Coarse-Fine Composite Control of Manipulator’s End-Effector Based on Stereovision

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Citation

Abstract
This paper deals with the coarse-fine composite closed-loop control method of position of the manipulator’s end-effectors. A fine position tracking mechanism of the manipulator’s end-effectors is proposed, which consists of three independent prismatic joints and is fixed at the traditional manipulator’s end-link. The traditional manipulator is used in the coarse tracking of the end-effector’s position, and the mechanism is used in the fine tracking of the end-effector’s position owing to its high positioning precision caused by its shorter links. Stereovision technology is introduced into the manipulator’s control system. The object to be grasped is imaged on two CCD fixed at the end prismatic joint of the mechanism, and the two images is processed by a fast method in order to obtain the position of the object to be grasped. Then the position difference between the object to be grasped and the mechanism’s end-effectors will be fed back to input of the controller as the error of coarse tracking. The above methods realize the coarse-fine composite closed-loop control of position at manipulator’s end-effectors based on stereovision.

1. Introduction

Mobile manipulators have recently been applied to useful aerospace tasks such as assembly, inspection, or part transfer. These arms must have links of small cross-section (compared to their length), but then they exhibit an undesirable flexibility, leading to mechanical vibrations, that make the control of their tip position extremely difficult. Furthermore, due to the existence of joint angle errors, the longer the arm is, the more difficult it is to control the tip position. For example, the Canadarm2 (SSRMS) successfully installed on the Space Station during STS-100 mission in April 2001, has two symmetrical booms connected by an elbow joint, and its overall arm is 17.6 meters long. Considering that the elbow joint is in the endpoint of the overall arm, an angle error of 1° in the elbow joint may cause at most 0.307 meters error of tip position.

Closed-loop feedback control is an elementary and commonly used method in control theory, its basic principle is that comparison between the reference input and the feedback signal results in an actuating signal that is the difference between these two quantities, the actuating signal then acts to maintain the output at the desired value. However, it is impossible for the convention manipulator to obtain the position and orientation of its end-effector directly because of the lack of some necessary sensors. Thus feedback signal of the position and orientation of its end-effector has to be obtained indirectly from the angle displacements of its joints in order to actualize closed-loop feedback control of the position and the orientation of its end-effector. This kind of closed-loop feedback control actually belongs to semi-closed-loop feedback control, and can not eliminate the position error and the orientation error caused by gear meshing clearance and mechanical

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looseness\textsuperscript{[3-9]} In reference\textsuperscript{[10]}, a camera is introduced to control system of a 2-DOF manipulator, and the coordinates of the manipulator’s end-effector are resolved by analyzing the image of motion area. This coordinates are subtracted from the anticipant coordinates to get an error as a compensation of the manipulator control system, then the full close-loop control of the system is realized and the problem of low control precision brought by semi-closed-loop control is resolved. Considering that it is impossible to measure the position and the orientation of a manipulator’s end-effector accurately using only one camera, a new simple stereovision model is proposed, and a fuzzy controller\textsuperscript{[11-14]} based on only visual information is implemented to a robot manipulator in reference\textsuperscript{[15]}. However, for the conventional manipulator, even if the errors of its position and the orientation have been obtained using stereovision method\textsuperscript{[16]}, it is still very difficult to eliminate these errors and improve control precision resulting from gear meshing clearance and mechanical looseness. Hence, there are two problems we must solve simultaneously in order to control the position and orientation of a manipulator’s accurately: the first is how to obtain the manipulator’s position and orientation directly, and the second is what additional mechanism is needed to eliminate the position and the orientation error.

However, as far as the position tracking control of the manipulator’s end-effector is concerned, the above two problems can be solved by mounting a stereovision system and a fine position tracking mechanism on extremity of the manipulator. The stereovision system is used to obtain the position and the orientation of the manipulator’s end-effector, and the fine position tracking mechanism is used to eliminate the coarse tracking errors produced by original position tracking system. After adding the fine position tracking mechanism, the two position tracking systems, along with the stereovision system, will form a coarse-fine composite closed-loop control system of position of the manipulator’s end-effector.

