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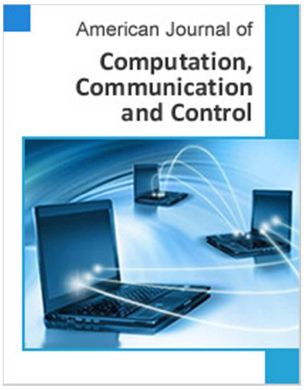

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Comparison of the Advanced D-Partitioning of Continuous and Digital Control Systems with Other Stability Analysis Methods

Kamen Yanev

Department of Electrical Engineering, Faculty of Engineering and Technology, University of Botswana, Gaborone, Botswana

Email address
yanevkm@yahoo.com, yanevkm@mopipi.ub.bw

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Abstract
This article contributes for the further advancement of an innovative broad-spectrum analysis tool that can determine the regions of stability, defined by the variations of the system's parameters and their interaction in the n-dimensional parameter space. The analysis of the interaction between the uncertain system parameters is bringing new ideas in the solution of the problem of stability. Control systems are discussed in the continuous and in the digital time-domain. This innovative broad-spectrum analysis tool, classified as Advanced D-Partitioning, is compared with other stability analysis methods applied to systems with multivariable parameters. The comparison reveals the considerable advantages of the Advanced D-Partitioning in terms of quick and graphical determination of the regions of stability defined by the variation of the system's parameters and their interaction in the parameter space.

1. Introduction

Most of the research on the matter of stability analysis of systems with variable parameters is limited to very specific cases. There is a shortage of universal analysis tool, procedure or algorithm that can show the variable parameter margins and their interaction for different type of control systems.

One of the main objectives of this research is to reveal a general stability analysis tool, rather than one attached to just few specific control systems, or systems with specific limitations. Following some initial ideas of Neimark categorized as D-Partitioning [1], this technique was better clarified and further developed by the author of this research in his previous published work [2], [3], [4], [5] and it was classified as the method of Advanced D-Partitioning. After this further expansion, the method of the Advanced D-Partitioning became a powerful tool for system analysis. The approach to develop this broad-spectrum analysis tool for linear control systems is transforming the Laplace s-plane into an n-dimensional parameter space with the aid of interactive MATLAB procedures. This introduces a clear graphical display of the system's parameters variation and their interaction. As a result, this determines transparent regions of stability and instability in the parameter space.

Further advancement of the D-Partitioning method is demonstrated in this research, by further expanding the system discretization, employing the Bilinear Transform, known also as the Tustin's Method [6]. The suggested approach converts the continuous-time system prototypes into its discrete-time equivalent. This gives a unique opportunity for further advancement of the D-partitioning analysis for its application in the discrete-time domain, considering the Euler's approximation.

The current research explores plants with multivariable parameters, inspired by cases of abnormal behaving systems, being stable at lower parameter values, further becoming unstable at a specific parameter range and turn out to be again stable at higher parameter values [3], [7]. This phenomenon was caused by the simultaneous variation of some of the system's parameters.

This research is also demonstrating the benefits of the Advanced D-Partitioning method for stability analysis of systems with uncertain parameters, compared to other well-known methods. The comparison with Nyquist and Bode stability criterion [8], [9] and the Kharitonov's Theorem assessment method [10], [11], [12] shows the considerable advantages of the Advanced D-Partitioning for stability analysis of control systems with variable parameters.

2. The D-Partitioning as Proposed by Neimark

Neimark suggested that the space of an *n*-order system's characteristic equation coefficients can be partitioned into a number of regions corresponding to the number of roots in the left-hand side of the s-plane. The position of the characteristic equation roots in the s-plane depends on the values of the system's parameters. This method was categorized as D-Partitioning [1]. In its initial ideas it was only theoretical, its applications were limited and it was rarely implemented because of its obscurity.

3. Advancement of the D-Partitioning in Case of One Variable Parameter

In previously published work [3] [4], the author of this research suggested further advancement of the method of the D-Partitioning stability analysis of continuous control systems in case of one variable parameter.

To facilitate the upgrade of the stability analysis, the system's characteristic equation can be presented in the format, exposing the variable parameter:

$$G(s) = P(s) + vQ(s) = 0 \tag{1}$$

The D-partitioning regions could be obtained if the following substitution is applied $s = j\omega$, from where the variable parameter is presented as a complex number, seen from equations (2) and (3):

$$G(j\omega) = P(j\omega) + vQ(j\omega) = 0 \tag{2}$$

$$v = -\frac{P(j\omega)}{Q(j\omega)} = X(\omega) + jY(\omega) \tag{3}$$

The D-partitioning regions are obtained graphically in the ν-plane for range of frequency variation $-\infty \le \omega \le +\infty$. Since, in the s-plane analysis, the region of stability is on the left-hand side of the plane, in the complex plane $v = X(\omega) + jY(\omega)$, the region of stability remains always on the left-hand side of the D-Partitioning curve for a change of frequency from $-\infty$ to $+\infty$. [3], [4].

The D-partitioning curve in terms of one variable parameter can be plotted in the complex plane facilitated by the MATLAB "nyquist" m-code. To avoid any misinterpretation of the D-Partitioning procedure, the "nyquist" m-code is modified into a "dpartition" m-code with the aid of the MATLAB Editor and a proper formatting. The "dpartition" m-code will plot the curve of a specific system parameter in terms of the frequency variation from $-\infty$ to $+\infty$ [3], [4].

