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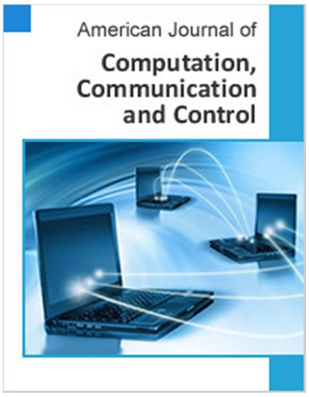

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Adaptive DPD Modeling in DSL Customer Premises Equipment

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Abstract
Digital subscriber line (DSL) systems transport high-bandwidth data, such as multimedia, to service subscribers over ordinary twisted pair copper wire telephone lines. One of the important components of the DSL system is the line driver (LD) or the power amplifier (PA). The linearity of the LD is crucial to have a high-speed internet. Any non-linearity in the LD will create intermodulation distortion and spectral regrowth that will become an obstacle to increase DSL data rate. Digital predistortion (DPD) is a known technique that can be used in order to linearize the LD and hence suppressing any nonlinear distortion. This paper considers direct predistortion of DSL-LD that is modelled using Volterra series by connecting in tandem an adaptive Volterra predistorter. The coefficients of the predistorter is recursively estimated using the Prediction Error Method (PEM). Simulation study on a LD which is modelled as 5th order Volterra system shows that the Nonlinear Filtered-x Prediction Error Method (NFxPEM) algorithm can significantly suppress spectral regrowth and converge much faster than the well-known Nonlinear Filtered-x Least Mean squares (NFxLMS) algorithm.

1. Introduction

In wired and wireless communication systems, the nonlinearity of high power amplifiers is an obstacle to increase the transfer data rate, see [1-2]. Adaptive linearization schemes for weakly nonlinear Volterra systems were proposed in [3]. These schemes require the existence of the inverse of the linear subsystem which can't be always guaranteed to be causal and stable. In [4], a direct predistortion technique was proposed based on Spectral Magnitude Matching (SMM) method that minimizes the sum squared error between the spectral magnitudes of the output signal of the nonlinear system and the desired signal. The coefficients of the predistorter were estimated recursively using the generalized Newton iterative algorithm. The drawback of this suggested SMM approach is the high computation complexity.

In [5], a linearization scheme for nonlinear systems was introduced as shown in Figure 1. The idea of the approach is to connect a nonlinear pth-order Volterra predistorter $C_{(p)}$ in tandem with the nonlinear system $H_{(q)}$ that can be described by qth-order Volterra series with M-tap memories. Then, adaptively adjusting the coefficients of the predistorter in order to reduce the error between the input and desired signals. These coefficients were estimated recursively using the NFxLMS algorithm [6-8]. The approach of [5], like the one introduced in this paper, requires an estimate for the nonlinear system $H_{(q)}$ which is denoted as $\widehat{H}_{(q)}$ in Figure 1 and assumed to be known. Otherwise, a kernel estimation technique for the nonlinear system $H_{(q)}$ based on the adaptive Volterra filter should be considered, see [9].

