beta \]  

We can see that relative quantities are functions of the anisotropy parameter \(\beta\) since \(\varphi_{ba}\) also depends on \(\beta\). This means that the anisotropy parameter is a very important quantity. We remind that the magnitude of the relative Raman
polarizability (tangent of the anisotropy parameter) defined in Eq.(25) is also called the anisotropy factor [6].

4. Experiment and Numerical Results

In this part, we present the Raman spectrum for the geometrical configuration in used and the results of the numerical determination of the anisotropy parameters, the rough and relative cross sections, the relative Raman polarizabilities and their phase differences.

4.1. Raman Spectra of a Pure LiIO$_3$

The experiments were done at a temperature of 296K with the integration time of 30s. The laser power at the sample was 1.30mW. This was measured using a Coherent Field Master GS power meter with model LM-2 power sensor. The 514.5nm line of an argon ion laser and a Jobin Yvon T64000 Raman spectrometer operated in triple subtractive mode was used to collect these spectra. The samples were polished on the two faces perpendicular to the x direction. The spectra are measured with 10˚ rotation step intervals around the x axis, thus in the yz plane. The recorded spectra are reported in Fig. 1 in backscattering configuration x(yy)x.

![Fig. 1. Raman spectra of a pure LiIO$_3$; rotation around the x axis (yz plane).](image)

The 4 frequencies of A modes have been observed in these spectra and are in agreement with the literature [5,15,16]. Especially in [16], the temperature dependence of the Raman spectra for A modes of the α-LiIO$_3$ has been studied to point out the nature and role of the defects. When we consider the Raman responses from 0 to 90˚, we observed the emergence of the peaks of E$_2$ mode at 95cm$^{-1}$ and that of A mode at 791.7cm$^{-1}$. Simultaneously, the peak of A mode at 233.5cm$^{-1}$ disappears.

4.2. Evaluation of the Integrated Areas

Table 1 displays the integrated areas obtained for the 4 vibrational modes of the A symmetry specie of the Lithium iodate crystal for some crystal rotation angles.

<table>
<thead>
<tr>
<th>Angle</th>
<th>$\omega_0$ (cm$^{-1}$)</th>
<th>$A_0$ (a.u)</th>
<th>$\Gamma$ (cm$^{-1}$)</th>
<th>$S_{max}$ (a.u)</th>
<th>$S$ (a.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>145.3</td>
<td>89.6</td>
<td>7.7</td>
<td>3337</td>
<td>32889</td>
</tr>
<tr>
<td></td>
<td>233.5</td>
<td>64.4</td>
<td>33.6</td>
<td>659</td>
<td>28078</td>
</tr>
<tr>
<td></td>
<td>356.2</td>
<td>161.0</td>
<td>16.8</td>
<td>4147</td>
<td>89229</td>
</tr>
<tr>
<td></td>
<td>791.7</td>
<td>175.7</td>
<td>5.1</td>
<td>27811</td>
<td>182817</td>
</tr>
<tr>
<td>30</td>
<td>145.3</td>
<td>73.5</td>
<td>7.1</td>
<td>2952</td>
<td>26990</td>
</tr>
<tr>
<td></td>
<td>233.5</td>
<td>116.9</td>
<td>15.4</td>
<td>512</td>
<td>20474</td>
</tr>
<tr>
<td></td>
<td>356.2</td>
<td>137.2</td>
<td>4.7</td>
<td>3285</td>
<td>64827</td>
</tr>
<tr>
<td></td>
<td>791.7</td>
<td>75.6</td>
<td>7.7</td>
<td>23664</td>
<td>142782</td>
</tr>
<tr>
<td>60</td>
<td>145.3</td>
<td>21.0</td>
<td>43.4</td>
<td>167</td>
<td>913</td>
</tr>
<tr>
<td></td>
<td>233.5</td>
<td>39.9</td>
<td>18.9</td>
<td>913</td>
<td>22080</td>
</tr>
<tr>
<td></td>
<td>356.2</td>
<td>163.8</td>
<td>5.0</td>
<td>26661</td>
<td>170458</td>
</tr>
</tbody>
</table>
4.3. Anisotropy Parameter at the Minimum Point of the Residue and Numerical Results

Fig. 2 displays the variation of the residue as a function of the anisotropy parameter for 791.7cm$^{-1}$ frequency. The anisotropy parameter can be found by considering the minimum point of the residue. With the use of a program written in Fortran 90, we have fitted the integrated areas listed in Table 1 to obtain the expected values of the cross sections defined in Eq.(20) and the residue defined in Eq. (23). The optical properties obtained as results of the fitting program, i.e. the anisotropy parameter $C$, the phase difference $\phi_{ba}$, the relative cross section and the magnitude of the relative polarizability are listed in Table 2. The relative errors are also listed.

