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Calculations for Crack Growth Rate in Whole Process Realized with Two Kinks of Methods for Elastic-Plastic Materials Contained Crack

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Abstract
In view of short crack and long crack behaviors there are distinctly differences, to use the theoretical approach, to adopt the simple stress-parameter, or the strain-parameter-method and the multiplication-method of two-parameters, to establish numerous new calculation models in whole crack propagation process for elastic-plastic steels, which are the crack growth driving forces, the crack growth rate equations for different stages, the crack-growth-rate-linking-equation in whole process; For transitional crack size and the crack growth rate at transitional point from short crack to long crack growth respectively to put forward a lot of expressions, to provide the concrete and detailed calculation process, the steps and the methods; For new discovering and there are functional relations of some key materials parameters, respectively to give the new calculable formulas, the new definitions, the new physical meanings and geometrical meanings for them. Thereby to make linking and communication between for the modern fracture mechanics and the traditional material mechanics; So that to realize calculations for the crack growth rate in whole process based on traditional material constants with two kinds of methods.

1. Introduction

In view of complexity of the elastic-plastic material properties contained crack, to consider the micro and macro crack of behaviors are obviously different under different loading conditions, so to research the problems of the driving forces under so many factors and conditions, to establish so many crack propagation rate models in the whole process, which are all very complicated problems.

As everyone knows for the traditional material mechanics, that is a calculable subject, and it has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the crack growth rate problems for some structures when it is pre-existing flaws and concentrated stress under repeated loading. In that it has no to contain such calculable parameters as crack variable a or as the damage variable D in its calculating models. But in the fracture mechanics and the damage mechanics, due to there are these variables, so they can all calculate above problems. Nowadays latter these disciplines are all subjects mainly depended on tests. So that, for above elastic-plastic materials and structures of contained defects, if want to solve the growth rate calculations for the whole process from the micro crack to macro crack growth, that are more difficult, to pay the manpower and money for experiments are more huge.

Author thinks, in the mechanics and the engineering fields, where are also to exist such a scientific law as similar to genetic elements and clone technology in life science. Author had used the theoretical approach as above the similar principles, proposed some calculation models, now sequentially discovers some new scientific laws, adopts the simple stress-parameter or the strain-parameter-method, and the multiplication-method of two parameters, provides some new calculable models for the crack growth driving force, and for the crack growth rate. Try to make the fracture mechanics, step by step become such calculable disciplines as the traditional material mechanics. That way, it may be having practical significances for decreasing experiments, for stinting man powers and funds, for promoting engineering applying and developing to relevant disciplines.

2. Crack Growth Rate Calculations in Whole Process for Elastic-Plastic Steels Contained Crack

For some elastic-plastic steels of pre-existed flaw, about its driving force[1], crack growth rate and life's calculation equations for short crack growth processes, for which some models have been proposed in reference[2-4]; And its driving force, crack propagation rate and life's calculating problems for long crack growth processes, some models had been also provided related references[5-6].

Inside this paper, from short crack to long crack, it uses a called as "the single stress or strain parameter method" and "the multiplication-method of two-parameters" for the crack propagating rates puts up the whole process calculations, that are by means of the stress σ and the strain ε as "genetic element" in first stage[7-8] or by the stress intensity factor K_1 and the crack tip open displacement range δ_t as "genetic element" in the second stage, to establish various calculable models for the driving forces and the crack growth rates, thereby achieve the calculations of crack propagating rates in whole process under low cycle fatigue loading.

2.1. The Calculations for Short Crack Growth Process

2.1.1. The Single Parameter Method

Under the work stress is more than yield stress $\sigma > \sigma_s (= \sigma_y)$ condition or low fatigue, the short crack growth rate equation corresponded to positive direction curve CC_1 in attached fig.1, here to adopt the strain range $\Delta\varepsilon_p$ to express that is as following form

$$da_1 / dN_1 = B_1 (\Delta I)^{m'_1} (mm/cycle) \tag{1}$$

Here

$$I_1 = (\varepsilon_p)^{m_1'} \cdot a_1, (\%^{m_1'} \cdot mm) \tag{2}$$

$$\Delta I_1 = (\Delta\varepsilon_p)^{m_1'} \cdot a_1, (\%^{m_1'} \cdot mm) \tag{3}$$

Where the a_{01} is an initial micro crack size, The I_1 is defined as short crack growth strain factor, that is driving force of short crack growth under monotonous load; ΔI_1 is defined as strain factor range, that is driving force under fatigue load, their units are "$(\%)^{m_1'} \cdot mm$". ε'_f is a fatigue ductility factor, m'_1 is fatigue ductility exponent, $m'_1 = -1/c_1'$, c_1' just is also a fatigue ductility exponent under low cycle fatigue. The B_1 is defined as comprehensive material constants. Author thinks, its physical meaning of the B_1 is a concept of the power, it is a maximal increment value to give out energy in one cycle before failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (attached fig. 1) that is a projection of corresponding to curve 2 on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m'_1 of the formula (4). And the B_1 because there is functional relation with other parameters, So the B_1 is a calculable comprehensive material constants,

Here

$$B_1 = 2[2\varepsilon'_f]^{-m'_1} \times (v_{eff})^{-1}, (\%)^{m_1'} \times mm/cycle \tag{4}$$