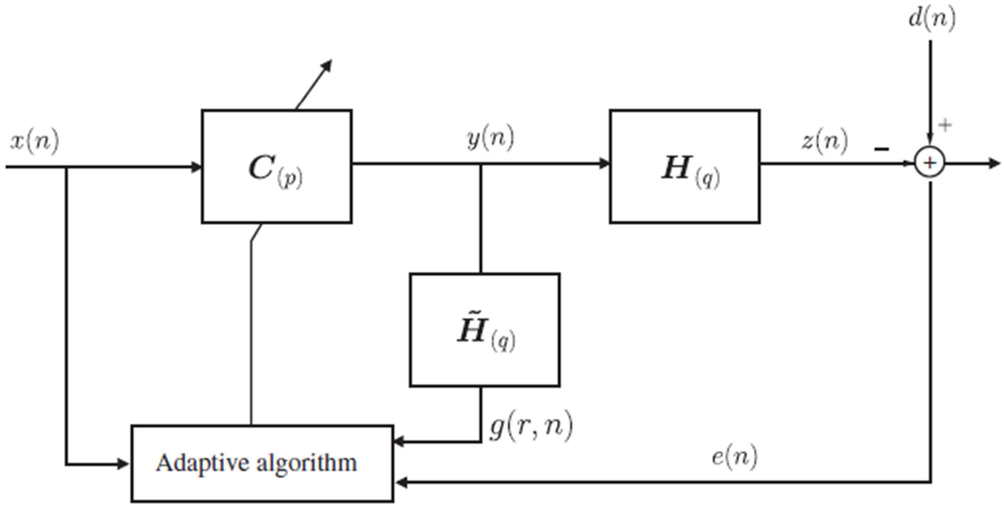

Figure 1. *Compensation of nonlinear distortion using nonlinear filtered-x algorithm.*

In this paper, the coefficients of the predistorter are estimated recursively using the Recursive Prediction Error Method (RPEM) algorithm, see [10, 11]. The RPEM algorithm gives consistent parameter estimates under weak conditions. Therefore, using the RPEM algorithm is expected to reduce the steady state mean square error and to minimize the total nonlinear distortion at the output of the nonlinear system as compared to the NFxLMS algorithm. Moreover, the RPEM algorithm is known of its high convergence speed.

This paper is organized as follows. In Section 2, a review for the NFxLMS algorithm is given. The NFxPEM algorithm is presented in Section 3. In Section 4, the DSL Customer Premises Equipment (CPE) – Analog Front End (AFE) is explained. In Section 5, a comparative simulation example between the NFxPEM and NFxLMS algorithms implemented in the DSL CPE-AFE is given. Conclusions are presented in Section 6.

2. The NFxLMS Algorithm

The NFxLMS algorithm, introduced in [5] and shown in Figure 1, assumes that the nonlinear system $H_{(q)}$ to be compensated is a discrete time-invariant causal system. The block diagram in Figure 1 consists of the nonlinear physical system $H_{(q)}$ to be compensated using a nonlinear predistorter $C_{(p)}$ and an adaptive algorithm to estimate the proper coefficients of the predistorter. The output of the nonlinear physical system $z(n)$ is compared to the desired output $d(n)$ in order to construct an error signal to be used in the adaptive algorithm in addition to a filtered version from the presdistorter's output signal denoted as $g(r;\, n)$. In Figure 1, the filter used to generate $g(r;\, n)$ is denoted as $\hat{H}_{(q)}$ and represents an estimate of the nonlinear physical system $H_{(q)}$. In case the nonlinear physical system is already known, $\hat{H}_{(q)} = H_{(q)}$. In case the nonlinear system is unknown, a system identification method should be used first to identify the system in order to be able to generate $g(r;\, n)$.

In this paper, the system $H_{(q)}$ with input and output signals $y(n)$ and $z(n)$ can be modeled by qth-order Volterra series with M-tap memories. Hence, the output $z(n)$ is given by

$$z(n) = \sum_{k=1}^{q}\left(\sum_{i_1=0}^{M-1} \dots \sum_{i_k=0}^{M-1} h_k(i_1, \dots, i_k)\, y(n-i_1) \dots . y(n-i_k)\right),$$

where $h_k(i_1, \dots \dots, i_k)$ are the kth-order kernels of the nonlinear system. Similarly, the relation between the input and output of the adaptive Volterra filter is given by

$$y(n) = \sum_{k=1}^{p}\left(\sum_{i_1=0}^{N-1} \dots \sum_{i_k=0}^{N-1} c_k(i_1, \dots, i_k; n)\, x(n-i_1) \dots . x(n-i_k)\right),$$

where N is the number of memories in the adaptive Volterra fillter and $c_k(i_1, \dots \dots, i_k;\, n)$ are the kth-order kernels of this filter. According to the pth-order Volterra theorem [10], the Volterra filter $C_{(p)}$ can remove nonlinearities up to pth-order provided that the inverse of the first-order Volterra system is causal and stable.

The kernels of the adaptive Volterra filter can be estimated by minimizing the mean square distortion defined as

$$E\{e^2(n)\} = E\{[d(n) - z(n)]^2\},$$

where E denotes the Expectation and $d(n)$ is the desired signal defined as

$$d(n) = x(n-\tau) + v(n).$$

Here τ is the time delay necessary to have a causal Volterra predistorter and $v(n)$ is zero-mean Additive White Gaussian Noise (AWGN).

