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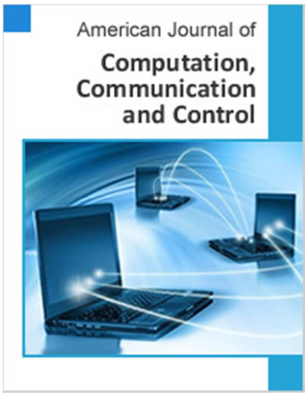

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Dynamic Set-Point Weighting of 2DOF/PID via Super Twisting Sliding Modes

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Abstract
The work deals with the set-point weighting of PID controllers with two degrees of freedom (2DOF/PI_D). A dynamic weighting method is proposed to overcome the limitations that these controllers usually present in complex and/or non-linear processes. Unlike the conventional procedure in which the set-point weight of the integral action is set to 1, in this work that weight is dynamically adjusted. The proposal complements ideas of high order sliding mode control (HOSM) with concepts of immersion of systems and manifold invariance (I&I). This allows achieving the target dynamics in finite time and, potentially, allows preserving the anti-reset-windup properties that 2DOF/PID controllers present in linear systems. The main features of the proposal are validated through an example.

1. Introduction

Over the years, several PID structures have been proposed. One of them is the so-called PID with set-point weighting or two degree of freedom PID controller (2DOF-PID). This structure results especially useful for accomplishing several specifications simultaneously [1]. Particularly, a well known feature of the 2DOF-PID structure is that responses of the system to both disturbances and changes in the set-point can be adjusted separately. The popularity of this structure is such that most real commercial PID controllers include set-point weighting. In spite of that, the common practice is to set the integral weight at one (by steady state error reasons) and the derivative weight at zero to avoid large transients in the control signal due to sudden changes in the set-point (i.e. the derivative kick effect) [2] [3] [4]. In this case, where only the proportional set point weight is tuned, the controller is known as 2DOF-PI_D.

The effects of set-point weighting are rather intuitive in most of the simple processes. For this reason, empirical tuning methods are extensively used. Several methods with theoretical support have also been proposed for both SISO and MIMO processes [5], between them some approaches include autotuning and dynamic weighting to improve the tracking behaviour or robustness [3] [6] [7] [8] [9] as well as to limit the coupling between variables in MIMO systems [10] [11]. References of the most popular methods for tuning 2DOF-PID can be found in Mudi & Dey (2011) [12] and O'Dwyer (2012) [13].

Although the lots of heuristic rules and analytic methods that have been proposed for the tuning of the set-point weight of these controllers, little has been written that explicitly deals with nonlinear processes. In this case, suitable constant weights for a given set-point could be inappropriate for other reference values. Additionally, constant values for the weight coefficient in all nonlinear range of operation could drive to excessively conservative behaviours. Then, when the processes are complex and/or highly nonlinear, variable weights should be considered. While in some works, this problem is addressed from a process of linearization [11], in this paper the problem is focused using concepts from the theory of nonlinear systems (I&I). More precisely, with concepts of HOSM [14] [15] as complement of the theory of immersion of systems and invariance manifolds [16]. In this framework, a new methodology to the dynamic tuning of the set-point weights is presented. Unlike the conventional procedure in which the set-point weight of the integral action is set to 1, in this proposal that weight is dynamically adjusted. The proposal allows to assign a reduced-order lineal dynamics for the tracking response independently of the set-point change. From a practical position, the adjustment is performed by a simple super twisting sliding mode regime that accepts a straightforward implementation.

The paper is organized as follows. Section 2 briefly reviews the basic concepts of immersion of systems and invariant manifolds and high order sliding mode control. In section 3, the new proposal for the dynamic adjustment of the set-point weights of 2DOF-PID controllers in nonlinear processes is presented. Then, the main features of the proposal are validated through an example. Finally, conclusions are summarized.

2. The I&I Philosophy and High Order Sliding Mode Properties

The concepts of system immersion and manifold invariance are closely linked to the control theory of nonlinear systems. Effectively, while the idea of system immersion is usually associated with the transformation of a system into another with specific properties, the notion of invariant manifold has been extensively used to deduce control actions in nonlinear systems. Recently, Astolfi et al. have formalized a new theoretical framework termed I&I (Immersion and Invariance) [17] that removes some constrains of the conventional definitions of system immersion and manifold invariance. The new framework has the attractive property of reducing the design problem of nonlinear controllers to subproblems which might be substantially easier to solve and that do not require knowledge of Lyapunov functions.

The basic idea of I&I philosophy consists in forcing the closed loop system dynamics to converge asymptotically into a desired behaviour. The desired behaviour, which presents a smaller number of variables than the original dynamical system, is immersed in the system original dynamics and is rendered “invariant”, i.e. all trajectories that enter in the state space of this smaller subsystem remain in it. Figure 1 illustrates this basic idea of I&I philosophy in the state space defined by the n state variables of the actual system. The shaded surface (manifold M) corresponds with the desired closed loop dynamic response of order $p<n$. In the general case the desired behaviour is set in terms of other p state variables ζ. Then, through an proper control action $u=v(x)$ any trajectory beginning in an arbitrary initial state $x(0)$ is forced to converge asymptotically to M to ensure the desired dynamic. The attractiveness of M is defined in terms of a distance function $\xi_1=dist(x,M)$ whose absolute value must be reduced to zero. This signal ξ_1 can be defined in different ways, which gives an additional degree of freedom to design.