In this research, a number of original examples of practical implementation of the method will be shown. As initial example, a real-life cruise control system is reduced to a third order system of Type 0 [12]:

The open loop linear transfer function of the system with variable parameter *K* is:

$$G_O(s) = \frac{1000K}{(s+10)(s+50)(s+100)} \tag{4}$$

The transfer function of the unity feedback control system can be represented as:

$$G_{CL} = \frac{1000K}{(s+10)(s+50)(s+100)+1000K} \tag{5}$$

The system gain is $0.02K$, while *K* is the gain factor, being a variable parameter due to some temperature effects within the environment of its operation. The characteristic equation of the feedback system is:

$$G(s) = (s+10)(s+50)(s+100) + 1000K = 0 \tag{6}$$

The variable parameter equation (6) is modified as:

$$G(s) = P(s) + KQ(s) = 0 \tag{7}$$

Where the polynomials of equation (7) are as follows:

$$P(s) = (s+10)(s+50)(s+100) \tag{8}$$

$$Q(s) = 1000 \tag{9}$$

Therefore, the variable parameter is presented as:

$$K(s) = -\frac{P(s)}{Q(s)} = -\frac{s^3 + 160s^2 + 6500s + 50000}{1000} \tag{10}$$

The D-partitioning curve in terms of one variable parameter *K* is plotted in the complex plane within the frequency range $-\infty \le \omega \le +\infty$, facilitated by the following code and illustrated in Figure 1.

```
>> K=tf([-1 -160 -6500 -50000],[1000])
Transfer function:
-s^3 - 160 s^2 - 6500 s - 50000
------------------------------
1000
>> dpartition(K)
```

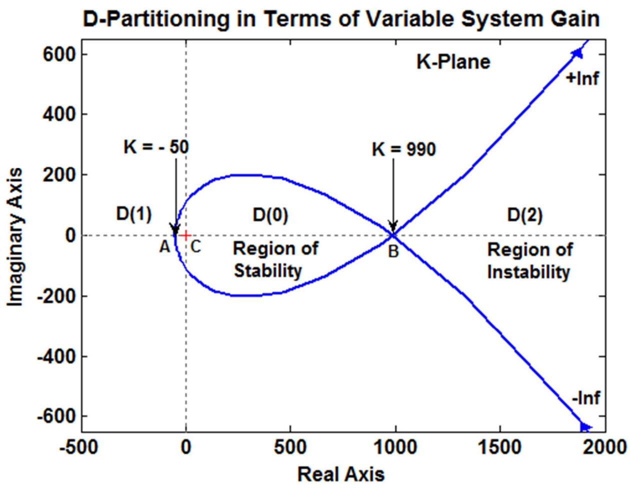

Figure 1. D-Partitioning Facilitated by the "dpartition" m-code.

The Advanced D-partitioning determines three regions on the *K*-plane: D(0), D(1) and D(2). Only D(0) is the region of stability, being the one, always on the left-hand side of the curve for a frequency variation from –∞ to +∞. The factor *K* range related to the segment *AB*, corresponds to a stable system being within the stable region D(0).

4. Comparison with Nyquist Stability Criterion (Continuous System and One Variable Parameter)

Ignoring the negative values of K, the results from the Advanced D-partitioning are compared and confirmed with the outcome from the Nyquist stability criterion for the cases of the positive gain factors $K \in [400, 1200]$ corresponding to system gains $0.02K \in [8, 24]$. This is facilitated by the following code and shown in Figure 2.

```
>> K=[400:50:1200];
>> for n=1:length(K)
Ao_array(:,:,n)=tf([1000*K(n)], [1 160 6500 50000]);
end
>> Ao1=tf([990000],
[1 160 6500 50000])
Transfer function:
990000
-----------------------------
s^3 + 160 s^2 + 6500 s + 50000
nyquist(Ao_array,Ao1)
>> nyquist(Ao_array,Ao1)
```

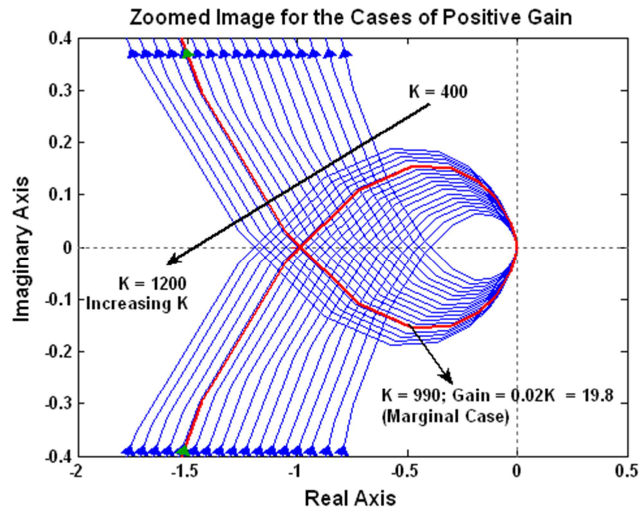

Figure 2. Zoomed Image of the Marginal Gain, Confirmed with the Aid of the Nyquist Stability Criterion.

The case of $K = 990$, or a system gain $0.02K = 19.8$ corresponds to a marginal case, while the case of $K = 1200$, corresponding to $0.02K = 24$ relates to an unstable state of the system. The results from the Linear Time Invariant (LTI) array model confirm exactly the results from the Advanced D-Partitioning analysis. Differing from the Nyquist criterion, the Advanced D-Partitioning detects directly the variable parameter marginal value, delivering much quickly and accurately results in a simpler approach.

