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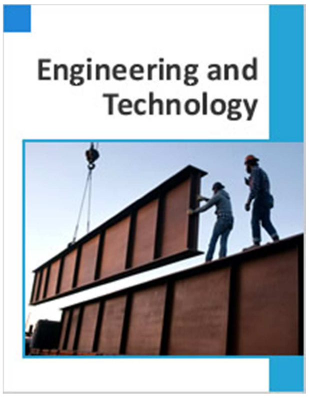

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Analytic-Imitation Model for Determination of the Natural Frequency of Oscillatory Systems and Its Research

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Abstract
This paper presents a specific analytical-imitation model for the devices that determines the natural frequency of oscillatory systems for two devices which was proposed by one of the co-authors, and also traditional devices that are closer in technical aspect to known (prototype) devices. During the researching process of the model additional factors affected on the accuracy of the final result are identified and the analysis of the basic error in determining the natural frequency of the OS, taking into account errors introduced by the composite blocks of the device, are provided. A structural diagram of the model, a flowchart of the algorithm and a tabular data of the research results are given.

1. Introduction

Various methods are used to test various devices of control systems [1-6]. In general, these methods can be subdivided into two categories: analytical and imitative.

There are a large number of books devoted to analytical research. Particularly, in papers [7] the issues of the estimation of productivity and optimization of computing systems are discussed and the issues of the estimation of computing processes on specialized calculators are examined.

Analytical methods allow to achieve good results, particularly, to describe the operation of various devices of control systems, to identify the main factors affecting the effectiveness of the system, to determine the degree of their influence, and also to define the relationship between them.

However, analytical methods have the following disadvantages:

a. The simplicity of analytical models and often artificial adjustment of the existing mathematical apparatus. Such an approach calls into question the results of analytical modeling. The results turn out to be incorrect during the imitation process;
b. The cumbersomeness of calculations for local models of control systems.
c. The complexity of the analytical description of processes, caused by the stochastic nature occurring inside the control systems;
d. The insufficient development of the analytical apparatus, which does not allow to choose the optimal characteristics of control devices in many cases.

Analytical studies lead to the need to use numerous simplifying assumptions about the number of phases of service in the system, the nature of the rods of various applications between functional nodes and their service period.

The consequence of such simplifications is a significant difference in the behavior of the mathematical model compared with the functioning of its technical prototype.

2. Method

In connection with the above mentioned disadvantages of analytical models, the analytical-imitation model becomes the main working device for analyzing. It allows better taking into account additional factors caused by the nonlinearity of the transformation of the individual nodes of the device. The relationship between the parameters of the system and the degree of the influence of the individual parameters of the composite nodes on the final result are especially important.

A specialized imitation model was constructed without the standard existing models to study the devices [8, 9], in order to estimate the basic error in determining the basic informative parameter of the oscillatory systems.

The general structure of the imitation model for determining the natural frequency of oscillatory systems and the functional algorithm

The general structural scheme of the simulation model for determining the natural frequency of oscillatory systems is shown in Figure 1. It contains the following main blocks: 1 - block of linear function formation, 2 - sampling-storage block, 3 – block for converting an analogue value into a harmonic frequency signal, 4 - object - oscillatory system, 5 - detection block, 6 - block for determination of amplitude value, 7 - comparison block, 8 - block of frequency determination.

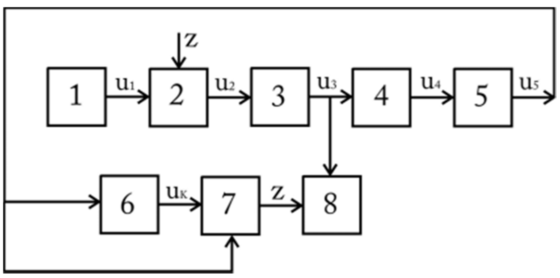

Figure 1. *Structural scheme.*

Block 1 is intended to form a linear function. The increasing of the linear function up to the value Um occurs in the first half-period (0-T/ 2), and decreasing of the linear function from U_m to 0 in the second half-period (T/2-T), where T is the repetition period of the linear function $U_i(t)$.

