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Investigation of Operational Reliability of Electric Locomotives on the Basis of Semi-Markov Models of the Recovery Systems

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Abstract: Based on the general theory of Markov renewal processes and semi-Markov processes with nonexponential distributions of the operating time for failure and recovery, the applicability of stationary reliability indicators to the reliability assessment of electric locomotives with an asynchronous motor is shown. Calculations are made of the availability factor of the average stationary operating time for failure and the average steady-state recovery time for electric locomotives of the Kazakhstan railway.

Keywords: Semi-Markov Process, Semi-Markov Model of the Recovery System, Restoring Devise, Process of Markov Recovery, Embedded Markov Chain, Cartesion Product, BorelSets, Phase-Enlargement Algorithm

1. Introduction

Semi-Markov processes (SMP), introduced independently and almost simultaneously in 1954-1955. P. Levi, V. Smith and L. Takach, represent a direct generalization of Markov chains that have been studied quite well in probability theory. A significant development of the theory of SMP was obtained in [1-4].

The semi-Markovian process, like the Markov chain, is a transition random process. The main thing that distinguishes the semi-Markov process from the Markov chain is the rejection of the requirement of an indication of theexpanential distribution of the residence time in each state. Instead of losing the Markov property of the trajectories of the process, it becomes possible to cover a wider class of systems, the description of which is impossible with the help of Markov chains.

In a number of works [5-13], constructions of semi-Markov processes (SMP) with special discrete-continual phase spaces were proposed. With the help of these constructions, it was possible to consider the classical schemes of recoverable systems under arbitrary (nonexponential) laws of distribution of fail-safe operation times of elements and their recovery.

In the monograph [13], SMP classes modeling the behavior of reconstructed systems with a fully accessible (unbounded) reconstruction and being realized as a superposition of independent semi-Markov processes with a finite number of states were considered, which implies the exponentiality of the initial distributions. In the case of systems with incomplete (limited) recovery, an additional assumption was made about a "quick" recovery (in comparison with the average time of failure-free operation), which allowed using limit theorems for asymptotic phase coarsening.

In the book [14] superpositions of both independent and dependent SMP with a common (in general, infinite number of states) are introduced as a class of SMP that simulate the functioning of recoverable systems with different types of redundancy, bounded and unlimited number of restoring devices (RD) phase space. In this class, one can investigate any of the systems given, for example, in the handbook [15], without additional restrictions on the type of exponentiality of the original distributions.

As in the well-known paper [16], the book [14] notes the limited applicability of SMP with a finite number of states. Attempts to model the behavior of complex technical systems in the classes of such SMPs inevitably lead to the fact that the initial data have to introduce restrictions on the exponential nature of the distributions. An analysis of the real streams of events shows that they are by no means always Poisson.

The privilege of exponential distribution in analyzing the reliability of recoverable systems is explained by the characteristic property of this class of distributions among all absolutely continuous distributions, namely: the absence of aftereffect, which immediately allows us to use the developed apparatus of Markov chains. The rejection of the application of Markov chains, even partial (see, for example, [17]) significantly complicates the corresponding mathematical apparatus. There exist such classes of random processes as semi-Markov processes with discrete phase space, processes with discrete interference of the case, piecewise linear processes (see the bibliography in [9]) and others that are rather successfully used as mathematical models of systems in theory reliability in those cases when the apparatus of Markov chains is insufficient.

One of the existing methods that make it possible to reduce the semi-Markovian model to the Markov model is that the initial nonexponential functions of the distribution of the time of failure-free operation and the recovery time are approximated by the Erlang ones. The resulting difficulties in realizing the increase in the dimension of the phase space are of a computational nature.

In this paper, an approach based on semi-Markov processes and Markov recovery processes is used to analyze the reliability of the operation of electric locomotives with an asynchronous motor, the investigation apparatus of which was developed in [2] and was further developed in [13, 14].

2. Method

2.1. Markov Recovery Processes

The processes of Markov recovery (PMR) appear as a result of analysis of the structure of the trajectories of Markov processes. Under natural conditions of regularity, the trajectories of transition Markov processes behave in such a way that in each state of the phase space the Markov process stays a random time having an exponential distribution with a parameter that depends on the state of the process; Then there is a jump (instantaneous transition) to the new state in accordance with the distribution of the probabilities of the transition of the Markov chain with discrete time, etc. Thus, a transition homogeneous Markov process can be structurally defined by a stochastic kernel that determines the transition probabilities of the Markov chain, which determines the changes in the states of the process, and not a negative function on states that specify the parameters of exponentially distributed random variables that determine the residence times in the process states between neighboring jumps.