Actually the idea of the coarse-fine composite closed-loop control has been actualized in photoelectrical tracking system, where the coarse tracking system is applied to control the gimbal, and fine tracking system is applied to control the fast steering mirror so as to refrain the remaining error in coarse tracking. Research and experiment results show that the method of the coarse-fine composite closed-loop control can improve the tracking precision and reaction speed in tracking systems, can also be applied to laser communication and beam stabilization, besides photoelectrical tracking\textsuperscript{[17]}. Unfortunately, the above method of the coarse-fine composite closed-loop control cannot be applied to the position control of the manipulator’s end-effector directly for the reason that in above-mentioned applications only orientation of light beam is need to control, but for the case of the position control of the manipulator’s end-effector we have to modulate the position while keeping the manipulator’s end-effector at one given orientation. For this reason, a fine position tracking mechanism of the manipulator’s end-effectors is presented, and the stereovision system is introduced in this paper. Based on the fine position tracking mechanism and the stereovision system, the correlative algorithms of the coarse-fine composite closed-loop control are discussed in the rest of this paper.

2. The Fine Tracking Mechanism

![Fig. 1. Appearance of the fine position tracking mechanism](image1)

![Fig. 2. Internal structure of the fine position tracking mechanisms](image2)

As shown in Fig.1, the fine position tracking mechanism consists of three translating joints, the first one of them is the parallel translating joint, i.e., expansion link, and the other two are the vertical translating joints with the same interior structure as the first one.

As shown in Fig.2, the crust of the parallel translating joint is a hollow cylinder with a motor mounted on its undersurface. The axis of the motor is connected to two screw-jointed screws with internal and external thread respectively. One end of the screw with internal thread extends out of the cylinder and is connected to the base of a vertical translating joint. The crust of the vertical translating joint is a hollow cylinder with a side notch and a motor mounted on its undersurface. The axis of the motor is connected to a screw. The sliding block
consists of two cross-connected columns. The one with bigger radius is hollow column with thread on its internal wall. The screw passes through the hollow column, and can drive the sliding block moving back and forth through its rotation. Another column extends out of the crust from the side notch, and is connected to the base of the second vertical translating joint. The sliding block of the second vertical translating joints connected to the end-effector.

The motors of the three translating joints can implement bidirectional rotation, thus the fine position tracking mechanism can drive the end-effector towards the desired direction.

As the main part of the stereovision system, two CCD cameras with optical axis parallel to the symmetry axis of the end-effector, are mounted on the crust of the second vertical translating joint. The sliding block of the second vertical translating joint is connected to the end-effector.

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As the main part of the stereovision system, two CCD cameras with optical axis parallel to the symmetry axis of the end-effector, are mounted on the crust of the second vertical translating joint.

The fine position tracking mechanisms is mounted on the end link of the manipulator, and the symmetry axis of its parallel translating joint coincides with that of the end link.

3. Measurement Coordinate System and the Positioning of the Object to be Grasped

After mounting the fine position tracking mechanisms and the stereovision system on the conventional manipulator, in order to provide positioning function, we need to define a measurement coordinate system and present correlative positioning algorithms of the object to be grasped orderly.

3.1. Measurement Coordinate System

The measurement coordinate system is defined with origin $O$ and axes $x$, $y$, $z$, where $O$ is the intersection point of common image plane of the two CCD cameras and the symmetry axis of the end-effector, axis $x$ points rightward, and coincides with the line passing through the intersection points of optical axes and common image plane of two CCD cameras, axis $y$ points upward and axis $z$ points forward.