Block 2 is intended for temporary storage of the current value of the input function $U_i(t)$ that varies linearly with time. It comes from the output of the Block 1. The current value of the input signal is stored according to the control signal Z when Z = 0; The linearly-varing function from the Block 1 is transmitted to the output without changing the signal parameters, and the signal at the output is similar to the signal at the input $U_z(t)$ and when Z= 1, the current value of the input signal Z=1 is determined and stored during the action of the signal. So, at the output the current value of the signal coming from Block 1 is fixed.

Block 3 is designed to obtain a harmonic signal $U_3(t)$ (with a constant amplitude) at the output. Its frequency is proportional to the input signal $f = f_H + \Delta f\, U_z(t) / Um$, which comes from Block 2.

The output frequency signal is transmitted to the object-oscillatory system 4 and to the frequency determination block 8.

Block 4 - the object - oscillatory system modulates the input harmonic signal according to the amplitude and at the output the amplitude-frequency modulated function U_4 (t) is formed and it is transmitted to the Block 5.

Block 5 – the detection allocates a DC component and forms a low-frequency function U_5 (t) at the output, the shape of which is determined by the amplitude-frequency characteristic of the oscillatory system (Block-4).

Block 6 is designed to detect and fix the extreme value of the input signal U_k.

Block 7 is designed to compare the signal U_k from the output of the Block 6 to the signal U_5 (t). The output signal z of the Block 7 is determined by the condition:

$$z = \begin{cases} 0, if\ U_k \le U_5(t) \\ 1, if\ U_k > U_5(t) \end{cases} \quad (1)$$

Block 8 is designed to calculate the half sums of two frequencies, which are determined by the method embedded in the model.

The algorithm of the imitation model, which imitates the functioning of the device for determining the natural frequency of the oscillatory system, is based on the use of the sampling at the time of the output functions, which are discussed above. A two-level quantization step is applied and the Lagrange interpolation formulas are used as the real output characteristics of the individual blocks of the model. The algorithm of the imitation model is implemented in the algorithmic language C^{++}. This program, called REZON, consists of two parts. The first part defines the parameters in the first half-period (from 0 to T / 2) with proportional increasing of the initial function U_i (t) (Block 1) from 0 to value Um. In this part, the first frequency f1, which is stored in memory, is determined.

The second part defines the parameters in the second half-period (from T/2 to T) with proportional decreasing of the initial function $U_1(t)$. In this case $U_i(t)$ is changed from U_m to 0.

The flowchart of the algorithm is shown in Figure 2.

