Mathematical Model of Air Quality in the Ahinsan-Atonsu-Agogo Industrial Area of Kumasi, Ghana

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Abstract
Air pollution poses a serious threat to socio-economic development, with the tendency of even adversely affecting the survival of an entire population. This study aimed at modeling the concentrations of SO₂, NO₂ and CO as well as some aerodynamic quantities in the Ahinsan-Atonsu-Agogo Industrial Area of Kumasi to assess the extent of air pollution and their growth rates using the statistical method of system identification based on autoregressive time series analysis, leading to deterministic discrete - time linear autonomous models. Results showed models of the first order which are unstable with generally escalating growth rates. For the CO at high levels, no model was obtained for the gas. Except in the horizontal direction in the morning, the speed and direction of the wind affected the movement of gases in all periods. The escalating growth rates may be due to vehicular traffic, clustering of industries and power plants in the area. Building of chimneys high enough for effective dispersal and dilution of pollutants, construction of bypasses around the industrial area to prevent concentration of vehicles, are some of the measures that could avert the associated devastating consequences in the near future.

1. Introduction

1.1. Background

Air is one of the essential gifts to man. It is a life supporting system without which life cannot exist. Air is truly the closest man can come to possess as an infinite and free resource. Over a long period of time, the atmosphere was in fact a free good. Until recently the earth’s living systems evolved fairly in a balance with the atmosphere; hydrosphere and lithosphere unaffected by humans’ activity.

Industrialization is central to the economies of modern societies and an indispensable motor of growth especially in developing countries, where it is needed to widen the development base and cope with growing needs. It has brought prosperity to man since the output of goods and services per man-hour has increased dramatically over the levels attained in earlier ages [1]. The Industrial Revolution, the movement in which machines changed people's way of life as well as their methods of manufacturing has witnessed the
use of products turned out swiftly by mass production, by people (and sometimes, robots) working on assembly lines using power-driven machines. This has occurred since machine work has replaced human toil. Then the energy - consumption per man – hour and also per capita has steadily increased. Predominantly, this energy has been generated by means of the combustion of fossil fuels such as coal and oil. The accompanying wastes cannot be discounted. Thus, waste products are released making the atmosphere unhealthy and degrading air quality in the neighbourhood of the industries. Thus, increasing industrialization and urbanization have created growing demands to use the atmosphere, whether consciously or not, as a waste disposal medium [2]. Naturally, cities are characterized by high concentrations of humans, materials and activities. These are normally associated with the highest levels of pollution and also the largest impact [3].

Atmospheric pollution is a threat to socio-economic development and could adversely affect the survival of the entire population. Thus, atmospheric pollution is the biggest environmental health risk [4]. Globally, it exhibits significant level of threat to the health of millions of humans, introducing fairly large financial burden on the general global society [5]. Conditions of deteriorating air quality have grown more severe as urbanization and mass consumption have become the standard way of life throughout the large part of the world. Economy and ultimately, mine wastes are the underlying factor behind the emergence of serious air pollution problems. The decline in air quality signals the end of the free air resource. Although the use of air largely remains free for waste disposal into the atmosphere, the social costs of poor air quality in the form of dangers to human health, property destruction, and many others (including presently unknown) accrue to society as a whole. Often the stacks of factories are built higher in order to disperse the pollutants over larger volumes of air, thus lowering the ambient concentration of pollutants [6]. According to [7], there is a close connection between population’s health and the quality of the atmosphere. As alarming as these are, the World Health Organization has identified ambient air pollution as a high public health priority, indicating the relationship of air pollution with increased mortality and shortened life expectancy [8].

Modeling, the conceptualization of a real process to characterize its behavior, is adopted by scientific to enhance the investigation of phenomena in order to reveal and better understand the cause-effect relationships. Epidemiological and animal model data indicate that primarily affected systems are the cardiovascular and the respiratory system. However, the function of several other organs can be also influenced [9, 10, 11, 12]. Under the conditions of the air polluter impact there is high morbidity and mortality from cardio-vascular diseases, respiratory disease, nervous system and sensory organ disturbances, gastrointestinal disease and circulatory disease. Poor air quality has been cited as a factor in these conditions [13].