The NFxLMS algorithm is obtained by applying the stochastic gradient algorithm, see [10, 11], as

$$C_k(n+1) = C_k(n) - \frac{\mu_k}{2}\,\Delta_k(n),$$

where μ_k is a small positive constant that controls stability and rate of convergence of the adaptive algorithm and usually is defined as the step-size parameter. Also,

$$C_k(n) = \left(c_k(0,\dots,0;n),\dots,c_k(N-1,\dots,N-1;n)\right)^T$$

and the gradient vector $\Delta_k(n)$ is defined as

$$\Delta_k(n) = \left(\frac{\partial e^2(n)}{\partial\, c_k(0,\dots,0;n)},\dots,\frac{\partial e^2(n)}{\partial\, c_k(N-1,\dots,N-1;n)}\right)^T$$

Straightforward analysis of the gradient components gives

$$\frac{\partial e^2(n)}{\partial\, c_k(i_1,\dots,i_k;n)} = -2e(n)\sum_{r=0}^{M-1} g(r;n)\, x(n-r-i_1)\ \dots\ x(n-r-i_k),$$

where

$$g(r;n) = \frac{\partial z(n)}{\partial y(n-r)} = h_1(r) + 2\sum_{i=0}^{M-1} h_2(r,i)\, y(n-i) + 3\sum_{i_1=0}^{M-1}\sum_{i_2=0}^{M-1} h_3(r,i_1,i_2)\, y(n-i_1)\, y(n-i_2) + \cdots$$

3. The NFxPEM Algorithm

The NFxPEM algorithm is derived by the minimization of the cost function [11]

$$V(C) = \lim_{\mathcal{N}\to\infty}\frac{1}{\mathcal{N}}\sum_{n=1}^{\mathcal{N}} E[e^2(n,C)],$$

where $e(n,C)$ is the prediction error which is defined as

$$e(n,C) = d(n) - z(n,C),$$

and C is defined as

$$C = \left(C_1^T(n), C_2^T(n),\dots,C_p^T(n)\right)^T.$$

The formulation of the NFxPEM algorithm requires the negative gradient of $e(n;C)$ w.r.t. C which is defined as

$$\psi = -\frac{de(n,C)}{dC} = \left(\psi_1^T(n),\psi_2^T(n),\dots,\psi_p^T(n)\right)^T,$$

where

$$\psi_k(n) = \left(\frac{\partial z(n)}{\partial c_k(0,\dots,0;n)},\dots,\frac{\partial z(n)}{\partial c_k(N-1,\dots,N-1;n)}\right)^T.$$

Straightforward analysis gives

$$\frac{\partial z(n)}{\partial c_k(i_1,\dots,i_k;n)} = \sum_{r=0}^{M-1} g(r;n)\, x(n-r-i_1)\ \dots x(n-r-i_k).$$

Hence, the NFxPEM algorithm follows as (*cf.* [10, 11])

$$e(n,C) = d(n) - z(n,C)$$

$$\lambda(n) = \lambda_0\lambda(n-1) + 1 - \lambda_0$$

$$S(n) = \psi^T(n)P(n-1)\psi(n) + \lambda(n)$$

$$P(n) = \frac{1}{\lambda(n)}\left(P(n-1) - P(n-1)\psi(n)S^{-1}(n)\psi^{T}(n)P(n-1)\right)$$

$$C(n) = C(n-1) + P(n)\psi(n)e(n, C).$$

Here $\lambda(n)$ is a forgetting factor that grows exponentially to 1 as $n \to 1$ where the rate λ_o and the initial value $\lambda(0)$ are design variables. The numerical values $\lambda_o = 0.99$ and $\lambda(0) = 0.95$ have proven to be useful in many applications. Also, $P(n) = nR^{-1}(n)$ where $R(n)$ is the Hessian approximation in the Gauss-Newton algorithm, see [10, 11]. The most common choice for the initial condition of $P(n)$ is $P(0) = \rho I$, where I is the identity matrix and ρ is a constant that reflects our trust in the initial parameter vector $C(0)$. In case of no prior knowledge, $C(0) = 0$, and ρ is large to speed up convergence to the true parameter vector.