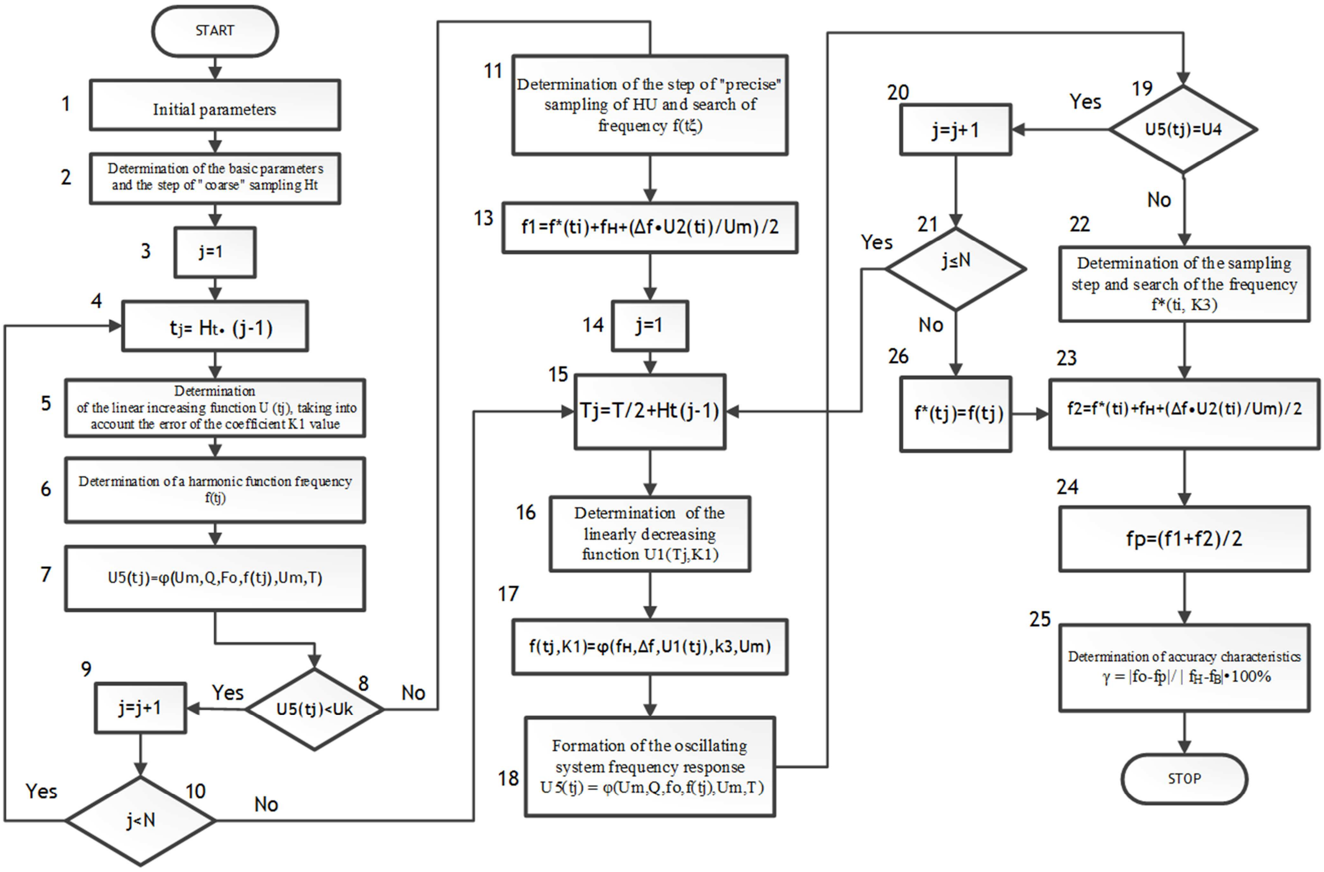

Figure 2. *Flowchart of the algorithm.*

Block 1 specifies the initial data defining the range of the lower and upper frequency values - f_H: f_B, the amplitude of the frequency response of the oscillatory systems - Um, the coefficients K1, K2, K3, K4 that determine the instrumental errors of the corresponding blocks, the number of sampling points in the entire frequency range - N, the value of the period - T, the linearly-varying initial function U_1 (t).

Block 2 determines the specific values of the main informative parameters of the oscillatory system, such as the Q factor and the natural frequency f_o, and also the step of "coarse" sampling is computed.

$$H_t = \frac{T}{2 \cdot N}$$

This allows to carry out a "rough" search for the extremum of the function of the frequency response of the oscillatory systems with a minimal time cost.

Block 4 determines the value of the time discrete tj in the first half-period (from 0 ± T/2).

Block 5 generates the initial linearly-varying function Ui(tj), particularly, this block sets the proportionally increasing function, taking into account the nonlinearity error specified by the coefficient K_1.

Block 6 is designed to convert the input function into the proportional frequency f (tj) of the harmonic function with constant amplitude.