A generalization of such a construction of transition processes was the concept of semi-Markov processes (SMP) introduced independently by P. Levi, V. Smith, and L. Takach. The change in the states of these processes is also controlled by a discrete Markov chain, called the embedded Markov chain (EMC), and the residence times in states between neighboring jumps have arbitrary distribution functions that depend on the state of the process. Thus, for the constructive specification of semi-Markov processes, the initial material is the sequence $\{\xi_n, \Theta_n, n \geq 0\}$ whose first component $\{\xi_n, n \geq 0\}$ forms a homogeneous Markov chain that fixes the state of the process at the nth step (after n jump), and the second component $\Theta_n \geq 0$ fixes the time of the system stay in the state ξ_n. The two-component sequence $\{\xi_n, \Theta_n, n \geq 0\}$ is a homogeneous Markov chain whose transition probabilities do not depend on the values of the second component (the residence time in the previous state does not affect the time of stay in this state and the change of this state). Such a Markov chain $\{\xi_n, \Theta_n, n \geq 0\}$ is called the process of Markov recovery (PMR). The pre-existence times Θ_n are conditionally independent random variables for a fixed trajectory of the embedded Markov chain $\{\xi_n, n \geq 0\}$ [13].

To model a system in the form of a PMR, it is necessary that the states of the system possess a semi-Markov property, i.e. the sequence $\{\xi_n, n \geq 0\}$ of changes in the states of the system must constitute a homogeneous Markov chain, and the intervals between the neighboring states $\{\Theta_n, n \geq 0\}$ should be conditionally independent on the Markov chain $\{\xi_n, n \geq 0\}$. At the same time, there is always the possibility of expanding the phase space of physical states to semi-Markov states [2].

At the simulation of recovery technical systems (RTS) by use of Markov chains and associated with them random processesmetodically substantiated the introduction of the phase space represented by the pair (Z, $\mathcal{Z}$), where Z is the space of semi-Markov physical states $\vec{e}$, where $\vec{e} = (e_1, e_2, \ldots, e_n)$ is a vector (tuple), the components of which conveniently encode the physical states of all n elements of the system, for example, 1 - the element after the restoration was included in the work; 0 - the element refused and began to recover; 2 - the element has failed and has queued for recovery (in the case of a limited number of restoring devices (RD)); $\mathcal{Z}$ is the set of all Borel subsets of the space of semi-Markov states Z. The elements z of the space Z have the form $z = (\vec{e}, \vec{x})$, or $z = (\vec{e})$, where the vector$\vec{x}$, one of whose components is zero, indicates the times elapsed since the last state changes of the corresponding elements [14]. The point is that if none of the distribution functions $F_d^{(i)}(t)$,$d = \overline{0,1}$, $i = \overline{1,n}$ ($F_1^{(i)}(t)$ and $F_0^{(i)}(t)$ are, respectively, the function of the distribution of the failure-free time (FFOT) and the recovery time (RT) of the i-th element) is not exponential, it is impossible to construct an embedded Markov chain (EMC) with a set of semi-Markov) of physical states Z. The possibility to construct a EMC appears after the expansion of states from Z to semi-Markov states from Z, introducing continuous components [14].

A random process with a fixed phase space, according to the Kolmogorov extension theorem (see, for example, [18]), is uniquely (in the probabilistic sense) determined by the assignment of all its finite-dimensional distributions. In the case of Markov chains with discrete time and phase space (Y, $\mathcal{Y}$), where (Y, $\mathcal{Y}$) is an arbitrary measurable space, all finite-dimensional distributions are computed using the initial distribution ρ_0(B), Bϵ $\mathcal{Y}$, and the transition probabilities P (y, B), yϵY, Bϵ $\mathcal{Y}$. And soit is sufficient to know only ρ_0(B), B$\epsilon\mathcal{Y}$, and P (y, B), yϵY, Bϵ $\mathcal{Y}$ in order to completely define a Markov chain with discrete time. We set Y = Z × [0, ∞), $\mathcal{Y}$=$\mathcal{Z}$×σ, where$\mathcal{Z}$-σ algebra of subsets of Z, and σ- is the σ algebra of Borel sets on [0, ∞); × - is the sign of the Cartesian product of spaces. Consider the two-component Markov chain $\{\xi_n, \Theta_n, n \geq 0\}$, $\xi_n \epsilon Z$, $\Theta_n \epsilon [0, \infty)$ for which

$$P(\xi_0 \epsilon B) = p_0(B), B \epsilon \mathcal{Z}; \tag{1}$$

$$P(\xi_{n+1} \epsilon B, \Theta_{n+1} \leq t | \xi_0, \Theta_0, \xi_1, \Theta_1, \dots, \xi_n = z, \Theta_n) == P\{\xi_{n+1} \epsilon B, \Theta_{n+1} \leq t | \xi_n = z\} \tag{2}$$

and

$$P\{\xi_{n+1} \epsilon Z, \Theta_{n+1} < \infty | \xi_n = z\} = 1 \tag{3}$$

where $p_0(\cdot)$- is some distribution on $\mathcal{Z}$. The characteristic feature of the Markov chain $\{\xi_n, \Theta_n, n \geq 0\}$is expressed in (2): the depending on probabil of the event $\{\xi_{n+1} \epsilon B, \Theta_{n+1} \leq t\}$ from the past is realized only as depending on the first component $\xi_n$at the previous moment.

Definition 1. A Markov chain $\{\xi_n, \Theta_n, n \geq 0\}$, defined by equalities (1), (2), is called (homogeneous if the right-hand side in (2) does not depend on n) the process of Markov recovery (PMR).