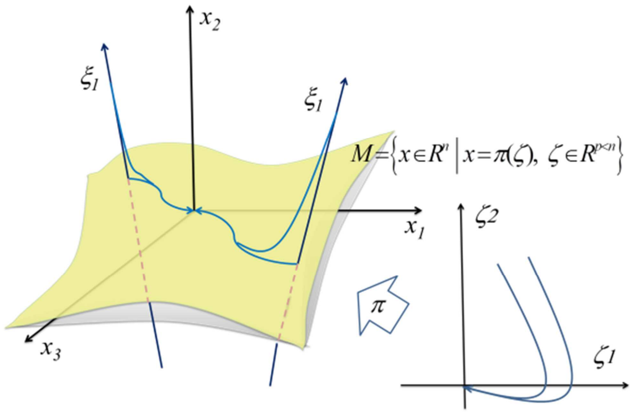

Figure 1. *Graphical interpretation of the I&I philosophy.*

Although I&I notions are used in a wide range of problems, they are usually introduced in the context of systems stabilization from the strict and long-established ideas of system immersion and manifold invariance [17]. The Appendix resumes this frequent explanation.

In this work, I&I ideas are combined with sliding mode concepts. Particularly, it is Sliding Mode control has proved to be an apt technique capable of coping with complex characteristics of nonlinear systems. Since its origin SM has evolved into a powerful design technique for a wide range of applications. The discontinuous nature of SM control action, provides excellent system performance, which includes insensitivity to certain parameter variations and rejection of disturbances. However, in practice, direct application of such discontinuous control action can be not adequate for some actual plants. In addition, it can generate output chattering, which deteriorates the robustness of conventional SM control. To attenuate this problem, the concept of higher order sliding modes was introduced. HOSM are an excellent option to control nonlinear uncertain systems operating in perturbed environments [15]. HOSM techniques allow zeroing the sliding variable and its first time derivative in finite time, through a continuous control $u(t)$ acting discontinuously on its time derivative, reducing strongly the chattering phenomenon. They result in controllers with several attractive characteristics

a. Robustness with respect to disturbances and model uncertainties,
b. Finite-time convergence.
c. Reduction of mechanical stresses and chattering (i.e., high-frequency vibrations of the controlled system),

compared to standard sliding mode strategies,

d. Relatively simple control laws, which entail low real-time computational burden.

e. Capability of dealing with nonlinear system, and therefore, wider ranges of operation are attained in comparison to design techniques based on model linearization.

One of the most powerful high order continuous sliding mode control algorithms is the super-twisting control law, which is used in this work for the set-point weighting in 2DOF/PID. [15] [18]

3. Dynamic Set-Point Weighting Based on Concepts of I&I and HOSM

Consider the state model of a nonlinear process

$$\dot{x} = f(x) + g(x)u \tag{1}$$

$$y = h(x)$$

and the PID controller

$$u = k_p\left[(b_p\, r - y) + \frac{k_i}{k_p}\int (b_i r - y)dt - \frac{k_d}{k_p}\frac{dy}{dt}\right] \tag{2}$$

where k_p, k_i and k_d are the proportional, integral and derivative gains respectively, which are tuned for the proper rejection of perturbations in the vicinity of $y=r$. bp and bi are the set-point weights for the proportional and integral actions.

It is important to note that unlike the conventional procedure in which the set-point weight of the integral action bi is fixed constant and equal to 1, in this proposal bi is an adjustment variable.

In complex and/or non linear processes, constant values of the set-point weights not always can solve the trade-off between the overshoot and settling time in the closed loop response, particularly when the non linear process works with set-points forcing the variables to operate in a wide range of values. In this respect, the present proposal deals with the dynamic tuning of the weights $bp(t)$ and $bi(t)$ so that the tracking system response presents a suitable behaviour, specifically a tracking response with reduced-order linear dynamics independently of the nonlinear characteristics of the system. As it will be shown, both the order of the desired tracking dynamics and the corresponding eigenvalues can be chosen without major difficulty from basic ideas of I&I and HOSM. For ease of presentation, in section 3.2 the tuning proposal is first introduced for the case in which a first-order tracking dynamics is specified.

3.1. Implementation of the Dynamic Weights *bp(t)* and *bi(t)*

Although $bp(t)$ and $bi(t)$ can be implemented through variable gains, in this work it is proposed to do it from the addition of signals $wp(t)$ and $wi(t)$ on the set-point. Then

$$u = k_p\left[(r + w_p - y) + \frac{k_i}{k_p}\int (r - w_i - y)dt - \frac{k_d}{k_p}\frac{dy}{dt}\right] \tag{3}$$

where

$$b_p(t) = \frac{r + w_p(t)}{r} \tag{4}$$

$$b_i(t) = \frac{r + w_i(t)}{r} \tag{5}$$

The implementation of the weights $bp(t)$ and $bi(t)$ according to equations (4) and (5) is similar to that used in the reference conditioning techniques to solve problems of windup, variable coupling, bump, transients in control gain scheduling, etc.. Depending on the nonlinear characteristics of the process, the determination of $bp(t)$ and $bi(t)$ may present difficulties. To avoid them, a practical procedure that actually simplifies this adjust is proposed in this section. That consists in forcing the attractiveness of M via a simple high order sliding mode.