Block 7 forms a function $U_5(t_j)$. Its shape coincides with the frequency response of the oscillatory system:

$$U_5(t_j) = \phi(U_m, Q, f_0, f(t_j), U_{0m}, T) = \frac{U_m}{\sqrt{1 + \frac{2 \cdot Q \cdot (f(t_j) - f_0)}{f_0^2}}} + U_{om} \cdot \sin(0.002 \cdot \pi \cdot f(t_i) \cdot T) \cdot K_4$$

Where U_m is the amplitude of the frequency response of the OS; Q is the sufficiency of OS; $f(t_j)$ is the current frequency defined in Block 6; f_0 is the given natural frequency of the OS; U_{0m} is the amplitude of the variable high-frequency component of the amplitude-frequency characteristic (residue detection); T-period of the linearly-varying function $u_i(t_j)$, K4 - coefficient determining the error of detection.

It should be noted that the first member of the function is defined as a symmetric function to describe the asymmetric function and it also can be replaced by another one

$$\frac{U_m}{\sqrt{1 + Q^2 \cdot (1 - \frac{f_0^2}{f^2})^2}}$$

Block 8 is used as a comparison block. This block compares the AFC function of the OS with its extreme value U_k.

When $U_5(t_j) \geq U_k$, then the transition is done to the Block 4, if $U_5(t_j) < U_k$; then the transition is made to Block 15.

Block 11 is designed to find and determine the frequency $f^*(t_j)$ with the minimum - "exact" step of the sampling at the moment t. When the value of the AFC function of the OS is equal to the extremum value and (thus, when $U_5^*(t_j) = U_k$), where t_i is one of the instants of time in the interval [t_{j-1}; t_j] (Figure 3).

In this interval, the "exact" sampling step is defined as:

$$H_u = \frac{t_{j-1} - t_j}{N} = \frac{H_i}{N}$$

(each step of the time discrete in this interval is numbered by i), where H_t is the "rough" step of the sampling in time, N is the number of sampling points.

To ensure the required accuracy of frequency determination, the "exact" step is selected from the condition:

$$\left|\frac{1}{H_u}\right| \leq \delta_f$$

Where δ_f is the discreteness error of frequency determination.

Block 13 is designed to refine the first frequency, taking into account the error K2 of the sampling-storage block, caused by the error of information storage and its determination. This frequency value is written into memory.

The blocks listed above determine the parameters of the first half of the half cycle, but in the second half the blocks 14 - 22 determine the parameters and they are similar to the above blocks.

In Block 23, the value of the second frequency is adjusted, determined and stored.

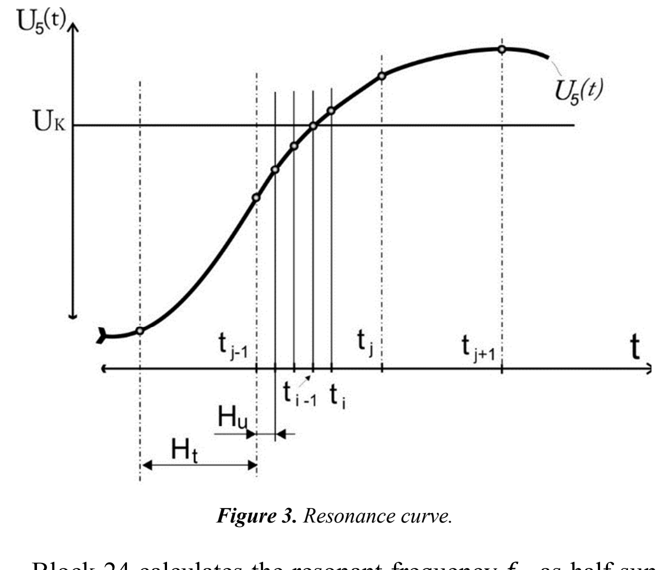

Figure 3. *Resonance curve.*

Block 24 calculates the resonant frequency f_p, as half sum of the frequencies f_1 and f_2.

