Journal of Materials Sciences and Applications
2016; 2(6): 39-50
http://www.aascit.org/journal/jmsa
ISSN: 2381-0998 (Print); ISSN: 2381-1005 (Online)

American Association for Science and Technology

Keywords
Principles of Genes,
Conventional Constants,
Threshold Value of Damage,
Critical Value of Damade,
Damage Factor,
Assessment Method

Received: August 26, 2016
Accepted: September 6, 2016
Published: December 6, 2016

Calculations for Damage Strengh to Linear Elastic Materials-The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

Yangui Yu

Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering, Hangzhou, China

Email address
gx_yyg@126.com

Citation
Yangui Yu. Calculations for Damage Strengh to Linear Elastic Materials-The Genetic Elements and Clone Technology in Mechanics and Engineering Fields. *Journal of Materials Sciences and Applications.* Vol. 2, No. 6, 2016, pp. 39-50.

Abstract
The author bases on the principles of similar to the genetic genes in the life sciences, discovers some new constants shown material properties from micro to macro damage, and proposes some new computing models which are the threshold values and the critical ones on damage to some metallic materials; That is to use the theoretical approach, to adopt the conventional material constants, to derive the new mathematical models and the stress factor of called damage strength, to provide simple assessment criterions on the damage strength and the calculating methods in each stage. In addition, it supplements again the comprehensive figure of the material behaviours; gives yet a detailed calculating example for a safety assessment. This works may be there are practical significances for make linking and communication between the modern fracture mechanics and the damage mechanics, for the decreasing experiments.

1. Introduction

As is well-known, in the traditional materials mechanics, in describing materials behaviours and their strength problems, its main calculating parameters are the stress σ, the strain ε and relevant material constants, e.g. yield stress $\sigma_s(\sigma_y)$, elasticity modulus E and reduction of area ψ, etc. And in the fatigue discipline, it also adopts the stress σ and the strain ε as calculating parameters to use the fatigue strength coefficient σ'_f and the fatigue ductility coefficient ε'_f, etc., as its material constants. In the damage mechanics, it is based on the damage parameter D as its variable to calculate life prediction problems. In the fracture mechanics, it describes the materials behaviours at the crack tip on the strength problems, which is based on the crack size a as its variable, to use the fracture toughness K_{1c} and the critical crack tip open displacement δ_c as its material constants.

To refer to the genes and clone technologies in the life science in [1-10], which traits consist in: they had both self-genetic properties, and had the transferable and the recombination properties. In fact, in the model $K_1 = \sigma\sqrt{\pi a}$ [11-13] of the stress intensity factor in fracture mechanics, in the crack tip open displacement δ_t, in their critical values

$K_{1c} = \sigma\sqrt{\pi a_c}$ and the δ_c, all include the parameters σ, ε, π and their material constants σ_s, ε_s and fracture stress σ_f etc. Here for the stress σ, the strain ε and its relevant material constants σ_s and E, etc, in the materials mechanics can be considered as the genetic elements; the parameters σ'_f, b'_1, ε'_f, c'_1, D, etc, in the fatigue discipline and the damage mechanics can also be considered as the genetic elements; and the crack size a in the fracture mechanics can also be considered as the genetic elements. If can make a link among the materials mechanics, the fatigue subject, the damage mechanics and the fracture mechanics, and if we can provide some conversion methods to make them also convert each other for their relations between the variables, between the material constants and between the dimensional units in the equations, then it would realize this goal. For example, here can consider them as genes for the stress σ and their material constants $\sigma_s, E, \psi, \sigma'_f, \varepsilon'_f$, to make them combine with the variable D_1 of micro-damage, which are together transferred into micro-damage-mechanics, and in combination with the variable D_2 of macro-damage, which are transferred into macro-damage-mechanics. In the same way, here can also consider them as genes for the stress σ and $\sigma_s, E, \psi, \sigma'_f, \varepsilon'_f$, to make them combine with the variable a_1 of short crack, which are together transferred into micro-fracture-mechanics, and combine with the variable a_2 of long crack, which are transferred into macro-fracture-mechanics. Then it is able by these parameters σ, ε, $\sigma'_f, \varepsilon'_f$, etc, to establish their renewing models for the driving forces, for the crack propagating rates and the life equations, or for the damage growth rates and the life equations. Even can also adopt the variable D or a to describe materials behaviours in the whole process.

Above the peculiarities of those parameters and material constants which they are as compared to those ones in the life sciences, they are in different disciplines, but for both all have own inheritable properties (similar to genetic elements), and for both all have the traits of the transferable and the recombination on the epistemology and on the methodology, which, in practice, are all very similar.

Based on the cognitions and the concepts mentioned above, the author draws a link among the engineering materials, the materials mechanics, the fatigue, the damage mechanics and the fracture mechanics, for relationships among their parameters are analysed, for their equations are derived, for their dimensional units convert each other; then to derive a lot of the new mathematical model, and for these newly made computing models are calculated, checked again and again; finally, to provide the calculable equations and expressions (1-17). This is to try to set up communications among many disciplines mentioned above and thereby solve those problems in crack (damage) growth process about which are the driving forces, the strength criteria, the rates of crack propagation and the life calculation, for which become the calculable ones, so that they would be applied in practical engineering. If can realize the goals, it will have practical significances for the design of machineries and structures and for the computational analysis of safe operations and assessments where they are widely distributed in communications and transportation, the aerospace industry, mechanical engineering and other fields.

