The Effect of Non-Linear Pivot Stiffness on Tilting Pad Bearing Dynamic Force Coefficients

Ali Abasabadarab, Seyed Alireza Hosseini, Mohammad Abasabadarab, Reza Allahyari

Mechanical Engineering, No19, East Gharmsar, South Shiraz St, Vanak Sq, Tehran

Email address
aabasabadi@pted-co.com (A. Abasabadarab)

Abstract
Journal bearings are used on a variety of different rotating and reciprocating machines to support shafts that are rotating inside a bearing. In contrast to anti-friction type bearings where the primary mechanism is rolling action of contacting components, journal bearing so pirate by supporting the load of the shaft on a pressurize do film. Approach of this paper is to describe static and dynamic loads and dynamic characterizes that effect on fluid film thickness and enhance tilting pad bearing model by including nonlinear pivot flexibility for rocker, spherical, and flexure type pivots.

1. Introduction

Figure 1 show a tilting pad journal bearing comprised of four pads. Each pad tilts about its pivot making a hydrodynamic film that generates a pressure reacting to the static load applied on the spinning journal. This type of bearing is typically installed to carry a static load on a pad (LOP) or a static load in between pads (LBP). Commercial tilting pad bearings have various pivot designs such as rocker pivots (line contact), spherical pivots (point contact) and flexure supported pivots.

Figure 1. Schematic views of a four pad tilting pad bearing [1].

Accurate prediction of tilting pad bearing forces and force coefficients is essential to design and predict the dynamic performance of rotor-bearing systems. Parameters affecting tilting pad bearing force coefficients include elastic deformation of the bearing pads and pivots, thermal effects affecting the lubricant viscosity and film clearance, etc. [2, 3].
Rocker and spherical pivots in tilting pad allow nearly frictionless pad rotation. An ideal rocker TPB, shown in Fig. 2(a), allows the pad to roll without slipping around a cylindrical pivot inside the curvature of the bearing. A spherical TPB, seen in Fig. 2(b), allows the pad to rotate about a spherical pivot fixed to the inside curvature of the bearing [4].

The flexure pivot TBP, depicted in Fig. 3, is a modern advancement in TBP designs. It is a two-piece configuration that uses electron discharge machining to manufacture the pad, connected by a flexure thin web to the bearing housing. This design eliminates tolerance stack ups that usually occur during manufacturing and assembly, pivot wear, and unloaded pad flutter problems which occur in conventional tilting pad bearings [4].

As seen in Fig. 4, pivot flexibility makes the pad to displace along the radial (ξ) and transverse (η) directions. The pad also tilts or rotates with angle (δ) [5, 6].

2. Analysis

Coordinate System and Fluid Film Thickness

Figure 6 shows the geometry and coordinate system for a tilting pad journal bearing. A local coordinate is placed on the bearing surface with the {x} axis in the circumferential direction and the {z} axis in the axial (in plane) direction. Inertial axes {X, Y, and Z} have origin at the bearing center. \( e_x, e_y \) Represent the journal center displacements along the X, Y axes. The position of a tilting-pad is referenced to the angular coordinate \( \theta = \frac{x}{R} \), with \( \Theta_\ell \) as the pad leading edge angle, \( \Theta_t \) as the pad trailing edge angle, and \( \Theta_p \) as the pad pivot point angle. \((\delta^k, \xi^k, \eta^k)\) denote the \( k^{th} \) pad rotation and radial and transverse displacements; \( k = 1, ..., N_{pad} \) [7].

The fluid film thickness in the \( k_{th} \) pad is [8].
\[h = C_p + e_X \cos(\theta) + e_Y \sin(\theta) + \left(\xi^k - \eta^k\right) \cos(\theta - \Theta) + (\eta^k + R\delta^k) \sin(\theta - \Theta)\]

Where \(C_p\) is the pad machined radial clearance, and \(r_p = C_p - C_m\) is the pad preload with \(C_m\) as the bearing assembled clearance. Presently, for simplicity, a bearing pad is assumed rigid[8].

### 3. Perturbation Analysis

Due to prepare an numerical analysis, Consider small amplitude journal and pad motions about static equilibrium position (SEP) and applying an external static load with components \((W_{X0}, W_{Y0})\) to the journal determines its static equilibrium position \((e_{X0}, e_{Y0})\) with fluid static pressure field \(P_{0x}, P_{0y}\), film thickness \(h_0^k\), and corresponding equilibrium \(k^{th}\) pad rotation and deflections \(\delta^k, \xi^k\). Small amplitude journal center motions \((\Delta e_{X0}, \Delta e_{Y0})\) of frequency \(\omega\) about the static equilibrium point. Hence:

\[e_X(t) = e_{X0} + \Delta e_X e^{i\omega t}\]
\[e_Y(t) = e_{Y0} + \Delta e_Y e^{i\omega t}\]

And for \(k = 1, \ldots, N_{pad}\) will have:

\[\delta^k(t) = \delta^k_0 + \Delta \delta^k e^{i\omega t}, \xi^k(t) = \xi^k_0 + \Delta \xi^k e^{i\omega t}, \eta^k(t) = \eta^k_0 + \Delta \eta^k e^{i\omega t}\]

The sum of the pads fluid film reaction forces must balance the external load \((W_{X0}, W_{Y0})\) applied on the journal. The external forces add a static (equilibrium) \((W_{X0}, W_{Y0})\) load to a dynamic part \((\Delta W_X, \Delta W_Y)\) \(e^{i\omega t}\) [8].