5. Comparison with Nyquist Stability Criterion (Digital System and One Variable Parameter)

The stability of the same system is examined in the discrete-time domain. Initially *K(s)* is introduced as a continuous-time function and next converted into its digital equivalent *K(z),* facilitated by the Tustin Transform [6], [7]. In accordance with the Euler's approximation [6], [13], observing that $T_s \leq (0.1T_{min}$ to $0.2T_{min})$, the sampling period is chosen as $T_s = 0.001$sec, since the continuous system minimum time-constant is $T_{min} = 0.01$sec. Then, the Advanced D-Partitioning in terms of the variable gain factor *K* is shown in Figure 3 and is achieved in the discrete-time domain by the code:

```
>> K=tf([-1 -160 -6500 -50000],[1000])
Transfer function:
-s^3 - 160 s^2 - 6500 s - 50000
------------------------------
1000
>> Kd = c2d(K,0.001,'tustin')
Transfer function:
-8.653e006 z^3 + 2.463e007 z^2 - 2.335e007 z + 7.373e006
-------------------------------------------------------
z^3 + 3 z^2 + 3 z + 1
Sampling time: 0.001
>> dpartition(Kd)
```

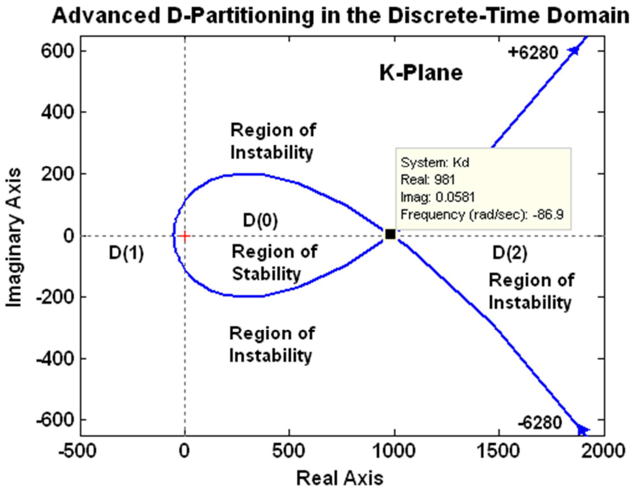

Figure 3. *Advanced D-Partitioning in Terms of the Variable Gain Factor K in the Discrete-Time Domain.*

If the sampling frequency is ω_S, the D-Partitioning curve at Figure 3 is plotted in the discrete-time domain within the frequency range $\omega = \pm\omega_s/2 = \pm 2\pi/2T_s = \pm 6280$rad/sec. There is a very close match between the system marginal results obtained in continuous-time domain ($K = 900$) and the results in discrete-time domain ($K = 981$). The minor difference is due to the Euler's approximation.

By already knowing the marginal value $K = 981$ from the Advanced D-Partitioning, the result can be confirmed in the discrete-time domain with the aid of the Nyquist stability criterion, as sown in Figure 4, by applying the code:

```
>> Gol=tf([0 981000],[1 160 6500 50000])
>> Gold=c2d(Gol,0.001,'tustin')
Transfer function:
0.0001134 z^3 + 0.0003401 z^2 + 0.0003401 z + 0.0001134
-------------------------------------------------------
z^3 - 2.846 z^2 + 2.698 z - 0.8521
Sampling time: 0.001
>> nyquist(Gold)
```

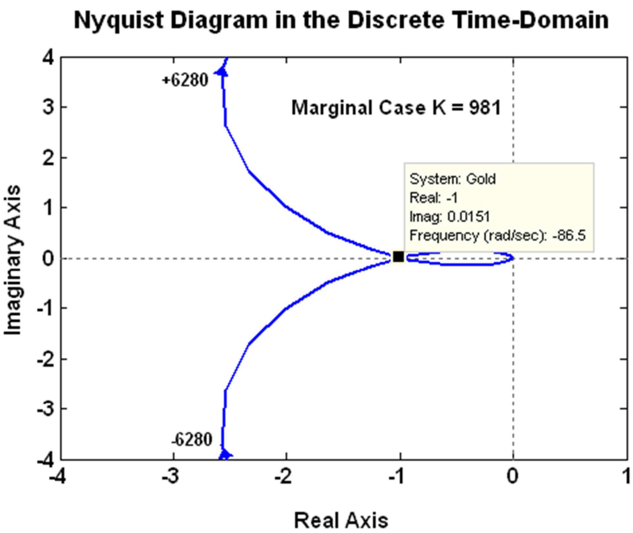

Figure 4. *Confirmation of the Result from the Advanced D-Partitioning with the Aid of the Nyquist Stability Criterion in the Discrete-Time domain.*

As is seen from Figure 4, the Nyquist curve of the digital open loop control system is passing exactly via the point (-1, j0), therefore the closed loop digital control system is marginal. This confirms precisely the results, obtained from the Advanced D-Partitioning stability method.

In case the marginal value $K = 981$ is not known, it would be a time-consuming effort to determine and verify it with the aid of the Nyquist stability criterion for digital control systems. This confirms the advantage of the Advanced D-Partitioning that promptly determines the marginal value.