2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the "Comprehensive figure of materials behaviors" as Figure 1 (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes O' I", O I', O_1 I, O_2 II, O_3 III, O_4 IV and a bidirectional ordinate axis $O'_1\ O_4$. For the area between the axes $0'$ I" and 0_1 I, it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes O I' and O_2 II, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the O_2 II, the O_3 III and O_4 IV where they are calculatied and applied by the macro-damage mechanics and the macro-fracture mechanic. But for between the axes 0_1 I and O_2 II, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

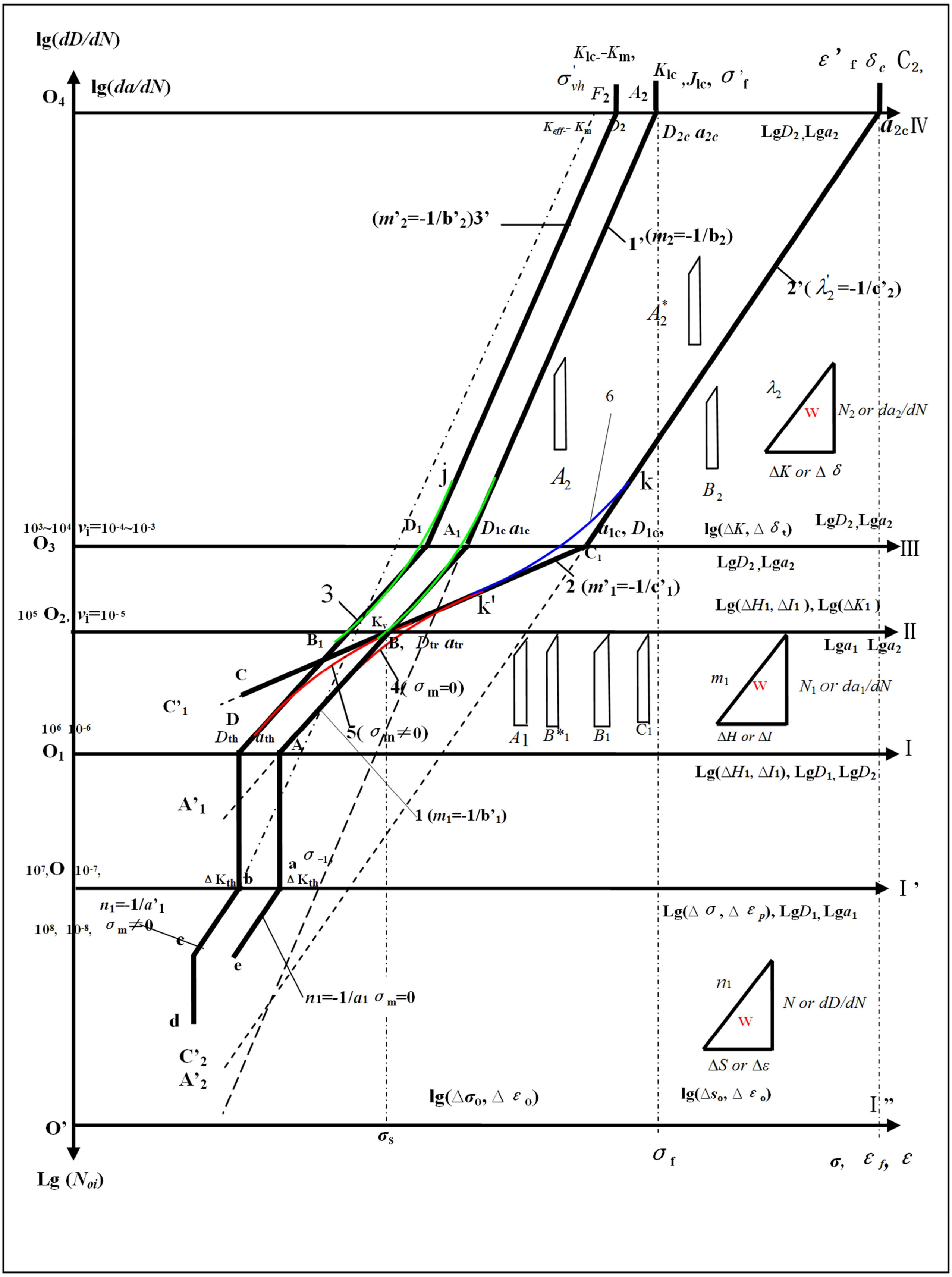

Figure 1. *Comprehensive figure of material behaviors 1 (Or called calculating figure of material behaviors or bidirectional combined coordinate system and simplified schematic curves in the whole process).*

On the abscissa axes $0'$ I" and 0_1 I, they are represented with parameters the stress σ and the strain ε as variables. On the abscissa axes O I' there are the fatigue limit σ_{-1} at point "a" $(\sigma_m = 0)$ and "b" $(\sigma_m \neq 0)$ that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axes O_1 I there are points "A" and "D" that just are the locations placed at threshold values to another materials. On the abscissa axes 0_1 I and 0_2 II that they could all represented as variables with the stress intensity factor range ΔH_1 of short crack, and the strain intensity factor ΔI, and the stress intensity factor range ΔK_1 of long crack. On the other hand, they both are yet represented as variables with the short crack a_1 and the long crack a_2 (or damage D_1 and D_2). And here there are materal constants of two that they are defined as the critical factor K_y of crack-stress-intensity and the critical factor K'_y of the damage-stress-intensity at the first stage, where that are just the transition parameters corresponded to the critical crack size $a_{tr}(=a_{1c})$ or the critical value of damage $D_{tr}(=D_{c1})$, they are just placed at point at the point B $(\sigma_m = 0)$ and at point B_1 $(\sigma_m \neq 0)$ corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors; but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level. On the abscissa axes O3 III, it is represented as variable with the stress intensity factor ΔK_1 (or $\Delta\delta_t$) of long crack; it is a boundary between the first stage and the second stage for some elastic-plastic materials. On this axes O3 III there are the critical points at D1, A1, and C1 (D1c, A1c). On abscissa 0_4 IV, the point A_2 is corresponding to the fatigue strength coefficient σ'_f, the critical stress intensity factor values $K_{1c}(K_{2fc})$ and the critical values D'_{2c} and a_{2c} for the mean stress $\sigma_m = 0$; the point D_2 is corresponding to the $\sigma_m \neq 0$; the point C_2 corresponding to the fatigue ductility coefficient ε'_f and critical crack tip open displacement value δ_c; the point F corresponding to a very high cycle fatigue strength coefficient σ'_{vhf}. In addition on the same O_4 IV, there are yet another critical values $J'_{1c}(J_{1c})$, etc. in the long crack propagation process.