\[W_X = W_{X0} + \Delta W_X e^{i\omega t} = \sum_{k=1}^{N_{pad}} F^k_X, \quad W_Y = W_{Y0} + \Delta W_Y e^{i\omega t} = \sum_{k=1}^{N_{pad}} F^k_Y\]

The equations of motion for the \(k^{th}\) pad are:

\[
\begin{bmatrix}
M^k_{pad} \\
\Delta M^k_{pad}
\end{bmatrix}
= \begin{bmatrix}
M^k_p & M^k
\\
F^k_p & F^k
\end{bmatrix}
\]

Where \(M^k_p, F^k_p, F^k_p\) are the pad pivot reaction moment and forces, and \(M^k, F^k, F^k\) are the fluid film forces acting on the \(k^{th}\) pad. The pad mass matrix is:

\[
[M^k_{pad}] = \begin{bmatrix}
m^k b^k & -m^k c^k \\
m^k c^k & m^k
\end{bmatrix}
\]

With \(b\) and \(c\) as the radial and transverse distances from the pad center of mass to the pad pivot, respectively. \(m^k\) and \(I^k_{G}\) are the pad mass and mass moment of inertia about the pad pivot. \(F^k_p = I^k_{G} + m^k (c^2 + b^2)\), where is \(I^k_{G}\) the pad moment of inertia about its center of mass [9].

### 4. Evaluation of Pivot Nonlinear Stiffness

The pivot stiffness is, in general, a nonlinear function of the applied (fluid film) load acting on a pad. Consider, as sketched in Figure 7, a typical radial force \(F_{PZ}\) versus pivot nonlinear radial deflection \(\zeta\) [9].

![Figure 7. Conceptual depiction of stiffness and damping coefficients in a fluid film journal bearing [8]](image)

![Figure 8. Typical force versus pivot (nonlinear) radial deflection [9].](image)
The assumption of small amplitude motions about an equilibrium position allows the pivot reaction radial force to be expressed as:

\[ F_{Pc} = F_{Pc0} + K_{Pc \Delta \zeta_P} \]

Where \( F_{Pc} = f (\zeta_P) \) is the static load on the pivot and \( K_{Pc \Delta \zeta_P} \) is the force due to radial displacement.\[9\]

5. Comparison Between Predicted Static and Dynamic Coefficients and Ref. \[10\]

Figure 9 depicts a schematic view of a five pad, rocker back, TPB tested by Carter and Childs \[10\]. Bearing force coefficients were experimentally obtained for shaft speeds from 4k-12k rpm and static loads from 0-19.5 kN.\[10\].

Mineral oil (Mobil DTE) ISO VG32 lubricated the bearing. The lubricant inlet supply pressure and temperature are 1.55 bar (gauge) and 43° C, respectively. The load applied to the bearing is along the -Y direction. Table 1 details the bearing geometry and fluid properties.

<table>
<thead>
<tr>
<th>Geometry, Operation and condition</th>
<th>Value</th>
<th>Fluid Properties, Ref. [10]</th>
<th>Mobile DTE ISO VG32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor diameter, D</td>
<td>101.587 mm</td>
<td>Viscosity @ 40° C</td>
<td>31 cSt</td>
</tr>
<tr>
<td>Pad axial length, L</td>
<td>60.32 mm</td>
<td>Viscosity @ 100° C</td>
<td>5.5 cSt</td>
</tr>
<tr>
<td>Pad number and arc length</td>
<td>(57.87°)</td>
<td>Density @ 15°C</td>
<td>850 kg/m³</td>
</tr>
<tr>
<td>Pivot offset</td>
<td>60%</td>
<td>Specific heat</td>
<td>1951 J/(kg-K)</td>
</tr>
<tr>
<td>Loaded radial pad clearance, C_p</td>
<td>110.5 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loaded radial bearing clearance, C_b</td>
<td>79.2 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pad preload, ( r_p = 1 - \frac{C_o}{C_p} )</td>
<td>0.283</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pad mass, ( m_p )</td>
<td>1.0375 kg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pad mass moment of inertia (at pivot), ( I_p )</td>
<td>0.00045 kg- m²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For load-between-pad configuration (LBP), Carter and Childs \[10\] present no synchronous force coefficients versus load. Figure 10 shows Ref. \[10\] predicted and experimental direct stiffness’s for a journal speed of 4000rpm. Experimental direct stiffness \( K_{YY} \) is over predicted by ~28% for a static load of 14.8 kN\[10\].

For load-between-pad configuration (LBP), Carter and Childs \[10\] present no synchronous force coefficients versus load. Figure 10 shows Ref. \[10\] predicted and experimental direct stiffness’s for a journal speed of 4000rpm. Experimental direct stiffness \( K_{YY} \) is over predicted by ~28% for a static load of 14.8 kN\[10\].

Figure 11 shows the predicted direct static force coefficients versus static load given a flexible rocker pivot and a rigid pivot for an isothermal flow case. The direct static stiffness’s decrease for a flexible pivot, the difference amounting to a large percentage, ~33% \[10\].

Figure 12 shows the predicted bearing static eccentricity versus applied load when considering both a rigid pivot and a flexible pivot.
6. Conclusion

This paper introduces a general framework to identify linear and nonlinear stiffness and damping coefficients on journal bearings. The article describe static and dynamic loads and dynamic characterizes that effect on fluid film thickness and enhance tilting pad bearing model by including nonlinear pivot flexibility for rocker, spherical, and flexure type pivots. Result show that an improvement in bearing force coefficient predictions is when pivot stiffness is placed in series with the bearing force coefficients derived from a rigid pivot model.

References