In the case of a homogeneous PMR,

$$Q(t, z, B) = P\{\xi_{n+1} \epsilon B, \Theta_{n+1} \leq t | \xi_n = z\} \tag{4}$$

For fixed zϵZ, $B \epsilon \mathcal{Z}$, the function (with respect to t) Θ (t, x, B) does not decrease and, by (4), for any zϵZ, Q (t, x, B) is a distribution function (DF) of some nonnegative random variable (RV). The function Q (t, x, B) is called a semi-Markov kernel (SM-kernel) [13].

2.2. Semi-Markov Processes

At the simulation of RTS evolution of interest is the possibility of calculating the characteristics not only at the moments of the change of states, but also at any current time t. With each PMR$\{\xi_n, \Theta_n, n \geq 0\}$, which is specified by the right-hand parts (1) and (2), there is associated (at least one) a random process with a continuous time ξ (t) with the same as y $\{\xi_n, n \geq 0\}$ phase space. In order to associate with indicated PMR a single random process with continuous time, we define on the traectiry$\xi_0, \xi_1, \xi_2 \dots$of the Markov chain embedded in given PMRa family of conditionally independent RV $\zeta_{\xi_n, \xi_{n+1}}$ with conditional DF [2]:

$$P\{\Theta_{n+1} \leq t | \xi_n, \xi_{n+1}\} = F_{\zeta_{\xi_n, \xi_{n+1}}}(t) \tag{5}$$

Definition 2. The PMR$\{\xi_n, \Theta_n, n \geq 0\}$, defined by the relations (1), (2), is called strongly regular if for any T> 0

$$\lim_{N \to \infty} \sup_x P\{\textstyle\sum_{n=0}^{N} \zeta_{\xi_n, \xi_{n+1}} < T | \xi_0 = x\} = 0. \tag{6}$$

In other words, the PMR$\{\xi_n, \Theta_n, n \geq 0\}$is strongly regular if for the letters of all the trajectories $\xi_0, \xi_1, \dots, \xi_n$, ...of the Markov chain given PMR the series $\sum_{n=0}^{\infty} \zeta_{\xi_n, \xi_{n+1}}$ diverge.

Using RV $\zeta_{\xi_n, \xi_{n+1}}$, the so-called counting process is introduced:

$$\nu(t) = max\left\{n: \sum_{k=0}^{n} \zeta_{k, \xi_{k+1}} \leq t\right\};$$

In the case of a regular PMW, the random process $\nu(t)$ taking values in {0, 1, 2,...} with probability 1 is finite for any tε [0, ∞) [13].

Definition 3. A random process $\xi(t) \stackrel{\text{def}}{=} \xi_{\nu(t)}$, where $\{\xi_n, n \geq 0\}$- the Markov chain embedded in the (regular) PMR $\{\xi_n, \Theta_n, n \geq 0\}$, is called a (regular) semi-Markov process (SMP).

Thus, since the counting process ν (t) is a monotonically increasing transition process with right-continuous trajectories, then the SMP ξ (t) is also transition process, whose jump instants coincide with the instants of the jumps of the process ν (t) and the trajectory ξ (t) are continuous from the right.

2.3. Superposition of Independent Semi-Markov Processes

We consider a system of n independently operating elements (subsystems), the operation of each of which is described by the SMP with $\xi^i(t)$ with an arbitrary phase space $\left(Z^{(i)}, \mathcal{Z}^{(i)}\right)$, $i = \overline{1, n}$ (the processes $\xi^i(t)$ are independent in the aggregate). For each $Z^{(i)}$ there is a partition $Z^{(i)} = Z_1^{(i)} \cup Z_0^{(i)}$, $Z_1^{(i)} \cap Z_0^{(i)} = \emptyset$, $i = \overline{1, n}$ where $Z_1^{(i)} \epsilon \mathcal{Z}^{(i)}$ is interpreted as the set of operable states of the ith element; $Z_0^{(i)} \epsilon \mathcal{Z}^{(i)}$ is the set of fault states of the ith element.

Let

$$\zeta_i(t) = \chi_{Z_1^{(i)}}\left(\xi^i(t)\right) = \begin{cases} 1, \text{если } \xi^i(t) \epsilon Z_1^{(i)}(t); \\ 0, \text{если } \xi^i(t) \epsilon Z_0^{(i)}(t); \end{cases}$$

$$\zeta(t) = \{\zeta_1(t), \zeta_2(t), \dots, \zeta_n(t)\}$$

The vector process $\zeta(t)$ characterizes the state of the system at the moment of time on the states (operability or inoperability) of its elements. The set of possible values of the process $\zeta(t)$ is the set of distinct binary vectors $d = (d_1, d_2, \dots, d_n)$ from $(0,0, \dots 0)$to $(1,1, \dots 1)$.