![Figure 2](image-url)

**Table 2.** Optical properties deduced from the fit.

<table>
<thead>
<tr>
<th>$\omega_0$ (cm$^{-1}$)</th>
<th>$A_0$ (a.u)</th>
<th>$\Gamma$ (cm$^{-1}$)</th>
<th>$S_{max}$ (a.u)</th>
<th>$S$ (a.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>145.3</td>
<td>79459</td>
<td>44737</td>
<td>82.76</td>
<td>1.29</td>
</tr>
<tr>
<td>233.5</td>
<td>37506</td>
<td>7254</td>
<td>53.58</td>
<td>0.25</td>
</tr>
<tr>
<td>356.2</td>
<td>104410</td>
<td>13703</td>
<td>42.48</td>
<td>0.15</td>
</tr>
<tr>
<td>791.7</td>
<td>458546</td>
<td>274198</td>
<td>78.70</td>
<td>1.49</td>
</tr>
</tbody>
</table>

*: relative cross section.

$: $magnitude of the relative polarizability

$: $relative error

**Table 3.** Relative tensors of the Raman cross sections and the Raman polarizabilities.

<table>
<thead>
<tr>
<th>$\omega_0$ (cm$^{-1}$)</th>
<th>233.5</th>
<th>356.2</th>
<th>791.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCS(A(z))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
</tr>
<tr>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>0 0</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
</tr>
<tr>
<td>1,29</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
</tr>
</tbody>
</table>

| RP(A(z))               |       |       |       |
| 100                    | 1 0   | 1 0   | 1 0   |
| 0 1                    | 0 0   | 0 0   | 0 0   |
| 0 0 1.13e$^{02.76i}$    | 0 0   | 0 0   | 0 0   |

<table>
<thead>
<tr>
<th>$\omega_0$ (cm$^{-1}$)</th>
<th>233.5</th>
<th>356.2</th>
<th>791.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>M.R.P$^b$</td>
<td>/</td>
<td>/</td>
<td>/</td>
</tr>
<tr>
<td>100</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
</tr>
<tr>
<td>0 1</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
</tr>
<tr>
<td>0 0</td>
<td>1 0</td>
<td>1 0</td>
<td>1 0</td>
</tr>
<tr>
<td>1.22e$^{78.72i}$</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
</tr>
</tbody>
</table>
The rough cross sections evaluated above \((S_{ab}, S_{a}, S_{b})\) and obtained for a particular spectrometer cannot be compared directly to those obtained from other spectrometers because they are given with arbitrary units. Meanwhile, relative quantities (cf. Table 3) obtained with this method are comparable with those obtained from other spectrometers.

Figures 3 and 4 represent theoretical simulations and experimental data of the Raman scattering cross sections for A modes as functions of the crystal rotation angle evaluated respectively at 356.2 cm\(^{-1}\) and 791.7 cm\(^{-1}\).

Fig. 3. Theory and experiment at 356.2 cm\(^{-1}\); rotation around the x axis (yz plane).

Fig. 4. Theory and experiment at 791.7 cm\(^{-1}\); rotation around the x axis (yz plane).

Figures 3 and 4 show that the NLLSM associated with the FPRM is in good agreement with the experiment. This method is preferable than the manual fitting (MF) associated with the FPRM presented earlier [6] because it allows the minimization of the residue. Especially in Fig. 4, we can note that the final experimental point 90° (for 791.7 cm\(^{-1}\)) is not good because it increases the error and modifies slightly the profile of the curve. This may be due to the term containing the phase difference in the cross section which does not contribute at the extremal points (0 and 90°). Table 2 shows
that the errors are less than 6%. In some previous studies on
LiNbO$_3$ not all quantities were provided (Kaminow and
Johnston [3]). In addition, in these studies, the cross sections
and the Raman polarizabilities were determined with very
large errors sometimes more than 50% (Johnston [4]). Useful
comparison with previous work on LiIO$_3$ is difficult due to
the nature of the relative method used.