Also more widespread and ultimately more damaging effects may accompany persistent human exposure to poor air quality over long time periods. Although medical evidence has established that a general relation exists between air quality and human health; an important problem is to quantify this because of the multitude of other factors that determine human health. The overall weight of this evidence leaves little doubt that widespread and adverse human effects are associated with poor air quality especially the criteria pollutant (mainly NO₂, SO₂ and CO) with a lifetime of several months in the atmosphere [14]. Other studies have pointed to an association of non-respiratory cancers with air quality indices and / or urban versus rural residency. Various infant mortality rates, total mortality rates, pneumonia and pulmonary tuberculosis diseases also show increases with higher levels of air pollution. Several studies indicate that work absenteeism is significantly higher when atmospheric conditions reflect poor air quality than when the air is relatively clean. Children are more susceptible than adults to the effects of air pollution because of environmental, behavioral and physiological factors [15]. Children spend more time outside engaged in physical activity, exposing them to larger doses of ambient air pollution [16, 17]. Air quality models play an important role in atmospheric science and air quality management since they enables a complete description of the deterministic nature of air quality problem extending it to cover causal factors such as emission sources, meteorological processes, and physical and chemical transformations and guidance on the implementation of control strategies.

1.2. Sulphur Dioxide (SO₂)

SO₂, a colourless corrosive gas, is directly damaging to plant, animals and property. When fossil fuel is burned, sulphur is converted to sulphur dioxide and sulphur trioxide (SO₃) each of which has serious health effects. SO₂ reacts with water vapour or dissolves in atmospheric droplets to form sulphuric acid which may corrode, deteriorate and weaken structures and result in considerable property damage. It also results in the formation of acid rain that causes many aquatic life forms to die. According to [18], SO₂ damage is second to only smoke as cause of air polluted-related health damage. The oxides of nitrogen are general represented by NOx because of their inter-convertibility. Organic substances in the atmosphere combine photochemically with nitrogen oxides to create ozone (O₃), a bluish gas that has devastating effect on vegetation and is responsible for a significant amount of other property damage. Satellite observations have been successfully used to verify trends of SO₂ emissions [19, 20, 21, 22] but not without significant difficulty. Monitored dispersed ambient air quality does not give highlight of the differences in localized pollution levels [23]. According to [24], the global emissions estimate largely falls within this expanded uncertainty range. The estimated trends in global SO₂ emissions are within the range of representative
concentration pathway projections and the uncertainty previously estimated for the year 2005 [25].

1.3. Carbon Monoxide (CO)

This is a colourless and odorless gas produced by inefficient combustion of carbon fuel. It can pose a serious health problem. It is highly toxic at significant levels of concentration and can cause decreased human efficiency in low but chronic doses. Although motor vehicles are the major sources of CO emissions from other economic activities are also responsible for significant contribution of this pollutant [26]. CO damage is also fatal in causing health hazards which are -related air pollution. It reduces oxygen delivery to the heart and brain [27]. Motor vehicles together with economic activities contribute significant amount of CO the atmosphere.

1.4. Nitrogen Dioxide (NO₂)

The major source of NO₂ in urban areas is traffic, with ambient concentrations fluctuating with the morning and evening rush hours [28]. Wood industries, power stations and domestic emissions such as burning plants, protein-base foods are also major emission sources of this gas [29]. Potential health hazards e. g. respiratory irritants and acid rains which in turn has a negative effect on the forest, aquatic habitats, corrosion of property, etc. as noted by [30] cannot be discounted. Studies on the toxicological profile of NO₂ indicate acute toxicity with dermal, oral and inhalation routes of exposure. The gradient in NO₂ concentration from the roadside to the more general urban background is steep [31]. Of particular concern are exposures of NO₂ near roadways due to the acuteness of their effects on susceptible individuals, such as people with asthma, children, and the elderly [32]. Under conditions such as of high doses or confined space, NO₂ has caused catastrophic injury to human, including even death.

1.5. Air Temperature

Conditions of the atmosphere including stabilities of wind and rain exert profound effect on the transportation of the air pollutant [33]. Studies have shown that reduction of exhaust gas temperature results also in reduction of criteria gases [34]. Geographical location of temperature, wind and other weather elements disperse pollutant differently [35, 36]. For instance, the wind and rain may effectively dilute pollutants to relatively safe concentrations despite a fairly high rate of emissions.

1.6. Wind Speed and Direction

Dispersal of air pollutants is affected by many factors including meteorological conditions especially wind speed, wind direction and atmospheric stability. [37] observed exhibition of extremely complex dispersion patterns by many air pollutants, especially in environments such as cities and towns where there are a large number of emission sources and major variations in environmental conditions. [38] found that wind speed and temperature cause high dispersion of atmospheric pollutants and also that, high wind speed is the meteorological parameters that affect the dispersion of atmospheric pollutants the most. In recent times, a variety of methods of wind speed prediction models including both simple and complex (combinations) ones have been developed. [39] reported short-term wind speed with a representative hourly time series for the site and had a model with good prediction accuracy.

Establishing monitoring programme for air quality is a very important part of air quality management. Air pollution control in the last decade has continued to have a positive impact on public health [40]. Air quality measurement and surveillance programme may be undertaking in respect of many objectives some of which are:

- a. facilitation of measurements of background concentrations,
- b. obtain data for research investigations,
- c. monitoring of current levels to form baseline for assessment,
- d. develop abatement strategies,
- e. check the air quality against the set standards or limit values,
- f. inform the public about the air quality and raise the awareness,
- g. develop warning systems for the prevention of undesired air pollution episodes, to support legislation in relation to the air quality limit values and guidelines.