4. DSL CPE-AFE

In general, the performance of any analog circuitry suffering from nonlinear behavior can be improved by predistortion techniques. In this section, the implementation of DPD in DSL Customer Premises Equipment (CPE) - Analog Front End (AFE) is demonstrated. The focus is adaptively predistort the transmitted signal, e.g., discrete multi-tone (DMT) signal, in order to reduce the nonlinear distortion caused by the weakly nonlinear transmit path including the LD, Hybrid circuit, etc. The simulation scheme is first introduced, then the implementation of the adaptive predistorter is discussed. The simulation results are given in next section.

Figure 2 shows the structure of the CPE-AFE in DSL systems. The components in the transmission and receiving paths are given in Table 1. The "Pofi" represents the linear filters between the Digital-to-Analog Converter (DAC) and LD. The "Rec. Filter" represents the linear filters between the Analog-to-Digital Converter (ADC) and the Hybrid. The "Hybrid" is an interface between two-wire and four-wire circuits, connecting the transmission and receiving paths to the twisted pair cable (telephone line).

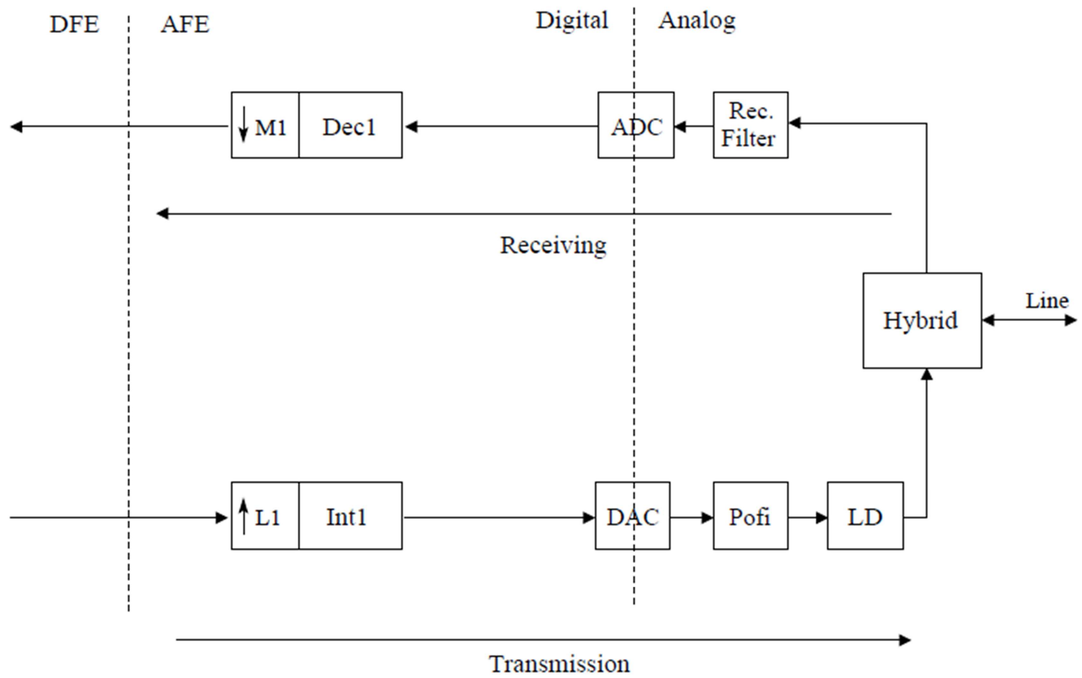

Figure 2. *Structure of DSL CPE-AFE.*

Table 1. *Components in the transmission and receiving paths.*

Tran./Rec. path component	Component name
↑L1	Up-sampler
Int1	Interpolation filter
DAC	Digital-to-analog converter
LD	Line driver
Rec. Filter	Receiver filter
ADC	Analog-to-digital converter
Dec1	Decimation filter
↓M1	Down-sampler