Figure 2 illustrates the scheme of the 2DOF-PID controller with a detail of the proposed tuning circuit for the variable set-point weighting. Note that simple algebraic operations convert the previous scheme into a higher order sliding mode algorithm known as super twisting (ST) [15] [17] acting on the reference r. Since, the set-point weighting is performed at the lowest power signal level, a digital implementation is also straightforward.

The conditioning actions proposed are

$$w_p = \begin{cases} -\lambda\,|\,s_0\,|^{\alpha}\ sign(s(\xi)) & \text{si}\ \ |s(\xi)| > s_0 \\ -\lambda\,|\,s(\xi)\,|^{\alpha}\ sign(s(\xi)) & \text{si}\ \ |s(\xi)| \le s_0 \end{cases} \tag{6}$$

$$w_i = -\gamma\,|\,s_0\,|\ sign(s(\xi)) \tag{7}$$

with $s(\xi)$ is a function for zeroing ξ_1 and γ, λ and s_0 design parameters.

According to the commented equivalence with respect to ST algorithms, α can be chosen in the interval (0; 0.5) in order that the trajectories of the controlled system converge to the origin of the plane ($s(\xi), \dot{s}(\xi)$) in finite time, however, α = 0.5 has proven to be the best option for the non-ideal case [15]. Then, the sufficient conditions of convergence in finite time and operation in HOSM are

$$\begin{aligned} &\gamma > \frac{C}{\Gamma_m} \\ &\lambda > \frac{\sqrt{2(\gamma\,\Gamma_M + C)}}{\Gamma_m} \end{aligned} \tag{8}$$

where C, Γ_m and Γ_M verify the differential inclusion

$$\ddot{s} \in [-C, C] + [\Gamma_m, \Gamma_M]\dot{u} . \qquad (9)$$

Then, taking advantage of the ST properties, the immersion of the real dynamics of the system in the desired one is achieved in finite time if it is chosen of relative degree 1.

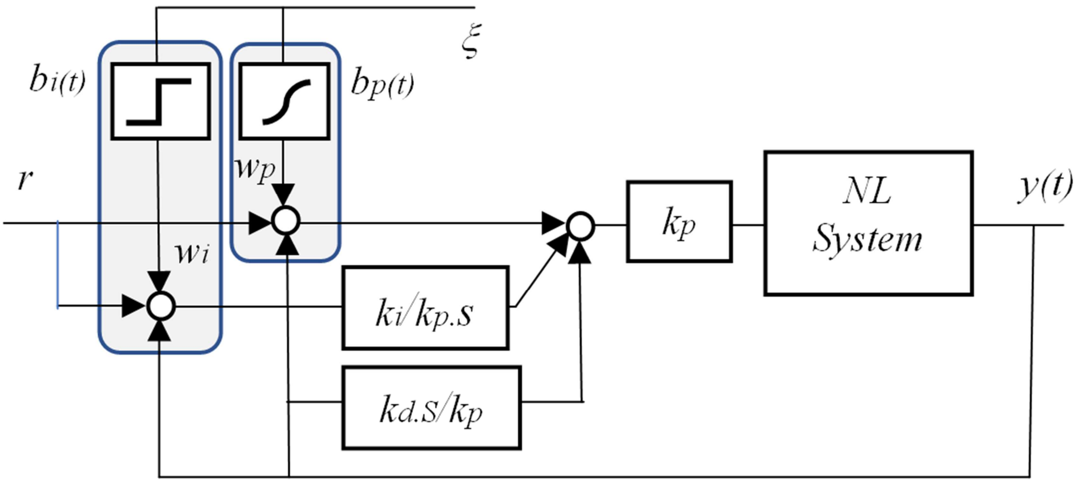

Figure 2. *2DOF-PID controller with the proposed tuning circuit for the weights bp(t) and bi(t).*

3.2. First-Order Tracking Dynamics

In this section, we adjust $bp(t)$ and $bi(t)$ based on I&I ideas to achieve asymptotic immersion of the tracking dynamics in a invariant manifold M parameterized by the solutions of

$$\dot{y} = -\lambda y + \lambda r \qquad \lambda > 0 . \qquad (10)$$

Among the different possibilities to select a signal for describing the discrepancy between the actual dynamics of the state $x(t)$ and the corresponding with the manifold M, can be defined

$$\xi_1(x) = dist(x, M) = L_{flc}h(x) + \lambda h(x) - \lambda r . \qquad (11)$$

where $L_{flc}h(x)$ is the Lie derivative of the output $h(x)$ along the closed-loop vector field flc.