We denote it by $D = \{d\}$. Consider the concept of a system failure: on a set of binary vectors D there is given a function g (d) such that

$$g(d) = \begin{cases} 1, \text{if the system is operable for a given combination of states of its elements} \\ 0, \text{otherwise} \end{cases}$$

The set of values of the vector d at which the system is operable is denoted by D_1, and the set of values at which the system is inoperable, - through D_0, i.e. $D_1 = \{d: g(d) = 1\}$, $D_0 = \{d: g(d) = 0\}$. By assumption, $D = D_1 \cup D_0$, $D_1 \cap D_0 = \emptyset$.. We investigate the reliability characteristics of such a system, in particular, the mean stationary (established) residence times of the process ξ (t) in the sets D_1 and D_0 (the mean time between failures and the average recovery time of the system, respectively) and the limiting value as t → ∞ of the probability to find the process ζ (t) in the set D_1 (stationary availability coefficient). The required reliability characteristics of a system are obviously related to the probabilistic characteristics of the residence times of the process: $\eta(t) = \{\xi^{(1)}(t), \xi^{(2)}(t), \dots, \xi^{(n)}(t)\}$ in the subsets of states $J_1 and J_0$, where

$$J_1 = \cup_{d \epsilon D_1} \prod_{i=1}^{n} Z_{d_i}^{(i)} , J_0 = \cup_{d \epsilon D_0} \prod_{i=1}^{n} Z_{d_i}^{(i)} \tag{7}$$

It is obvious that $J_0 \cap J_0 = \emptyset$. The sign of $\prod_{i=1}^{n} Z_{d_i}^{(i)}$ means the Cartesian product $Z_{d_1}^{(1)} \times Z_{d_2}^{(2)} \times \dots \times Z_{d_n}^{(n)}$.

Using the method of introducing additional components $u^{(i)}(t)$, we can reduce the problem to the study of the characteristics of SMP.

Let $u^{(i)}(t)$ –be a non-stop of the process $\xi^{(i)}(t)$ at time t, i.e. $u^{(i)}(t) = t - sup\{u: u \le t, \xi^{(i)}(u) \ne \xi^{(i)}(t)\}$. Then the non-stop of the process $\xi^{(i)}(t)$ at the instant of the nearest "in last" to instant of the jump of one of the processes, at the instant t, $\xi^{(i)}(t), k = \overline{1,n}$, is equal to $v^{(i)}(t) = u^{(i)}(t) - u(t)$.

Definition 4. A superposition of (semi-Markovian) processes $\xi^{(i)}(t)$, $i = \overline{1,n}$, is called the a 2n-component process

$$\xi(t) = \left(\xi^{(1)}(t), \dots, \xi^{(n)}(t);\ v^{(1)}(t), \dots, v^{(n)}(t)\right) \tag{8}$$

Process (8) is semi-Markovian. In fact, it does not change its value between two neighboring jumps in the processes $\xi^{(i)}(t), i = \overline{1,n}$, i.e. is a process with piecewise-constant trajectories. At the instant of its jumps, which coincide with the instants of the jumps of the initial SMS, $\xi^{(i)}(t), i = \overline{1,n}$ it has the Markov property. This follows from the fact that at these instant the process ξ (t) coincides with the process $\left(\xi^{(1)}(t), \dots, \xi^{(n)}(t);\ u^{(1)}(t), \dots, u^{(n)}(t)\right)$, and the pairs $\left(\xi^{(i)}(t), u^{(i)}(t)\right)$ are Markov processes. At the time of the jump ξ (t), one of its components $v^{(i)}(t), i = \overline{1,n}$, is zero.

We introduce the following notation: $z = (z_1, z_2, \dots, z_n)$ is an n-dimensional vector whose components are elements of the sets $Z^{(i)}$; $dz = (dz_1, dz_2, \dots, dz_n)$, $\vec{x}^{(k)} = (x_1, \dots, x_{k-1}, 0, x_{k+1} \dots, x_n) - n$ is an n-dimensional vector, the k-th component of which is equal to zero, and the remaining are non-negative real numbers; $d\vec{x}^{(k)} = dx_1 dx_2 \dots dx_{k-1}, dx_{k+1}, \dots, dx_n$; $J = J_1 \cup J_0 = \prod_{i=1}^{n} Z^{(i)}$; $\mathcal{A} = \mathcal{Z}^{(1)} \times \mathcal{Z}^{(2)} \times \dots \times \mathcal{Z}^{(n)}$; $R_+^{n,k} = \{\vec{x}^{(k)}\}-$ is the set of possible values of the vector $\vec{x}^{(k)}$; $R_t^n = \cup_{k=1}^{n} R_+^{n,k}$; σ_+^n- σ-algebra of Borel sets on R_+^n.

The phase space of the SMP ξ (t) is (Z, $\mathcal{Z}$), where $Z = \cup_{k=1}^{n} J \times R_+^{n,k}, \mathcal{Z} = A \times \sigma_+^n$.

In accordance with the accepted assumptions, the set of states Z of the processξ (t) can be represented in the form

$$Z = Z_1 \cup Z_0, Z_1 \cap Z_0 = \emptyset,$$

where $Z_1 = J_1 \times R_+^n$ is the set of operable states of the system, and $Z_0 = J_0 \times R_+^n$ is the set of fault states of the system.