6. Comparison with Bode Stability Criterion (Digital System and One Variable Parameter)

By already knowing this marginal value $K = 981$, the result can be verified in the discrete-time domain also by the Bode stability criterion by applying the code:

```
>> Gol=tf([0 981000],[1 160 6500 50000])
Transfer function:
981000
-----------------------------
s^3 + 160 s^2 + 6500 s + 50000
>> Gold=c2d(Gol,0.001,tustin')
Transfer function:
0.0001134 z^3 + 0.0003401 z^2 + 0.0003401 z + 0.0001134
-------------------------------------------------------
z^3 - 2.846 z^2 + 2.698 z - 0.8521
Sampling time: 0.001
>> margin(Gold)
```

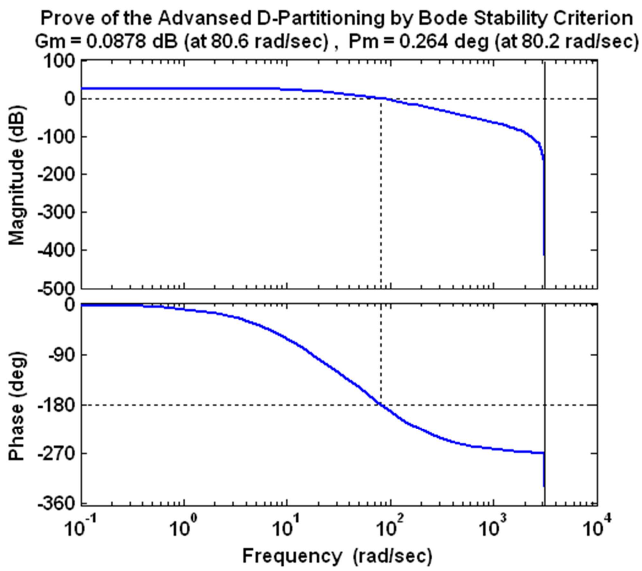

Figure 5. *Confirmation of the Result from the Advanced D-Partitioning with the Aid of the Bode Stability Criterion in the Discrete-Time Domain.*

As is seen from Figure 5, the Gain and the phase margins are accordingly GM = 0.0878 ≈ 0 and PM = 0.264 ≈ 0, that proves the marginal state of the digital control system. The operating frequency is restricted to $\omega = \omega_s/2$.

7. Advancement of the D-Partitioning in Case of Two Variable Parameter

Further advancement of the D-Partitioning for systems with two simultaneously variable parameters is suggested by the author in previous publication [3]. The general characteristic equation of a system can be presented as:

$$G(s) = \mu P(s) + \gamma Q(s) + R(s) = 0 \tag{11}$$

where *P(s), Q(s),* and *R(s)* are polynomials of *s*

μ and γ are system's variables parameters

The border of the D-Partitioning regions in the plain (μ, γ) is determined by [3]:

$$G(j\omega) = \mu P(j\omega) + \gamma Q(j\omega) + R(j\omega) = 0 \tag{12}$$

A case is demonstrated for a control system of the armature-controlled dc motor and a type-driving mechanism. The gain *K* and one of the time-constants *T* are uncertain and simultaneously variable. Initially, the system is presented in the continuous time-domain and further is converted into its digital equivalent. The open-loop transfer function of the continuous system is:

$$G_{O2}(s) = \frac{K}{(1+Ts)(1+0.5s)(1+0.8s)} \tag{13}$$

The characteristic equation of the feedback system is:

$$K + (1+Ts)(1+0.5s)(1+0.8s) = 0 \tag{14}$$

By substituting $s = j\omega$ equation (14) is modified to:

$$K = -1 + (1.3T + 0.4)\omega^2 + j\omega(0.4T\omega^2 - 1.3 - T) \tag{15}$$

The imaginary term of equation (15) is set to zero, since the gain *K* may obtain only real values. Therefore:

$$\omega^2 = \frac{1.3 + T}{0.4T} \tag{16}$$

After the result of (16) is substituted into equation (15):

$$K = \frac{1.3T^2 + 1.69T + 0.52}{0.4T} = 3.25T + 4.225 + \frac{1.3}{T} \tag{17}$$

The D-Partitioning curve $K = f(T)$, shown in Figure 6, is plotted with the aid of the following code:

```
>> T = 0:0.1:5;
>> K = 3.25.*T+4.225+1.3./T
K =
Columns 1 through 10
Inf 17.5500 11.3750 9.5333 8.7750 8.4500 8.3417 8.3571 8.4500
8.5944
Columns 11 through 20
8.7750 8.9818 9.2083 9.4500 9.7036 9.9667 10.2375 10.5147
10.7972 11.0842
Columns 21 through 30
11.3750 11.6690 11.9659 12.2652 12.5667 12.8700 13.1750
13.4815 13.7893 14.0983
Columns 31 through 40
14.4083 14.7194 15.0313 15.3439 15.6574 15.9714 16.2861
16.6014 16.9171 17.2333
Columns 41 through 50
17.5500 17.8671 18.1845 18.5023 18.8205 19.1389 19.4576
19.7766 20.0958 20.4153
Column 51
20.7350
>> plot(T,K)
```