For an ordinate axis, an upward direction along the ordinate axis is represented as crack growth rate da/dN or damage growth rate dD/dN in each stage and the whole process. But a downward direction is represented as life N_{oi}, N_{oj} in each stage and the whole lifetime ΣN.

In the area between axes $0'$ I" and 0_2 II, it is the fatigue history from un-crack to micro-crack initiation. In the area between axes O_1 I' and 0_2 II, it is the fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_2 - O'$ on ordinate axis is as the history relating to life N_{mac} from grains size to micro-crack initiation until macro-crack forming; the distance O_4 - O' is as the history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

In the crack forming stage, the partial coordinate system made up of the upward and the ordinate axes O O_4 and the abscissa axes 0 I', 0_1 I and 0_2 II is represented as the relationship between the crack growth rate dD_1/dN_1 (or the short crack growth rate da_1/dN_1) and the crack-stressfactor range ΔH_1 (or the damage strain factor range ΔI_1). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis O_2 0_4 and abscissa O_2 II, O_3 III and 0_4 IV at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range ΔK , J -integral range ΔJ and crack tip displacement range $\Delta\delta_t$ (da_2/dN_2 - ΔK , ΔJ and $\Delta\delta_t$). Inversely, the coordinate systems made up of the downward ordinate axis O_4O_1 and the abscissa axes O_4 IV, O_3 III, O_2 II,, O_1 I, and O I' are represented respectively as the relationship between the ΔH-, ΔK- range and each stage life N_{oi}, N_{oj} and the lifetime $\sum N$ (or between the $\Delta\varepsilon_p$-, $\Delta\delta_t$- range and the life $\sum N$).

2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve ABA_1 is represented as the varying laws as the behaviours of the elastic materials or some elastic-plastic ones under high cycle loading in the macro-crack-forming stage (the first stage): positive direction ABA_1 represented as the relations between dD_1/dN_1 (or da_1/dN_1)-ΔH; inverted A_1BA, between the $\Delta H_1 - N_{oi}$. The curve CBC_1 is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction CBC_1 is represented as the relations between da_1/dN_1 - ΔI_1; inverted C_1BC, the relations between the $\Delta\varepsilon_p - N_{oi}$.

The curve A_1A_2 in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction A_1A_2 showed as da_2/dN_2 - ΔK (ΔJ); inverted A_2A_1, between the ΔK_2, $\Delta J - N_{oj}$. The C_1C_2 is showed as: the positive, relation between the da_2/dN_2 - $\Delta\delta_t$ under low-cycle loading, inverted C_2C_1, between $\Delta\delta_t$ (ΔJ)- N_{oj}. By the way, the curves $'Dbcd', (\sigma_m = 0)$ and the $'Aae'$ ($\sigma_m = 0$) are represented as the laws under the very high cycle fatigue.

It should yet point that the curve AA_1A_2 (1-1') is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, $da/dN \le 10^{-6}$); the curve DD_1D_2 (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, ($da/dN \le 10^{-6}$). The curve CC_1C_2 (2-2') is depicted as the rate curve under low cycle loading. The curve $eaABA_1A_2$ is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ($\sigma_m = 0, da/dN < 10^{-7}$), the curves $dcbDD_1D_2$ and $dcbF_2$ are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ($\sigma_m \neq 0, da/dN < 10^{-7}$). Inversely, the curve A_2A_1A is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, $N \le 10^6$), the curve D_2D_1D, as the lifetime curve under unsymmetrical cycle loading ($N \le 10^6$). The curve C_2C_1C is depicted as the lifetime curve under low cycle loading ($N \le 10^5$). On the other hand, the curve A_2A_1BAae is as the lifetime one in whole process included very high cycle fatigue ($\sigma_m = 0, N > 10^7$), the curves D_2D_1Dbcd and F_2bcd are all depicted as the lifetime ones in whole process ($\sigma_m \neq 0, N > 10^7$).

It should also be explained that the comprehensive figure 1 of the materials behaviours may be a complement as a fundamental research; that is a tool to design and calculate for different structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.

3. Strengh Calculations on Damage Under Monotonic Loading

Here the damage variables D for describing the damage growth process that are defined as follows:

1). From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage. If applying the concept of the damage mechanics, it is defined in the micro-damage stage where it adopts variable D_1 called the micro-damage variable, which is corresponded to the variable a_1 of a short crack that it is corresponding curve AA1 in figure 2;

