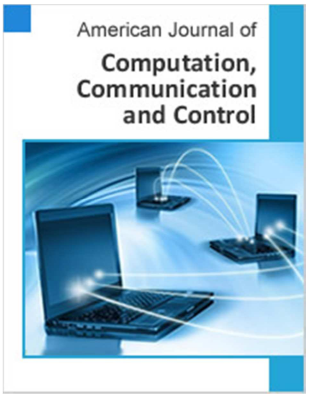
American Journal of Computation, Communication and Control
2017; 4(6): 46-50
http://www.aascit.org/journal/ajccc
ISSN: 2375-3943

American Association for Science and Technology



Keywords
Urban Traffic,
Robustness,
User Equilibrium,
Traffic Networks,
Relative Total Cost

Received: October 17, 2017
Accepted: November 14, 2017
Published: December 26, 2017

The Study About Transportation Network Assignment Model and Robustness

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Citation
Zhao Chun-xue. The Study About Transportation Network Assignment Model and Robustness. *American Journal of Computation, Communication and Control.* Vol. 4, No. 6, 2017, pp. 46-50.

Abstract
The robustness is a characteristic that measures the network system performance under interference. The investigation of the robustness about the traffic network is an important branch of the traffic assignment. In this paper, the robustness of the congested traffic network is invested by building the traffic assignment model when a link or node of the network is paralyzed. The paper assumes link cost is affected by the flow of this link and all adjacent links, the traffic network assignment model is constructed, the total cost of the network is given under user equilibrium. In addition, the robustness of the traffic network is analyzed when a link or node is paralyzed; the robustness ranking of each link and node is given by calculating the relative cost index value. The results show that robustness of the network components is different, and the robustness varies with traffic demand changing. The research method and conclusions can provide theoretical guidance for traffic network planning.

1. Introduction

Robustness is the characteristic that the network can maintain its performance under the partial degradation. The stronger the robustness is, the stronger the resistance of the network to emergencies is. Robustness, as an important indicator of evaluating complex networks [1], [2], also used to evaluate the merits of the traffic network. For example, Hoogendoorn et al. [3] took into account the uncertainty in the predicted traffic condition and the system performance based on the controlled Markov process, the new control methodology shows how to control for the reliability in the condition of the generic control inputs and objectives; Sakakibara et al. [4] used a topological index to quantify the road network dispersiveness. This approach can be used to evaluate the robustness of an urban highway network subject to catastrophic disaster; Scott et al. [5] presented a method to identify the critical link and evaluated the network performance. Moreover, he compared with the traditional volume/capacity (V/C) ratio; Tizghadam et al. [6] presented a self-organizing management system of a network, in which, the requirement of the network is translated to a graph-theoretic metric, and the management system automatically evolves to a stable and robust control point by optimizing the metric. Mendes et al. [7] used the electrical model and herding model to describe the emerging of the traffic jam up to the traffic gridlock and found the distributions of both the avalanche size and the flux follow a paw-law. Many different methods have been developed to study the system robustness and to identify the critical network components.

As an important branch of the robustness of the transportation network, the research about robustness of traffic network components (section or node) has also made some progress [8], [9], [10], [11], [12], [13]. In this paper, we assume that the link cost is affected by the flow of the link and all adjacent links, and the relative cost index [13] is used to study the robustness of the traffic network when a node or section is paralyzed.

The paper is organized as follows. We first review the traffic assignment models and the robustness measure. Then we deduce the user equilibrium solution when the link cost is affected by the flow of the link and all adjacent links. Finally, the relative cost index of each component of the network is calculated, and the robustness of each component is evaluated by sorting.

2. Traffic Assignment Models and the Robustness Index

The traffic assignment model presents a decision-making process, which includes static models and dynamic models. Depending on whether a traveler's road choice involves randomness or not, static model is divided into the following two classes: (i) deterministic approach, which includes user equilibrium and system optimal, (ii) stochastic approach, namely stochastic user equilibrium model. In user equilibrium, each traveler chooses the path of minimum travel time between the starting point and destination. On the other hand, the system optimal model is to minimizes the total travel time of the system. In stochastic user equilibrium, each traveler chooses to minimize his/her perceived travel time.