Then, if the signal $\xi_1(x)$ is zeroed through $bp(t)$ and $bi(t)$, the dynamics of the set-point response will be asymptotically immersed in the desired one (ec. (10)).

To provide the present proposal with theoretical support it is useful to reformulate the system model in the normal form [19] considering the variable $\xi_1(x(t))$ as the first state. Then, if the relative degree of $\xi_1(x(t))$ with respect to $w(t) = w_p(t) + \int w_i(t) k_i \, dt$ is ρ, we proceed to model the complete system ($n+1$ states: n states of the open-loop system and one of the PID) from the following state variables:

$$x_e = \begin{bmatrix} \xi \\ \eta \end{bmatrix}_{(n+1)x1} , \qquad (12)$$

Where, the elements of the first subset ξ of states are the signal $\xi_1(x)$ and its ρ-1 successive derivatives and the surplus states η can be freely chosen. For simplicity, the first state (η_1) of the set η is chosen as the actual process output $y=h(x)$. Then, the new closed loop model results

$$\begin{bmatrix} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = \xi_3 \\ \vdots \\ \dot{\xi}_{\rho-1} = \xi_\rho \\ \dot{\xi}_\rho = a(\xi) + b(\xi)w \\ \dot{\eta}_1 = q_1(\xi,\eta) + p_1(\xi,\eta)w = \xi_1 - \lambda\eta_1 + \lambda r \\ \dot{\eta}_2 = q_2(\xi,\eta) + p_2(\xi,\eta)w \\ \vdots \\ \dot{\eta}_{n+1-\rho} = q_{n+1-\rho}(\xi,\eta) + p_{n+1-\rho}(\xi,\eta)w \end{bmatrix}_{(n+1)x1} . \qquad (13)$$

being our objective to force the fast convergence of the output $\xi_1(t)$ to zero by the discontinuous action $w(t)$. To this end, i.e. to guarantee the attractiveness of M and, as a consequence, the asymptotic immersion of the tracking dynamic response in the target system, it is proposed a sliding mode regime on the control surface

$$s(\xi) = k_1\xi_1 + k_2\xi_2 + k_3\xi_3 + ... + k_\rho\xi_\rho = 0 \qquad (14)$$

where the coefficients k_i define the convergence speed of ξ_1 to zero [20]. The selected surface $s(\xi)$ has relative degree $\rho=1$ with respect to the signal $w(t)$ fulfilling the *condition for* the sliding motions of the ST.

Then, once achieved the sliding regime, from equation (14), results,

$$\xi_\rho = \dot{\xi}_{\rho-1} = -\frac{1}{k_\rho}(k_1\xi_1 + k_2\xi_2 + k_3\xi_3 + ... + k_{\rho-1}\xi_{\rho-1}) \qquad (15)$$

then substituting (15) in (13), the reduced-order state model (order $n+1$) is obtained

$$\left[\begin{array}{l} \dot{\xi}_1=\xi_2 \\ \dot{\xi}_2=\xi_3 \\ \vdots \\ \dot{\xi}_{\rho-1}=-\dfrac{1}{k_\rho}(k_1\xi_1+k_2\xi_2+k_3\xi_3+.....+k_{\rho-1}\xi_{\rho-1}) \\ \dot{\eta}_1=\dot{y}=\underbrace{L_{f+gu}h(x)+\lambda y-\lambda r}_{\xi_1(x)}-\lambda y+\lambda r \\ \dot{\eta}_2=q_2(\xi,\eta)+p_2(\xi,\eta)w \\ \vdots \\ \dot{\eta}_{n+2-\rho}=q_{n+2-\rho}(\xi,\eta)+p_{n+2-\rho}(\xi,\eta)w \end{array}\right]_{(n+1-1)x1} . \quad (16)$$

where the extinction speed of the variable ξ_1 is defined by a linear dynamics whose (ρ-1) eigenvalues are assigned from the characteristic polynomial

$$\frac{k_1}{k_\rho}+\frac{k_2}{k_\rho}\lambda_\xi+\frac{k_3}{k_\rho}\lambda_\xi^2+.....+\frac{k_{\rho-1}}{k_\rho}\lambda_\xi^{\rho-1}=0, \quad (17)$$

existing a degree of freedom for choosing the k_i coefficients. Then, making this speed fast enough compared with the corresponding to the target system (defined by the eigenvalue λ), the dynamic equation of state η_1,

$$\dot{\eta}_1=\underbrace{\dot{h}(x)+\lambda y-\lambda r}_{\xi_1(x)\to 0}-\lambda\eta_1+\lambda r \quad (18)$$

approaches to the first order desired dynamics $\dot{y}\to-\lambda y+\lambda r$.

Obviously, all the remaining states dynamics η_i must meet the stability requirements (i.e. minimum phase zero dynamics).