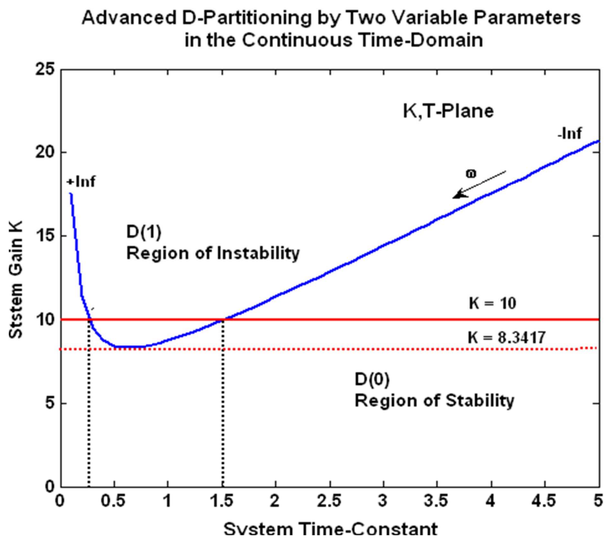

Figure 6. *Advanced D-Partitioning by Two Variable Parameters in the Continuous-Time domain.*

As seen from Figure 6, if the gain is $K = 10$, for variation of the time-constant within the range $0 < T < 0.25$sec and $T > 1.5$sec, the system is stable, but it becomes unstable in the range 0.25 sec < T < 1.5 sec. Higher values of *K*, enlarges the range of *T* at which the system will fall into instability, but if $K < 8.3417$ the system will be stable for any value of the *T*. It is evident that the system stability depends on the interaction between the two simultaneously varying parameters.

8. Comparison with Bode Stability Criterion (Digital System and Two Variable Parameters)

The examined control system with the two variable parameters is converted into its digital equivalent and is tested in the discrete-time domain for any of the simultaneous marginal values of the variables T and K. Equation (13) is modified by substituting any two simultaneous marginal values of the two variable parameters, for instance, T = 0.25 sec and K = 10. Taking into consideration the Euler's approximation, $T_s \le (0.1T_{min}$ to $0.2T_{min})$, the sampling period is chosen as $T_s = 0.05$sec, since the system's minimum time-constant is $T_{min} = 0.5$sec. The system's stability assessment in the discrete-time domain for this case is achieved by the code:

```
>> Go21=tf([0 10],[0.1 0.725 1.55 1])
Transfer function:
10
--------------------------------
0.1 s^3 + 0.725 s^2 + 1.55 s + 1
>> God21 = c2d(Go21,0.05,'tustin')
Transfer function:
0.001312 z^3 + 0.003935 z^2 + 0.003935 z + 0.001312
---------------------------------------------------
z^3 - 2.662 z^2 + 2.359 z - 0.6954
Sampling time: 0.05
>> margin(God21)
```

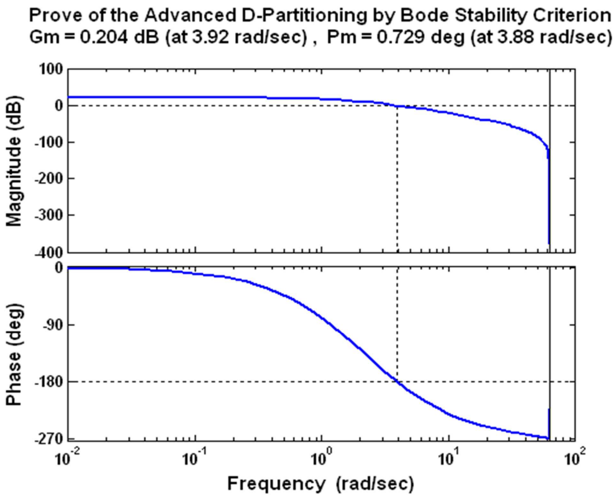

Figure 7. *System's Marginal Result from the Advanced D-Partitioning in Case of Two Variable Parameters, confirmed by the Bode Stability Criterion in the Discrete-Time Domain.*

As seen from Figure 7, if the simultaneously marginal values of the variable parameters are T = 0.25 sec and K = 10, the results for the digital system gain margin is GM = 0.017 dB ≈ 0 dB and the system phase margin is PM = 0.028° ≈ 0°, proving that the system is marginal.

Further, the system stability is explored for stability in the discrete-time domain, allocating T = 1 sec and K = 5, by applying the following code:

```
>> Go22=tf([0 5],[0.4 1.7 2.3 1])
Transfer function:
5
----------------------------
0.4 s^3 + 1.7 s^2 + 2.3 s + 1
>> God22 = c2d(Go22,0.05,'tustin')
Transfer function:
0.000176 z^3 + 0.0005279 z^2 + 0.0005279 z + 0.000176
-----------------------------------------------------
z^3 - 2.795 z^2 + 2.604 z - 0.8085
Sampling time: 0.05
>> margin(God22)
```

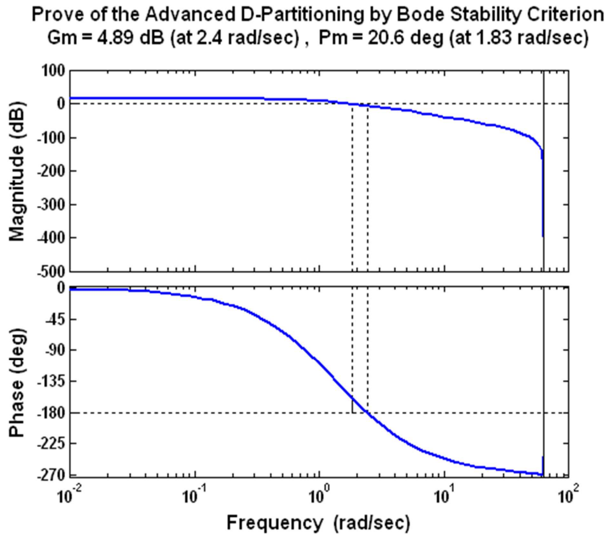

Figure 8. *System's Stability Result from the Advanced D-Partitioning in Case of Two Variable Parameters, confirmed by the Bode Stability Criterion in the Discrete-Time Domain.*

The positive gain and phase margins (GM = 4.89 dB and PM = 20.6°), prove that the digital system is stable.