In 1952, Wardrop first proposed Wardrop's first principle [14]: All travelers make decisions that make their travel time minimum, that is, That is, the time of the used path time for the starting point / end point (O / D) pair is equal to or less than the time of the unused path. This principle of equilibrium is also called user equilibrium, and its mathematical expression is given as follows:

$$C_p(x^*) = \lambda_w, \quad if \quad x_p^* > 0,$$
$$C_p(x^*) \geq \lambda_w, \quad if \quad x_p^* = 0,$$

where $C_p(\cdot)$ is the cost on the path under the user equilibrium, λ_w is the minimal path cost, x^* is an equilibrium flow and meet the traffic flow conservation criteria.

The user equilibrium is equivalent to the mathematical programming problem as follows:

$$\min \sum_{a \in L} \int_0^{f_a} c_a(y)dy \ ,$$

The link flow and path flow satisfy the following conditions:

$$\sum_{p \in P_w} x_p = Q, \forall w \in W,$$

$$x_p \geq 0, \forall p \in P_w,$$

$$f_a = \sum_{p \in P_w} x_p \delta_{ap}, \forall a \in L,$$

where f_a is the flow on link a; $c_a(\cdot)$ is the cost of link a; L is the set of links; W is the set of O/D pairs; Q is the total traffic demand of O/D pairs; x_p is the flow of path p; P_w is the path set of joining O/D pairs w; if link a is a part of path p; otherwise, $\delta_{ap} = 0$.

The system optimal model is to minimizes the total travel time of the system. As we have known, system optimal is obtained by charging users the marginal cost of traveling. About the relationship between total costs under different kinds distributions, researchers have done a large number of studies [15], where we know that, under the static traffic assignment, the solution between user equilibrium and system optimal is approximative in the free flow state; as the traffic becomes more congested, the difference between the solution under user equilibrium and that under system optimal becomes greater.

In stochastic user equilibrium [16], each traveler chooses to minimize his/her perceived travel time. Concretely, In the traffic network $G(N, L)$, where N and L are the collection of nodes and links, respectively, there exist a set of O/D pairs W with n_W elements and a set of the path P joining O/D pairs. It is assumed that

$$T_k^{rs} = t_k^{rs} + \xi_k^{rs}, \forall k, r, s$$

where T_k^{rs} represents the perceived travel time on path k between origin r and destination s, which is a random variable; t_k^{rs} is the actual travel time on path k between r and s; ξ_k^{rs} is a random error term associated with the path under the consideration and $E[\xi_k^{rs}]$ is expected to 0. Then the path will be chosen if its travel time is perceived to be the lowest among all the alternative paths. The probability of choosing such a path can be expressed as follows:

$$P_k^{rs} = \Pr(T_k^{rs} \leq T_l^{rs}, \forall l \in P_{rs}), \forall k, r, s$$

where P_k^{rs} is the probability of choosing path k from r to s; P_{rs} is the set of paths from r to s. Different distributions of the perceived travel time result in different models of the stochastic network loading. Once the distribution is specified, the probability of selecting each alternative path and the path flow assigned accordingly can be calculated. The path flow is given by

$$f_k^{rs} = q_{rs} P_k^{rs}, \forall k, r, s$$

Then the link flow is calculated as

$$f_a = \sum_{rs} \sum_k f_k^{rs} \delta_{a,k}^{rs}, \forall a$$

where f_k^{rs} is the flow on path k from r to s; q_{rs} is the travel demand from r to s; f_a is the flow on link a;

$\delta_{a,k}^{rs} = 1$, if link a is a part of path k from r to s; otherwise, $\delta_{a,k}^{rs} = 0$.

The Logit model has widely been used in the discrete choice models such as the modal split model and the trip distribution model. In this paper, the same Logit model is utilized, in which the distribution of utility terms is assumed to be independently and identically distributed Gumbel variates. The choice probability is then given by

$$P_k = \frac{\exp(V_k)}{\sum_{l=1}^{K} \exp(V_l)}$$

where P_k is the probability that path k is chosen, V_k is the measured utility on path k. In the traffic network, it usually implies that $V_k = -t_k$, where, t_k is the congestion on path k. In this paper, the travel time is regarded as the congestion. K is the total number of the paths.