In the CPE-AFE, the LD is the main source of nonlinear distortion during transmission. In our system, the nonlinearity can be described precisely using a 5th order Volterra system with 50 non-zero parameters [13]. Also here, the predistorter can be modeled as another 5th order Volterra system and the coefficients of the predistorter can be estimated using the NFxLMS or the NFxPEM algorithms which were described in Sections 2 and 3, respectively. DPD in DSL CPE-AFE is shown in Figure 3. Here, H_1 is a linear filter representing "DAC" and "Pofi", H_2 is another linear filter representing "Hybrid", "Rec. Filter" and "ADC". The reference model H consists of $\widehat{H}_1$, $\widehat{H}_2$ and the linear subsystem of the line driver model, see Figure 4. Here, $\hat{H}_1$ and $\hat{H}_2$ are the estimates of H_1 and H_2, respectively.

Since the Hybrid can be switchable during the startup phase of the DSL system and connects the transmission path to the receiving path, a predetermined DMT signal can be sent as an input training sequence $u(n)$ and the corresponding output signal $z(n)$ can be measured at the receive end. Here, $v(n)$ denotes the quantization noise of the "ADC". The reference signal $r(n)$ is generated from the upsampled training sequence, $x(n)$, filtered by the reference model H. Comparing the received signal $z(n) + v(n)$ with the reference signal $r(n)$, the error signal $e(n)$ is obtained and the time domain adaptive algorithms, such as the NFxLMS and NFxPEM algorithms, can be used to estimate the coefficients of the predistorter.