The system's instability is proven in the discrete-time domain by allocating T = 1 sec and K = 14, by the code:

```
>> Go23=tf([0 14],[0.4 1.7 2.3 1])
Transfer function:
14
----------------------------
0.4 s^3 + 1.7 s^2 + 2.3 s + 1
>> God23 = c2d(Go23,0.05,'tustin')
Transfer function:
0.0004927 z^3 + 0.001478 z^2 + 0.001478 z + 0.0004927
-----------------------------------------------------
z^3 - 2.795 z^2 + 2.604 z - 0.8085
Sampling time: 0.05
>> margin(God23)
```

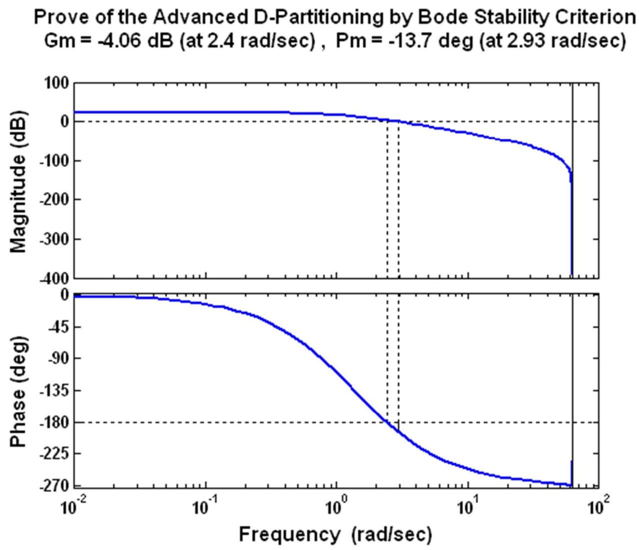

Figure 9. *System's Instability Result from the Advanced D-Partitioning in Case of Two Variable Parameters, confirmed by the Bode Stability Criterion in the Discrete-Time Domain.*

As seen from Figure 9, the negative values of the digital system margins (GM = –4.06 dB and PM = – 13.7°), prove that the digital closed loop system is unstable and is operating within the D-Partitioning region of instability D(1).

9. Comparison with Kharitonov's Theorem Assessment (Case of Two Variable Parameters)

Further, the method of the Advanced D-Partitioning is compared with the Kharitonov's theorem assessment [9], [10], [14], [15]. The comparison will be demonstrated for the two-variable parameter system of the armature-controlled dc motor and a type-driving mechanism with the open-loop transfer function shown in equation (13).

The Kharitonov's theorem assessment is used in the case where the coefficients are known to be within specified ranges. It provides a test of stability for a so-called interval polynomial that is the family of all polynomials:

$$P(s) = a_0 + a_1 s^1 + a_2 s^2 + ... + a_n s^n = 0 \qquad (18)$$

This polynomial is the characteristic equation of the control system with variable parameters, where each of its coefficients a_i can take any value in the specified intervals $a_i \in [a_i^-, a_i^+]$. The notations a_i^- and a_i^+ represent the lower and the upper limits of the variable coefficients.

The interval polynomial is considered as stable if the four Kharitonov polynomials represented in the set of equations (19) are stable [8], [12], [16].

$$\begin{aligned} P_1(s) &= a_0^- + a_1^- s^1 + a_2^+ s^2 + a_3^+ s^3 + ... = 0 \\ P_2(s) &= a_0^+ + a_1^+ s^1 + a_2^- s^2 + a_3^- s^3 + ... = 0 \\ P_3(s) &= a_0^+ + a_1^- s^1 + a_2^- s^2 + a_3^+ s^3 + ... = 0 \\ P_4(s) &= a_0^- + a_1^+ s^1 + a_2^+ s^2 + a_3^- s^3 + ... = 0 \end{aligned} \qquad (19)$$

There is a specific arrangement of the lower limit and upper limit coefficients at each one of four these polynomials. Each of the four Kharitonov polynomials is tested for stability with the aid of the Routh-Hurwitz stability criterion [17], [18]. The results are placed in tables for final assessment of the system's stability.

For the discussed system, its characteristic equation (13) is presented as an interval polynomial, where the variable gain K and the variable time-constant T are defined within specific limits.