The ability to obtain a comprehensive estimate of the overall urban air pollution helps in decision-making and assists in the implementation of preventive actions to reduce emissions. In designing the air quality monitoring programme, giving consideration to the air pollution problem definition, in concert with the resource (personnel analysis, budget and available equipment) are eminent. Autoregressive Integrated Moving Average (ARIMA) models have been used in a great number of time series prediction problems, because they are robust, as well as easy to understand and implement [41].

The citing of manufacturing industries which exert impact on the quality of air by their characteristic tendency to liberate pollutants into the atmosphere, together with residences and hospitals in the same locality is a disturbing phenomenon. This is the situation at the Ahinsan-Atosu-Kaase Industrial Area. The situation has becomes more worrisome as it has been left in the limbo.

There is the urgent need to understand and assess the various parameters such as source, rate of emission and growth of the major pollutants Ahinsan-Atosu-Agogo Industrial Area. This study aimed to set up a mathematical model to assess air pollutant dynamics for SO₂, NO₂, CO, air temperature, speed and direction and the possible growth rates of these quantities in the Ahinsan-Atosu-Agogo Industrial Area. This will enable policy makers to formulate the required policy for effective control measures to be observed by stakeholders and therefore contribute their quota towards the realization of pollution free environment. In this
respect, resource and environmental considerations must be integrated into the industrial planning and decision-making processes of government and industry.

2. Methodology

[42] observed that a cost effective air quality monitoring programme will be achieved through scientific approach. This actually begins with careful selection of project sites, a prerequisite for obtaining useful data. The study was undertaken at the Ahinsan-Atousu-Kaase Industrial Area. This an enclave of suburbs in Kumasi with a characteristic cluster of industries including the Kumasi Abattoir, Guinness Ghana Breweries Limited (GGBL) a merged company from (Guinness Ghana Limited and Ghana Breweries Limited), the Kumasi South Hospital and a host of wood processing industries. Explanations and illustrations of the dynamic behaviour of systems and the main methods of analyses were the foci. The statistical method of system identification using autoregressive time series analysis and solution of system differential equations models [43] were employed. It has been indicated that several indicators, mainly NO\textsubscript{2}, SO\textsubscript{2}, CO, particles with aerodynamic diameter less than 10 µm (PM\textsubscript{10}) and 2.5 µm (PM\textsubscript{2.5}) and ozone (O\textsubscript{3}) [44, 45] which are the priority pollutants [46] and meteorological data are involved in assessment of air pollution in space and time to address several issues including climate change, depletion of ozone layer, acidification, contamination by toxins, urban air quality and traffic air pollution. Among these NO\textsubscript{2}, SO\textsubscript{2} and CO are used for the limit values for ambient air pollution as prescribed by the World Bank. It is important to observe adequacy of sampled data and frequency in order to obtain the critical information about the processes being investigated. This informed the choice of these gases for the study.

The specific quantities used to determine the best models were: probability, residual variance and Q-statistics. The data describes the level of the SO\textsubscript{2}, NO\textsubscript{2}, CO liberated into the atmosphere: It also shows the temperature of the air and the direction and speed of the wind. This data was obtained using the The Rover Environmental Monitoring Station (REMS) sited at Ahinsan, Kumasi. The REMS device captured and recorded the selected quantities at an interval of one hour. An annual data for each quantity was obtained and were equally grouped into four under the headings: Morning, Afternoon, Evening and Night and their averages calculated. The results obtained as the averages of the levels of the parameters recorded were run on the SPSS using autoregressive model and displayed in the tables and graphs (Tables 1 to 10). The mean values for fortnights were used in the graphical representation of the phenomena (Figures 1 and 2).

3. Results

3.1. Identification of Models

In Tables 1 to 4, the model selected as the best has been marked with the asterisk (*) in all cases.

### 3.1.1. SO\textsubscript{2} Model

**i. Generation**

The results obtained as the averages of the levels of the parameters involving SO\textsubscript{2} recorded as run on the SPSS using autoregressive model are as follows:

<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Variance</th>
<th>Probability</th>
<th>Parameter</th>
<th>P Q – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00280614</td>
<td>0.89010104</td>
<td>-0.00342299</td>
<td>0</td>
</tr>
<tr>
<td>* 110</td>
<td>0.00230814</td>
<td>0</td>
<td>-0.44623523</td>
<td>0</td>
</tr>
<tr>
<td>101</td>
<td>0.00240784</td>
<td>0</td>
<td>0.75409260</td>
<td>0</td>
</tr>
<tr>
<td>111</td>
<td>0.00281305</td>
<td>0.24397407</td>
<td>0.03019034</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0.00277193</td>
<td>0.86934266</td>
<td>-0.00405087</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0.00406262</td>
<td>0</td>
<td>-0.58676521</td>
<td>0</td>
</tr>
</tbody>
</table>

**ii. Observations**

The Autoregressive model (AR (1,1)) was identified as the best in each period of the day when the recorded concentrations of SO\textsubscript{2} for morning, afternoon, evening and night were run with SPSS. This model was found to be the least in terms of probability of least error, residual variance and the Q – value.