As shown in [14], the availability coefficient, the mean time between failures and the average recovery time in the stationary mode of the SMP ξ (t) can be determined respectively by the following formulas:

$$K_\Gamma = \sum_{d \epsilon D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} \left\{\prod_{i=1}^{n} \left(T_1^{(i)} + T_0^{(i)}\right)\right\}^{-1} = \sum_{d \epsilon D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} / \sum_{d \epsilon D} \prod_{i=1}^{n} T_{d_i}^{(i)}; \tag{9}$$

$$T_1 = \sum_{d \epsilon D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} \left\{\sum_{d \epsilon D_1'} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \epsilon G(d)} \frac{1}{T_1^{(j)}}\right\}^{-1} = \sum_{d \epsilon D_1} \prod_{i=1}^{n} T_{d_i}^{(i)} \left\{\sum_{d \epsilon D_0'} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \epsilon I(d)} \frac{1}{T_0^{(j)}}\right\}^{-1} \tag{10}$$

$$T_0 = \sum_{d \epsilon D_0} \prod_{i=1}^{n} T_{d_i}^{(i)} \left\{\sum_{d \epsilon D_1'} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \epsilon G(d)} \frac{1}{T_1^{(j)}}\right\}^{-1} = = \sum_{d \epsilon D_0} \prod_{i=1}^{n} T_{d_i}^{(i)} \left\{\sum_{d \epsilon D_0'} \prod_{i=1}^{n} T_{d_i}^{(i)} \sum_{j \epsilon I(d)} \frac{1}{T_0^{(j)}}\right\}^{-1}, \tag{11}$$

where $T_1^{(i)}$ –is the mean time between failures of the i-th element; $T_0^{(i)}$ –is the average recovery time of the i-th element;

$$T_{d_i}^{(i)} = \begin{cases} T_1^{(i)}, if\, d_i = 1; \\ T_0^{(i)}, if\, d_i = 0; \end{cases} \tag{12}$$

D_1' – is the set of vectors $d \epsilon D_1$ such that changing the value of some one component from one to zero takes the vector d to the set D_0; D_0' – is the set of vectors $d \epsilon D_0$ such that changing the value of some one component from zero to one translates the vector d into the setD_1; $G(d)$ –is the set of numbers of the components of the vector $d \epsilon D_1'$, changing the value of each of which from one to zero takes the vector d to the set D_0; $I(d)$ –is the set of numbers of the components of the vector $d \epsilon D_0'$, changing the value of each of which from zero to one translates the vector d into the set D_1.

Similar formulas for K_Γ,T_1 and T_0 were obtained in [13] for stationary distributions of the SMP's describing a superposition of a finite number of alternating independent reconstruction processes. To construct a stationary distribution of such a superposition, the phase-enlargement algorithms (PEA) are used, which result in the splitting of the phase space of states and the gluing of states belonging to the same class. The PEA reveals a natural launlness in the functioning of complex systems: the transitions of a complex system between rich (enlarged) classes of states obey the Markov property even in the absence of this property in the initial assumptions because of the imposed constraints. If we ignore the transitions of the system within a particular class, then if the system stays in each class for a long time, the dependence of the transitions between the enlarged classes on the evolution of the system within the classes disappears, and the time of stay in each class accumulates as a sum of a large (random) number of random variables - time of stay in separate states. This allows us, under natural assumptions, to consider the time of stay in each class as distributed according to exponential law and to consider the superposition of a finite number of resulting (enlarged) alternating Poisson recovery processes with exponential distributions of residence times in the operable and inoperative state of the aggregate classes. A simplified description of semi-Markov systems by enlarged SMP's is possible provided that a stationary distribution of the real or reference (obtained after phase space splitting) of the embedded Markov chain (EMC) is established. At the same time, the main computational difficulty consists in determining the stationary distribution of the EMC.

Formulas (9) - (11) give exact relations for calculating stationary indicators of the reliability of systems with independently functioning elements, and [11]

$$K_\Gamma = T_1/(T_1+T_0) \tag{13}$$

3. Result

3.1. Semi-Markov Models of Recoverable Systems with Disconnection of Elements

Consider a complex recoverable system S of $n(n \geq 1)$ elements. The FFOT of the i-th element is the random variable (RV) $\alpha_1^{(i)}$ with the distribution function (DF) $F_1^{(i)}(t)$ (t), and RT is RV$\alpha_0^{(i)}$ with the DF$F_0^{(i)}(t)$, $i=\overline{1,n}$. Suppose that the DF $F_k^{(i)}(t)$, $k=\overline{0,1}$, $i=\overline{1,n}$ are absolutely continuous functions with respect to Lebesgue measure with the corresponding distribution densities, and RV$\alpha_k^{(i)}$,$k=\overline{0,1}$, $i=\overline{1,n}$are independent in the aggregate and have bounded averages $\left(0 \leq M_{\alpha_k}^{(i)} = T_k^{(i)} < \infty\right)$. Recovery is assumed to be unlimited, i.e. the queue for restoration is not formed; each of the elements can be in one of the following states: a working non-disabled, working disabled and restored disabled.

The discennectionof an element (a collection of elements) occurs due to the failure of any element of the system in the event that as a result of the failure the element (the totality of the elements) does not belong to any workable path. Under the workable way we will understand a set of interrelated elements, the functioning of which entails the viability of the system as a whole. At the time of the system failure as a whole, all the working elements that remain in the system are disconnected; in this case, only the restoration of the failed elements occurs in the system.