9.1. Case of Instability

The characteristic equation (13) is modified to the interval polynomial, shown in equation (20), now being a family of all polynomials:

$$P_A(s) = K_1 + 1 + (T_1 + 1.3)s + (1.3T_1 + 0.4)s^2 + 0.4T_1 s^3 \qquad (20)$$

Where:

$K_1 \in [8, 10]$; $T_1 \in [1, 2]$;

Considering equation (20), the lower and the upper limits of the suggested variable parameters are to be substituted. Accordingly, the following results are achieved for the set of the Kharitonov polynomial coefficients (21):

$$\begin{aligned} &a_0^- = 8 + 1 = 9; && a_0^+ = 10 + 1 = 11; \\ &a_1^- = 1 + 1.3 = 2.3; && a_1^+ = 2 + 1.3 = 3.3; \\ &a_2^- = 1.3\times 1 + 0.4 = 1.7; && a_2^+ = 1.3\times 2 + 0.4 = 3; \\ &a_3^- = 0.4\times 1 = 0.4; && a_3^+ = 0.4\times 2 = 0.8; \end{aligned} \qquad (21)$$

These coefficients are substituted in the set of equations (19), from where the set of four Kharitonov's polynomials (22) is presented in the final proper state for assessment:

$$\begin{aligned} k_1(s) &= 27.5 + 5.75s + 4.25s^2 + s^3 \\ k_2(s) &= 13.75 + 4.125s + 3.75s^2 + s^3 \\ k_3(s) &= 11.25 + 4.125s + 3.75s^2 + s^3 \\ k_4(s) &= 22.5 + 5.75s + 4.25s^2 + s^3 \end{aligned} \qquad (22)$$

In this case, the interval polynomial $P_A(s)$ is stable, if the four Kharitonov's polynomials (22) are stable. To verify their stability, the following supporting table is created to apply the Routh-Hurwitz stability criterion [17], [18]:

Table 1. *Routh-Hurwitz stability test (case of a third order system).*

k_i (s)			
s^3	a_n	a_{n-2}	0
s^2	a_{n-1}	a_{n-3}	0
s^1	b_1	0	0
s^0	c_1	0	…

where

$$b_1 = \frac{(a_{n-1} \times a_{n-2}) - (a_n \times a_{n-3})}{(a_{n-1} \times a_{n-2})}; c_1 = \frac{(b_1 \times a_{n-3}) - (a_{n-1} \times 0)}{b_1}$$

The Routh-Hurwitz stability criterion is applied to all Kharitonov's polynomials k_i *(s)*, where *(i = 1, 2, 3, 4)*.

Table 2. *Results from the four Kharitonov polynomials (case of instability).*

k_1 (s)		k_2 (s)		k_3 (s)		k_4 (s)	
1	**5.75**	**1**	**4.13**	**1**	**4.13**	**1**	**5.75**
4.25	27.5	3.75	13.75	3.75	11.25	4.25	22.5
-0.72		0.46		1.13		0.46	
27.5		13.75		11.25		22.5	

The first column of the Routh array for the three polynomials k_2 *(s)*, k_3 *(s)* and k_4 *(s)* are all positive (that is, there is no change of sign in the first column).

It is seen from Table 2 that the polynomial k_1 *(s)* has change of sign in the first column of the Routh array. Therefore the closed-loop system will be unstable for the suggested set of coefficients variations. This result will be confirmed by the Advanced D-Partitioning method.

9.2. Case of Stability

It is demonstrated that if the set of parameter variations is changed, the closed-loop system may become stable.

$$P_B(s) = K_2 + 1 + (T_2 + 1.3)s + (1.3T_2 + 0.4)s^2 + 0.4T_2 s^3 \tag{23}$$

Where

$K_2 \in [4, 6]$; $T_2 \in [2, 4]$;

Considering equation (23), the lower and the upper limits of the suggested variable parameters are to be substituted. In this case, the following results are achieved for the set of the Kharitonov polynomial coefficients (24):

$$\begin{aligned} a_0^- &= 4 + 1 = 5; & a_0^+ &= 6 + 1 = 7; \\ a_1^- &= 2 + 1.3 = 3.3; & a_1^+ &= 4 + 1.3 = 5.3; \\ a_2^- &= 1.3\times 2 + 0.4 = 3; & a_2^+ &= 1.3\times 4 + 0.4 = 5.6; \\ a_3^- &= 0.4\times 2 = 0.8; & a_3^+ &= 0.4\times 4 = 1.6; \end{aligned} \tag{24}$$

These coefficients are substituted in the set of equations (19), from where the set of four Kharitonov's polynomials (25) is presented in the final state for assessment:

$$\begin{aligned} k_1(s) &= 8.75 + 2.875s + 3.75s^2 + s^3 \\ k_2(s) &= 4.375 + 3.3125s + 3.5s^2 + s^3 \\ k_3(s) &= 3.125 + 3.3125s + 3.5s^2 + s^3 \\ k_4(s) &= 6.25 + 2.875s + 3.75s^2 + s^3 \end{aligned} \tag{25}$$

The Routh-Hurwitz stability criterion is applied to all these polynomials k_i *(s), where (i = 1, 2, 3, 4)*.

Table 3. *Results from the four Kharitonov polynomials (case of stability).*

k₁ (s)		**k₂ (s)**		**k₃ (s)**		**k₄ (s)**	
1	**2.88**	**1**	**3.31**	**1**	**3.31**	**1**	**2.88**
3.75	8.75	3.5	4.38	3.5	3.13	3.75	6.25
0.54		2.06		2.42		1.21	
8.75		4.38		3.13		6.25	

The first column of each Kharitonov's Polynomial, in Table 3, contains no change in sign and all its components are positive.

The conclusion is that all of the roots of each k_i *(s), (i = 1, 2, 3, 4)* polynomial have negative real parts and the closed-loop control system is stable for all parameter values in the specified ranges. Again, this result will be confirmed by the Advanced D-Partitioning method.