The AR (1,1) model is defined as (as explained in chapter one)
where
\[ W_k = \alpha w_{k-1} + \mu + \epsilon_k \] (1)

\[ \alpha_1 = \text{parameter to be determined} \]

\[ W_k = X_k - X_{k-1} \]

\[ \mu = \text{Mean of pollutant level} \]

\[ \epsilon_k = \text{error term or white noise.} \]

When (1) is integrated the following Autoregressive model of the form

\[ X_k = \alpha_1 X_{k-1} + \mu_k + c + \epsilon_k \] (2)

is obtained, where \( \mu \) and \( c \) are a constant and the concentration of pollutants in the air respectively.

### Table 5. Parameters of the AR (1,1) model estimated (SO\textsubscript{2}).

<table>
<thead>
<tr>
<th>Period</th>
<th>( \alpha )</th>
<th>( \mu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>-0.446</td>
<td>0.13</td>
</tr>
<tr>
<td>Afternoon</td>
<td>-0.464</td>
<td>0.28</td>
</tr>
<tr>
<td>Evening</td>
<td>-0.269</td>
<td>0.32</td>
</tr>
<tr>
<td>Night</td>
<td>-0.375</td>
<td>0.20</td>
</tr>
</tbody>
</table>

iii. Remark: In all cases, the integration constant was found to be zero to two decimal places.

Therefore an established equation as defined by The AR (1,1) model

\[ X_k = \alpha_1 X_{k-1} + \mu_k + \epsilon_k \]

is:

\[ X_k = 0.446X_{k-1} + (0.13)k + \epsilon_k \] for (morning)
\[ X_k = 0.464X_{k-1} + (0.28)k + \epsilon_k \] for (Afternoon)
\[ X_k = 0.269X_{k-1} + (0.32)k + \epsilon_k \] for (Evening)
\[ X_k = 0.375X_{k-1} + (0.20)k + \epsilon_k \] for (Night)

The resulting deterministic dynamics difference equations are all first order difference equations with constant coefficients. The reason being that confidence levels (probability) for these models were found to be zero to two decimal places (Tables 1, 2, 3 and 4). The level of air pollution in terms of SO\textsubscript{2} in the Ahinsan-Atomsu-Agogo Industrial Area area of Kumasi is found to be growing linearly at mean growth rates of -0.446, -0.464, -0.269 and -0.375 in the order of Morning, Afternoon, Evening and Night respectively.

\[ X_k = \alpha^k (X_0) + a + bk \] (4)

Where \( X_0 \) = initial concentration of SO\textsubscript{2} during the various periods. So we have the following:

\[ X_k = \left[ X_0 + \frac{0.13}{-0.446} \right] e^{-0.446k} - \left[ \frac{0.13}{-0.446} \right] \times 1 - 0.446k \text{ for Morning} \]
\[ X_k = X_0 + 0.6535e^{-0.446k} - 0.6535(1 - 0.446k) \]
\[ X_k = \left[ X_0 + \frac{0.28}{-0.446} \right] e^{-0.446k} - \left[ \frac{0.28}{-0.446} \right] \times 1 - 0.446k \text{ for Afternoon} \]
\[ X_k = X_0 + 1.30e^{-0.446k} - 1.30(1 - 0.446k) \]
\[ X_k = \left[ X_0 + \frac{0.32}{-0.269} \right] e^{-0.269k} - \left[ \frac{0.32}{-0.269} \right] \times 1 - 0.269k \text{ for Evening} \]
\[ X_k = X_0 + 4.42e^{-0.269k} - 4.42(1 - 0.269k) \]
\[ X_k = \left[ X_0 + \frac{0.20}{-0.375} \right] e^{-0.375k} - \left[ \frac{0.20}{-0.375} \right] \times 1 - 0.375k \text{ for Night} \]
\[ X_k = X_0 + 1.42e^{-0.375k} - 1.42(1 - 0.375k) \]
3.1.2. NO$_2$ Model

i. Generation

### Table 6. Models for NO$_2$ (Morning).

<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Variance</th>
<th>Probability</th>
<th>Parameter</th>
<th>P Q – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00500226</td>
<td>0.0002303</td>
<td>-1.0529411</td>
<td>0</td>
</tr>
<tr>
<td>* 110</td>
<td>0.00447954</td>
<td>0.0000000</td>
<td>-0.5244957</td>
<td>0</td>
</tr>
<tr>
<td>101</td>
<td>0.0274500</td>
<td>-0.9992610</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>111</td>
<td>0.0045228</td>
<td>0.0007399</td>
<td>-0.5244957</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0.00499537</td>
<td>0.0005930</td>
<td>-0.9608986</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0.00489800</td>
<td>0.0000000</td>
<td>-0.7854083</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 7. Models for NO$_2$ (Afternoon).