Disconnected elements are included in the system with the same level of performance and recovery, on which they found the disconnection or failure of the system. Inclusion occurs at the time of the end of the recovery of any element, i.e. the resumption of its work, provided that, together with the restored element, a workable path is formed.

Recovery of the failed element starts instantly, and repair completely restores its original reliability characteristics. Inclusion in the system of the restored element (a group of previously disabled elements) and shutdown occurs instantly.

The concept of system failure is introduced based on its functional structure: it can occur due to the failure of one or a combination of elements.

The functioning of the i-th element of the system, $i=\overline{1,n}$ is a sequence of consecutive working and restoring periods (with possible shutdown periods), and the functioning of the entire system S is an analog of the superposition n of independent alternating processes recovery, discussed in Section 3.

Let $\xi_i(t)$, $i=\overline{1,n}$ – be alternating recovery processes with possible shutdown periods, simulating the evolution of the corresponding element with initial distributions $P\{\xi_i(0)\} = 1$. If at the moment ti-th element, $i=\overline{1,n}$ is operable or disabled in the operable state, then $\xi_i(t)=1$, if it is restored, then $\xi_i(t)=0$.

Following the reasoning in Section 3, we consider the SMP $\xi(t)=\{\xi_1(t),\dots,\xi_n(t);\ u_1(t),\dots,u_n(t)\}$, which simulates the evolution of the system. Here $u_i(t)$for $\xi_i(t)=0$ is the recovery time (without taking into account the possible shutdown time due to the failure of the system as a whole) of the i-th element from the moment of its last failure to the nearest in last jump of one of the processes$\xi_j(t)$,$j=\overline{1,n}$, and $u_i(t)$ for $\xi_i(t)=1$ is the time of operation of the i-th element (without taking into account the possible shutdown time) from the moment of its last recovery to the nearest in last jump of one of the processes$\xi_j(t)$,$j=\overline{1,n}$.

The phase space of the system has the form (Z, $\mathcal{Z}$), where Z $=\left\{z=\left(d;x^{(i)}\right)::d\epsilon D; x^{(i)}\epsilon R_+^{(i)}\right\}$, $D=\{d: d=(d_1,\dots,d_n), d=\overline{0,1}, i=\overline{1,n}\}$, $R_+^i=\{x^{(i)}:x^{(i)}==(x_1,\dots,x_{i-1},0,x_{i+0},\dots,x_n), x_k\geq 0, k=\overline{1,n}, k\neq i\}$; $\mathcal{Z}$ is the σ-algebra of Borel sets in Z.

We denote by $I=\{i: d_i=0, i\bar{\epsilon}I_d\}$; I_d – is the set of numbers of disconnected elements in the state $\left(d;x^{(i)}\right)$.

The stationary reliability indices K_Γ, T_1, T_0 of the system S under consideration are determined by the formulas [14];

$$K_\Gamma = \sum_{d\epsilon D_1}\prod_{i=1}^n T_{d_i}^{(i)} \Big/ \sum_{d\epsilon D}\prod_{j=1}^n T_{d_j}^{(i)}\,; \tag{14}$$

$$T_1 = \sum_{d\epsilon D_1}\prod_{i=1}^n T_{d_i}^{(i)} \Big/ \sum_{d\epsilon D_0}\sum_{s\epsilon I}\prod_{\substack{j=1\\ j\neq s}}^n T_{d_j}^{(i)}\,; \tag{15}$$

$$T_0 = \sum_{d\epsilon D_0}\prod_{i=1}^n T_{d_i}^{(i)} \Big/ \sum_{d\epsilon D_0}\sum_{s\epsilon I}\prod_{\substack{j=1\\ j\neq s}}^n T_{d_j}^{(i)}\,; \tag{16}$$

3.2. System "p of n" with Shutdown After Failure

Consider a system of n independently functioning elements. Suppose that the operation of each i-th element is described by an alternating recovery process with the uptime

FFOT $\alpha_1^{(i)}\left(P\left\{\alpha_1^{(i)} \le t\right\} = F_1^i(t)\right)$ and restoration time (RT)$\alpha_0^{(i)}\left(P\left\{\alpha_0^{(i)} \le t\right\} = F_0^{(i)}(t)\right)$. The system is considered operable when at least p (1≤p≤n) elements work (in the example of the operation of the railway locomotive p = n). For definiteness, we assume that at the initial time t = 0 all elements begin to work. Define the main reliability characteristics of the system: stationaryavailabilitycoefficient K_Γ, mean time between failures$T_1$and average recovery time T_0.

Suppose that the following conditions are satisfied:

A1. The random variables (RV) $\alpha_1^{(i)}$ –FFOT elements, $\alpha_0^{(i)}$ – RT elements$i = \overline{1,n}$(n- number of elements in the system) are independent in aggregate and have limited mean $0 \le M\alpha_1^{(i)} = T_1^{(i)} < \infty$, $0 \le M\alpha_0^{(i)} = T_0^{(i)} < \infty$.

A2. The distribution functions (DF) of the RV$\alpha_1^{(i)}$and$\alpha_0^{(i)}$,$i = \overline{1,n}$, $F_1^{(i)}(t)$ and $F_0^{(i)}(t)$ respectively, are absolutely continuous with respect to the Lebesgue measure.