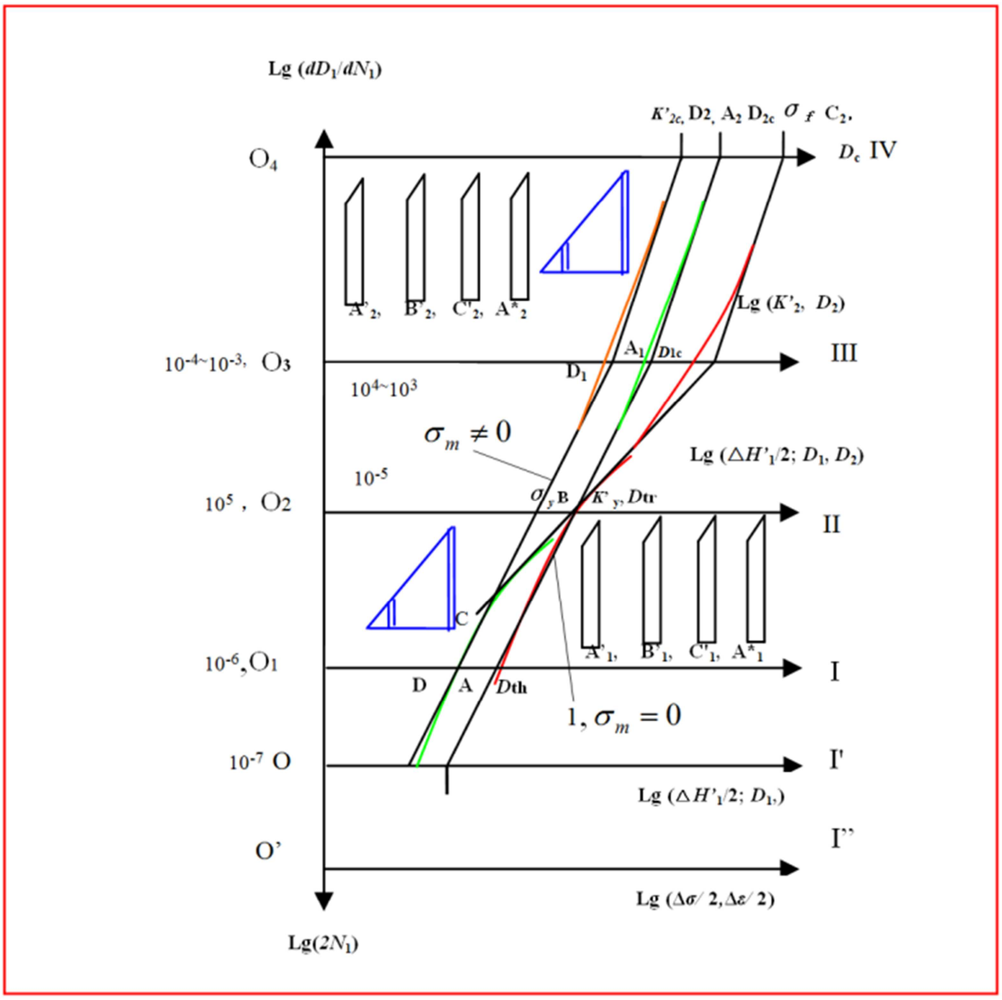

Figure 2. Figure of material damage behaviors in whole process.

2). From the macro-crack propagation to the fracture process is defined in the crack growth stage, or defined in the second stage, here is also applying the concept of the damage mechanics, it is defined in the macro-damage stage. The damage variable D_2 of this stage is called in the macro damage variable, it corresponds to the variable a_2 of the long crack that it is corresponding curve A_1A_2 in figure 2;

3). From micro-damages to full failure of a material, to adopt the parameter D as the variable in the whole process, it corresponds to the crack variable a in the whole process from short crack to long crack growth until full fracture that it is corresponding curve AA_1 A_2 in figure 2.

3.1. About the Driving Force and theThreshold Value on Damage

In the figure 2, it can be seen that differences with the loading ways and the stress levels, for the general steels, their behaviours were always shown defferences in the each stages, but they are all to exist the threshold values D_{th} of the damage, only depended on the exponents b_1 related to the material character in table 1.

Table 1. *Data of threshold values of damage.*

Materials [14-15]	Heat treatment	σ_b, MPa	σ_s, MPa	b_1	D_{th}, damage – units
BHW35	Normalizing 920°C, temper 620°C	670	538	-0.0719	0.2626
QT450-10	As cast condition	498.1	393.5	-0.1027	0.237
QT800-2	Normalizing	913.0	584.32	-0.0830	0.253
ZG35	Normalizing	572.3	366.27	-0.0988	0.240
60Si2Mn	Quench and medium-temperature tempering	1504.8	1369.4	-0.1130	0.228
45	Normalizing 850°C	576~624	377	-0.123	0.219
40Cr	Oil quenching 850°C, temper 560°C	845~940		-0.120	0.222
16MnL	Hot rolling	570		-0.1066	0.233
20	Hot rolling	432	307	-0.12	0.222
40CrNiMoA	Oil quenching 850°C, temper 580°C	1167		-0.061	0.271
BHW35	Normalizing 920°C, temper 620°C	670	538	-0.0719	0.262
30Cr2MoV	Normalizing 940°C, oil cooling 840°C, furnace cooling 700°C	719		-0.0731	0.261
30CrMnSiNi2A	Heat 900°C, isothermy 245°C, air cooling, temper 270°C	1655	1334	-0.1026	0.237
2A12CZ	Natural aging (CZ)	545		-0.0638	0.269
2A50 CS	Artificial aging (CS)	513		-0.0845	0.252
Ti6Al4V (TC4)	Air cooling 800°C	989		-0.07	0.264

It should point the location of the threshold value D_{th} of damage is at the point A where it is at the intersection one between the straight line AA1 and the abscissa axis O_1I in figure 2. And the threshld D_{th} can be calculable one with as following formula under the monotonous loading, it should be [16]

$$D_{th}=\left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5+b_1}}=(0.564)^{\frac{1}{0.5+b_1}} \quad (damage-units) \qquad (1)$$

Or

$$D_{th}=\left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5-(1/m_1)}} \quad (damage-units) \qquad (2)$$

The range of the D_{th} is in 0.21~0.275 (damage-units), it is equivalent to the lengths 0.21~0.275 (mm) of short crack. For linear elastic materials, to make the D_{th} is combined with the stress σ, so that it can form a model of the driving force that is as below,

$$H'_1=\sigma\cdot D_1^{1/m_1}=\sigma^{m_1}\cdot D_{th}=H'_{th}\,[MPa\cdot(damage-units)^{1/m_1}] \qquad (3)$$

In the formula (3), $m_1=-1/b_1$; The H'_1 is defined as the stress intensity factor of micro damage. Because the variable D_1 is a dimensionless value, it is equivalent to the short crack size a_1. Here it must be defined in "1mm length of crack " equivalent to "1-unit damage value", in "1m length of crack" equivalent to "1000 damage units" [17-19]. In an ordinary way, the $\sigma\cdot D_1^{1/m_1}$ may be: the $\sigma\cdot D_1^{1/m_1}<\sigma^{m_1}\cdot D_{th}=H'_{th}$ or $\sigma\cdot D_1^{1/m_1}\geq\sigma^{m_1}\cdot D_{th}=H'_{th}$, the strength criterions for them is as below,

$$H'_1=\sigma\cdot D_1^{1/m_1}\leq H'_{th}\,[MPa\cdot(damage-units)^{1/m_1}] \qquad (4)$$

Or

$$H'_1=\sigma\cdot D_1^{1/m_1}\geq H'_{th}\,[MPa\cdot(damage-units)^{1/m_1}] \qquad (5)$$

Where the H'_{th} is defined as the threshold factor of damage. If to take the yield stress σ_s to replace the σ in the equaton (4), it is come as following form

$$H'_{th-y} = \sigma_s \cdot D_{th}^{1/m_1} = \sigma_s^{m_1} \cdot D_{th} \ ,[MPa \cdot (damage\text{–}units)^{1/m_1}], \quad (6)$$

Then the H'_{th-y} is defined as the threshold factor of the yield stress, so that the H'_{th-y} must be the only the constant showing a material property; And the damage of a material is sure to grow if a $H'_1 \geq H'_{th-y}$.