<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Variance</th>
<th>Probability</th>
<th>Parameter</th>
<th>P Q – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.0027606</td>
<td>0.3758764</td>
<td>-0.10529411</td>
<td>0</td>
</tr>
<tr>
<td>* 110</td>
<td>0.00264302</td>
<td>0</td>
<td>-0.5244957</td>
<td>0</td>
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<tr>
<td>101</td>
<td>0.00224803</td>
<td>-0.82261863</td>
<td>0</td>
<td>0</td>
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<tr>
<td>111</td>
<td>0.00228754</td>
<td>0.1615049</td>
<td>0.03521618</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0.00224404</td>
<td>0.31674109</td>
<td>0.02463182</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0.00331251</td>
<td>0</td>
<td>-0.55613930</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 8. Models for NO$_2$ (Evening).

<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Variance</th>
<th>Probability</th>
<th>Parameter</th>
<th>P Q – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.00229391</td>
<td>0.00598615</td>
<td>0.6801792</td>
<td>0</td>
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<td>* 110</td>
<td>0.00218717</td>
<td>0</td>
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<td>101</td>
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<td>111</td>
<td>0.00233710</td>
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<tr>
<td>200</td>
<td>0.00222549</td>
<td>0.00258751</td>
<td>0.98108639</td>
<td>0</td>
</tr>
<tr>
<td>210</td>
<td>0.00315509</td>
<td>0</td>
<td>-0.55680736</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 9. Models for NO$_2$ (Night).

<table>
<thead>
<tr>
<th>Model</th>
<th>Residual Variance</th>
<th>Probability</th>
<th>Parameter</th>
<th>P Q – value</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.2711360</td>
<td>0.2711360</td>
<td>-0.02726171</td>
<td>0</td>
</tr>
<tr>
<td>* 110</td>
<td>0.00262714</td>
<td>0</td>
<td>-0.47068731</td>
<td>0</td>
</tr>
<tr>
<td>101</td>
<td>0.00307829</td>
<td>0</td>
<td>0.65262994</td>
<td>0</td>
</tr>
<tr>
<td>111</td>
<td>0.00224990</td>
<td>0.9241116</td>
<td>0.00243946</td>
<td>0</td>
</tr>
<tr>
<td>200</td>
<td>0.00224990</td>
<td>0.22848617</td>
<td>0.9949046</td>
<td>0</td>
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<tr>
<td>210</td>
<td>0.00340801</td>
<td>-0.580704</td>
<td>-0.24709199</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 10. Parameters of the AR (1,1) model estimated (NO$_2$).

<table>
<thead>
<tr>
<th>Period</th>
<th>$\alpha$</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Morning</td>
<td>-0.524</td>
<td>0.19</td>
</tr>
<tr>
<td>Afternoon</td>
<td>-0.427</td>
<td>0.28</td>
</tr>
<tr>
<td>Evening</td>
<td>-0.416</td>
<td>0.31</td>
</tr>
<tr>
<td>Night</td>
<td>-0.471</td>
<td>0.21</td>
</tr>
</tbody>
</table>

ii. Observation

The equation as defined by the AR(1.1) Model

$$X_k = \alpha X_{k-1} + \mu k + \epsilon_k$$

becomes:

$$X_k = -0.524X_{k-1} + (0.19)k + \epsilon_k \text{ for (morning)}$$

$$X_k = -0.4627X_{k-1} + (0.28)k + \epsilon_k \text{ for (Afternoon)}$$

$$X_k = -0.416X_{k-1} + (0.31)k + \epsilon_k \text{ for (Evening)}$$

$$X_k = -0.471X_{k-1} + (0.21)k + \epsilon_k \text{ for (Night)}$$

iii. Remark: In all cases, the integration constant was found to be zero to two decimal places.

iv. Deterministic NO$_2$ concentration dynamics model

The AR (1.1) model defined by the equation

$$X_k = \alpha X_{k-1} + \mu k + \epsilon_k$$

indicating a stochastic model that leads to a deterministic dynamics difference equation since the error term, $\epsilon_k$ is zero.
This deterministic dynamics difference is equation (3):

\[ X_k = -0.524X_{k-1} + (0.19) \text{ for (morning)} \]
\[ X_k = -0.427X_{k-1} + (0.28)k \text{ for (Afternoon)} \]
\[ X_k = -0.416X_{k-1} + (0.31)k \text{ for (Evening)} \]
\[ X_k = -0.471X_{k-1} + (0.21)k \text{ for (Night)} \]