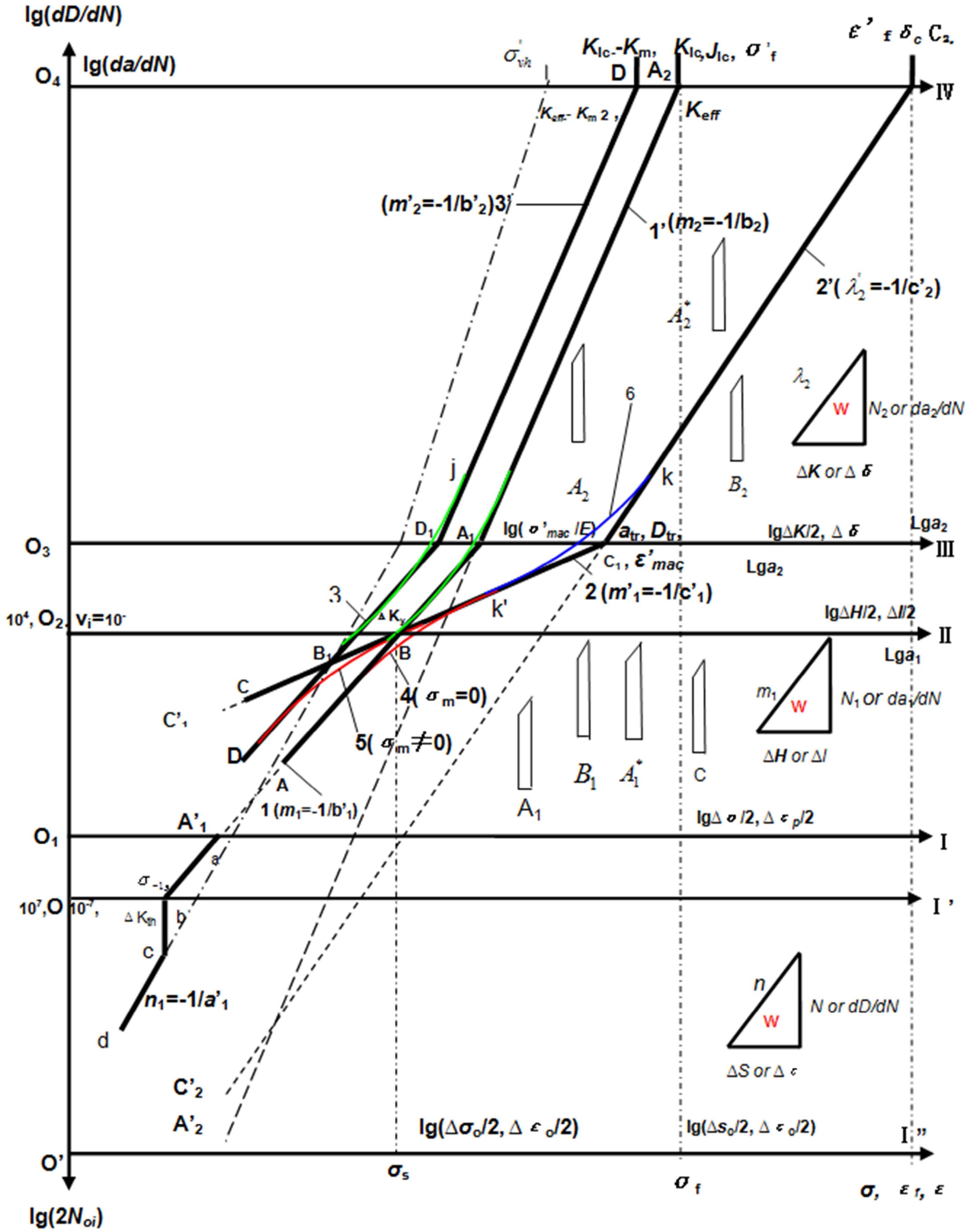
$$\begin{aligned} v_{eff} &= \ln(a_{1f} / a_0) / N_{1fc} - N_{01} \\ &= [\ln(a_{1f}/a_0)\text{-}\ln a_1/a_{01})] / N_{1f} - N_{01}, (mm/\text{cycle}) \end{aligned} \tag{5}$$

or

$$v_{eff} = [a_{1f} \ln(1/1\text{-}\psi)] / N_{1f} - N_{01}, (mm/\text{cycle}) \tag{6}$$

The v_{eff} in eqn (4-6) is defined as an effective rate correction factor in first stage, its physical meaning is the effective damage rate to cause whole failure of specimen material in a cycle, its unit is $mm/cycle$. ψ is a reduction of area. a_0 is pre-micro-crack size which has no effect on fatigue damage under prior cycle loading [9]. a_{01} is an initial micro crack size, a_f is a critical crack size before failure, N_{01} is initial life in first stage, $N_{01} = 0$; N_{1f} is failure life, $N_{1f} = 1$. Such, its final expansion equation for (1) is as following form,

$$da_1 / dN_1 = 2[2\varepsilon'_f]^{-m'_1} \times (\Delta\varepsilon_p)^{m_1'} \cdot a/v_{eff} (mm/cycle) \tag{7}$$



Attached figure 1. *Comprehensive figure of material behaviors (Bidirectional combined coordinate system and simplified schematic curves in the whole process) [1-3].*

If the materials occur strain hardening, and want to via the stress σ to express it, due to plastic strain occur cyclic hysteresis loop effect, then the life predicting equation corresponded to positive direction curve CC_1 in Fig1should be

$$da_1/dN_1 = A_1(\Delta H_1/2)^{m_1}, (mm/cycle), (\sigma > \sigma_s) \quad (8)$$

Where

$$H_1 = \sigma \cdot a_1^{1/m_1} \quad (9)$$

$$\Delta H_1 = \Delta\sigma \cdot a_1^{1/m_1} \quad (10)$$

H_1 is defined as the short crack stress factor, the $\Delta H_1/2$ is stress factor amplitude. Same, that H_1 is driving force of short crack growth under monotonous load, and the ΔH_1 is driving force of under fatigue loading. Its physical and geometrical meaning of the A_1 are similar to the B_1. The A_1 is also calculable comprehensive material constant, for $\sigma_m = 0$, it is as below

$$A_1 = 2(2\sigma_f^{'})^{-m_1}(v_{eff})^{-1}, (\sigma_m = 0) \quad (11)$$

But if $\sigma_m \neq 0$, here for the eqn (8) to adopt the correctional method for mean stress by in reference [10] as follow

$$A_1 = 2[2\sigma_f^{'}(1-\sigma_m/\sigma_f)]^{-m_1}(v_{eff})^{-1}, (\sigma_m \neq 0) \quad (12)$$

Or

$$A_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1} \ (\sigma_m \neq 0) \quad (13)$$

Where the $\sigma_f^{'}$ is a fatigue strength coefficient, K' is a cyclic strength coefficient. $m_1 = -1/b'_1$, m_1 and b'_1 are the fatigue strength exponent. $m_1 = -1/c'_1 \times n'$, $n' = b_1^{'}/c_1^{'}$, n' is a strain hardening exponent. So that, its final expansion equation for (8) is as below form,

$$da_1/dN_1 = 2(2\sigma_f^{'})^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), (\sigma > \sigma_s, \sigma_m = 0) \quad (14)$$

$$da_1/dN_1 = 2[2\sigma_f^{'}(1-\sigma_m/\sigma_f)]^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), (\sigma > \sigma_s, \sigma_m \neq 0) \quad (15)$$

If to take formula (13) to replace A_1 into eqn. (8), its final crack rate expansion equation is as below forming

$$da_1/dN_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma_f)]^{1/c'}(0.5\Delta\sigma)^{m_1} \cdot a_1/v_{eff}, (mm/cycle), (\sigma > \sigma_s, \sigma_m \neq 0) \quad (16)$$

Here, when $\sigma >> \sigma_s$, influence of mean stress in eqn (15-16) can be ignored.