3.2. Strength Calculation on Damage at the First Stage

When the damage growth gets to the micro damage stage where it is corresponding to the curve 1 (AB) between abscissa axis O1 I and the O2 II in figure 2. If the stress inside a structure component is loaded to the yield stress (at point B on abscissa axis O2 II) or over this level to the A1 (at point A1 on abscissa axis O3 III), then it can set up a criterion of the damage strength for it in the first stage, that is as below form [6]

$$H'_1 = \sigma \cdot D_1^{1/m_1} \leq [D_1] = H'_{1c} \ / n_1, (MPa \cdot damage\text{–}units^{1/m_1}) \quad (7)$$

$$H'_{1c} = \sigma_s \times \sqrt[m_1]{D_{1c}}, (MPa \cdot damage - units^{1/m_1}) \quad (8)$$

Where the damage value D may be to take the size of preexisting flaw in a component, it can also be calculated by a designer in designing. Then when the design stress is less than the yield stress ($\sigma_s = \sigma_y$), the damage value can be adopted with following formula,

$$D_1 = \left(\frac{\sigma}{\sigma_{pr}}\right)^{m_1} \approx \left(\frac{\sigma}{\sigma_s}\right)^{m_1}, \quad (9)$$

Where the $\sigma_{pr} \approx \sigma_e$ is a stress value of proportional limit (approximating to the elastic limit, it can also approximatively be took for the yield stress as the data is to lack. The H'_{1c} in (7) is defined as a critical value of the stress intensity factor on damage, the H'_{1c} is a value corresponded to the critical value K_y and the transition value Dtr of damage, also is the boundary between the short crack and the long crack. Their locations are respectively at points B on abscissa axis O2-II (in Fig. 2). For some cast iron, brittle materials and low toughness steels, which could be happened to fracture when their stresses are loaded to this level.

As is well know the mathematic model to describe a long crack in fracture mechanics that it is to adopt these "genes" σ and π and crack variable a, thereby to make the stress intensity factor $K_1 = \sigma\sqrt{\pi a}$; Here to take the macro damage variable D in the name of macro damage mechanics to displace the crack size a inside the K_1 , then it can still derive the equation of driving force for the describing behavior of it, that is as following form [10].

$$K'_1 = \sigma \times \sqrt{\pi D_1}, (MPa \cdot \sqrt{1000\text{ - }damage\text{ - }units}) \quad (10)$$

Here is sure to explain, the area between the abscissa axis O1-I and the O2-II in fig. 2, the D -value from the threshold D_{th} to D_{1c} ($D_{th} \leq D_1 \leq D_{mac} = D_{tr} = D_{1c}$), there are the mathematic models of the stress factors of two kinds, which are all suited in the section. In addition to above equations (6-8) can be applied, in theory another mathematic models (9-13) are still suitable in the first stage.

Where the K'_1 is a stress intensity factor of the macro damage that it is equivalent to H'_1, but their dimensions and units are differences at this same point. For that corresponding to size $a_{mac} (\approx a_{tr})$ of forming macro crack, that is the very that damage factor K'_{tr} of corresponded to the damage-value D_{tr} at transition point, also a the critical value D_{1c} in the first stage, where is just at point B corresponding yield stress σ_s on abscissa axis O2-II, and is on that boundary between the first stage and the second stage in fig. 2. Then the model of driving force at this point should be as follow

$$K'_y = \sigma_s \cdot \sqrt{\pi D_{tr}}, (MPa \cdot \sqrt{damage - unit - number}) \quad (11)$$

Here it need yet explain, this factor K'_y should theoretically be equivalent to above mentioned the H'_{1c} in first stage, although the dimensions and units between them are differences. Therefore the strength criterion of its damage should be calculated as following form,

$$K'_1 = y(a/b) \cdot \sigma \cdot \sqrt{\pi D_1} \leq [K] = K'_{1c} / n_1, (MPa\sqrt{1000 damage\text{–}units}) \quad (12)$$

$$K'_{1c} = \sigma_s \cdot \sqrt{\pi D_{1c}} \quad (MPa\sqrt{1000 damage - units}) \quad (13)$$

Where the $y(a/b)$ [20-21] is a correcting factor related with the shape and the size of a crack. K'_{1c} ($= K'_y$) is a the critical value of damage, they are all corresponding to the yield stress σ_s and the critical value D_{1c} of damage. It shoud point, because the yield stresses σ_s is the constant of uniquenesses for a material, the critical values of the damage D_{1c} and the factor K'_{1c} related the σ_s should also be considered as the only ones, and can also be applied as an important parameters showed its property. In practice, the critical value D'_{1c} could be calculated by means of below formula:

$$D_{1c} = \frac{K^2}{\sigma_s^2 \times \pi}, (damage - units) \quad (14)$$

Where K is a strength coefficient under monotonic loading. It has to point the calculating equations merntioned above are only suitable for some brittle materials and strain hardening ones, it does not suit the materials of strain softening.

In the table 2, here are listed to the critical values D_{1c} of damage for 13 kinds of materials.