The resulting deterministic dynamics difference equations are all first order difference equations with constant coefficients. The reason being that confidence levels (probability) for these models were found to be zero to two decimal places (Tables 6, 7, 8 and 9). There were linear growths of NO\textsubscript{2} pollutant in the study area with respective mean rates of -0.524, -0.427, -0.416 and -0.471 for Morning, Afternoon, Evening and Night.

From equation (4) i.e.

\[ X_k = \alpha^k (X_0) + a + bk, \]

with \(X_0 = \) initial concentration of NO\textsubscript{2} during the various sampling periods, with \(X_0 = \) initial concentration of NO\textsubscript{2} during the various sampling periods.

\[ X_k = \begin{cases} X_0 + \frac{0.19}{-0.524^2} e^{-0.524k} - \frac{0.19}{-0.524^2} \times 1 - 0.524k & \text{for Morning} \\ X_0 + 0.692 e^{0.446k} - 0.692(1 - 0.524k) & \end{cases} \]
\[ X_k = \begin{cases} X_0 + \frac{0.28}{-0.427^2} e^{-0.427k} - \frac{0.28}{-0.427^2} \times 1 - 0.427k & \text{for Afternoon} \\ X_0 + 1.54e^{0.446k} - 1.54(1 - 0.427k) & \end{cases} \]
\[ X_k = \begin{cases} X_0 + \frac{0.31}{-0.416^2} e^{-0.416k} - \frac{0.31}{-0.416^2} \times 1 - 0.416k & \text{for Evening} \\ X_0 + 1.85e^{0.416k} - 1.85(1 - 0.416k) & \end{cases} \]
\[ X_k = \begin{cases} X_0 + \frac{0.21}{-0.471^2} e^{-0.471k} - \frac{0.21}{-0.471^2} \times 1 - 0.471k & \text{for Night} \\ X_0 + 0.947e^{0.446k} - 0.947(1 - 0.471k) & \end{cases} \]

Figure 2. The fortnight concentration of NO\textsubscript{2} at Ahinsan-Atonsu-Agogo Industrial Area.

3.1.3. CO Model

The values recorded for carbon monoxide CO were found to be zero throughout and therefore no models were obtained for CO in any of the periods (morning, afternoon, evening or night).

3.1.4. Air Temperature

\[ X_k = -0.524X_{k-1} + (25.09)k + \varepsilon_k \text{ (morning)} \]
\[ X_k = -0.427X_{k-1} + (25.57)k + \varepsilon_k \text{ (Afternoon)} \]
\[ X_k = -0.416X_{k-1} + (25.83)k + \varepsilon_k \text{ (Evening)} \]
\[ X_k = -0.471X_{k-1} + (25.51)k + \varepsilon_k \text{ (Night)} \]

3.1.5. Stability of the System (Homogeneous)

The stability of the system depends on the eigenvalues of the matrix. The estimated parameters with their respective probabilities indicate diagonal matrices, implying that the
eigenvalues are the diagonals elements with their magnitudes are less than one (Tables 11 to 14).

**Table 11. Morning.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>-0.0206*</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-0.0718</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>-0.1814</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>-0.2751*</td>
</tr>
</tbody>
</table>

**Table 12. Afternoon.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>-0.4769*</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-0.4421</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>-0.2048</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>-0.4149*</td>
</tr>
</tbody>
</table>

**Table 13. Evening.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>-0.4817*</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-0.6164</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>-0.2048</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>-0.4979*</td>
</tr>
</tbody>
</table>

**Table 14. Night.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_{11}$</td>
<td>-0.5053*</td>
</tr>
<tr>
<td>$a_{12}$</td>
<td>-0.4316</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>-0.2048</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>-0.4986*</td>
</tr>
</tbody>
</table>

The parameters with probability greater than $\alpha = 0.05$ are zeros. From the above, with the exception of the parameters with asterisk, all the other parameters are zeros because they have probabilities greater than 0.05.

These generate matrices of the order morning, afternoon, evening and night respectively:

$$
\begin{pmatrix}
-0.0206 & 0.0000 \\
0.0000 & -0.2751
\end{pmatrix},
\begin{pmatrix}
-0.4769 & 0.0000 \\
0.0000 & 0.4149
\end{pmatrix},
\begin{pmatrix}
-0.4817 & 0.0000 \\
0.0000 & -0.4979
\end{pmatrix},
\begin{pmatrix}
-0.5053 & 0.0000 \\
0.0000 & -0.4986
\end{pmatrix}
$$

These are indicative of stability in terms of wind speed and direction.