2.1.2. The Two Parameter Multiplication Method

Same, under $\sigma > \sigma_s$ condition, if to adopt the two parameter multiplication method to express the crack rate equation corresponded to positive direction curve CC_1, it is as following

$$da_1/dN_1 = A_1^*(0.25\Delta Q'_1)^{\frac{m_1 m'_1}{m_1+m'_1}}, (mm/Cycle) \quad (17\text{-}1)$$

or

$$da_1/dN_1 = A_1^*(0.25\Delta\sigma \times \Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times a_1, (mm/Cycle) \quad (17\text{-}2)$$

Where the Q_1 is defined as the short crack stress intensity-Q_1 -factor of two-parameter, the ΔQ_1 is defined the short crack stress intensity Q_1 -factor range of two-parameter,

$$Q_1 = (\varepsilon \cdot \sigma)\ a_1^{1/\frac{m_1 m'_1}{m_1+m'_1}} \quad (18)$$

$$\Delta Q_1 = (\Delta\varepsilon \cdot \Delta\sigma)\ a_1^{1/\frac{m_1 m'_1}{m_1+m'_1}} \quad (19)$$

$$A_1^* = 2[4(\sigma'_f \varepsilon'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1}, (MPa^{\frac{m_1 m'_1}{m_1+m'_1}} mm/cycle), \ (\sigma_m = 0) \quad (20)$$

$$A_1^* = 2[4(\sigma'_f \varepsilon'_f)(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1}, (MPa^{\frac{m_1 m'_1}{m_1+m'_1}} mm/cycle) \ (\sigma_m \neq 0) \quad (21)$$

Here the eqn (18) is driving force of short crack growth under monotonic loading, and the eqn (19) is driving force under fatigue loading. It should be point that, the parameter A_1^* in eqn (17) is also a comprehensive material constant. Its physical and geometrical meaning of the A_1^* is similar to above the A_1. And Its slope of micro-trapezium bevel edge just is corresponding to the exponent $m_1 m'/(m_1 + m'_1)$ of the formula (20-21). By the way, here is also to adopt those material constants $\sigma'_f, b_1', \varepsilon'_f, c_1'$ as "genes" in the fatigue damage subject. Therefore, for the eqn (17), its final expansion equation corresponded to positive direction curves 2' (CC_1) (attached fig. 1) is as below form:

$$da_1/dN_1 = 2(4\sigma'_f\varepsilon'_f)^{-\frac{m_1m'_1}{m_1+m'_1}}\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}\times a_1/v_{eff},(mm/Cycle),(\sigma_m=0) \tag{22}$$

$$da_1/dN_1 = 2[4(\sigma'_f\varepsilon'_f)(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}\times a_1/v_{eff},(mm/Cycle),(\sigma_m\neq 0) \tag{23}$$

Where, influence of mean stress in eqn (23) can also be ignored. But it must point that the total strain range $\Delta\varepsilon$ in eqn (22-23) should be calculated by Masing law as following eqn.[11]

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \tag{24}$$

2.2. The Calculations for Long Crack Growth Process

2.2.1. The Single Parameter Method

Under $\sigma>\sigma_s$ condition, due to the material behavior comes into the long crack growth stage, the exponent in crack growth rate da_2/dN_2 equation also to show change from m_1' to λ_2; and due to it occurs cyclic hysteresis loop effect, its rate model corresponded to positive direction curve C_1C_2 in figure 1 is as below form

$$da_2/dN_2 = B_2'[y_2(a/b)\Delta\delta'_t/2]^{\lambda_2},(mm/Cycle) \tag{25}$$

Where

$$\delta_t = 0.5\pi\times\sigma_s\times a_2(\sigma/\sigma_s+1)/E, \tag{26}$$

$$\Delta\delta_t = 0.5\pi\times\sigma_s\times a_2(\Delta\sigma/2\sigma_s+1)/E, \tag{27}$$

Where δ_t is a crack tip open displacement, $\Delta\delta_t$ is a crack tip open displacement range [12]. The $y_2(a/b)$ is correction factor [13] related to long crack form and structure size. Here should note the B_2 is also a calculable comprehensive material constant,

$$B_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s+1)a_{2eff}/E)]^{1-\lambda_2}\times v_{pv},(\sigma_m=0) \tag{28-1}$$

$$B_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)a_{2eff}/E)]^{1-\lambda_2}\times v_{pv},(\sigma_m\neq 0) \tag{28-2}$$

$$v_{pv} = \frac{(a_{2pv}-a_{02})}{N_{2eff}-N_{02}} \approx 3\times10^{-5}\sim 3\times10^{-4} = v^*(mm/Cycle) \tag{29}$$

Where λ_2 is defined to be ductility exponent in long crack growth process, $\lambda_2=-1/c_2'$, c_2' is a fatigue ductility exponent under low cycle in second stage. the v_{pv} is defined to be the virtual rate, its physical meaning is an effective rate to cause whole failure of specimen material in a cycle in the second stage, the unit is $mm/Cycle$, its dimension is similar to the v^*-value in reference[14], but both units are different, where is the "$m/Cycle$".

So that, the conclusive expansion equations is derived from above mentioned eqn. (25) as follow

For $\sigma_m=0$,

$$da_2/dN_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s+1)a_{2eff}/E)]^{1-\lambda_2}v_{pv}\times\left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s+1)a_2}{E}\right]^{\lambda_2},(mm/cycle) \tag{30}$$

For $\sigma_m\neq 0$, it should be

$$da_2/dN_2 = 2[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)a_{2eff}/E)]^{1-\lambda_2}v_{pv}\times\left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s+1)a_2}{E}\right]^{\lambda_2},(mm/cycle) \tag{31}$$

Where, influence for mean stress usually can be ignored in the eqn (31). a_{2eff} is an effective crack size, it can be calculated from effective crack tip opening displacement δ_{2eff}

$$a_{2eff} = \frac{E\times\delta_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s+1)},(mm) \tag{32}$$

And

$$\delta_{2eff} = (0.25 \sim 0.4)\delta_c, (mm) \quad (33)$$

Here the δ_c is critical crack tip open displacement.