Table 2. *The critical values* D_{1c} *of damage.*

Materials [14-15]	σ_b, MPa	σ_s, MPa	K, MPa	D_{1c},
Hot rolled sheet 1005-1009	345	262	531	1.31
Steel: 1005-1009 Cold-draw sheet	414	400	524	0.546
RQC-100, Hot rolled sheet	931	883	1172	0.561
4340, quench and tempering	1241	1172	1579	0.578
Aluminum 2024-T3	469	379	455	0.46
30CrMnSiA, ① Hardening and tempering	1177	1104.5	1475.76	0.568
LC4CS, ① Heat treatment-CS	613.9	570.8	775.05	0.587
40Cr ③	940	805	1592	1.25
60Si2Mn, quench, medium-temperature tempering ③	1504.8	1369	1721	0.503
QT800-2, ② normalizing	913	584.3	1777	2.94
QT600-2, (B), ② normalizing	748.4	456.5	1440	3.167
QT600-2, (A) ② normalizing	677	521.3	1622	3.08
ZG35 ② normalizing	572.3	366.3	1218	3.51

Note: σ_b is a strength limit; σ_s is an yield limit;
(A)-Bar $\phi = 30$; (B)-Y-type test specimen;
①---The Masing's materials; ②---The cycle-harden material ③-Cyclic softening.

It could see from table 2 where the materials from number 1 to 9 are the steels, their critical values of damage are 0.43~1.42 damage-units in first stage (equivalent to 0.43~1.42mm of the crack sizes); The materials from number 10 to 13 are the nodular cast irons and a cast iron respectively, their critical values of damage are 2.94~3.51 damage-units. In practice, because they get already the critical values of the fracture at the first stage under yield stress, then those materials will occur the failures.

3.3. Strength Calculation on Damage at the Second Stage

When the damage growth gets to the macro damage stage, where it is corresponding to the curve BA_1A_2 in figure 2. In this stage, for the behaviour of some materials corresponding curve BA_1 between the abscissa axis OII and the O3III, they form the critical values D_{1c} of macro damage are usually later than those brittle materials, their life are also longer, so the transition points between two stages in damage process are on the abscissa axis O3III that just is as the boundary of them. In this case that strength criterion (11-12) on damage in first stage can still be sutied for calculations in the second stage.

By the way, when a structure is calculating in design, if the work sress greater than the yield stress, then the damage value D_1 in the equation (11) can also be calculated by following formula

$$D_1 = \frac{K^2}{\sigma^2 \times \pi}, (damage-units) \tag{15}$$

When the damage growth over the abscissa axis O3III in figure 2, the strength criterion of damage at later time in the second staege should be as following form

$$K'_2 = \sigma \cdot \sqrt{\pi D} \leq [K] = K_{2c}/n, (MPa\sqrt{1000 damage-units}) \tag{16}$$

$$K'_{2c} = \sigma_f \cdot \sqrt{\pi D_{2c}}, (MPa\sqrt{1000 damage-unit-number}) \tag{17}$$

Where the K'_2 is defined as the stress factor of damage in the second, the K'_{2c} is a critical factor of damage that it is equivalent to the critical stress intensity factor K_{1c} in fracture mechanics. The σ_f is a fracture stress, the D_{2c} is a critical value of momentary fracture where it is at the crossing point A2 on the abscissa axis O4-Ⅳ and the straight line 1 (A_1A_2) in fig. 2.

It should yet explain because the K'_{2c} is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by mens of the fracture stress σ_f (table 2). So that the critical value of damage D_{2c} under corresponding to the true stress σ_f should also be the only data. In theory, it must be there is as following functional relationship,

$$D_{2c} = \frac{K^2}{\sigma_f^2 \times \pi}, (damage-units) \tag{18}$$

In the table 3 to include the critical values D_{2c} of some materials.

Table 3. *The critical values* D_{2c} *of momentary fracture.*

Materials [14-15]	σ_b, MPa	σ_s, MPa	K, MPa	σ_f, MPa	D_{2c} damage – units
Hot rolled sheet 1005-1009	345	262	531	848	0.125
Steel: 1005-1009 Cold-draw sheet	414	400	524	841	0.124
RQC-100, Hot rolled sheet	931	883	1172	1330	0.247
4340, quench and tempering	1241	1172	1579	1655	0.280
Aluminum 2024-T3	469	379	455	558	0.212
30CrMnSiA, ① Hardening and tempering	1177	1104.5	1475.76	1795.1	0.215
LC4CS, ① Heat treatment-CS	613.9	570.8	775.05	710.62	0.379
40Cr ③	940	805	1592	1305	0.474
60Si2Mn, quench, medium-temperature tempering ③	1504.8	1369	1721	2172.4	0.20
QT800-2, ② normalizing	913	584.3	1777	946.8	1.121
QT600-2, (B), ② normalizing	748.4	456.5	1440	856.5	0.90
QT600-2, (A) ② normalizing	677	521.3	1622	888.8	1.06
ZG35 ② normalizing	572.3	366.3	1218	809.4	0.721

Note: σ_b is a strength limit; σ_s is an yield limit;

(A)-Bar $\phi = 30$; (B)-Y-type test specimen;

①---The Masing's materials; ②---The cycle-harden material ③-Cyclic softening.

4. Calculating Example

A test specimen made of nodular cast iron, its strength limit $\sigma_b = 913MPa$, yield limit $\sigma_s = 584.3MPa$, its material constant $b_1 = -0.083$, the strength coefficient $K = 1777MPa$, fracture stress $\sigma_f = 946.8MPa$. To suppose the working stress $\sigma_{\max} = 550MPa$, the y(a/b)=1 when it is calculated in a design for the material, to try to calculate respectively following data:

(1) Calculate the damage value D, the threseld value D_{th} of damage, the critical value D_{1c} and the D_{2c} of damage for the material, respectivaly;

(2) Calculate the the H'_1, threshold factor H'_{th}, H'_{1c}, critical factors K'_{1c} and K'_{2c} of damage, respectivaly;

(3) To use the assessment method of the damage factor to do an assessment for it.

The processes and steps of calculations are as below.

(1) Calculate each critical value D_{th}, D_{1c} and D_{2c} of damage, and to do an assessment for the material

According to the formulas (8) and (1) their damage and threshold values in the first stage are calculated respectively as below,

Here $m_1 = -1/b_1 = -1/-0.083 = 12.048$

1). $D_1 = \left(\dfrac{\sigma}{\sigma_y}\right)^{m_1} = \left(\dfrac{550}{584.3}\right)^{12.048} = 0.4825(damage-units)$

$$D_{th} = \left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+(-0.083)}} = 0.253(damage-units);$$

$$D_1 = 0.483 > D_{th} = 0.253(damage-unite)$$

So the damage in the material is necessarily to grow.