3.2. The coefficients of determination

The coefficients of determination ($R^2$) of $SO_2$ and $NO_2$, as established from their regression models graphs (Figures 1 and 2 and also Table 11) indicate that the models of great strengths in explaining the variability of the response data around their means.

**Table 15. Summary of $SO_2$ and $NO_2$ models with their respective coefficients of determination.**

<table>
<thead>
<tr>
<th>Time of day</th>
<th>$SO_2$</th>
<th>$NO_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Regression model</td>
<td>$R^2$</td>
</tr>
<tr>
<td>Morning</td>
<td>$y = 0.132x + 0.6723$</td>
<td>0.9770</td>
</tr>
<tr>
<td>Afternoon</td>
<td>$y = 0.2075x + 1.0535$</td>
<td>0.9770</td>
</tr>
<tr>
<td>Evening</td>
<td>$y = 0.2317x + 1.1698$</td>
<td>0.9774</td>
</tr>
<tr>
<td>Night</td>
<td>$y = 0.1511x + 0.7657$</td>
<td>0.9773</td>
</tr>
</tbody>
</table>

4. Discussion

4.1. $SO_2$

Results showed that the average concentration of $SO_2$ in the morning is 0.13ppm this value increases to 0.28ppm in the afternoon and reaches its peak in the evening at a value of 0.38ppm in the night. It then dropped to 0.20ppm in the night. The observed increasing order may be due to the order of increasing activities associated with emission of the gas. The major source of this gas emission are the industries such as the wood industries, power stations and domestic emissions such as burning plants, protein – base foods [29].

The drop in $SO_2$ concentration in the night is a reflection of the decline in these $SO_2$ activities in the night. The peak hourly average concentrations in a 24-hour period are quite significant according to the ambient air quality guidelines by the EPA, Ghana, which indicates an average of 0.9ppm as the maximum permissible $SO_2$ level for the industrial area. The values recorded for these periods are 14.4%, 31%, 42% and 22% morning, afternoon, evening and night respectively of the Ghana EPA Guideline Level. Similarly they were quite significant based on the WHO standard of 0.5ppm (a value described as health guideline) and the Department of Environment of USA standard of < 0.16ppm which is described as a very good quality air (while 1.06ppm is described as a very poor quality air).

Moreover the growth rates of the $SO_2$ emission in the respective periods signals alarming situation. The rates of growth of $SO_2$ in the atmosphere during the morning, afternoon, evening and night respectively were high. There are therefore potential health hazards e. g. respiratory irritants, acid rains which in turn has a negative effect on the forest, aquatic habitats, corrosion of property, etc. as note by [30].

4.2. $NO_2$

The mean concentration of $NO_2$ recorded also increased from morning (0.19ppm) through the afternoon (0.28ppm) to its peak in the evening (0.31ppm). It also dropped to 0.21ppm in the night, a trend similar to that observed in the case of $SO_2$. When compared to standards (the maximum
permissible NO\textsubscript{2} level for the industrial area set by EPA, Ghana, which indicates an average of 0.4ppm), the values recorded for these periods were 14.4%, 31%, 42% and 22% respectively higher for morning, afternoon, evening and night respectively. On the account of WHO, NO\textsubscript{2} concentration should be < 0.2ppm, a value described as health guideline, while the department of environment of USA has < 0.096 and 0.57 described as a very good quality air and very poor quality air respectively. These observations indicate that values recorded in this research work are higher.

The growth rates of the NO\textsubscript{2} emission during these periods indicated an unbounded growth and for which reason the capable of the gas exerting its health and other negative effects, if the necessary steps are not taken to minimize the growth rate, cannot be ruled out. The growth rates of NO\textsubscript{2} in the atmosphere were infinitely high during the morning, afternoon, evening and night. The observed high growth rate may be due to clustering of vehicular traffic, industries and power plants in the area.

The steps to minimize the growth rate of NO\textsubscript{2} are very important because the gas contributes to the formation of low-level ozone (O\textsubscript{3}). Other striking attributes of this gas are that it is poisonous even at lower concentrations, brings about respiratory irritants and also promotes the formation of acid rains.

### 4.3. CO

The levels of CO recorded were zero throughout. This may be due to the high instability of the gas which encourages subsequent oxidation to CO\textsubscript{2} at lower level before reaching the REMS censor device.

### 4.4. Air Temperature

The trend exhibited by the air temperature did not differ from those observed for the concentrations of SO\textsubscript{2} and NO\textsubscript{2} all rising from morning through the afternoon to the evening and dropping in the night. The means for air temperature were 25.09°C, 25.05°C, 25.83°C and 25.51°C in the order of morning, afternoon, evening and night. However the values remained fairly constant, with the only fluctuation around 25°C. The growth rate, however, indicate the possibility faster growth if the necessary steps are not taken.