The advanced D-Partitioning analysis, presented in this research, has considerable advantages, compared with the Kharitonov's theorem assessment. The advanced D-Partitioning analysis does not need a specified set of limits of parameter variations. It is applicable generally and can deliver results representing the exact marginal values of the multivariable parameters.

The D-Partitioning analysis results are obtained easily with the aid of the interactive MATLAB procedure. As already demonstrated, the D-Partitioning curve in terms of the two variable parameters is plotted by the simple MATLAB code. The clear graphical display of the regions of stability and instability is another significant advantage of the Advanced D-partitioning. A graphical comparison between the two methods, as seen from Figure 10, is confirming the considerable advantage of the Advanced D-Partitioning analysis compared with the Kharitonov's theorem assessment.

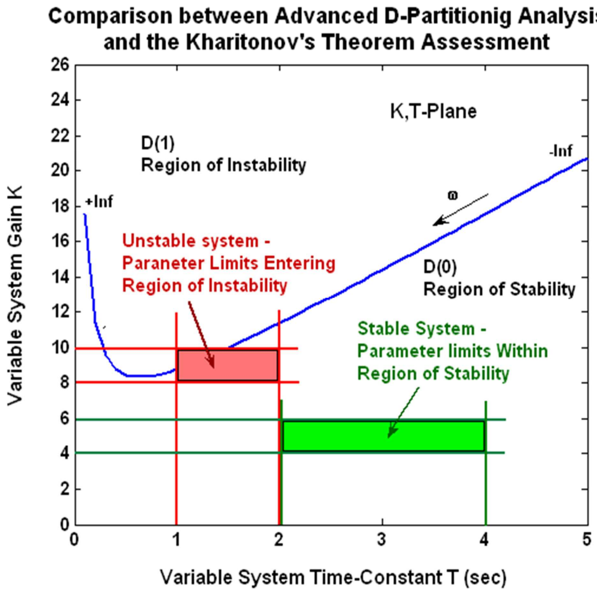

Figure 10. *Comparison between the Advanced D-Partitioning Analysis and the Kharitonov's Theorem Assessment in Terms of Two Simultaneously Variable Parameters.*

The graphical result of the Advanced D-Partitioning stability analysis is illustrating straight away and directly the region of stability D(0) and the region of instability D(1) that can be used for the entire general assessment of the closed-loop system stability.

By applying the Advanced D-Partitioning for two variable parameters, by plotting the D-Partitioning curve and directly applying the limits of the simultaneously variable parameters, the system's stability assessment can be promptly established.

When the two variable parameters are within the limits $K_1 \in [8, 10]$ and $T_1 \in [1, 2]$, these parameter variations are entering the region of instability D(1) and therefore the feedback control system will be unstable.

When the two variable parameters are within the limits $K_2 \in [4, 6]$ and $T_2 \in [2, 4]$, these parameter variations are entirely within the region of stability D(0) and therefore the feedback control system will be guaranteed asymptotically stable.

This exceptional phenomenon is demonstrating the considerable advantage of the D-Partitioning analysis in comparison with the Kharitonov's assessment. By applying the D-Partitioning analysis and implementing a simple interactive MATLAB procedure, the system's asymptotic stability can be swiftly determined and it can be graphically demonstrated, avoiding the significant calculations needed for the Kharitonov's theorem assessment.

10. Conclusion

The main contribution of this research is further upgrade of the method of the D-Partitioning as a stability analysis tool for systems with multivariable parameters and comparing it with other stability analysis methods.

In case of control systems with one variable parameter, the Advanced D-Partitioning method directly exposes the transparent images of regions of stability and instability, as well as the system's margins of stability in the complex plane of the variable parameter. It is also demonstrated that the Advanced D-Partitioning can be successfully applied to digital control systems. The Advanced D-Partitioning is compared with other well known methods for stability assessment, like the Nyquist and the Bode stability criterion. Even if both these methods are applicable for continuous and for digital control systems as well, their disadvantage is that they cannot directly establish the marginal value of a variable parameter. In this case, they are used only to confirm the results from the Advanced D-Partitioning method.

In case of control systems with two simultaneously variable parameters, the Advanced D-Partitioning method directly exposes the transparent images of regions of stability and instability in the parameters' plane.

Each point on the D-Partitioning curve represents the marginal values of the two simultaneously variable parameters, being a unique property of the advanced D-Partitioning stability analysis that is not offered by any other known stability analysis method.

A comparison is demonstrated between the Advanced D-Partitioning method and the Kharitonov's theorem assessment for control systems with simultaneously variable parameters.

The only advantage of the Kharitonov's assessment is that it can determine system's stability in the cases of variation of large number of the system's parameters, defined within specific limits.

At the same time the Kharitonov's assessment has substantial disadvantages. It is short of determination of the parameter marginal values of stability, the results are achieved after considerable calculations and there is lack of any graphical display visualizing these results. Another major disadvantage of this method is that the Kharitonov polynomials deal with the coefficients variations of the Kharitonov characteristic interval polynomial, rather than directly with the system's parameter variations. The variations of the system's parameters remain in a hidden mode and cannot be directly observed from the four Kharitonov polynomials.

Alternatively, the Advanced D-Partitioning analysis has considerable advantages compared with the Kharitonov's assessment. It does not need a specified set of limits of parameter variations. It is applicable generally and can deliver results representing the exact marginal values of the multivariable parameters.

This research is achieving new advances in knowledge. It may have also considerable practical application in analysis of industrial control systems with variable and uncertain parameters due to various process conditions.

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