Comments

a. Note that the HOSM tools are used in the context of I&I philosophy for zeroing the signal ξ_1 and not for forcing an SM on *M*. This procedure allows to choose the desired dynamic order $p<<n$ (for example $p=1$).
b. Observer that, in the previous explanation, the process output is one of the zero dynamics of the normal model (13). Effectively, the tracking objective is achieved in an indirect way actuating on the dynamic weights *bp(t)* and *bi(t)* for zeroing the variable ξ_1. In this sense, the proposed adjusting action could be interpreted as a special case of reference conditioning via sliding mode.
c. The HOSM tuning of the weights avoids chattering problems and guarantees that the dynamics of the system is immersed in the desired one in finite time.

3.3. Higher-Order Tracking Dynamics

Subsection 3.2 considered the asymptotic immersion of the tracking response in a target manifold parameterized by the trajectories of a first order dynamics. The extension of the previous ideas to the general case with higher order dynamics is straightforward. Indeed, it is enough to consider a new manifold parameterized by the solutions of the target tracking dynamics in the phase space

$$\overset{m}{y}=-(a_1y+\cdots+a_{m-1}\overset{m-2}{y}+a_m\overset{m-1}{y})+a_1r \quad (19)$$

with m verifying $m\leq n+1-\rho$, and a signal ξ_1 defining a separation between the actual state trajectory $x(t)$ and M, for example

$$\xi_1(x)=dist(x,M)=\overset{m}{h}(x)+a_1y+\cdots+a_m\overset{m-1}{y}-a_1r\cdot \quad (20)$$

Again, it is useful to state the normal model of the complete system, choosing ξ_1 as the first state, i.e. a model where the first ρ states are ξ_1 and its successive ρ-1 derivatives and where the first m states of the subset η are selected according to (16) as the output variable $y=h(x)$ and its m-1 derivatives, i.e.

$$\begin{aligned} &\eta_1=h(x) \\ &\eta_2=\dot{h}(x) \\ &\vdots \\ &\eta_m=\overset{m-1}{h}(x) \end{aligned} \quad (21)$$

then, when the trajectories are forced to converge asymptotically to M by a HOSM regime on (14), the zero dynamics verifies the target tracking dynamics (16):

$$\begin{aligned} &\dot{\eta}_1=\eta_2 \\ &\dot{\eta}_2=\eta_3 \\ &\vdots \\ &\dot{\eta}_m=\overset{m}{y}=\underbrace{\overset{m}{h}(x)+a_1y+\cdots+a_m\overset{m-1}{y}-a_1r}_{\xi_1(x)\to 0}-(a_1\eta_1+\cdots+a_m\eta_m)+a_1r \end{aligned} \quad (22)$$

4. Example

Consider a simplified nonlinear model of a laboratory thermal system for testing thermal properties of materials

$$\dot{x}=\begin{bmatrix}0,1\,x_2^2-x_1 \\ -x_2+u\end{bmatrix}, \quad y=\begin{bmatrix}1 & 0\end{bmatrix}x \quad (23)$$

where x_1 represent the sample temperature and x_2 the voltage applied to the electrical heater. Consider also a PID controller as (2), where the gains k_i=1.31 and k_p=17.76 and kd=0.89 have been tuned from the linearized model (in the proximity of the steady state point of regulation) to suitable disturbances rejection. In this case, the disturbance rejection has a characteristic close to that known as "quarter decay" which is considered adequate for many chemical processes (anyway it is important to keep in mind that the present proposal is independent of the PID gains tuning). Obviously, due to the nonlinear characteristics of the system, this type of response is not obtained in other operation points without the proper readjustment of the controller gains.

Figure 3 shows system output *y(t)* in response to different inputs (two changes of the set-point (0.6 and 0.75) and a constant disturbance) when constant set-point weights $bi = 1$ (conventional case) and $bp = 1$, 0.75 and 0.5 are used. It is observed a clear trade-off between overshoot and large settling time. The poor tracking responses contrast with the suitable disturbance rejection. This result is not surprising since the PID controller gains were tuned based on a linearized model that is valid on the surrounding of the steady state point and not for the tracking response. Part b of the figure shows the corresponding control action. Note that the controller saturates when the set-point changes at $t = 0$sec leading to an important windup.

The curves highlight 2DOF/PID constrains when constant weightings are used in nonlinear systems like this one, particularly to

1) overcome the trade off between regulation and tracking responses, effectively the reduction of bp to attend the overshoot of the controlled variable leads to a marked degradation of the settling time.
2) overcome restrictions problems in the control action. That is, in non-linear systems, it is not always possible to preserve the capabilities presented by the 2DOF/PID in linear systems to overcome problems such as windup without using additional correction actions. [21]

Due to the limitations for obtaining an acceptable tracking response from constant weight values, we proceed to evaluate the proposed dynamic tuning. The dynamics of the closed loop system, including the integral state of the PID controller, is given by the state differential equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} 0{,}1\, x_2^2 - x_1 \\ -k_d\left(0{,}1\, x_2^2 - x_1\right) + k_p\left[(r + w_p - x_1) + \dfrac{k_i}{k_p} x_i\right] \\ r + w_i - x_1 \end{bmatrix}. \qquad (21)$$