#### 4.4.1. Effect of Wind Speed and Direction on the Movement (Distribution) of the Gases

The system of equations is simplified with all the estimated parameters and are dealt with separately to bring out their interpretations as below.

#### 4.4.2. Morning

The equation in the system of difference equation

\[ X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1} \]

with \( a_{11} = -0.2751 \) and \( a_{12} = 0 \)

becomes

\[ X_{1k} = -0.2751 X_{1k-1} + 0X_{2k-1} \]

So \( X_{1k} = -0.2751 X_{1k-1} \) (horizontal direction),

And from \( X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1} \)

with \( a_{21} = -0.4986 \) and \( a_{22} = 0 \)

we have \( X_{2k} = -0.4986 X_{1k} + 0X_{2k-1} \)

So \( X_{2k} = -0.4986 X_{1k-1} \) (vertical direction).

The effect of the wind speed and direction is quite significant on the spread of the gases in both the horizontal and vertical directions.

#### 4.4.3. Afternoon

The system of difference equation

\[ X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1} \]

with \( a_{11} = -0.4769 \) and \( a_{12} = 0 \)

becomes

\[ X_{1k} = -0.4769 X_{1k-1} + 0X_{2k-1} \]

So \( X_{1k} = -0.4769 X_{1k-1} \) (horizontal direction),

And from \( X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1} \)

with \( a_{21} = -0.4149 \) and \( a_{22} = 0 \)

we have \( X_{2k} = -0.4149 X_{1k} + 0X_{2k-1} \)

So \( X_{2k} = -0.4149 X_{1k-1} \) (vertical direction).

The speed and direction wind affect the spread of the gases in both the horizontal and vertical directions significantly.

#### 4.4.4. Evening

The system of difference equation

\[ X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1} \]

with \( a_{11} = -0.4877 \) and \( a_{12} = 0 \)

becomes

\[ X_{1k} = -0.4877 X_{1k-1} + 0X_{2k-1} \]

So \( X_{1k} = -0.4877 X_{1k-1} \) (horizontal direction),

And from \( X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1} \)

with \( a_{21} = -0.4986 \) and \( a_{22} = 0 \)

we have \( X_{2k} = -0.4986 X_{1k} + 0X_{2k-1} \)

So \( X_{2k} = -0.4986 X_{1k-1} \) (vertical direction).

The effect of the wind speed and direction on the spread of the gases is in both the horizontal and vertical directions significantly.

#### 4.4.5. Night

The system of difference equation

\[ X_{1k} = a_{11}X_{1k} + a_{12}X_{2k-1} \]

with \( a_{11} = -0.5053 \) and \( a_{12} = 0 \)

becomes

\[ X_{1k} = -0.5053 X_{1k-1} + 0X_{2k-1} \]

So \( X_{1k} = -0.5053 X_{1k-1} \) (horizontal direction),

And from \( X_{2k} = a_{21}X_{1k} + a_{22}X_{2k-1} \)

with \( a_{21} = -0.4984 \) and \( a_{22} = 0 \)

we have \( X_{2k} = -0.4984 X_{1k} + 0X_{2k-1} \)

So \( X_{2k} = -0.4984 X_{1k-1} \) (vertical direction).

The effect of the wind speed and direction on the spread of the gases in both the horizontal and vertical directions is quite significant.

The direction and speed of the wind have been shown generally to affect the movement of the gases in both horizontal and vertical directions during the afternoon.
evening and night. However during the morning it affected only the vertical direction without the horizontal direction. This may generally be due to the clarity of that skies are clear in the night, allowing rapid radiant heat loss.

5. Conclusion

The level of pollution in terms of SO\(_2\) and NO\(_2\) are at the moment within acceptable Ghana EPA limits. However, the escalating growth rate indicates the possibility of exceeding the limits in the near future. CO level during the study period was not detectable at heights above 20 meters above the ground due to its prone to oxidation to CO\(_2\) at the at the high heights. Air temperature grew at an unbounded rate. Except in the horizontal direction in the morning, the speed and direction of the wind affected the movement of gases in all the periods. The above observations suggest that the prevailing conditions of unbounded growth rates of the studied atmospheric gases (except CO) as influenced by the aerodynamic variables point to possible rise to unacceptable levels if the necessary precautions are not taken. Measures including building of chimneys high enough to ensure effective dispersal and dilution of pollutants emitted to levels that are not harmful to human and animal health and damaging to property. Again, provision of social amenities such as schools, hospitals, markets, entertainment centers, etc around the industrial area will prevent oscillating of people between the industrial area and the central business district, which is in turn linked with heavy vehicular traffic and its associated pollution.

References