2.2.2. The Two Parameter Multiplication Method

In the two parameter multiplication method to calculate the crack growth rate in second stage, it can yet use two kinds of methods: the Q_2 -factor method and the σ -stress method.

(1) Q_2 -factor method

To use Q_2 -factor method calculating the long crack growth rate, here its effective models corresponded to positive direction curve C_1C_2 in figure 1 is as below form

$$da_2 / N_2 = B_2^* \times (0.25 y_2(a/b)\Delta Q_2)^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}}, (mm/cycle), \quad (34)$$

Where

$$Q_2 = y_2(a/b)K_1\delta_t, (MPa\cdot\sqrt{m}\cdot mm), \quad (35)$$

$$\Delta Q_2 = y_2(a/b)(\Delta K_2 \cdot \Delta\delta_t), (MPa\cdot\sqrt{m}\cdot mm) \quad (36)$$

$$K_2 = K_1 = \sigma\sqrt{\pi a_2}, (MPa\sqrt{m}) \quad (37)$$

The Q_2 -factor and ΔQ_2 are all long crack growth driving force, which are respectively under monotonous and repeated loading, their unit are all the "$MPa\cdot\sqrt{m}\cdot mm$". The K_2 is the stress intensity factor, the δ_t is a crack tip open displacement. B_2^* is also calculable comprehensive material constant, on the exponent as compared with above eqn. (20) and (21) that is not different.

$$B_2^* = 2[4(K_{2c}\delta_{2c})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \times v_{pv}, (MPa\sqrt{m}\times mm)^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \cdot mm/cycle, (\sigma_m = 0) \quad (38\text{-}1)$$

$$B_2^* = 2[4K_{2fc}\delta_{2fc}(1-K_{2m}/K_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \times v_{pv}, (MPa\sqrt{m}\times mm)^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \cdot mm/cycle(\sigma_m \neq 0), \quad (38\text{-}2)$$

Where the m_2 is an linear elastic exponent in long crack growth process in second stage, $m_2 = -1/b'_2$, and λ_2 is a ductility exponent, $\lambda_2 = -1/c'_2$, c'_2 is an exponent in second stage under low cycle fatigue. K_{2fc} is the critical stress intensity factor, K_{2m} is mean stress intensity factor, δ_{2fc} is the critical crack tip open displacement, which are critical values under fatigue loading.

So the conclusive crack growth rate expanded equation corresponded to positive direction curve C_1C_2 in fig.1 should be

For $\sigma_m = 0$

$$da_2 / N_2 = 2[4K_{2eff}\delta_{2eff}]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \times [0.25 y_2(a/b)\Delta K_2 \cdot \Delta\delta_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \; (\sigma_m = 0) \quad (39\text{-}1)$$

For $\sigma_m \neq 0$,

$$da_2 / N_2 = 2[4K_{2eff}\delta_{2eff}(1-K_{2m}/K_{2fc})]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv} \times [0.25 y_2(a/b)\Delta K_2 \cdot \Delta\delta_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \; (\sigma_m \neq 0) \quad (39\text{-}2)$$

In reference [15-16] refer to the effective stress intensity factor in fracture mechanics, same, here there is also an effective value K_{2eff} to propose as follow,

$$K_{2eff} \approx (0.25 - 0.4)K_{2fc} \quad (40\text{-}1)$$

$$K_{2fc} = \sigma'_f\sqrt{\pi a_{fc}} \quad (40\text{-}2)$$

$$K_{2eff} = \sigma'_f\sqrt{\pi a_{2eff}} \quad (40\text{-}3)$$

Where K_{2fc} and K_{2eff} are respectively the critical stress intensity factor and mean stress intensity factor under fatigue loading in the second stage; The δ_{2eff} is the effective crack tip open displacement. The a_{fc} in (40-2) is the critical crack size, the a_{2eff} in (40-3) is an effective crack size, for which is obtained and calculated from eqns (32-33), (37) and (40) under fatigue loading, and it should take less value.

(2) σ -stress method

For the ΔQ_2 and B_2^* in eqn (34), if adopt stress calculations, it should all be expressed by the stress σ, it is as following forms

$$Q_2 = 0.5\, y_2(a/b)\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s + 1)]/E \quad (41)$$

$$\Delta Q_2 = 0.5\, y_2(a/b)\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\Delta\sigma/2\sigma_s + 1)]/E \quad (42)$$

For $\sigma = 0$

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2f}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}, \{[MPa(\sqrt{m})^3]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle\}, \tag{43}$$

For $\sigma \neq 0$

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2f}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}, \{[MPa(\sqrt{m})^3]^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle\}, \tag{44}$$

Therefore the crack growth rate equation of corresponded to positive direction curve C_1C_2 in fig.1, its conclusive expansion equation is as below form,

For $\sigma_m = 0$,

$$da_2/dN_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}\times\Big([y_2(a/b)0.5\,\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\Big)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}, (mm/cycle) \tag{45}$$

For $\sigma_m \neq 0$,

$$da_2/N_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv}\times\Big([y_2(a/b)0.5\,\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\Big)^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \tag{46}$$

2.3. Calculations for the Crack Growth Rate in Whole Process

2.3.1. The Single Parameter Method

Due to short crack behaviors and long crack ones there are distinctly different, but from the short crack growth to the long crack growth process, it must exist a same crack size at the transition point, and the crack propagation rate at this point must be equal. According to this reasoning, with the help of the same location at transition point as the linking point, therefore to establish the crack- growth rate-linking-equation between the first and the second stage in whole process, this is just the theory basis of crack rate equation as whole process. So for availing to the crack rate calculation in whole process, author proposes an important research result and method: that is to be the problem for the transition crack size a_{tr} at transition point from short crack to long crack growth process. It can be derived between two stages to make equal by both crack growth rate equations. The calculating model is as follow:

$$(da_1/dN_1)_{a_{01}->a_{tr}} <= da_{tr}/dN_{tr} =< (da_2/dN_2)_{a_{tr}->a_{eff}} \tag{47}$$

Here the equation (47) is defined as the crack- growth-rate-linking-equation in whole process, the da_{tr}/dN_{tr} a_{tr} in (47) is crack growth rate at transition point.