In Block 25, the main output parameters of the imitation model are determined. Particularly, the resulting relative error γ is determined, the actual Q factor is calculated, as well as the numerical value of the resonant frequency for a given frequency range.

Results of the research of the analytical-imitation model and its analysis

The results of the research of the analytical-imitation model for devices [2, 3] for determining the natural frequency of the oscillatory systems are given in Tables 1 - 5. In these tables, the following notations are introduced;

K1 – is a coefficient determining the error of nonlinearity of the block of the linear function formation and taking the values 0; 0.1; 1 (%).

K2 – is a logical parameter that takes the values 0 or 1. (K2 = 1 means the presence of an error caused by the decrement of the output signal of the sampling-storage block in the storage mode, or the absence of this error at K2 = 0);

K3 - is a coefficient that determines the error of nonlinearity of the modulation characteristic of the block for converting an analogue value into a harmonic frequency signal. It takes the values 0; 0.02; 0.1; 1 (%).

K4 - is a logical parameter that takes the values 0 or 1. When K4 = 1, a detection error and, as a consequence, also a comparison error, which has a random character, exist. For this reason, its worst case was taken into account - the maximum error of the detection. So the interest is the resultant fundamental error in determining the natural frequency for low Q systems, the Q factor of OS was set in the range 3<Q<I5 at the points Q = 3, 5, 8, 10, the overlapping range 10 <w <50 MHz. In the indicated range, the resonance frequencies were set at the points $w_0 = 15, 20, 25, 35, 45$ MHz. The frequency quantization step was 1 kHz., which is quite acceptable in practice and it is possible to determine the required parameters with high accuracy. In addition, the initial data were set, particularly, the amplitude of the amplitude-frequency characteristic of the OS and the amplitude of the linearly-varying function, Um; The threshold of triggering of the comparator is Uk,

The frequency response of the OS was specified as an asymmetric function.

$$U(w) = \frac{U_m}{\sqrt{1 + Q^2(1 - \frac{w_0^2}{w^2})^2}}$$

The model was designed in such a way that it determined the required parameters for both the proposed and the known methods for determining the natural frequency of the oscillatory system, which made it possible to compare the proposed parameters with the known method and to estimate the obtained results more accurately.

2.1. Simulation of the Traditional Method

Table 1 gives the research results of the model for the known method. It is seen from this table that for 3 <Q <10 the relative fundamental error of determining the natural frequency of the OS is 0.347% <γ <4.573%, with Q = 10 all the results are acceptable, but at Q≤5 the error exceeds 1.5%

and increases 4 -13 times compared with Q = 10.

Table 1. Research results of the model for the known method.

№	K1	K2	K3	K4	γ (%)			
	(%)	-	(%)	-	Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	4.541	1.546	0.593	0.378
2	0.1	0	0	0	4.542	1.547	0.594	0.379
3	0	1	0	0	4.513	1.517	0.564	0.348
4	0	0	0.1	0	4.543	1.547	0.593	0.380
5	0	0	0	1	4.573	1.554	0.600	0.380
6	0.1	0	0.1	0	4.544	1.547	0.593	0.378
7	0.1	1	0.1	0	4.513	1.516	0.564	0.349
8	0.1	1	0.1	1	4.543	1.524	0.570	0.355
9	1	0	1	0	4.542	1.547	0.594	0.378
10	0	1	0.02	0	4.513	1.517	0.564	0.349
11	1	1	1	1	4.516	1.527	0.577	0.347

These results are practically not acceptable. They indicate the limited possibilities of the known method with the deterioration of the quality factor (in Table 1 in line №1, when K1 = K3 = 0% and K2 = K4 = 0). In the range 3 <Q <I0, the error is 0.378≤ γ ≤4.541.

This means that even with the ideal output characteristics of main blocks of the model, the asymmetry of the resonance curve of the frequency response of the OS effects on the result of determining the natural frequency, which is spreaded on the rest results of this table. In addition, it should be noted that the known devices in the dynamic mode have additional errors, which are defined in the second chapter of this thesis, causing an even greater deterioration in the accuracy of determining the natural frequency of the oscillatory system.