We denote the focal length and the pixel size of the two CCD cameras by $f$ and $l$ respectively. The two image plane coordinate system is defined with origin $O_i$, axes $m_i$, $n_i$ ($i = 1, 2$) and unit $l$, where $O_i$ is the intersection point of the image plane and the optical axis of the $i$th CCD camera, axis $m_i$ is horizontal and points rightward, axis $n_i$ is vertical and points upward. We denote the coordinates of the centroid $A$ of the object to be grasped in measurement coordinate system and two image plane coordinate systems by $A(x, y, z)$, $A(m_1, n_1)$, $A(m_2, n_2)$ respectively. By the theory of pinhole imaging we have

$$x = \frac{m_1 + m_2}{m_1 - m_2} d$$

$$y = \frac{2n_1}{m_1 - m_2} d$$

$$z = \frac{2f}{l(m_1 - m_2)} d$$

In coarse position tracking process, we need to keep each translating joint of the fine position tracking mechanism fixed at its middle position, so that the symmetry axis of the end-effector coincides with that of the parallel translating joint. Therefore in this process, the fine position tracking mechanism can be regarded as a link with length $d$.

3.2. Positioning Algorithms

Without loss of generality we assume that the fine position tracking mechanisms is mounted, for example, on a general 6-axis manipulator shown in Fig.3, where $O_i$ and $\theta_i$ are origin of the $i$th coordinate system and angle displacement of the $i$th joint, $i = 0, 1, \cdots, 6$, and

$$|O_0A| = a_1, |O_2O_3| = a_2$$

$$|O_3O_4| = a_3, |O_5O_6| + d = a_4$$

The position of the manipulator’s end-effector can be expressed as

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} c_1(a_2c_2 + a_3c_3) + a_4(c_1c_23c_4s_5 - s_1s_5) \\ s_1(a_2c_2 + a_3c_3) + a_4(s_1c_23c_4 + c_1s_5) \\ d_1 - a_2s_2 - a_3s_3 - a_4s_23c_4s_5 \end{pmatrix}$$

where

$$\theta_{23} = \theta_2 + \theta_3$$

$$c_i = \cos \theta_i, s_i = \sin \theta_i (i = 1, 2, \cdots, 6)$$

The transformation matrix from the measurement coordinate system to the base coordinate system can be
expressed as \[ T = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \]

where

\[
\begin{align*}
t_{11} &= c_1c_5c_{234} - s_1s_5 \\
t_{12} &= s_1c_5c_{234} + c_1s_5 \\
t_{13} &= -c_5s_{234} \\
t_{21} &= -c_1s_5c_6c_{234} + c_1s_6s_{234} - s_1c_5e_6 \\
t_{22} &= -s_1s_5c_6c_{234} + s_1s_6s_{234} + c_1c_5e_6 \\
t_{23} &= s_5c_6s_{234} + s_6c_{234} \\
t_{31} &= c_1s_5s_6c_{234} + c_1c_6s_{234} + s_1c_5s_6 \\
t_{32} &= s_1s_5c_6s_{234} + s_1c_6s_{234} - c_1c_5s_6 \\
t_{33} &= -s_5s_6s_{234} + c_6c_{234}
\end{align*}
\]

Consequently, the coordinates of the object to be grasped in the base coordinate system can be expressed as

\[
\begin{pmatrix} u \\ v \\ w \end{pmatrix} = T \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}
\]

### 3.3. The Elimination of Position Error

After mounting the stereovision system, the manipulator can be applied to measure the position of the object to be grasped in the base coordinate system. Considering the existence of the manipulator’s end-effector position error caused by gear meshing clearance and mechanical looseness, the above coordinates of the object to be grasped can be expressed as

\[
\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{w} \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix}
\]

where \( \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} \) and \( \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \) are the real coordinates of the object to be grasped the real coordinates of the object to be grasped in the base coordinate system and the random vector representing the measurement error. We can eliminate the position error and obtain the real coordinates of the object to be grasped through many times of measurements in coarse position tracking process. For \( i = 1, 2, \ldots, n \), expressing the coordinates of the object to be grasped in the \( i \) measurement by

\[
\begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} + \begin{pmatrix} \eta_{1i}^{(i)} \\ \eta_{2i}^{(i)} \\ \eta_{3i}^{(i)} \end{pmatrix}
\]

where \( \begin{pmatrix} \eta_{1i}^{(i)} \\ \eta_{2i}^{(i)} \\ \eta_{3i}^{(i)} \end{pmatrix} \) is the random vector representing the measurement error in the \( i \) th measurement. Adding these \( n \) inequalities leads to

\[
\frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} = \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} + \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} \eta_{1i}^{(i)} \\ \eta_{2i}^{(i)} \\ \eta_{3i}^{(i)} \end{pmatrix}
\]