For $\sigma_m \neq 0$, to select driving force equations (10) and (27), to select formula (13) and (28-2) for relative comprehensive material constant A_1 and B_2, for above related parameters are substituted into eqn (47), then to derive its expanded crack growth rate-linking-equation for eqn (47)corresponded to positive curve CC_1C_2 is as following form

$$\begin{aligned}&\frac{da_1}{dN_1} = \Big\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'}\times(v_f\times a_{tr})^{-1}\times(\Delta\sigma/2)^{m_1}\times a\Big\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}} =< \\ &\frac{da_2}{dN_2} = \left\{2\big[(\pi\sigma_s(\sigma'_f/\sigma_s+1)a_{eff}/E)\big]^{\lambda_2}\times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)a}{E}\right]^{\lambda_2}\right\}_{a_{tr}->a_{eff}}, (mm/cycle), (\sigma \neq 0)\end{aligned} \tag{48}$$

2.3.2. The Two Parameter Multiplication Method

For the multiplication method of two parameter, if in $\sigma_m \neq 0$ as example, its expanded crack-growth-rate-linking-equation corresponded to positive curve CC_1C_2 is as following form

$$\frac{da_{1\text{tr}}}{dN_1}=\left\{2[4\sigma'_f\,\varepsilon'_f\,(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v_{eff})^{-1}\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}a\right\}_{a_{01}->a_{tr}}<=\frac{da_{\text{tr}}}{dN}$$

$$=\frac{da_2}{dN_2}=<\left\{\begin{array}{l}2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times\\ \times\left([0.5\,\sigma\cdot\sigma_s(\sqrt{\pi a_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\end{array}\right\}_{a_{tr}->a_{eff}},mm/cycle,(\sigma\ \neq 0) \quad (49)$$

It should point that the calculations for the crack growth rate in whole process should be according to different stress level and loading condition, to select appropriate calculable equation. And here have to explain that its meaning of the eqns (47-48) is to make a linking for the crack growth rate between the first stage and the second stage, in which before the transition-point crack size, its crack growth rate should be calculated by the short crack growth rate equation; and after the transition-point crack size a_{tr} it should be calculated by the long crack growth rate equation. Note that it should not been added together by the crack growth rates for two stages. About calculation method, it can be calculated by means of computer doing computing by different crack size [17-18].

3. Calculating Example

3.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b=573MPa$, yield limit $\sigma_s=361MPa$, fatigue limit $\sigma_{-1}=267.2MPa$, reduction of area is $\psi=0.51$, modulus of elasticity $E=200000MPa$; Cyclic strength coefficient $K'=1165MPa$, strain-hardening exponent $n'=0.187$; Fatigue strength coefficient $\sigma'_f=947.1MPa$, fatigue strength exponent $b'_1=-0.111$, m_1=9.009; Fatigue ductility coefficient $\varepsilon'_f=0.464$, fatigue ductility exponent $c'_1=-0.5395$, $m'_1=1.8536$. Threshold value $\Delta K_{th}=8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c}=K_{1c}=92.7\text{MPa}\sqrt{\text{m}}$, critical damage stress intensity factor $K_{1c}(K_{2c})$. Its working stress $\sigma_{\max}=450MPa$, $\sigma_{\min}=0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b)=1$. Other computing data are all included in table 1.

Table 1. *Computing data*

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv}	m_2	δ_c, mm	λ_2	$y_2(a/b)$	a_{th}, mm
92.7	28.23	8.6	2×10^{-4}	3.91	0.18	2.9	1.0	0.07

3.2. Required Calculation Data

Try to calculate respectively by calculating methods of two kinds as following different data and depicting their curves:

(1) To calculate crack size a_{tr} at the transitional point between two stages;

(2) To calculate the crack growth rate da_{tr}/dN_{tr} at transitional point;

(3) To calculate the short crack growth rate da_1/dN_1 in first stage from micro crack $a_{01}=0.02mm$ growth to crack $a=2mm$;

(4)To calculate the long crack growth rate da_2/dN_2 in second stage from $a_2=0.2mm$ to long crack effective size $a_{2\text{eff}}=5mm$;

(5)Calculating for crack growth rate da/dN in the whole process;

(6)To depict the curves of the crack growth rate da/dN in whole process.

3.3. Calculating Processes and Methods

3.3.1. Calculations for Relevant Parameters

The concrete calculation methods and processes are as follows,

1) Calculations for stress range and mean stress:

Stress range calculation:
$\Delta\sigma=\sigma_{\max}-\sigma_{\min}=450-0=450(MPa)$;

Mean stress calculation:
$\sigma_m=(\sigma_{\max}+\sigma_{\min})/2=(450-0)/2=225MPa$.

2) Calculation for effective damage value a_{eff}

For the effective crack size a_{1eff} in first stage and the second stage, both can be calculated respectively, and can take smaller one of both. According to formulas (32), calculation for effective crack size a_{2eff} in second stage is as follow,

$$a_{2eff} = \frac{E\times\delta_{eff}}{\pi\sigma_s(\sigma_f/\sigma_s+1)} = \frac{200000\times0.25\times0.18}{\pi361(947.1/361+1)} = 2.1(mm),$$

Take $a_{2eff} = 2.0mm$, here for a_{1eff} in first stage to take same value by the second stage, $a_{1eff} = a_{2eff} = 2mm$.

3) According to formulas (6), to calculate correction coefficient v_{eff} in first stage:

$$v_{eff} = a_{eff}\ln[1/(1-\psi)] = 2\times\ln[1/(1-0.51)] = 1.43,(mm/\text{cycle}).$$

4) By eqn (29), to select virtual rate v_{pv} in second stage, here take:

$$v_{pv} = \frac{a_{2eff}-a_{02}}{N_{2f}-N_{02}} \approx 2.0\times10^{-4}(mm/Cycle),\ (N_{2f}=1,\ N_{02}=0).$$

Here by means of two kinds of methods to calculate respectively as below:

3.3.2. The Single Parameter Method

(1) To calculate crack size a_{tr} at the transitional point between two stages

1) By the crack-rate-link formulas (47-48), to select relevant equation for short crack growth rate calculating:

At first, calculation for comprehensive material constant A_1 by eqn (13)

$$\begin{aligned}A_1 &= 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'}\times(D_{ef}\times v_f)^{-1}\\ &= 2\times1165^{-9.01}\times[2\times0.464(1-225/947.1)]^{1/-0.5395}(2\times0.713)^{-1}\\ &= 6.28\times10^{-28},(MPa\sqrt[m_1]{mm})^{-m_1}\times mm/cycle\end{aligned}$$

Here select the crack growth rate equations (8) and (16) in first stage to simplify calculations as follow form,

$$\begin{aligned}da_1/dN_1 &= A_1\times(\varDelta\sigma/2)^{m_1}\times a_1 = 3.193\times10^{-28}\times(450/2)^{9.01}\times a_1\\ &= 6.28\times10^{-28}\times1.56\times10^{21}\times a_1 = 9.8\times10^{-7}\times a_1\end{aligned}$$