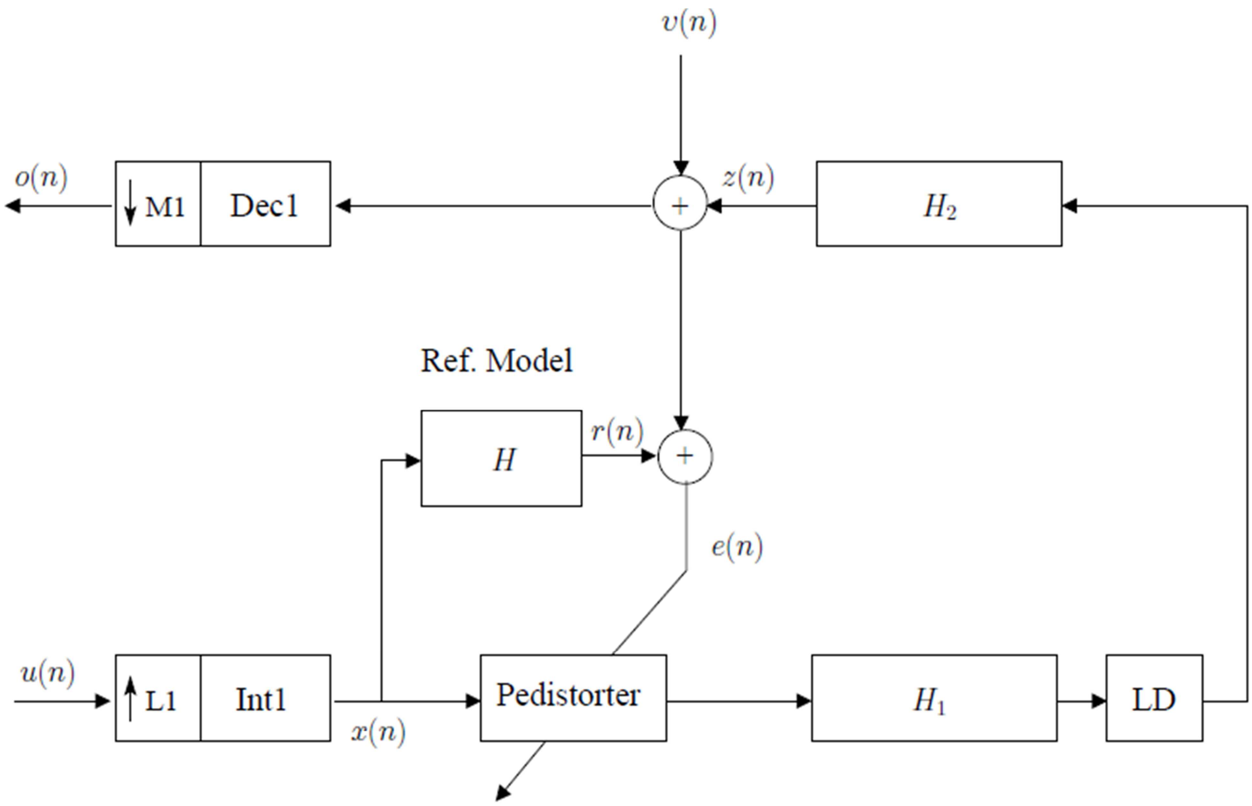

Figure 3. *DPD in DSL CPE-AFE.*

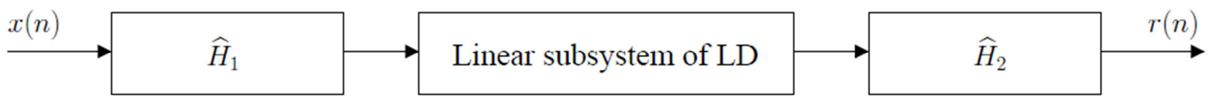

Figure 4. *The reference model.*

5. Simulation Results

In this simulation, the DSL CPE-AFE shown in Figure 3 is modeled using Matlab Simulink in Figure 5. The line driver is modeled as a 5th-order Volterra system. The memory lengths of the 1st to 5th order kernels are 15, 0, 5, 0 and 2, respectively. The Volterra predistorter is also assumed to be a 5th-order Volterra system with same memory lengths. The training sequence $u(n)$ is a DMT signal which is defined as

$$u(n) = \sum_{k=0}^{K} 2|U_K| exp\left[j\left(2\pi\frac{f_{max}}{K}kn + \varphi_k\right)\right],$$

where U_k are user-defined amplitudes, φ_k are random phases with uniform distribution and $E\{e^{j\varphi_k}\} = 0$. The number of tones is $K = 64$ and $f_{max} = 4312.5$ Hz. The data length is 2^{13} samples, the crest factor of the signal is 3 and the root mean square (RMS) value is 0.25. The up-sampling and down-sampling factors are $L_1 = 5$ and $M_1 = 5$, respectively.

The filters used in this simulation, are given in Table 2. Here, H_1 and H_2 are approximated using Butterworth filters, also $\hat{H}_1$ and $\hat{H}_2$ are Butterworth filters but with ±10% mismatch of the normalized cut-off frequency. This mismatch will cause a group delay difference and hence the signals $r(n)$ and $z(n)$ will not be synchronized to each other. Therefore, this group delay difference needs to be compensated in the simulation. The adaptive algorithms were initialized with $\mu = 10^{-4}$, $P(0) = 10^{-3} I$, $\lambda_0 = 0.99$ and $\lambda(0) = 0.95$.

The Mean Multi Tone Power Ratio (MMTPR) [14] is used to measure the performance. The MMTPR for each tone of the output signal $o(n)$ is defined as

$$\mathrm{MMTPR}_k = \frac{\hat{E}\{T_k\}}{\hat{E}\{\eta_k + \sum_{j,j\neq k} I_j\}}, k = 1,2,\dots,K.$$

Here $\hat{E}\{.\}$ is the mean obtained by 100 independent experiments, T_k and η_k stand for the transmitted and noise powers of the kth tone, respectively, and $(\sum_{j,j\neq k} I_j)$ is the inter-modulation power of the kth tone from the other $K-1$ tones.

In order to evaluate $(\eta_k + \sum_{j,j\neq k} I_j)$ for each experiment, $u(n)$ can be chosen as a DMT signal without the kth tone, then the measured power at the kth tone of $o(n)$ can be considered as $(\eta_k + \sum_{j,j\neq k} I_j)$.