The error vector caused by representing the real coordinates \( \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} \) by \( \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \) is

\[
\frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} \eta_{1i}^{(i)} \\ \eta_{2i}^{(i)} \\ \eta_{3i}^{(i)} \end{pmatrix}
\]

with the variance

\[
D\left( \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} \eta_{1i}^{(i)} \\ \eta_{2i}^{(i)} \\ \eta_{3i}^{(i)} \end{pmatrix} \right) = \frac{1}{n} D\left( \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \end{pmatrix} \right)
\]

This means that the variance of the random vector \( \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix} \) is only \( 1/n \) of the random vector \( \begin{pmatrix} u \\ v \\ w \end{pmatrix} \). If \( n \) is bigger enough, the real coordinates \( \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} \) can represented by the average of the \( n \) measurement coordinates:

\[
\begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} = \frac{1}{n} \sum_{i=1}^{n} \begin{pmatrix} u_i \\ v_i \\ w_i \end{pmatrix}
\]

### 4. The Coarse-Fine Composite Closed Loop Control

The coarse-fine composite closed loop control involves the coarse position tracking control and the fine position tracking control. In coarse position tracking process, three translating joints of the fine position tracking mechanism are fixed at their middle positions, so the manipulator still can be regarded as a general 6-axis manipulator. For any desired position and orientation of its end-effector to grasp an object, the angle displacements of the six joint can be calculated by solving the associated inverse kinematics equations. \([13]\)
Whereas in fine position tracking process, six revolute joints maintain their original angle displacement. By the images of the object to be grasped from the two CCD cameras, the position \((x, y, z)\) of the object to be grasped in measurement coordinate system, i.e., the position tracking error vector in coarse position tracking process, can be obtained using the formulas in section 2. In order to eliminate the position tracking error we have to control the three translating joints of the fine position tracking mechanism so that each component of the error vector approaches zero as close as possible.

Let us denote by \(L\) the pitch of the three screws of the fine position tracking mechanism, then the angle displacements of screws of the two vertical translating joints are \(\frac{2\pi L}{4}\) (Rad) and \(\frac{2\pi}{L}\) (Rad) respectively, and the angle displacement of the screw of the parallel translating joint is \(\frac{2\pi}{L}\) (Rad). Positive value of the angle displacement means positive rotation of the screw, and negative value of the angle displacement means inverse rotation of the screw.

**5. The Motion Range of the Translating Joint**

We mount the fine position tracking mechanism on the conventional manipulator for the purpose of eliminating position error of the manipulator’s end-effector caused by gear meshing clearance and mechanical looseness, thus the motion range of the translating joint lies on the maximum of position error of the manipulator’s end-effector in coarse position tracking process.

In the case where the error caused by gear meshing clearance and mechanical looseness is ignored, the end-effector position of the manipulator shown in Fig. 3 can be expressed as

\[
\begin{bmatrix}
\mathbf{X} \\
\mathbf{Y} \\
\mathbf{Z}
\end{bmatrix} = \begin{bmatrix}
c_1(a_2c_2 + a_3c_3) + a_4(c_3c_2a_2 + s_3s_2) \\
s_1(a_2c_2 + a_3c_3) + a_4(s_3c_2a_2 + c_3s_2) \\
d_1 - a_2s_2 - a_3s_3 - a_4c_3c_5
\end{bmatrix}
\]