2) Calculating for long crack growth rate in second stage by (31):

Calculation for comprehensive material constant B_2 by eqn (28)

$$\begin{aligned}B_2 &= 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)a_{eff}/E)\right]^{-\lambda_2}\times v_{pv}\\ &= 2\left[2(3.1416\times361(947.1/361+1)(1-225/947.1)\times2/200000)\right]^{-2.9}\\ &\times2\times10^{-4} = 9.1988,\ (mm)^{-\lambda_2}\times mm/Cycle\end{aligned}$$

To calculate the crack growth rate in second stage, and to simplify calculation equation as follow form,

$$\begin{aligned}da_2/dN_2 &= B_2\left[\frac{0.5\pi\sigma_s y_2(\varDelta\sigma/2\sigma_s+1)a_2}{E}\right]^{\lambda_2}\\ &= 9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)a_2}{E}\right]^{2.9}\\ &= 9.1988\times1.6698\times10^{-7}a_2^{2.9}\\ &= 1.5384\times10^{-6}a_2^{2.9}(mm/cycle)\end{aligned}$$

3) Calculation for crack size a_{tr} at transitional point:

According to the equations (47) and (48), to do calculation for crack size a_{tr} at the transitional point; then, to take brief crack-rate-linking-calculating-formulas as follow form,

$$6.28\times10^{-28}\times1.56\times10^{21}\times a_{tr} = 9.1988\times1.6698\times10^{-7}\times a_{tr}^{2.9}$$

$$a_{tr} = (0.638)^{\frac{1}{1.9}} = (0.638)^{0.5263} = 0.789(mm)$$

So to obtain the transitional point crack size $a_{tr} = 0.789(mm)$.

(2) To calculate the crack rate at transitional point a_{tr}

$$\begin{aligned}da_1/dN_1 &= da_{tr}/dN_{tr} = 9.8\times10^{-7}a_1\\ &= 9.8\times10^{-7}\times0.789 = 7.74\times10^{-7}(mm/cycle)\end{aligned}$$

$$\begin{aligned}da_2/dN_2 &= da_{tr}/dN_{tr} = 1.5384\times10^{-6}a_{tr}^{2.9}\\ &= 1.5384\times10^{-6}\times(0.79)^{2.9} = 7.74\times10^{-7}(mm/cycle)\end{aligned}$$

Here it can be seen, the crack-rate at the transition point ($a_{tr} = 0.789mm$) is same.

(3)Calculations for the crack growth rates da/dN in whole process

To select eqn (48),the da/dN from micro-crack $a_{01} = 0.02mm$ to transitional point $a_{tr} = 0.789mm$, again to long-crack $a_{eff} = 5mm$ is as follow:

1) To select eqn (48)and to put into relevant data above mentioned as below

$$\begin{aligned}\frac{da_1}{dN_1} &= \left\{3.193\times10^{-28}\times(450/2)^{9.01}\times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}}\\ &=< \frac{da_2}{dN_2} = \left\{9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)a_2}{E}\right]^{2.9}\right\}_{a_{tr}->a_{eff}} ,(mm/cycle),(\sigma\neq0)\end{aligned}$$

2) To calculate and simplify,

$$\frac{da_1}{dN_1} = \left\{6.28\times10^{-28}\times1.56\times10^{21}\times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}}$$
$$=< \frac{da_2}{dN_2} = \left\{9.1988\times1.6698\times10^{-7}a_2^{2.9}\right\}_{a_{tr}->a_{eff}}, (mm/cycle), (\sigma \neq 0)$$

3) For above formulas, it can derive more simplified crack-rate-linking-equation in whole process corresponded to different crack size as follow form

$$\frac{da_1}{dN_1} = \left\{9.8\times10^{-7}\times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{tr}}{dN_{tr}} =< \frac{da_2}{dN_2} = \left\{1.5384\times10^{-6}a_2^{2.9}\right\}_{a_{tr}->a_{eff}}, (mm/cycle), (\sigma \neq 0)$$

4) According to above the simplified rate-linking-equation, by means of a computer, to do the crack growth rate computing in whole process from micro crack $a_{01} = 0.02mm$ to transitional point size $a_{tr} = 0.789mm$, again to long crack size $a_{eff} = 5mm$. The crack growth rate data corresponded to different crack sizes is all included in table 2~4.

(4)To depict the crack growth rate curves in the whole process

By the data in tables 2-4, the crack growth rate curves for two stages and whole process are depicted respectively in figure 2 and 3.

3.3.3. The Two Parameter Multiplication Method

(1) To calculate the crack size a_{tr} at the transitional point between two stages

1) Calculation for comprehensive material constant A_1^* in first stage by eqn (21)

$$A_1^* = 2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(a_{eff}\times v_f)^{-1}$$
$$= 2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}}\times(2\times0.713)^{-1}$$
$$= 2.216\times10^{-5}(MPa^{\frac{m_1m'_1}{m_1+m'_1}}mm/cycle)$$

2) Calculation for comprehensive material constant B_2^* in second stage by eqn(44)

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv} =$$
$$= 2\{[\frac{947.1\times361(947.1/361+1)}{200000}(\sqrt{\pi\times2})^3](1-225/947.1)\}^{-\frac{3.91\times2.9}{3.91+2.9}}\times2\times10^{-4}$$
$$= 2\{[6.1945\times\pi^{1.5}2^{1.5}]0.7624\}^{-1.665}\times2\times10^{-4} = 2\{74.381\}^{-1.665}\times2\times10^{-4} = 3.0625\times10^{-7}, (MPa^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle)$$

3) Calculation for crack size a_{tr} at transitional point

According to the crack growth rate-linking-equations (47) and (49), to input the relevant data mentioned above, it is calculated as following

a) The first step: to set up equal expression at transitional point between the first stage and the second stage,

$$A_1^*\times(0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1m'_1}{m_1+m'_1}}\times a_{tr} = B_2^*[(\Delta\sigma/2)\cdot0.5\sigma_s(\sqrt{\pi a_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}};$$

b) The second step: to input the relevant data,

$$2.216\times10^{-5}\times(0.25\times2.553\times10^{-3}\times450)^{\frac{9.009\times1.8536}{9.009+1.8536}}\times a_{tr}$$
$$= 3.0625\times10^{-7}\times[0.5(450/2)\cdot361(\sqrt{\pi a_{tr}})^3(450/2\times361+1)/E]^{\frac{3.91\times2.9}{3.91+2.9}};$$

c) Third step:to make simplified equal equation:

$$3.22\times10^{-6}a_{tr} = 2.6695\times10^{-6}\times a_{tr}^{2.4975};$$

d) Take result: $a_{tr} = 1.2062^{0.6678} = 1.133(mm)$;

So obtain the transitional point crack size $a_{tr} = 1.133mm$ between two stages.

