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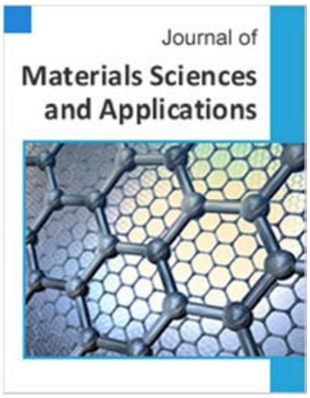

The Duration of Production with Two Related Materials

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Abstract
In the manufacturing process, firms should estimate the duration of materials demand to make sure the productions scheduling and output. Most products manufacturing are consist from more than one material and the relations of each material may be substitutes or complements. The downstream manufactures may order materials from different sources of upstream. Thus to predict the duration of more than one relative material demand is an important topic. This paper bases on Farlie-Gumbel-Morgenstern family model to predict the duration of two relative materials (substitutes or complements). In the proposed model, the parameter of correlation can reflect the material relations of complementary or substitute. Thus, this joint model can describe not only complementary but also substitute context. For more practical, the empirical data are used to estimate the parameters of the proposed model and to test the model validation. The results show the correlations of these two materials are negative. It means they are substitutes relations. And there is good fitness between empirical data and simulation data in the proposed model. Finally the discussion and conclusions are made for practical application.

1. Introduction

The materials or goods can be separated into substitutes and complements [1]. Differences of these two classifications are according to their relations [2]-[3]. The substitute materials (goods) are two materials which are competitive relations. The increase sales of one material can cause the decrease sales of another. The complements materials (goods) refers to the inter-materials (goods) consumption is combined together and one's demand will affect another's. If the price of a materials (goods) increases, it will lead to a decline in demand for another materials (goods) [4] [5]. In the manufacturing process, firms should estimate the duration of materials demand to make sure the productions scheduling and output. Most products manufacturing are consist from more than one material and the relations of each material may be substitutes or complements. The downstream manufactures may order materials from different sources of upstream [6]. Thus to predict the duration of more than one relative material demand is an important topic.

Huang [7] explore the duration between two different kinds of complementary materials and consider the bivariate exponential distribution to model two related manufacturing durations of these complement materials. She provides both MLE (maximum likelihood estimate) and moment methods to estimate the parameters of proposed model. But there are no empirical data used to demonstrate the results of parameter estimations and to make model validation.

This paper extends Huang's research [7], author considers two materials are not dependent but doesn't limit their relations to be complementary products. Base on

Farlie-Gumbel-Morgenstern family model [8] [10] in which the parameter of correlation can reflect the complementary orsubstitute relations. Thus, our joint model can describe not only complementary but also substitute context.

The Farlie-Gumbel-Morgenstern family model is used in the research [11] to predict the multi-source materials demand in the manufacturing process. The total materials demand quantities are composed by "ordering quantity of past" and "recency of ordering time". And the variable "ordering quantity of past" is assume from two different upstream sources which is the Farlie-Gumbel-Morgenstern family model and follow two different normal distributions. Thus, its joint density can be calculated.

This paper use Farlie-Gumbel-Morgenstern family model to portray the two relative (substitutes or complements) materials but we focus on the correlation of material not material from multi-sources.

Beside Huang [7], she considers the duration of two materials are both exponential. But the different kinds of materials may be from different categories even different forms. In this research, the different distributions (exponential and gamma distributions) are considered to demonstrate the various durations of materials. This paper also uses empirical data to estimate parameters and make model validation.

This paper is organized as follows: first the probability density function and cumulative distribution function of two materials will be demonstrated. Their durations of production are respectively exponential and gamma distributions. Secondly, the joint model is derived. Base on the Farlie-Gumbel-Morgenstern family model, the joint probability density function (p.d.f.) and cumulative distribution function (c.d.f.) are calculated. Thirdly, the empirical data will be used to estimate the parameters. The data will be divided into two parts. One is for parameter estimation. Another is for model validation. MLE methods are conduct to estimate the parameters. Then, the results of parameter estimation to simulate data according the proposed model (the joint p.d.f. and c.d.f.) are used. The comparison of half of empirical data and simulation data are made. The root-mean-square deviation (RMSD) are used to calculate the fitness between these two kinds of data. The model validation is shown in this section. Finally, the discussions and conclusions are made.