We simulate the functioning of each i-th element by a semi-Markov process (SMP) $\xi^{(i)}(t)$ (t) with the set of states $E^{(i)} = \{0,1\}$ and a semimarkov matrix

$$Q^{(i)}(x) = \begin{pmatrix} 0 & F_0^{(i)}(x) \\ F_1^{(i)}(x) & 0 \end{pmatrix}$$

The initial distribution by the condition of the problem $P\{\xi_i(0) = 1\} = 1$. The process $\xi_i(t)$ takes the value 1 if the i-th element at the time t works, and the value 0, if it is restored at time t.

By the skipping, non-stops $u_i(t) = t - sup\{u: u \le t, \xi^{(i)}(u) \ne \xi^{(i)}(t)\}, i = \overline{1,n}$,, the processes $\xi_i(t), i = \overline{1,n}$, we determine at each instant of time the minimum non-stop $u(t) = \min_{i=1,2,\dots,n}\{u_i(t)\}$ and the non-stops of the initial processes at the moment the nearest jump in the past of one of them $\upsilon_i(t) = u_i(t) - u(t)$. We consider the semi-Markov superposition of the processes $\xi_i(t)$, i.e. The SMS is $\xi(t) = \{\xi_1(t), \dots, \xi_n(t); \upsilon_1(t), \dots, \upsilon_n(t)\}$ with the initial distribution $P\{\xi(0) = (1,1,\dots,1; 0,0,\dots,0)\} = 1$.

Let $d = (d_1, \dots, d_n)$ be an n-dimensional binary vector such that $dd_j = \overline{0,1}$, $j = \overline{1,n}$. We denote by $|d| = \sum_{j=1}^n d_j$ that is,| d | is equal to the number of units in the vector d. Let D be the set of all possible n-dimensional binary vectors. We set $x^{(k)} = (x_1, \dots, x_{k-1}, 0, x_{k+1}, \dots, \xi_n)$, i.e. $x^{(k)}$ is an n-dimensional vector with nonnegative components, one of which (k-n) is necessarily zero. We denote the set of all possible vectors $x^{(k)}$by $R_+^{n,k}$.

The set of states of the process ξ (t) can be written as $Z = \left\{\left(d: x^{(k)}\right): d\epsilon D, x^{(k)}\epsilon R_+^{n,k}, k = \overline{1,n}\right\}$. Proceeding from the conditions of the problem, $Z = Z_1 \cup Z_0$ where $Z_1 = \left\{\left(d: x^{(k)}\right): d\epsilon D, |d| \ge p, x^{(k)}\epsilon R_+^{p,k}, k = \overline{1,n}\right\}$, corresponds to the set of states of the system's operability, and $Z_0 = \left\{\left(d: x^{(k)}\right): d\epsilon D, |d| < p, x^{(k)}\epsilon R_+^{n,k}, k = \overline{1,n}\right\}$, to the set of failure states of the system.

When the conditions A_1, A_2 are satisfied, the following expressions for K_Γ, T_1 and $T_0$are valid, respectively [14]:

$$K_\Gamma = \sum_{|d|\ge p} \prod_{i=1}^n T_{d_i}^{(i)} / \sum_{|d|\ge p-1} T_{d_i}^{(i)}; \tag{17}$$

$$T_1 = \left(\sum_{|d|\ge p} \prod_{i=1}^n T_{d_i}^{(i)}\right) / \left(\sum_{|d|=p-1} \sum_{\{j:d_j=0\}} \prod_{i\ne j} T_{d_i}^{(i)}\right); \tag{18}$$

$$T_0 = \left(\sum_{|d|=p-1} \prod_{i=1}^n T_{d_i}^{(i)}\right) / \left(\sum_{|d|=p-1} \sum_{\{j:d_j=0\}} \prod_{i\ne j} T_{d_i}^{(i)}\right), \tag{19}$$

where$T_{d_i}^{(i)}$ are determined by the relation (12).

It is obvious that the sets of vectors d with the capacity | d | ≥ p and | d | <p coincide with the sets D_1 and D_0, respectively, and the sets with the capacity | d | = p-1, consisting of the vectors obtained from the vectors $d\epsilon D_0$, in each of which the change of only one component with the number$j\epsilon I(d)$from zero to one takes the vector d to the set D_1, coincides with D_0', and consists of the vectors $d\epsilon D_1$, in each of which the change of only one is associated with the number $j\epsilon G(d)$ from one to zero, coincides with D_1'. In this case, formulas (17) - (19) coincide with formulas (14) - (16), respectively.

Let us show the application of formulas (17) - (19) for the calculation of stationary reliability indicators of electric locomotives with an asynchronous traction motor.

3.3. Example

Let for the KZ electric locomotives with an asynchronous traction motor of alternating current operated on the Kazakhstan railway there are data on unscheduled repairs in 2016, related to violations of the small gear wheel (SGW)) - i = 1, compressor (K) -i = 2, the pantograph (P) -i = 3 and the traction motor (TM) -i = 4:

a) KZ 12:12.02.2016 – SGW (40h.), 01.03.2016 – SGW (46h.), 21.03.2016 – K (14h.),01.04.2016 – P (5h.), 20.06.2016 – TM (24h.);

б) KZ 17:08.01.2016 – TM (15h.), 11.01.2016 – TM (11h.), 07.06.2016 – P (4h.),06.09.2016 – SGW (200h.).