(2) Calculations for the crack growth rate at transitional point a_{tr}

$$da_1 / dN_1 = da_{tr} / dN_{tr} = 3.22\times10^{-6}\times a_{tr} = 3.22\times10^{-6}\times1.133 = 3.648\times10^{-6}(mm/cycle)$$

$$da_2 / dN_2 = da_{tr} / dN_{tr} = 2.6695\times10^{-6}\, a_{tr}^{2.4975} = 2.6695\times10^{-6}\times1.133^{2.4975} = 3.646\times10^{-6}(mm/cycle)$$

Thus it can be seen, the crack growth rate at the transition point crack size $a_{tr} = 1.113(mm)$ is corresponding, that is 3.646×10^{-6} $(mm/cycle)$.

(3) Calculations for the crack growth rates in whole process by crack growth sizes

Select the crack growth rate equation (49), the calculations for the crack growth rates da/dN in whole process from micro crack $a_{01} = 0.02mm$ to transitional point $a_{tr} = 1.113mm$, again to long crack size $a_{eff} = 5mm$ are as follow,

1) According to the crack growth rate-linking-equations (49),and to input the above relevant data:

$$\frac{da_{1\mathrm{tr}}}{dN_1} = \left\{ 2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}}\times(2\times0.7133)^{-1}\times(0.25\times\Delta\varepsilon\cdot450)^{\frac{m_1 m'_1}{m_1+m'_1}}\, a \right\}_{a_{01}->a_{tr}} <= \frac{da_{\mathrm{tr}}}{dN}$$

$$= \frac{da_2}{dN_2} =< \left\{ \begin{array}{l} 2\{[\dfrac{947.1\times361(947.1/361+1)}{200000}(\sqrt{\pi\times2})^3](1-225/947.1)\}^{-\frac{3.91\times2.9}{3.91+2.9}}\times2\times10^{-4} \\ [0.5(450/2)\times361(\sqrt{\pi a_2})^3(450/2\times361+1)/200000]^{\frac{3.91\times2.9}{3.91+2.9}} \end{array} \right\}_{a_{tr}->a_{eff}} ,mm/cycle,(\sigma \neq 0)$$

2) From above calculations, we can derive simplified the crack growth rate-linking-equations in whole process corresponded to different crack size as follow form

$$\frac{da_1}{dN_1} = \left\{3.22\times10^{-6}\times a_1\right\}_{a_{01}->a_{tr}} <= \frac{da_{\mathrm{tr}}}{dN_{\mathrm{tr}}} =< \frac{da_2}{dN_2} = \left\{2.6695\times10^{-6}\, a_2^{2.4975}\right\}_{a_{tr}->a_{eff}} ,mm/cycle,(\sigma \neq 0)$$

According to above the simplified rate-linking-equation, computing by means of a computer, to do computing in whole process which are for different sizes from micro crack $a_{01} = 0.02mm$ to transitional point size $a_{tr} = 1.113mm$, again to long crack size $a_{eff} = 5mm$ .The data calculated with the two parameter multiplication method is included in tables 2-4.

(4) To depict the crack growth rate curves in the whole process

By the data in tables 2-4, the crack growth rate curves for two stages and whole process are depicted respectively in figure 2 and 3.

From tables 2~4, it is observed that the micro crack size from 0.02 mm to macro crack 2mm (as the first stage) , comparison for result data calculated by the single parameter method and the two-parameter multiplication-method, both ratio is all 3.25/1; Comparison for their result data calculated from the crack size 0.5 mm to 5mm (as the second stage) , both ratio is gradually to reduce from 2.3/1 to 0.91/1. These data are shown as so long history in whole crack growth process from micro crack to long crack, for the calculating results data can all be accepted for two of mothers. Here looked from the overall trend, the two-parameter multiplication-method of crack rate data in the whole is more moderate.

Here comparisons for crack growth rate data of calculating results by two kinds of methods are also included in in table 2, 3 and 4.

Table 2. *Comparisons for crack growth rate data in two stages by two kinds of methods*

Data point of number	**1**	**2**	**3**	**4**	**5**
Crack size (mm)	0.02	0.04	0.1	0.2	0.4
Rate by single-parameter in first stage	1.96×10^{-8}	3.92×10^{-8}	9.8×10^{-8}	1.96×10^{-7}	3.92×10^{-7}
Rate by two-parameter in first stage	6.44×10^{-8}	1.29×10^{-7}	3.22×10^{-7}	6.44×10^{-7}	1.29×10^{-6}
Ratio	3.25/1	3.25/1	3.25/1	3.25/1	3.25/1
Rate by single-parameter in second stage	Invalid section			1.446×10^{-8}	1.079×10^{-7}
Rate by two-parameter in second stage	Invalid section			4.79×10^{-8}	2.71×10^{-7}

Table 3. *Comparisons for crack growth rate data in two stages by two kinds of methods*

Data point of number	5	6	7	Transition point	8-Transition point
Crack size (mm)	0.5	0.6	0.7	0.789	1.133
Rate by single-parameter in first stage	4.95×10^{-7}	5.88×10^{-7}	6.869×10^{-7}	7.732×10^{-7}	1.11×10^{-6}
Rate by two-parameter in first stage	1.61×10^{-6}	1.93×10^{-6}	2.25×10^{-6}	2.54×10^{-6}	3.65×10^{-6}
Ratio	3.25/1	3.25/1	3.25/1	3.25/1	3.25/1
Rate by single-parameter in second stage	2.06×10^{-7}	3.497×10^{-7}	5.468×10^{-7}	7.732×10^{-7}	2.21×10^{-6}
Rate by two-parameter in second stage	4.73×10^{-7}	7.45×10^{-7}	1.1×10^{-6}	1.48×10^{-6}	3.65×10^{-6}
Ratio	3.23/1	2.13/1	2.01/1	1.91/1	1.65/1

Table 4. *Comparisons for crack growth rate data in two stages by two kinds of methods*