Therefore, if we denote by \(\xi_i\) the angle error of the \(i\) th joint caused by gear meshing clearance and mechanical looseness,

\[
\begin{align*}
C_1C_2 &= c_1c_2\cos\xi_1 - s_1c_2\sin\xi_1 \\
C_2C_3 &= s_1c_2\cos\xi_2 - s_1s_2\sin\xi_2 - s_1c_2\sin\xi_2 \\
C_3C_4 &= s_1c_2\cos\xi_3 - s_1s_2\sin\xi_3 - s_1c_2\sin\xi_3 \\
C_4C_5 &= s_1c_2\cos\xi_4 - s_1s_2\sin\xi_4 - s_1c_2\sin\xi_4 \\
C_5C_6 &= s_1c_2\cos\xi_5 - s_1s_2\sin\xi_5 - s_1c_2\sin\xi_5 \\
C_6C_7 &= s_1c_2\cos\xi_6 + c_1c_2\sin\xi_6 + s_1c_2\cos\xi_6 + c_1c_2\sin\xi_6 \\
C_7C_8 &= s_1c_2\cos\xi_7 - s_1s_2\sin\xi_7 + s_1c_2\sin\xi_7 - s_1c_2\sin\xi_7 \\
C_8C_9 &= s_1c_2\cos\xi_8 - s_1s_2\sin\xi_8 + s_1c_2\sin\xi_8 - s_1c_2\sin\xi_8 \\
C_9C_{10} &= s_1c_2\cos\xi_9 - s_1s_2\sin\xi_9 + s_1c_2\sin\xi_9 - s_1c_2\sin\xi_9
\end{align*}
\]

Regarding \(\xi_i\) as an infinitesimal, using the first order Taylor expansion we obtain

\[
\cos\xi = 1 \\
\sin\xi = \xi
\]

Ignoring the terms with order higher than one we have

\[
\begin{align*}
C_1C_2 &= c_1c_2\cos\xi_1 - s_1c_2\sin\xi_1 + c_1c_2\sin\xi_1 \\
C_2C_3 &= s_1c_2\cos\xi_2 - s_1s_2\sin\xi_2 + s_1c_2\sin\xi_2 \\
C_3C_4 &= s_1c_2\cos\xi_3 - s_1s_2\sin\xi_3 + s_1c_2\sin\xi_3 \\
C_4C_5 &= s_1c_2\cos\xi_4 - s_1s_2\sin\xi_4 + s_1c_2\sin\xi_4 \\
C_5C_6 &= s_1c_2\cos\xi_5 - s_1s_2\sin\xi_5 + s_1c_2\sin\xi_5 \\
C_6C_7 &= s_1c_2\cos\xi_6 + c_1c_2\sin\xi_6 + s_1c_2\cos\xi_6 + c_1c_2\sin\xi_6 \\
C_7C_8 &= s_1c_2\cos\xi_7 - s_1s_2\sin\xi_7 + s_1c_2\sin\xi_7 - s_1c_2\sin\xi_7 \\
C_8C_9 &= s_1c_2\cos\xi_8 - s_1s_2\sin\xi_8 + s_1c_2\sin\xi_8 - s_1c_2\sin\xi_8 \\
C_9C_{10} &= s_1c_2\cos\xi_9 - s_1s_2\sin\xi_9 + s_1c_2\sin\xi_9 - s_1c_2\sin\xi_9
\end{align*}
\]
\( S_1 C_{23} = s_1 c_{23} \cos \xi \cos (\xi_2 + \xi_3) - s_1 s_2 c_{23} \cos \xi \sin (\xi_2 + \xi_3) + c_1 c_{23} \sin \xi \cos (\xi_2 + \xi_3) = s_1 c_{23} - s_1 s_{23} (\xi_2 + \xi_3) + c_1 c_{23} \xi \)

\( S_{234} C_{5} = s_{234} \cos (\xi_2 + \xi_3 + \xi_4) c_5 \cos \xi_5 - s_{234} \cos (\xi_2 + \xi_3 + \xi_4) s_5 \sin \xi_5 + c_{234} \sin \xi_5 (\xi_2 + \xi_3 + \xi_4) c_5 \cos \xi_5 \)

\( = s_{234} c_5 - s_{234} s_5 c_5 + c_{234} c_5 (\xi_2 + \xi_3 + \xi_4) \)

\( C_{234} C_{5} = c_{234} c_5 \cos (\xi_2 + \xi_3 + \xi_4) \cos \xi_5 - c_{234} s_5 \cos (\xi_2 + \xi_3 + \xi_4) \sin \xi_5 - s_{234} c_5 \sin (\xi_2 + \xi_3 + \xi_4) \cos \xi_5 \)