We propose as target tracking dynamics a first order response with time constant τ=5sec

$$\dot{\zeta} = -\lambda\zeta + \lambda r = -.2\zeta + .2r \qquad (22)$$

with $\zeta = y$ and a signal

$$\xi_1(x) = 0{,}1\, x_2^2 - x_1 + \lambda y - \lambda r \qquad (23)$$

as a measure between the actual trajectories and the manifold defined from the solutions of the target dynamics (22). This signal ξ_1 has relative degree $\rho = 2$ with respect to the discontinuous signal $w(t)$. Then its absolute value can be reduced in a controlled way forcing a sliding mode on the surface

$$s(\xi) = k_1\xi_1(x) + k_2\xi_2(x) = 0 \qquad (24)$$

with $k_1/k_2 >> \lambda$ to guarantee a convergence speed faster than corresponding to the selected for the tracking response. In the present case we choose $k_1/k_2=2$ (i.e. the time constant of the extinction speed of ξ_1 ten times less than the corresponding to the target dynamics $1/\lambda$=5sec).

Figure 4a shows the response of the non linear process with the 2DOF-PID controller with the proposed dynamic weighting. From a practical point of view, this tracking response presents the target dynamics (22) with a much better performance than the corresponding to constant weights. In particular, a lesser settling time is observed.

In Figure 4b, it is observed that the dynamic conditioning of the reference allows to take better advantage of the degrees of freedom of the 2DOF/ PID to avoid the windup without adding additional corrections. This is possible because the choice of the tracking dynamics can limit its derivative when the change of the set-point occurs. Thus, the adjustment variable here is related to the target selected dynamics. Indeed, in part b of this figure it is seen that the control action is substantially smaller avoiding the constraints of the actuator. In addition, it is appreciated that there is not chattering in control signal $u(t)$.

Figures 5a and 5b show the dynamic evolution of the set-point weights that ensure the desired dynamics verifying that the steady state value of bi is 1, necessary condition to avoid the error of steady state. Figures 5c and 5d show the extinction of the ξ_1 (function chosen to define the discrepancy between the actual system dynamics and the desired one) through $s(\xi)$ and its derivative. This fact can also be verified in the figure 5d where is observed, in the plane ($s(\xi), \dot{s}(\xi)$). Then, after a finite time t = 0.9sec, the response of the original system is the desired one.

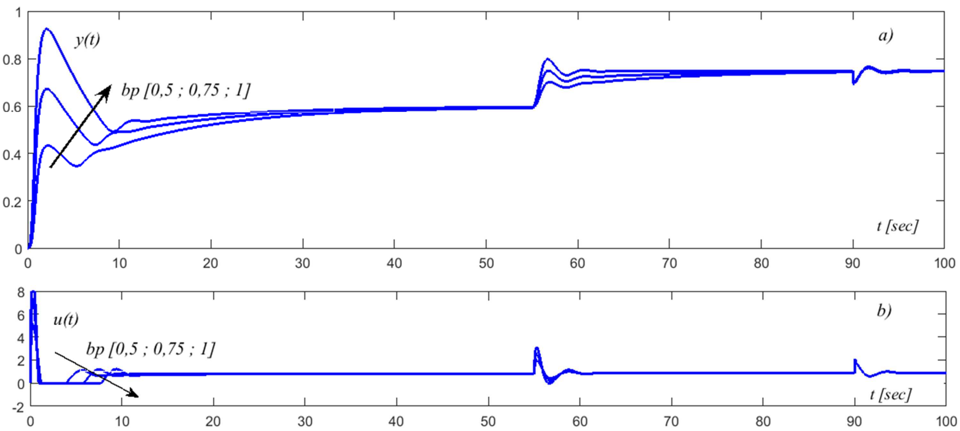

Figure 3. *Controlled variable (upper part) and control input (lower part) for 2DOF/PID with constant weights: bp=(0.5; 0.75; 1) and conventional bi=1.*

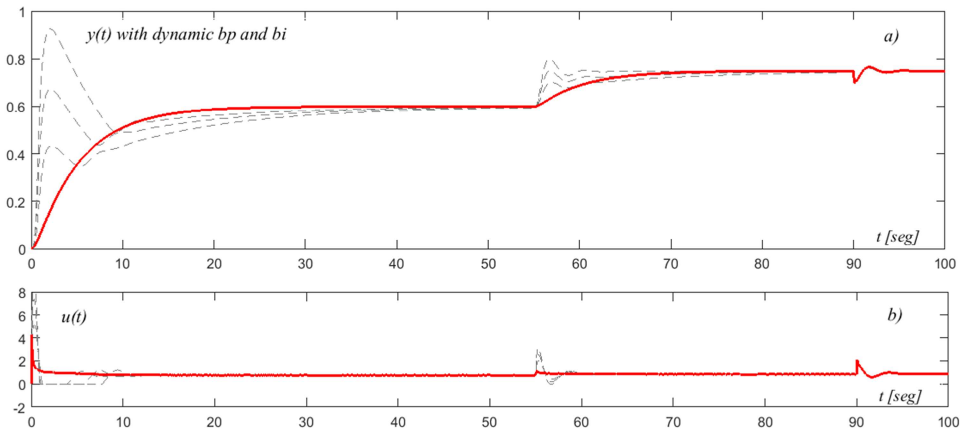

Figure 4. *Controlled variable (a) and control input (b) for 2DOF/PID with the proposed dynamic set-point weighting.*