5. Conclusion

In this work, a systematic method for the determination of
the relative tensor of the Raman scattering cross sections,
the relative tensor of the Raman polarizabilities is presented.
The theory of the crystal rotation presented earlier has been
improved. A new approach for the determination of the
anisotropy parameter and the cross section using the nonlinear
least square method associated with the fit parameter
reduction method has been developed. It appears in this work
that the anisotropy parameter is the most important
parameter for the determination of some optical properties of
crystals. This method is the appropriate approach for the theo-
retical analysis of the Raman scattering cross sections obtained
from the theory of the crystal rotation on the one hand and the
quantitative analysis of the integrated areas obtained from the
Raman spectra on the other hand. This work can be extended
to the determination of the longitudinal cross section. Other
crystals can be investigated as well as other laser excitation
wavelengths.

Acknowledgment

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discussions of this work.

Appendixes

A. Rotation Around the Z Axis: Case of E$_1$ Modes

A rotation around the z axis allows us to express the Raman
cross section for E$_{1x}$ symmetry:

$$
S_{E_{1x}(θ)}^{E_{1x}(θ)} = S_{E_{1x}} = \frac{1}{2} S_{cd} \left( 1 + \cos(2\alpha) \cos(2θ) + \sin(2\alpha) \cos(2θ) \right)
$$

(A.1)

$$
S_{E_{1y}(θ)}^{E_{1y}(θ)} = S_{E_{1y}} = \frac{1}{2} S_{cd} \left( 1 - \cos(2\alpha) \cos(2θ) - \sin(2\alpha) \cos(2θ) \right)
$$

(A.2)

Where the anisotropy parameter $\alpha$ is given by:

$$
\cos \alpha = \frac{|e|}{\sqrt{|e|^2 + |f|^2}}, \quad \sin \alpha = \frac{|f|}{\sqrt{|e|^2 + |f|^2}}
$$

(A.3)

$$
S_{cd} = \sigma_{30} \left( |e|^2 + |f|^2 \right) = S_x + S_y = S_c \left( 1 + \tan^2 \alpha \right)
$$

(A.4)

$S_c = S_{cd} \cos^2 \alpha$

(A.5)

$S_d = S_{cd} \sin^2 \alpha$

(A.6)

The cross sections given by Eq. (A.1) and Eq. (A.2) are
measurable for propagation directed along the y axis and the x
axis respectively in the backscattering geometry. Finally, Eq.
(A.1) and Eq. (A.2) allow the choice of the E$_{1x}$ or E$_{1y}$
symmetries.

B. Rotation Around the Z Axis: Case of E$_2$ Modes

For E$_2$ modes, the results are the following:

$$
S_{E_{2x}(θ)}^{E_{2x}(θ)} = S_{E_{2x}} = \frac{1}{2} S_{ef} \left( 1 + \cos(2γ) \cos(4θ) - \sin(2γ) \cos(4θ) \right)
$$

(B.1)

$$
S_{E_{2y}(θ)}^{E_{2y}(θ)} = S_{E_{2y}} = \frac{1}{2} S_{ef} \left( 1 - \cos(2γ) \cos(4θ) + \sin(2γ) \cos(4θ) \right)
$$

(B.2)

Where $\cos γ = \frac{|e|}{\sqrt{|e|^2 + |f|^2}}, \quad \sin γ = \frac{|f|}{\sqrt{|e|^2 + |f|^2}}$

(B.3)

$$
S_{ef} = \sigma_{30} \left( |e|^2 + |f|^2 \right) = S_x + S_y = S_c \left( 1 + \tan^2 γ \right)
$$

(B.4)

$S_c = S_{ef} \cos^2 γ$

(B.5)

$S_d = S_{ef} \sin^2 γ$

(B.6)

For the propagation along the z axis direction, the cross
sections given by equation Eq. (B.1) and Eq. (B.2) are
measurable in the backscattering geometry for parallel and
perpendicular polarizations, respectively. Due to the presence
of an ionic conductivity when an electric field propagates
along the optical axis (z) of an LiIO$_3$ crystal, its optical
properties are not of interest [17] for this case of propagation.

References