In recent years, the dynamic traffic assignment model is widely investigated [17], [18], [19], [20]. It includes dynamic user equilibrium, dynamic user optimal, dynamic system optimal, dynamic stochastic user equilibrium. In dynamic models, the time factor is considered. In this paper, we discuss the robustness of traffic network under static user equilibrium, so the dynamic traffic assignment model is not listed one by one.

In order to calculate the robustness of network components, then we review a few concepts [11]. The total congestion on link a is denoted as follows:

$$\hat{c}_a(f_a) = c_a(f_a) f_a,$$

Where $\hat{c}_a(f_a)$ is the total congestion on link a; $c_a(f_a)$ is the unit congestion on the link a; f_a is the traffic flow on the link a. Therefore, the total congestion TC of the network is given by:

$$TC = \sum_{a \in L} \hat{c}_a(f_a),$$

Where L is the set of links. The link flow f satisfies nonnegative and conservation conditions. Then the relative total congestion index of the link l can be defined as follows:

$$\Gamma^l = \frac{TC(G-l) - TC}{TC}.$$

Where $TC(G-l)$ denotes the total congestion when link l is removed from the network; we define Γ^l as the importance indicator of link l under user equilibrium. Similarly, the importance indicator Γ^M of node M is denoted as follows:

$$\Gamma^M = \frac{TC(G-M) - TC}{TC}$$

Where $TC(G-M)$ denotes the total congestion when node M is removed from the network.

3. Research About Robustness

In this paper, we take Braess' five-link network (Figure 1) as an example to study the robustness of traffic network components under user equilibrium. It is assumed that the link cost is affected not only by the flow of this link, but also by the flow of all other adjacent sections. There are three different paths between the starting point o and the end point r in Figure 1: opr, oqr and $opqr$.

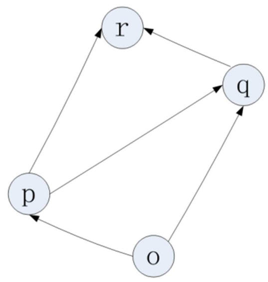

Figure 1. *The five-link network.*

We assume that the cost of each link is respective

$$c_{op} = 3(\gamma f_{op} + f_{pr} + f_{pq} + f_{oq}) + 10,$$

$$c_{qr} = 3(\gamma f_{qr} + f_{oq} + f_{pq} + f_{pr}) + 10,$$

$$c_{oq} = (\gamma f_{oq} + f_{qr} + f_{pq} + f_{op}) + 20,$$

$$c_{pr} = (\gamma f_{pr} + f_{op} + f_{pq} + f_{qr}) + 20,$$

$$c_{pq} = 2(\gamma f_{pq} + f_{op} + f_{pr} + f_{oq} + f_{qr}) + 5.$$

The cost of three paths is as follows:

$$C_1 = c_{op} + c_{pr},$$
$$C_2 = c_{oq} + c_{qr},$$
$$C_3 = c_{op} + c_{pq} + c_{qr},$$

The flow of each link and path satisfies the following relationship:

$$f_{op} = x_1 + x_3,$$

$$f_{pr} = x_1,$$

$$f_{oq} = x_2,$$

$$f_{qr} = x_2 + x_3,$$

$$f_{pq} = x_3,$$

Path flow and traffic demand meet the following conservation relationship:

$$Q = x_1 + x_2 + x_3 .$$

In order to solve the flow of each path for the five-link traffic network under the user equilibrium, let $C_1 = C_2 = C_3$, we get the path flow of the network under user equilibrium:

if $Q \le \frac{5}{5\gamma+4}$, $x_1 = x_2 = 0, x_3 = Q$,

if $\frac{5}{5\gamma+4} \le Q \le \frac{5}{\gamma+6}$, $x_1 = x_2 = \frac{(5\gamma+4)Q-5}{4(2\gamma-1)}, x_3 = \frac{(-\gamma-6)Q+5}{2(2\gamma-1)}$,

if $Q \ge \frac{5}{\gamma+6}$, $x_1 = x_2 = \frac{Q}{2}, x_3 = 0$

Otherwise

$$T^5 = x_1C_1 + x_2C_2 + x_3C_3 ,$$

Where T^5 is the total cost of five-link network. There are different path flow allocation because of different values of Q. We calculate the total cost as follows:

$$T^5 = (8\gamma+10)Q^2 + 25Q, \text{ if } Q \le \frac{5}{5\gamma+4},$$

$$= Q[\frac{(7\gamma^2+4\gamma-20)Q+5(\gamma+2)}{2(2\gamma-1)} + 30], \text{ if } \frac{5}{5\gamma+4} \le Q \le \frac{5}{\gamma+6},$$

$$= 2(\gamma+2)Q^2 + 30Q, \text{ if } Q \ge \frac{5}{\gamma+6}.$$

Let $\gamma = 2$, we study the importance index of the different components of the traffic network under user equilibrium and sort the importance. In the process of traffic distribution, the change of traffic demand changes the flow of each component of the traffic network, which leads to the change of the importance index and the importance ranking. In different ranges, the relative total congestion of each link of the network is listed in table 1.

Table 1. *Importance value of each link in different ranges.*

	$Q \in [0, \frac{5}{14})$	$Q \in [\frac{5}{14}, \frac{5}{8})$	$Q \in [\frac{5}{8}, +\infty)$
$\frac{5-14Q}{26Q+25}$	$\frac{5-14Q}{26Q+25}$	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
oq	0	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
pr	0	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
qr	$\frac{5-14Q}{26Q+25}$	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
pq	$\frac{5-18Q}{26Q+25}$	$\frac{8Q-5}{4Q+50}$	0

When $Q \in [0, \frac{5}{14})$, if $Q \in [0, \frac{5}{18})$, $\Gamma^{pq} > \Gamma^{oq}, \Gamma^{pq} > \Gamma^{pr}$,

if $Q \in [\frac{5}{18}, \frac{5}{14})$, $\Gamma^{pq} < \Gamma^{oq}, \Gamma^{pq} < \Gamma^{pr}$,

Then the importance ranking of each link in different ranges is listed in table 2.

Table 2. *Importance ranking of each link in different ranges.*

	$Q \in [0, \frac{5}{18})$	$Q \in [\frac{5}{18}, \frac{5}{14})$	$Q \in [\frac{5}{14}, \frac{5}{8})$	$Q \in [\frac{5}{8}, +\infty)$
1	1	1	1	1
oq	3	2	1	1
pr	3	2	1	1
qr	1	1	1	1
pq	2	3	2	2

The importance value and the importance ranking of each node in different ranges are listed respectively in table 3 and table 4.

Table 3. *Importance value of each node in different ranges.*

	$Q \in [0, \frac{5}{14})$	$Q \in [\frac{5}{14}, \frac{5}{8})$	$Q \in [\frac{5}{8}, +\infty)$
o	$+\infty$	$+\infty$	$+\infty$
p	$\frac{5-14Q}{26Q+25}$	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
q	$\frac{5-14Q}{26Q+25}$	$\frac{14Q-5}{4Q+50}$	$\frac{2Q}{4Q+15}$
r	$+\infty$	$+\infty$	$+\infty$

Table 4. *Importance ranking of each node in different ranges.*

	$Q \in [0, \frac{5}{14})$	$Q \in [\frac{5}{14}, \frac{5}{8})$	$Q \in [\frac{5}{8}, +\infty)$
o	1	1	1
p	2	2	2
q	2	2	2
r	1	1	1

From Table 1 to 4, it can be seen that the importance index and ranking of different components of the network are different, and the importance index and the ranking of the components change with the traffic demand changing. It provides some theoretical guidance for transportation network planning.

4. Results

In this paper, we review the traffic assignment models and study the robustness of the traffic network. In the study, it is assumed that the link impedance is not only related to the traffic flow in this link, but also to the flow of all adjacent links. We get the ranking of the components by calculating the relative impedance of each component of the traffic network under the user equilibrium. Because we take into account the influence on the link cost of the flow of all adjacent link during the research process, it can reflect the actual traffic phenomenon and provide some guidance for the research on the robustness of the traffic network. The robustness of more complex traffic networks remains to be explored.

Acknowledgements

The research was supported by National Natural Science Foundation of China (71171124, 11401011), it was also supported by Natural Science Research Project of the Education Department in Henan Province (14B110032) and Research Development Project of Anyang Normal University (AYNU-KP-A02).

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