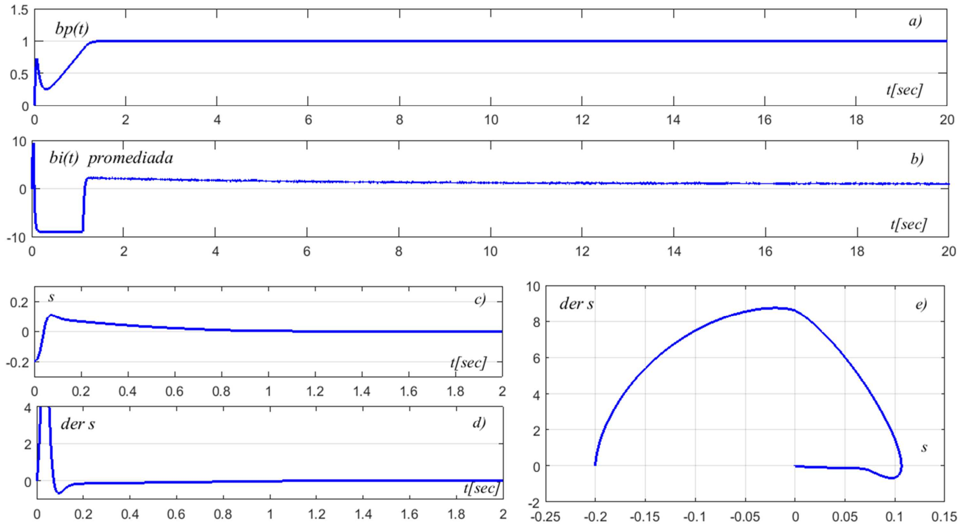

Figure 5. *a) and b) Dynamic adjustment of bp(t) and bi(t) from I&I and HOSM ideas; c) and d) evolution of the function $s(\xi)$ and its derivative $\dot{s}(\xi)$; d) trajectory in the plane ($s(\xi), \dot{s}(\xi)$).*

5. Conclusion

2DOF-PID controllers are widely used in industrial environments. Although lots of heuristic rules and analytic methods have been proposed for tuning the set-point weighting, little has been written that explicitly deals with nonlinear processes. In these cases, constant weight coefficients for suitable tracking of a given set-point are usually inappropriate for other set-point values (and/or other initial conditions). This fact encourages the use of dynamic weights. In this way, this paper has developed a new method for dynamic tuning of the set-point weighting of 2DOF-PID controllers, suitable for nonlinear processes. The proposal is focused using concepts from the theory of nonlinear systems, more precisely, with concepts of high order sliding mode control as complement of the theory of immersion of systems and manifold invariance. Unlike the conventional procedure in which the set-point weight of the integral error is set at 1, the proposal includes its adjustment. The proposed adjusting action can be interpreted as a special case of reference conditioning via high order sliding modes. From a practical point of view, the dynamic weighting is performed via a simple super twisting algorithm that accepts a straightforward implementation, which makes it suitable for industrial applications. In addition, this algorithm avoids chattering problems and guarantees that the dynamics of the system is immersed in the desired one in finite time. The main features of the proposal are validated through an example.

Appendix

I&I Basics

As was commented earlier, for simplicity reasons, the *I&I* basics are usually introduced in the context of the systems stabilization making use of the strict concepts of system immersion and manifold invariance [17]. To this end, consider the nonlinear system,

$$\dot{x} = f(x) + g(x)u \tag{A.1}$$

with $x \in R^n$ and $u \in R^m$, and where we are interested in getting a feedback control law $u = v(x)$ so that the controlled system presents an asymptotically stable equilibrium at the origin. Based on conventional ideas of system immersion and manifold invariance, the problem is addressed by finding:

1. a target system with reduced-order dynamics

$$\dot{\zeta} = \alpha(\zeta) \quad \zeta \in R^{p<n}, \tag{A.2}$$

asymptotically stable at the origin;

2. a smooth mapping

$$x = \pi(\zeta), \tag{A.3}$$

3. a state feedback control $u = v(x)$ such that

$$\pi(\zeta(0)) = x(0), \tag{A.4}$$

$$\pi(0) = 0, \tag{A.5}$$

$$f(\pi(\zeta)) + g(\pi(\zeta))v(\pi(\zeta)) = \frac{\partial \pi}{\partial \zeta}\alpha(\zeta). \tag{A.6}$$

If the previous problem can be solved, any state trajectory x of the closed loop system can be seen as a mapping π of a trajectory ζ of the target system. As this target system is asymptotically stable at the equilibrium, $x(t)$ converges to the origin. From a geometric point of view all closed loop trajectories $x(t)$ live in the invariant manifold

$$M = \left\{ x \in R^n \mid x = \pi(\zeta),\ \zeta \in R^{p<n} \right\} \tag{A.7}$$

with internal dynamics $\dot{\zeta} = \alpha(\zeta)$.