From the data for KZ 12 we find

$$\alpha_i^{(i)} = 1032; 390; 434; 250; 1915; 4646\ (i = \overline{1,4}); \alpha_0^{(1)} = 40; 46; \alpha_0^{(2)} = 14; \alpha_0^{(3)} = 5; \alpha_0^{(4)} = 24.$$

Therefore, for $T_m^{(i)} = M_{\alpha_m}^{(i)} (m = \overline{0,1})$we have:

$$T_1^{(i)} = 8667/6 = 1444{,}5(\text{ч})\ (i = \overline{1,4}); T_0^{(1)} = 43(\text{ч}), T_0^{(2)} = 24(\text{ч}), T_0^{(3)} = 5(\text{ч}), T_0^{(4)} = 24(\text{ч}).$$

Applying the formulas (17) - (19), we obtain:

$$K_{\Gamma} = \frac{(1444,5)^4}{(1444,)^4 + (43+14+5+24)\cdot(1444,5)^3} = \frac{1444,5}{1530,5} = 0,944;$$

$$T_1 = \frac{(1444,5)^4}{4\cdot(1444,5)^3} = \frac{1444,5}{4} = 361,125(h);$$

$$T_0 = \frac{(43+14+5+24)(1444,5)^3}{4\cdot(1444,5)^3} = \frac{86}{4} = 21,5(h).$$

From the data for KZ 17 we find:

$$\alpha_1^{(i)} = 107; 57; 3541; 2180; 2608(i=\overline{1,4}); \alpha_0^{(1)} = 200; \alpha_0^{(2)} = 0; \alpha_0^{(3)} = 4; \alpha_0^{(4)} = 15; 11.$$

Consequently,

$$T_1^{(i)} = 1698,8(ч)(i=\overline{1,4}); T_0^{(1)} = 200(ч), T_0^{(2)} = 0(ч), T_0^{(3)} = 4(ч), T_0^{(4)} = 13(ч).$$

Calculations according to formulas (17) - (19) give:

$$K_{\Gamma} = \frac{(1698,8)^4}{(1698,8)^4 + (200+0+4+13)(1698,8)^3} = \frac{1698,8}{1915,8} = 0,886;$$

$$T_1 = \frac{(1698,8)^4}{4\cdot(1698,8)^3} = \frac{1698,8}{4} = 424,7(h);$$

$$T_0 = \frac{(200+0+4+13)(1698,8)^3}{4\cdot(1698,8)^3} = \frac{1698,8}{4} = 424,7(h).$$

4. Discussion

It is easy to verify that the obtained values of K_{Γ}, T_1 and T_0 for KZ 12 and KZ 17 satisfy equality (13).

Averaging K_{Γ}, T_1 and T_0 for all KZ electric locomotives, we obtain the average values (for all KZ electric locomotives operated on the railway in question) of $\overline{K}_{\Gamma}, \overline{T}_1$ and $\overline{T}_0$ of K_{Γ}, T_1 and T_0.

Thus, in terms of indicators $\overline{K}_{\Gamma}, \overline{T}_1$ and $\overline{T}_0$, one can judge the reliability of each brand of electric locomotives operated on a given railway and choose the best of them both in terms of their preparation $\overline{K}_{\Gamma}$ by the end of the study time period and in the mean time $\overline{T}_1$ and the average repair time $\overline{T}_0$.

In addition, in terms of $\overline{K}_{\Gamma}, \overline{T}_1$ and $\overline{T}_0$, it is possible to compare the reliability levels of AC and DC electric locomotives.

Formulas (17) - (19) can be applied to calculate stationary reliability indicators of various types of complex systems with failures that have non-exponential functions of the distribution of the MTBF and the recovery time.

5. Conclusion

The functioning of electric locomotives with an asynchronous motor based on data on unscheduled repairs related to violations of the separated elements of an electric locomotive in the past period is presented in the work in the form of a rebuilt system with the disconnection after failure of one element (or several elements) of all the remaining elements.

After extending the states of the system to semi-Markovian, by introducing continuous components, the superposition of alternating independent processes with, in general, non-exponential distribution functions (DF), the residence time in each state becomes, under natural initial assumptions, a system of enlarged alternating independent processes with exponential distributions of residence times in the states representing the semi-Markov process (SMP), describing the process of Markov recovery (PMR).

Under the conditions of independence of the times of stay in each state of individual elements and the continuity of their RF for the obtained PMW, the formulas for calculating the reliability indices in the steady-state mode are applicable: availability factor ($K_г$), mean time to failure (T_1), and average system recovery time (T_0).

Based on the averaged (for the totality of electric locomotives of each brand), the reliability indexes $\overline{K_г}$, $\overline{T_1}$ and $\overline{T_0}$ can be used to compare the reliability levels of a given brand of AC and DC electric locomotives. The above analysis of the reliability of electric locomotives can be applied to various types of complex technical systems with failures and nonexponential FRs of residence times in states.

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