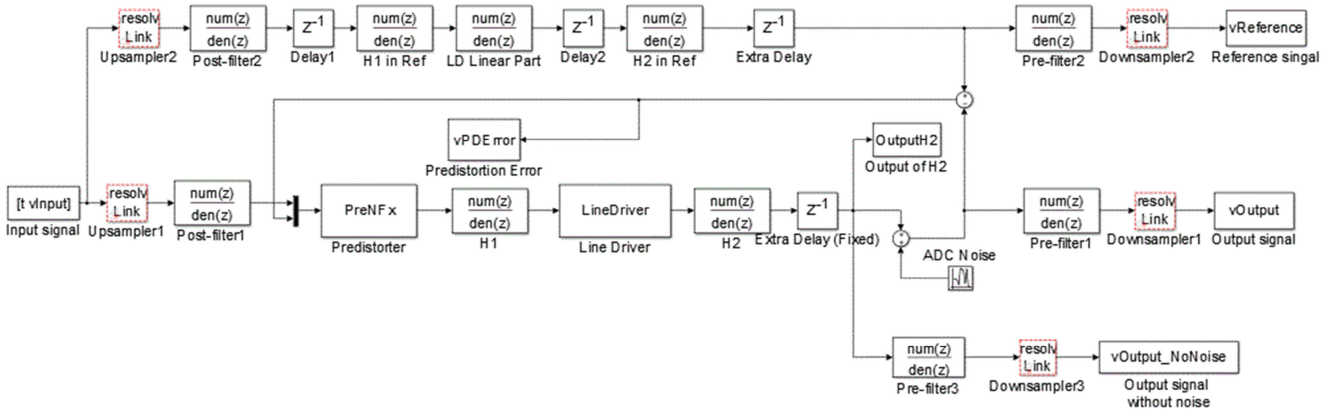

Figure 5. *Simulink platform for the block diagram in Figure 3.*

Table 2. *Filters in the simulation.*

Component	Filter Type	Order	Normalized cut-off frequency
Int1	FIR, low pass	200	0.15π
Dec1	FIR, low pass	200	0.15π
H_1	Butterworth, low pass	8	0.5π
H_2	Butterworth, low pass	8	0.5π
$\hat{H}_1$	Butterworth, low pass	8	0.5π±10%
$\hat{H}_2$	Butterworth, low pass	8	0.5π±10%

Figure 6 shows the MMTPR values of the system output with and without the predistorter. The MMTPR values of the tones with index $\{k|k = 7, 17, 27, 37, 47\}$ were measured. From this figure, we can see that the adaptive predistorter using the NFxPEM algorithm can achieve much better MMTPR values as compared to the NFxLMS algorithm and hence more effectively compensate the nonlinear distortion than the NFxLMS algorithm for DSL systems.

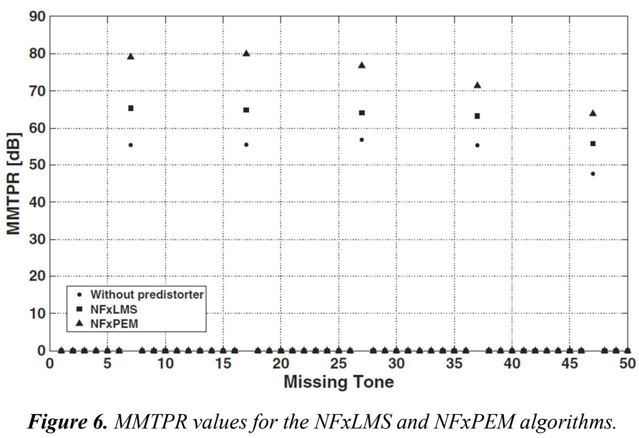

Figure 6. *MMTPR values for the NFxLMS and NFxPEM algorithms.*

6. Conclusions

High speed data rate DSL systems can be developed by digital predistortion of transmitted data. The digital predistorter is implemented in the CPE-AFE and its parameters can be estimated recursively in the start-up phase of the DSL modem. A detailed comparative simulation study between two algorithms: the NFxLMS and NFxPEM algorithms, is given in this paper to estimate the predistorter parameters taking into consideration all components in both transmitting and receiving paths of the CPE-AFE. Simulation results show that the NFxPEM algorithm achieves much better MMTPR values as compared to the NFxLMS algorithm – hence more capable to suppress nonlinear distortion and spectral regrowth.

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