2. The Model

2.1. The Probability Density Function of Two Materials

To consider two materials A and B, their durations of production are respectively random variable t_a and t_b. The random variable t_a follows exponential distribution with the parameter λ (see equation (1)). The random variable t_b follows gamma distribution with the parametersα and β (see equation (2)).

$$f_A(t_a) = \lambda e^{-\lambda t_a} \tag{1}$$

$$f_B(t_b) = \frac{\beta}{\Gamma(\alpha)} t_b^{\alpha-1} e^{-\beta t_b} \tag{2}$$

2.2. The Cumulative Distribution Function of Two Materials

According equation (1) and (2), we consider the c.d.f. of the durations of production$t_1$and t_2 are demonstrated respectively by equation (3) and (4)

$$F_A(t_a) = 1 - \lambda e^{-\lambda t_a} \tag{3}$$

$$F_B(t_b) = 1 - \frac{1}{\Gamma(\alpha)} \Gamma(\alpha, \beta t_b) \tag{4}$$

2.3. The Joint Model of Two Materials

Base on Farlie-Gumbel-Morgenstern Distributions [12]-[13], the joint distribution of materials A and Bare calculated. The joint p.d.f. of the durations of production t_a and t_b is demonstratedas (5).

$$f_{A,B}(t_a, t_b) = \frac{\lambda\beta}{\Gamma(\alpha)} t_b^{\alpha-1} e^{-(\lambda t_a + \beta t_b)} \left[1 - \gamma\left(1 + 2\lambda e^{-\lambda t_a}\right)\left(1 + \frac{2}{\Gamma(\alpha)} \Gamma(\alpha, \beta t_b)\right) \right] \tag{5}$$

The joint c.d.f. of the durations of production t_a and t_b is demonstrated as (6).

$$F_{A,B}(t_a, t_b) = \left(1 - \lambda e^{-\lambda ta}\right)\left(1 - \frac{1}{\Gamma(\alpha)} \Gamma(\alpha, \beta t_b)\right)\left[1 + \frac{\gamma\lambda e^{-\lambda t_a}}{\Gamma(\alpha)} \Gamma(\alpha, \beta t_b)\right] \tag{6}$$

γ is the parameter of correlation between A and B. The range of γ is -1 to 1.

3. The Analysis Method

The empirical data are conduct to estimate the parameters of the joint model. 20153 pair samples are from a computer engineering manufacturing firm. They are two kinds of different production materials. The information of data includes the duration of ordering time from two upstream manufactures such as material A is 19 days and material B is 23 days. We separate our data into two parts. One is for parameters estimation and another is for model validation. Our analysis process is as following:

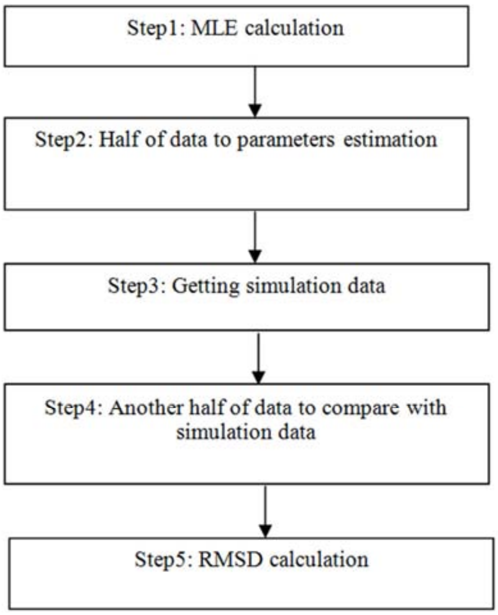

Figure 1. *The analysis process.*

According to the figure 1, the analysis process includes five steps.

Step1: MLE method is used to estimate the parameters of the joint model. The author calculates the likelihood function and differentiate the likelihood function regarding four parameters (λ, α, β, γ)and set them equal to zero.

Step2: There are 10076 pair samples (about half of the total samples size) used to estimate these four parameters. The results are demonstrated in "The Results of Estimation" section.

Step3: The results of parameters are used to make simulation data.10077 data are simulated in order to make comparison with empirical data.

Step4: Another half of empirical data (10077 data) is used to compare with simulation data.

Step5: The RMSD (root-mean-square deviation) are calculated to make the model validation.

The next section follows these analysis processes.

Parameters Estimation

MLE is used to estimate the parameters of model. Let Ω is the parameter vector, there are *n* pair samples (t_{a1}, t_{b1}), (t_{a2}, t_{b2}),…(t_{an}, t_{b2}), the likelihood function is:

$$\begin{aligned} & L(\Omega|(\mathrm{t_{a1}},\mathrm{t_{b1}}),(\mathrm{t_{a2}},\mathrm{t_{b2}}),...,(\mathrm{t_{an}},\mathrm{t_{bn}})) \\ & = \prod_{i=1}^{n} f_{A,B}(\mathrm{t_{ai}},\mathrm{t_{bi}};\Omega) \\ & = \left[\frac{\lambda\beta}{\Gamma(\alpha)}\right]^n e^{-\sum_{i=1}^{n}(\lambda t_{ai}+\beta t_{bi})} \prod_{i=1}^{n} t_{b_i}{}^{\alpha-1}\left[1-\gamma\left(1+2\lambda e^{-\lambda t_{ai}}\right)\left(1+\frac{2}{\Gamma(\alpha)}\Gamma\left(\alpha,\beta t_{bi}\right)\right)\right] \end{aligned} \tag{7}$$

We differentiate $L(x|(\mathrm{t_{a1}},\mathrm{t_{b1}}),(\mathrm{t_{a2}},\mathrm{t_{b2}}),...,(\mathrm{t_{an}},\mathrm{t_{bn}}))$ respectively regarding λ, α, β, γ and set them equal to zero.