According to the formula (13) its critical value of damage at the first stage is

2). According to the formula (14), its value of macro damage in the second stage is

$$D_1 = \frac{K^2}{\sigma^2 \times \pi} = \frac{1777^2}{550^2 \times \pi} = 3.323(damage-units);$$

3). By the formula (13), its critical value of macro damage is as below

$$D_{1c} = \frac{K^2}{\sigma_s^2 \times \pi} = \frac{1777^2}{584.3^2 \times \pi} = 2.944(damage-units);$$

So that $D_1 = 3.323 > D_{1c} = 2.944(damage-units)$

4). According to the formula (17), its critical value of momentary fracture is

$$D_{2c} = \frac{K^2}{\sigma_f^2 \times \pi} = \frac{1777^2}{946.8^2 \times \pi} = 1.1216(damage-units).$$

(2) Calculate the stress intensity factor H'_1 and the critical value H'_{1c} of damage in the first stage, respectivaly;

Its stress factor of damage in the first stage is

According to the formula (6) and (7), the factor H'_1 is

$$H'_1=\sigma \cdot D_1^{1/m_1}=550\times\sqrt[12.048]{4.83\times10^{-4}}=291.8, (MPa\cdot1000damage-units^{1/m_1}).$$

The critical factor of damage in the first stage is as below,

$$H'_{1c}=\sigma_s\cdot D_{1c}^{1/m_1}=584.3\times\sqrt[12.048]{2.944\times10^{-3}}=360.73, (MPa\cdot1000damage-units^{1/m_1}).$$

Its permited value should be,

$$[H_1']=H'_{1c}/n=360.73/n=120$$
$$(MPa\cdot1000damage-units^{1/m_1}).$$

So that $H'_1=291.8>[H'_1]=120$ $(MPa\cdot1000damage-units^{1/m_1})$,

Therefore, the calculating result by the criterion in the first stage, that is not safe.

(3) Calculate the stress intensity factor K and the critical value K'_{2c} by macro damage, respectivaly;

1). According to the formulas (9) ~ (12), the factor K'_1, the threshold value corresponding the yield stress σ_s and the critical one of damage in second stage are respectively as follow,

a) For the stress factors K_1' of the damage, here there are tow of calculating data, that are as follow,

$$K'_1=y(a/b)\sigma\times\sqrt{\pi D}=1\times550\times\sqrt{\pi4.82\times10^{-4}}=21.4(MPa\sqrt{1000\text{-}damage-units})$$

$$K'_1=y(a/b)\sigma\times\sqrt{\pi D}=1\times550\times\sqrt{\pi3.23\times10^{-4}}=17.52(MPa\sqrt{1000\text{-}damage-units}).$$

In two of calculating data, it should take larger one.

b). The threshold values K_{th-s}' of the damage corresponding to the yield stress is as below

$$K'_{th-s}=D_s\times\sqrt{\pi D_{th}}=584.3\times\sqrt{\pi2.53\times10^{-4}}=16.47(MPa\cdot\sqrt{1000-damage-units}),$$

So $K'=17.52 and 21.4>K'_{th-y}=16.47(MPa\cdot\sqrt{1000-damage-units})$.

On the other hand, the critical factor on macro damage is

c). The critical factor of damage in this stage is

$$K'_{1c}=\sigma_s\times\sqrt{\pi D_{1c}}=584.3\times\sqrt{\pi2.994\times10^{-3}}=56.64(MPa\cdot\sqrt{1000-damage-units}),$$

Its permited value should be,

$$[K']=K'_{1c}/n=56.64/3=18.9(MPa\cdot\sqrt{1000-damage-units}),$$

So that, the $K'=21.4$ mentioned above, it is already greater than the permited value [K'],

$$K'_1=21.4>[K'_1]=18.9(MPa\cdot\sqrt{1000-damage-units}),$$

Therefore, the result calculated by the criterion on macro-damage, that is still not safe.

2) The critical factor in second stage is as below:

According to the strength criterion (15-16), the critical value of the momentary fracture is

$$K'_{2c}=\sigma_f\times\sqrt{\pi D_{2c}}=946.8\times\sqrt{\pi1.122\times10^{-3}}=56.21(MPa\cdot\sqrt{1000-damage-units}),$$

3) Its permissible value of damage factor is

$$[K]=K'_{1c}/n=56.64/3=18.89(MPa\cdot\sqrt{1000-damage-units}),$$

The result is also as bellowing case,`

$$K'=21.4>[K]=18.89(MPa\sqrt{1000-damage-units})$$

So that the damage value for the material is not in range of the permissible value for a design.

It can see from the above calculations, for the critical factors of damage, the $K'_{1c}=K'_{2c}$, because corresponding to end point of the K'_{1c}-value just is the starting point of the K'_{2c}-value where they are at same point A2 on abscissa axis O4 IV; but for their critical values of damages, $D_{2c}\neq D_{1c}$. So when to take the value for the $[K]$ it must only be caculated by the K'_{1c}/n or K'_{2c}/n with the safe factor n .

5. Conclusions

(1) The new threshold value D_{th} of damage which it can show own inherent property, that is depended on the sole material constant b_1, is a calculable one.
(2) For some materials of the brittle and happened strain hardening under monotonous loading, their critical damage values D_{1c} in the first stage could be calculated with corresponded the yield stresses σ_y ; their critical values D_{2c} of the momentary fracture in the secodt stage could also be calculated with related the fracture stresses σ_f .
(3) The critical D_{1c} and D_{2c} of damage are inherent constants shown the materials'characters; so the critical stress factors K_{1c} and K_{2c} based on D_{1c} and D_{2c} are also sole values, and are all calculable ones; Their computing models can be used to calculate both for the safe assessment to materials preexistted a flaw and for the predicting damage in design process; But the error of calculating data is larger for the shown strain softening's ones.
(4) Because the yield stresses σ_s is the constant of uniquenesses for a material, the critical values of the damage D_{1c} and the factor K'_{1c} related the σ_s should also be considered as the only ones, and can also be applied as an important parameters showed its property; And for some cast iron, brittleness and low toughness materials, which are all the more so.
(5) The factor-value at end point of the K'_{1c} is the very one at starting point of the K'_{2c}; but for their critical values of damages, $D_{2c} \neq D_{1c}$. So for some materials of the brittle and happened strain harding, if to take the value for the $[K]$ it must only be caculated by the K'_{1c}/n or K'_{2c}/n with the safe factor n .
(6) In those computing models are proposed in the paper, if readers want to apply in engineering calculations, it must yet be verified by combined experiments, and it have to consider the influences for the shape and the size to a crack and a structure.