2.2. Simulation of the Developed Methods

2.2.1. Simulation of the First Method

The research results of the model in tables 2 and 3 correspond to the first and second developed methods and the values of the basic relative error in determining the natural frequency of the OS are given. It can be seen from the tables that in the range 3 < Q < 10, the error caused by the asymmetry of the resonance curve of the AFC OS does not exceed 0.996% for ideal characteristics of the main blocks of the model.

Table 2. Research results of the model for the first developed method [8].

№	K1	K2	K3	K4	γ I (%)			
	(%)	-	(%)	-	Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	0.996	0.354	0.138	0.088
2	0.1	0	0	0	0.997	0.355	0.138	0.089
3	0	1	0	0	0.974	0.3257	0.110	0.060
4	0	0	0.1	0	0.996	0.355	0.139	0.089
5	0	0	0	1	1.023	0.360	0.145	0.090
6	0.1	0	0.1	0	0.998	0.355	0.138	0.088
7	0.1	1	0.1	0	0.967	0.325	0.108	0.059
8	0.1	1	0.1	1	1.000	0.330	0.104	0.046
9	1	0	1	0	0.997	0.355	0.139	0.088
10	0	1	0.02	0	0.967	0.325	0.109	0.058
11	1	1	1	1	0.974	0.332	0.119	0.051

Table 3. Research results of the model for the second developed method [9].

№	K1	K2	K3	K4	γ II(%)			
	(%)	-	(%)	-	Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	0.085	0.032	0.013	0.010
2	0.1	0	0	0	0.085	0.030	0.013	0.008
3	0	1	0	0	0.057	0.036	0.053	0.056
4	0	0	0.1	0	0.083	0.032	0.012	0.009
5	0	0	0	1	0.145	0.035	0.026	0.016
6	0.1	0	0.1	0	0.086	0.031	0.012	0.007
7	0.1	1	0.1	0	0.056	0.035	0.053	0.059
8	0.1	1	0.1	1	0.093	0.025	0.050	0.056
9	1	0	1	0	0.086	0.028	0.012	0.007
10	0	1	0.02	0	0.055	0.036	0.053	0.056
11	1	1	1	1	0.062	0.057	0.015	0.049

2.2.2. Simulation of the Second Method

The second, third and fourth lines indicate that the introduced errors of individual blocks are insignificant, and can be said they have almost no effect on accuracy, except of the results at points a) Q = 5, K1 = 0.1, K2 = K4 = 0, K3 = 0%; and b), Q = 3, K1 = K3 = 0%, K2 = 0, K4 = 1.

Point a) is the error caused by the decrement of the output signal of the sampling-storage block (this is a systematic error, it is constant and can be eliminated by subtraction) and point b) is caused by a strong asymmetry of the resonance curve of the amplitude-frequency characteristic of the OS, as well as a detection error which is not investigated at this stage.

Line №6 and №9 determines the influence on the results of determining the main informative parameter of two factors simultaneously for different values of the coefficients KI and K3 (K2 = K4 = 0).

Line №7 determines the effect on the final result of three factors simultaneously, when KI = K3 = 0,1, K2 = 1.

Lines №8 and №11 determine the influence of all four factors at two values of the coefficients K1 = 0,1, 1%; and K3 = 0.1, 1%.

The worst result was expected at the maximum values of the coefficients, but the obtained results (№11) are not particularly different from the results with the minimum values of the coefficients.

Table 3 shows the research results of the model for the second developed method. The results given in the Table 3 differ from the results of Table 2 approximately by one order of magnitude.

Such results are explained by the fact that the results for the first method slightly depend, and for the second method they almost do not depend on the influencing factors, and the error is mainly caused by the asymmetry of the resonance curve of the frequency response of the OS. This proves the theoretical validity and acceptability of the developed methods and devices [2] [3].