That is

$$\begin{cases} \frac{\partial\lambda}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) = 0 \\ \frac{\partial\alpha}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) = 0 \\ \frac{\partial\beta}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) = 0 \\ \frac{\partial\gamma}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) = 0 \end{cases} \tag{8}$$

In which

$$\frac{\partial\lambda}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) = \left[\frac{\beta}{\Gamma(\alpha)}\right]^n \prod_{i=1}^{n} t_{b_i}{}^{\alpha-1}\left[\lambda\cdot\frac{d}{d\lambda}R + R + 2\gamma\lambda^2\cdot\frac{d}{d\lambda}S + 4\gamma\lambda S + \frac{4\lambda^2\Gamma(\alpha,\beta t_{bi})}{\Gamma(\alpha)}\cdot\frac{d}{d\lambda}S + \frac{8\lambda\Gamma(\alpha,\beta t_{bi})}{\Gamma(\alpha)}S\right] \tag{9}$$

where

$$R = e^{-\sum_{i=1}^{n}(\lambda t_{ai}+\beta t_{bi})}$$

$$S = e^{-\sum_{i=1}^{n}(2\lambda t_{ai}+\beta t_{bi})}$$

And

$$\frac{\partial\gamma}{\partial} L(x(\mathrm{t_{an}},\mathrm{t_{bn}})) == \left[\frac{\lambda\beta}{\Gamma(\alpha)}\right]^n e^{-\sum_{i=1}^{n}(\lambda t_{ai}+\beta t_{bi})} \prod_{i=1}^{n} t_{b_i}^{\alpha-1}\left(1+2\lambda e^{-\lambda t_{ai}}\right)\left(1+\frac{2}{\Gamma(\alpha)}\Gamma(\alpha,\beta t_{bi})\right) \tag{10}$$

4. The Results of Estimation

4.1. The Parameters Estimation

According to the analysis process, the results of parameters estimation is in table 1.

Table 1. *The results of parameters estimation.*

Parameters	λ	α	β	γ
	2.32	3.22	2.032	-0.323

It can be found that γ is a negative value which means material A and B are negative correlative. Thus material A and B are substitute materials. When the duration of material A increases, the duration of material B will decrease. Finally, these results are used to simulate 10077data.

4.2. The Model Validation

We calculate root-mean-square deviation (RMSD) [14] to make comparison between empirical data and simulation data.

$$\text{RMSEA=}\sqrt{\frac{\sum_{j=1}^{m}(y_{aj}-\hat{y}_{aj})^2+(y_{bj}-\hat{y}_{bj})^2}{m}} \tag{11}$$

In equation (11), y_{aj} is the j-th empirical data from material A, y_{bj} is the j-th empirical data from material B, $\hat{y}_{aj}$ is the j-th empirical data from material A and $\hat{y}_{bj}$ is the j-th empirical data from material B. There are 10077 samples (half of empirical data), thus m=10077. The results show RMSD=0.817 which is larger than 0.5. Thus the simulation data is closed to empirical data.

5. The Conclusion and Discussion

This research consider two relative material demand durations. The results show joint model this paper proposes has a better fitness with the real data. The model supposes the durations of these two materials are exponential and gamma density. But in the manufacturing industry, the distribution types may be various and depend on the product categories or its property. Thus, other density such as Weibull or Erlang can be used to find the fitness of duration of materials demand. The researchers can also try different composition such as gamma-Weibull or Weibull-exponential to find their joint distribution.

The results of this research show that these two materials are substitute relationship (based on the correlation is negative). It means the duration of one material ordering increases then another material ordering duration will decrease. In the future, other relations such as complements can be test. The other topic such as the price between different correlations (substitute or complements) can be included into the duration question to discuss the balance of manufactory ordering strategy. And this model can also be used to test other kinds of material duration demand.

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Biography

Hui-Hsin Hiang. Was born in Taipei in 1978. She received the Ph. D. in management sciences from Tamkang University, New Taipei City in 2008. Now, she is an associate professor with the Department of Business Administration, Aletheia University, New Taipei City. She has published papers on International Journal of Information and Management Sciences (IJIMS), Information and Knowledge Management, and Advanced Materials Research. Her research interests include marketing model, consumer behavior, and applied statistics.

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