\( = c_{234} c_5 - c_{234} s_5 c_5 - s_{234} c_5 (\xi_2 + \xi_3 + \xi_4) \)

\( C_1 C_{234} C_5 = c_1 \cos \xi_2 c_{234} c_5 - c_1 \cos \xi_2 c_{234} s_5 \cos \xi_5 - c_1 \cos \xi_2 c_{234} s_5 \sin \xi_5 \) \( \sin \xi_5, c_1 \cos \xi_2 c_{234} c_5 \)

\( = c_1 c_{234} c_5 - c_1 c_{234} s_5 c_5 - c_1 c_{234} s_5 (\xi_2 + \xi_3 + \xi_4) \)

\( S_1 C_{234} C_5 = s_1 \cos \xi_2 c_{234} c_5 - s_1 \cos \xi_2 c_{234} s_5 \cos \xi_5 - s_1 \cos \xi_2 c_{234} s_5 \sin \xi_5 - c_1 \sin \xi_2 c_{234} c_5 \)

\( = s_1 c_{234} c_5 - s_1 c_{234} s_5 c_5 - c_1 c_{234} c_5 (\xi_2 + \xi_3 + \xi_4) \)

Now we substitute all the foregoing expressions into the two equations about \( \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \) and \( \begin{bmatrix} \tilde{X} \\ \tilde{Y} \end{bmatrix} \), and obtain

\[
X - \tilde{X} = X - (a_1 c_2 + a_1 c_3 + a_1 c_{23} + a_1 c_{23} C_5 - a_4 S_2 S_3) = a_1 [c_1 s_2 \xi + c_1 s_2 \xi] + a_1 [c_1 s_2 (\xi_2 + \xi_3) + s_1 c_{23} \xi]
\]

\[
+ a_1 [c_1 c_{23} s_1 \xi + s_1 c_{23} c_1 \xi] + a_4 (s_1 c_{23} \xi + c_1 s_1 \xi)
\]

\[
= a_1 [s_1 c_{23} \xi + s_1 c_{23} \xi] + a_1 [c_1 c_{23} \xi + c_1 c_{23} \xi] + a_4 (s_1 c_{23} \xi + c_1 s_1 \xi)
\]

\[
= (a_1 s_1 \xi + a_1 c_1 \xi + s_1 c_{23} \xi) + c_1 (s_1 + c_1) \xi + a_4 (s_1 c_{23} + c_1 s_1) \xi
\]

\[
+ a_1 (s_1 \xi + c_1 \xi + c_1 \xi) \xi
\]

\[
Y - \tilde{Y} = Y - S_1 (a_1 C_2 + a_1 C_3) - a_4 (S_2 C_{23} + C_1 S_3) = a_2 s_2 s_3 \xi - a_2 c_2 c_3 \xi + a_1 s_1 s_2 (\xi_2 + \xi_3) - a_1 c_1 c_2 \xi
\]

\[
+ a_2 s_2 c_{23} \xi - a_2 c_2 c_{23} \xi - a_2 c_2 c_3 \xi + a_1 s_1 s_2 (\xi_2 + \xi_3) - a_a c_1 c_2 \xi
\]

\[
= (a_1 s_1 \xi - a_2 c_2 c_3 \xi - a_2 c_2 c_3 \xi + a_1 c_1 \xi + s_1 a_1 s_1 \xi + a_2 a_1 s_1 \xi + a_2 a_1 c_1 \xi + s_1 a_1 s_1 \xi + a_2 a_1 c_1 \xi + a_2 a_1 c_1 \xi)
\]

\[
Z - \tilde{Z} = Z - d_4 + a_2 S_2 + a_2 S_2 + a_2 S_2 + a_2 S_2 = a_2 c_2 \xi + a_2 c_2 \xi + a_2 (\xi_2 + \xi_3) + a_1 [c_2 c_1 (\xi_2 + \xi_3) + s_2 s_3 (\xi_2 + \xi_3)]
\]

\[
= a_1 c_2 c_3 + a_1 c_2 c_3 \xi + a_1 c_2 c_3 \xi - a_2 s_3 s_3 \xi
\]