Acknowledgments

Author thanks sincerity the Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering gives to support and provides research funds.

References

[1] Yan Gui Yu. Damage Growth Rate Calculations Realized in Whole Process with Two Kinks of Methods. American Journal of Science and Technology. Vol. 2, No. 4, 2015, pp. 146-164.

[2] Yangui Yu. The Life Predicting Calculations in Whole Process Realized from Micro to Macro Damage with Conventional Materials Constants. American Journal of Science and Technology. Vol. 1, No. 5, 2014, pp. 310-328.

[3] Yangui Yu. Life Predictions Based on Calculable Materials Constants from Micro to Macro Fatigue Damage Processes. American Journal of Materials Research. Vol. 1, No. 4, 2014, pp. 59-73.

[4] Yangui Yu. The Life Predicting Calculations Based on Conventional Material Constants from Short Crack to Long Crack Growth Process. International Journal of Materials Science and Applications. Vol. 4, No. 3, 2015, pp. 173-188. doi: 10.11648/j.ijmsa.20150403.15.

[5] Yangui Yu. The Life Predicting Calculations in Whole Process Realized with Two kinks of Methods by means of Conventional Materials Constants under Low Cycle Fatigue Loading. Journal of Multidisciplinary Engineering Science and Technology (JMEST) ISSN: 3159-0040 Vol. 1 Issue 5, December – 2014.

[6] Yangui Yu. Calculations for Crack Growth Rate in Whole Process Realized with Two Kinks of Methods for Elastic-Plastic Materials Contained Crack. Journal of Materials Sciences and Applications. Vol. 1, No. 3, 2015, pp. 100-113.

[7] Yangui Yu. The Life Predicting Calculations in Whole Process Realized by Calculable Materials Constants from short Crack to Long Crack Growth Process. International Journal of Materials Science and Applications. Vol. 4, No. 2, 2015, pp. 83-95. doi: 10.11648/j.ijmsa.20150402.13.

[8] Yangui Yu. The Calculations of Crack Propagation Rate in Whole Process Realized with Conventional Material Constants. Engineering and Technology. Vol. 2, No. 3, 2015, pp. 146-158.

[9] Yangui Yu. Calculations for Crack Growth Rate in Whole Process Realized with the Single Stress-Strain-Parameter Method for Elastic-Plastic Materials Contained Crack. <AASCIT>Journal of Materials Sciences and Applications. Vol. 1, No. 3, 2015, pp. 98-106.

[10] Yu Yangui, Sun Yiming, MaYanghui and XuFeng. The Computing of intersecting relations for its Strength Problem on Damage and Fracture to Materials with short and long crack, In: *International Scholarly Research Network ISRN. Mechanical Engineering,* Volume, Article ID 876396. http://www.hindawi.com/isrn/me/(2011).

[11] David Broek Elementary Engineering Fracture Mechanics, Third revised edition Martinus Nijhoff Publishers, 1982, 6~14.

[12] Michael F. Ashby and David R. H. Jones, Engineering Materials, An introduction to their propertied and applications, Pergamon Press, 1980, P. 145-147.

[13] B. P. Kogaev, H. A. Maxymov, A. P. Gycenkov. Calculations on Strength and Durability for the Structure and Mechanical parts, Handbook, Machinery Industry Press, Moscow, 1985 70~75. (in Russian) (В. П. Когаев, Н. А. Махутов, А. П. Гусенков. Расчеты Деталей Машин и Конструкций на Прочность и долговечность, Справочник, Машиностроение 1985, Москва, 70-75).

[14] China Machine Press, Mechanical design handbook, New Edition, Volume 5.31-124-135; 31-57-60; 31-134-136.(in Chinese).

[15] Shaobian Zhao and Zhongbao Wang, Anti-Fatigue Design---Methods & Data, China Machine Press, P. 90-109, 469-489.

[16] Yangui Yu. Multi-Targets Calculations Realized for Components Produced Cracks with Conventional Material Constants under Complex Stress States. <AASCIT>Engineering and Technology. Vol. 3, No. 1, 2016, pp. 30-46.

[17] Yangui Yu. The Calculations of Evolving Rates Realized with Two of Type Variables in Whole Process for Elastic-Plastic Materials Behaviors under Unsymmetrical Cycle. Mechanical Engineering Research. Canadian Center of Science and Education 2012; 2. (2): 77-87; ISSN 1927-0607 (print) E-ISSN 1927-0615 (Online).

[18] Yangui Yu, The life predictions in whole process realized with different variables and conventional materials constants for elastic-plastic materials behaviors under unsymmetrical cycle loading, Journal of Mechanics Engineering and Automation. 5 (2015) 241-250. doi: 10.17265/2159-5275/2015.04.006.

[19] Yangui Yu. The Predicting Calculations for Lifetime in Whole Process Realized with Two Kinks of Methods for Elastic-Plastic Materials Contained Crack. <AASCIT>Journal of Materials Sciences and Applications. Vol. 1, No. 2, 2015, pp. 15-32.

[20] S. V. Doronin, et al., Ed. RAN U. E. Soken, Models on the fracture and the strength on technology systems for carry structures, (Novosirsk Science, 2005), PP. 160-165.(in Russian).

[21] U. Zerbst, S. Beretta, G. Kohler, A. Lawton, M. Vormwald, H. Th. Beier, C. Klinger, I. C erny´, J. Rudlin, T. Heckel a, D. Klingbeil, Safe life and damage tolerance aspects of railway axles – A review. Engineering Fracture Mechanics. 98, 214–271 (2013).