Data point of number	9	10	11	12	13
Crack size (mm)	1.5	2.0	3.0	4	5
Rate by single-parameter in first stage	1.47×10^{-6}	1.96×10^{-6}	Invalid section		
Rate by two-parameter in first stage	4.83×10^{-6}	6.44×10^{-6}	Invalid section		
Ratio	3.25/1	3.25/1			
Rate by single-parameter in second stage	4.986×10^{-6}	1.148×10^{-5}	3.72×10^{-5}	8.57×10^{-5}	1.64×10^{-4}
Rate by two-parameter in second stage	7.35×10^{-6}	1.51×10^{-5}	4.15×10^{-5}	8.51×10^{-5}	1.49×10^{-4}
Ratio	1.47/1	1.32/1	1.12/1	0.99/1	0.91/1

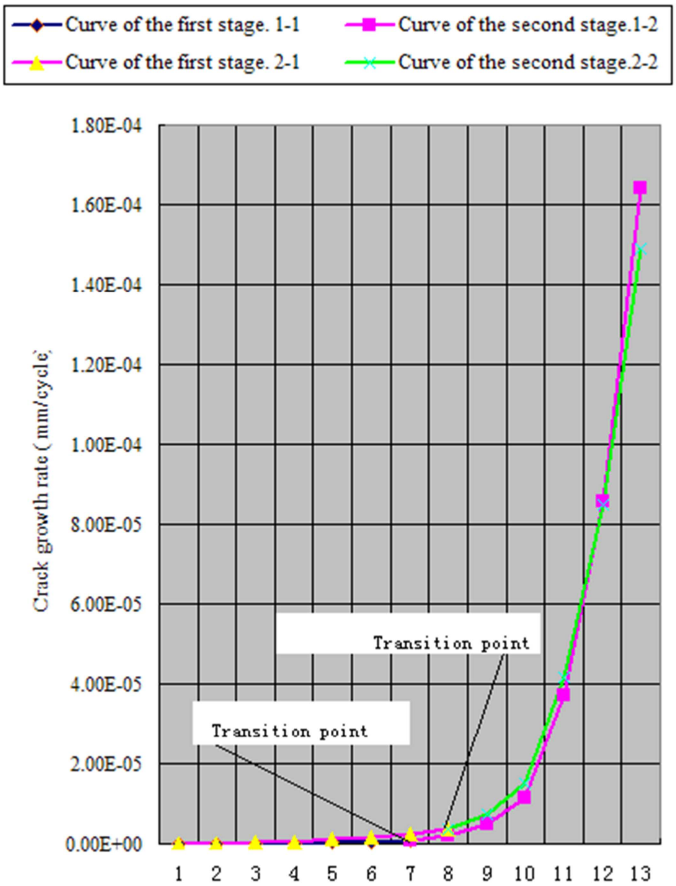


Figure 2. *Comparison of life curves in whole process (in decimal coordinate system)*

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;
(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789*mm*).
(D) 2-1---Curve in first stage depicted by two-parameter calculating data;
(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;
(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size 1.113*mm*).

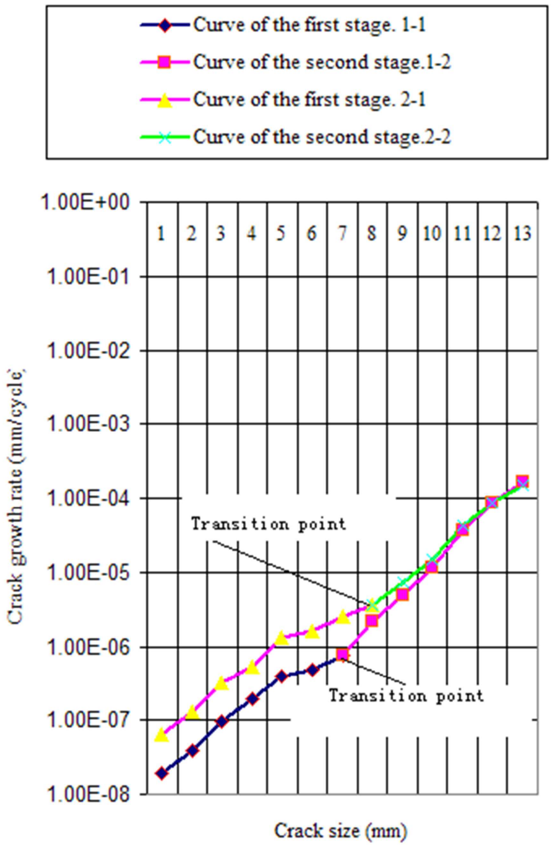


Figure 3. *Comparison of crack growth rate curves in whole process (in logarithmic coordinate system)*

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;
(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size $0.789mm$).
(D) 2-1---Curve in first stage depicted by two-parameter calculating data;
(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;
(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size $1.113mm$).

4. Conclusions

(1) About comparison for calculating methods of two kinds: Looked from the overall trend for the crack growth rate curves, the result data calculated by two methods is basically closer in whole process; especially both crack rate data in second stage is closer. For the single stress-strain-parameter method, its calculation model is simpler; for the two-parameter method, its calculation in whole process is more moderate, but its calculation models are more complex.

(2) About the theory basis of the whole process crack-growth-rate model: From the short crack growth to the long crack growth process it must exist a same crack size at the transition point, the crack propagation rate at this point must be equal. According to this reasoning, with the help of the transition point, to establish the crack- growth-rate-linking-equation between the first and the second stage in whole process, this is just the theory basis of crack rate equation as whole process.

(3) About new cognition for some key material constants: For some new material constants A_1 , A_1^* and B_2, B_2^* about the crack growth rate equations in the fracture mechanics, in practice there are functional relations with other parameters, so they are all calculable parameters by means of the relational expressions (11-13), (17,20), (28), (38), etc.

(4) About cognitions to the physical and geometrical meanings for key parameters: The parameters A_1 , A_1^* in the first stage and the B_2, B_2^* in the second stage, their physical meanings are all a concept of power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

(5) About the calculating methods for the crack-growth rate-linking-equation in whole process: Before the transition size a_{tr} , the crack growth rate should be calculated by the short crack growth rate equation, after the transition point a_{tr} it should be calculated by the long crack growth rate equation.

(6) Total conclusion: Based on the traditional material mechanics is a calculable subject, in consideration of the conventional constants there are “the hereditary characters”, In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions (1)~(5), then make the

fracture mechanics disciplines become calculable subjects, that will be to exist the possibility.

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