3. Results and Discussion

The results of a comparative evaluation given in Tables 4 and 5 are in exact compliance with the first and second methods compared with the known method. Table 4 shows that the accuracy of the first proposed method, taking into account the errors introduced by individual blocks, exceeds 4-7 times and the high accuracy characteristics are developed in the worst way, when all influencing factors have the maximum values (line №11).

Table 4. *Accuracy characteristics of the first proposed method.*

№	K1	K2	K3	K4	γ/γ_I			
	(%)	-	(%)	-	Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	4.557	4.366	4.280	4.271
2	0.1	0	0	0	4.553	4.358	4.280	4.257
3	0	1	0	0	4.630	4.658	5.129	5.800
4	0	0	0.1	0	4.559	4.357	4.260	4.250
5	0	0	0	1	4.466	4.316	4.118	4.222
6	0.1	0	0.1	0	4.550	4.356	4.288	4.270
7	0.1	1	0.1	0	4.665	4.653	5.211	5.892
8	0.1	1	0.1	1	4.543	4.614	4.482	7.552
9	1	0	1	0	4.554	4.352	4.248	4.276
10	0	1	0,02	0	4.525	4.272	4.035	4.184
11	1	1	1	1	4.633	4.588	4.843	6.767

Table 5. *Accuracy characteristics of the second proposed method.*

№	K1	K2	K3	K4	γ/γ_{II}			
	(%)	-	(%)	-	Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	52.979	48.333	44.511	37.800
2	0.1	0	0	0	52.900	50.861	43.528	47.110
3	0	1	0	0	78.965	41.384	10.580	6.151
4	0	0	0.1	0	54.099	47.503	47.400	42.650
5	0	0	0	1	31.380	43.505	22.497	23.750
6	0.1	0	0.1	0	52.463	49.437	46.296	49.336
7	0.1	1	0.1	0	80.152	43.147	10.529	5.841
8	0.1	1	0.1	1	48.499	48.839	11.320	6.202
9	1	0	1	0	52.707	54.483	47.871	49.276
10	0	1	0.02	0	81.174	41.442	10.584	6.248
11	1	1	1	1	72.832	26.523	37.590	6.964

Table 5 shows that the accuracy characteristics of the second proposed method which exceed the accuracy characteristics of the known method by several orders of magnitude, and this is especially noticeable at low Q values (for Q<5, the accuracy of the second developed method increases in 41-79 times compared to the known method).

The obtained results of the comparative evaluation prove the advantage of the developed (proposed) methods in comparison with those known methods under the conditions of the low Q and the asymmetry of the resonance curve of the ACF of the OS.

4. Conclusion

The research of the analytical-imitation model and the comparative analysis of the obtained results were carried out for the proposed and well-known methods, taking into account the instrumental error introduced by the individual blocks:

1) The advantages of the proposed methods for determining the natural frequency of the OS are proved in comparison with the known methods under conditions of low Q and the asymmetry of the shape of the resonance curve of the AFC of the OS.

2) The main influencing factors and the degree of influence on the accuracy of determining the basic informative parameter of the OS are revealed.

3) The research has shown that for the low Q of the OS (3<Q <15), the resulting basic error in determining the main informative parameter of the first proposed method does not exceed 1% (γ <1%) for the entire range of variation and for all values of the coefficients KI, K2, K3, K4,γ, and for the second method, the indicated error is $\gamma \leq 0{,}095$ when with the known method the resulting error is 1.5% $\leq \gamma \leq$ 5%. In addition, the known method for determining the natural frequency additionally contains a dynamic error caused by the rate of change of the controlled oscillator frequency. This also worsens the accuracy [10]. The proposed methods do not have such a disadvantage.

The results of the comparative evaluation of the proposed two methods for determining the basic informative parameter of the OS allow us to recommend the development of devices based on the second method (higher accuracy) without taking into account the hardware costs, and, if we reduce the last one (cost), it can be based on the first method.

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