Even though this approach is theoretically precise, it is not always practical since both the mapping $x = \pi(\zeta)$ and the control $u = v(x)$ depend on the initial conditions, which complicates the calculus (actually, in many applications, it could be impossible to be solved). From a practical standpoint, these limitations of the conventional definitions of system immersion and manifold invariance can be overcome by the I&I ideas determining a solution for (A.5) and (A.6) (i.e. without requiring (A.4)), and modifying the control action $u = v(x)$ such that M is attractive, i.e. for any initial condition, the system trajectories of the closed loop system

$$\dot{x} = f(x) + g(x)v(x) \tag{A.8}$$

converge to the manifold M. The attractiveness of M is defined in terms of a distance function

$$\xi_1 = dist(x, M) \tag{A.9}$$

whose absolute value is reduced to zero. This signal can be defined in different ways, which gives an additional degree of freedom to design.

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References

[1] Araki M., Taguchi H.. Two-Degree-of-Freedom PID Controllers. *Int. J. of Control, Automation, and Systems.* 2003. 1, 401-411.

[2] Åström, K., Hagglund, T. *Advanced PID control.* ISA. Research Triangle Park, USA. 2006.

[3] Vilanova R., Visioli A.. *PID Control in the Third Millennium: Lessons Learned and New Approaches (Advances in Industrial Control) 2012th Edition.* Springer-Verlag London Limited. 2012.

[4] Guzmán J., Åström K., Dormido S., Hägglund T., Berenguel M., Piguet Y.. "Interactive Learning Modules for PID Control: Using interactive graphics to learn PID control and develop intuition", *IEEE Control Systems Magazine*. 2008, 28, 118-134.

[5] O'Dwyer, A.. *Handbook of PI and PID controller tuning rules.* 3rd Edition. Imperial College Press. London, UK. 2009.

[6] Hast, M., Hägglund T.. Optimal proportional–integral–derivative set-point weighting and tuning rules for proportional set-point weights. *IET Control Theory & Applications.* 2015, 9, 2266-2272.

[7] Dey, C., Mudi, R., Lee, T.. A PID controller with dynamic set-point weighting. *In Proc. of the IEEE Int. Conf. on Industrial Technology, ICIT 2006.* 18-21 December. 1071–1076. Bhubaneswar, India.

[8] Vilanova R., Alfaro V. M., Arrieta O.. Simple robust autotuning rules for 2-DoF PI controllers. *ISA Transactions.* 2012, 51, 30-41.

[9] Mantz R.. "A PI controller with dynamic set-point weighting for nonlinear processes*". IFAC Conference on Advances in PID Control. PID'12.* 2012. Brescia, Italy.

[10] Åström, K., Johansson, K., Wang, Q.. Design of decoupled PID controllers for two-by-two systems. *IEE Proc. on Control Theory and Applications*. 2002, 49, 74-81.

[11] Bianchi, F., Mantz, R., Christiansen, C.. Multivariable PID control with set-point weighting via BMI optimisation. *Automatica*. 2008. 44, 472-478.

[12] Mudi R. K., Dey Ch.. Performance Improvement of PI Controllers through Dynamic Set-point Weighting. *ISA Transactions.* 2011. 50, 220-230.

[13] O'Dwyer, A.. *An Overview of Tuning Rules for the PI and PID Continuous-Time Control of Time-Delayed Single-Input, Single-Output (SISO) Processes.* Chapter 1 in book PID Control in the Third Millennium Lessons Learned and New Approaches. Editors: Vilanova R. and Visioli A.. Springer-Verlag London Limited. 2012.

[14] L. Fridman and A. Levant, "Higher order sliding modes as a natural phenomenon in control theory," in *Robust Control via Variable Structure and Lyapunov Techniques,* vol. 217. London, U.K.: Springer-Verlag, 1996, ch. 1, pp. 107-133.

[15] Shtessel Y., Edwards C., Fridman L., Levant A. *Sliding Mode Control and Observation.* Birkhauser. London. UK. 2014.

[16] Astolfi A., Ortega R.. Immersion and invariance: a new tool for stabilization and adaptive control of nonlinear systems. *IEEE Trans. on Automatic Control.* 2003, 4, 590-606.

[17] Astolfi, A., Karagiannis, D., Ortega, R. *Nonlinear and Adaptive Control with Applications.* Springer-Verlag. London. UK. 2008.

[18] Shtessel Y., Moreno J., Plestan F., Fridman L., Poznyak A. "Super-twisting Adaptive Sliding Mode Control: a Lyapunov Design". *49th IEEE Conference on Decision and Control.* Atlanta, USA, 2010. 5109-5113.

[19] Isidori, A.. *Nonlinear Control Systems II.* Springer-Verlag London Limited, UK. 1999.

[20] Utkin V., Guldner J., Shi. J.. *Sliding Mode Control in Electromechanical Systems.* Taylor and Francis. London. 1999.

[21] Puleston P., Mantz R. J. Proportional plus Integral MIMO Controller for Regulation and Tracking with Anti- Wind-Up Features. *Ind. Eng. Chem. Res.* 1993, 32, 2647-2652.