Accordingly we have

\[
\|X - \tilde{X}\| \leq (a_1 + a_4) \|\xi\| + (a_1 + a_3) \|\xi_2 + \xi_3\| + (a_1 + a_4) \|\xi_2 + \xi_3\| + 2 a_4 \|\xi_5\| \|Y - \tilde{Y}\| \leq (a_2 + a_3 + a_4) \|\xi\| + (a_2 + a_3 + a_4) \|\xi\|
\]

\[
+ (a_3 + a_4) \|\xi\| + 2 a_4 \|\xi_5\| \|Z - \tilde{Z}\| \leq (a_2 + a_3 + a_4) \|\xi_2 + \xi_3\| + (a_2 + a_3 + a_4) \|\xi_5\| + a_4 \|\xi_4\| + a_4 \|\xi_5\|
\]

By considering the roundedness of these the joint angle errors, we denote by \( \xi_0 \) the common upper bound of their absolute value, i.e.,

\[
\|\xi\| \leq \xi_0 (i = 1,2, \cdots 5)
\]

then

\[
\|X - \tilde{X}\| \leq (2a_2 + 3a_3 + 7a_4) \xi_0
\]

\[
\|Y - \tilde{Y}\| \leq (2a_2 + 3a_3 + 7a_4) \xi_0
\]

\[
\|Z - \tilde{Z}\| \leq (a_2 + 2a_3 + 4a_4) \xi_0
\]

\[
\sqrt{\|X - \tilde{X}\|^2 + \|Y - \tilde{Y}\|^2 + \|Z - \tilde{Z}\|^2}
\]

\[
\leq \sqrt{2(2a_2 + 3a_3 + 7a_4)^2 + (a_2 + 2a_3 + 4a_4)^2} \xi_0
\]
For example, in the case of
\[ a_2 = 2.5 \text{ (m)}, \quad a_3 = 1.5 \text{ (m)}, \quad a_4 = 0.1 \text{ (m)} \]
we have
\[ \sqrt{x^2 + y^2 + z^2} \leq 15.58 \xi_0 \]

Furthermore, if \( \xi_0 = 1^\circ = 0.0174 \text{ (rad)} \), as long as the translating length of the translating joints is bigger than 0.54 meter, the fine position tracking mechanism can be able to eliminate the residual position error of the manipulator’s end-effector in coarse position tracking process.

6. Conclusions

After mounting the fine position tracking mechanism and the stereovision system on the conventional manipulator’s end-effectors, the coarse-fine composite closed-loop control method can be applied to the new manipulator for the purpose of eliminating the position error of the end-effector caused by gear meshing clearance and mechanical looseness. The application of the stereovision system in the manipulator not only makes it possible for the closed-loop feedback control of the manipulator to be implemented, but also provides the manipulator with positioning function.

Actually, as far as conventional manipulator is concerned, the gear meshing clearance and mechanical looseness cause not only the position error of the end-effector, but also the orientation error of the end-effector. Moreover, from the analysis of the position error of the end-effector in section 4, we know that position error of the end-effector will diminish as the length of the links diminishes. Therefore if the orientation error of the end-effector may be neglected, the 6-axis manipulator can improve its position tracking precision by using shorter links, but the orientation precision of its end-effector cannot be improved by using shorter links. Thus, how to implement the fine orientation tracking of the end-effectors a problem needs to be resolved.

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References

Biography

**Kai An** was born in 1957. He received his B.S. degree in applied mathematics from Shanxi University, Taiyuan, China, in 1982, M.S. degree in applied mathematics from Northwestern Polytechnical University, Xi’an, China, in 1989, and Ph.D. degree in control theory and control engineering from Xi’an Jiaotong University, Xi’an, China, in 2000. From 2000 to 2002 he was a Postdoctoral Associate at the Institute of Optics and Electronics, Chinese Academy of Sciences, Chengdu, China. He has published more than 100 conference and journal papers, and his more than 30 inventions have been patented. His research interests include intelligent control, optical engineering, and space robot. He may be contacted at an@163.com.