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Magnitude of the photon momentum in matter

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Abstract
We show that the total momentum of a photon in matter increases by n times as compared with that in free space where n is the refractive index of the matter. The photon is considered as a wave train which total momentum consists of two components. The electromagnetic component is smaller by n^2 times than the total momentum. The mechanical component that is distributed uniformly between leading and trailing edges of the train and is $(1\text{-}1/n^2)$ part of the total momentum. Our analysis is based on results of two well-know unambiguous but contradictory experiments. No preliminary assumption about kinds and physical origin of optically induced force responsible for a change of the photon momentum in matter is made. It is shown that these experiments can be matched if the Abraham force arising in the regions where leading and trailing edges of the wave train are propagating is taken into account.

1. Introduction

So far, there is a debate about a magnitude of the photon momentum p in matter. In accordance with many theoretical considerations [1-3], in particular, in accordance with relation $p=h/\lambda$, where h is the Planck constant, λ is the wavelength of the photon in matter, the momentum of a photon in matter is greater by n times than that in free space. The same result has been obtained by means of a direct measurement of the pressure produced by the light on a mirror located inside a liquid transparent dielectric [4-5]. However, there is the Balazs thought experiment [6] that is described many times [2, 7-9]. It is shown theoretically on the basis of the law of conservation of momentums in a closed system without any assumptions about properties of the photon that a transparent block into which the photon is launched should move in a direction of a propagation of the photon. In this case a part of the momentum of the photon is transferred to the block. Therefore, the momentum of the photon propagating inside the block becomes smaller than that of the same photon propagating in free space p_0. It is shown [2, 7, 8, 9] that the block acquires the momentum given by

$$p=p_0(1\text{-}1/n) \qquad (1)$$

Then the momentum of the photon within the block is equal $p_0\text{-}\ p_0(1\text{-}1/n)=p_0/n$. This result corresponds to the momentum proposed by Abraham [10]. As is pointed in review [7], “If argument advanced in favor of the Abraham momentum were to be incorrect, than that would bring into question uniform motion of an isolated body as expressed in the Newton’s first law of motion”.

Hitherto all attempts to match rival results of the experiments failed because a reason of this discrepancy is not found out. Recognition that both results are correct [9] because they correspond to experiments is insufficient. An explanation is required why the momentums are different. We present our explanation and show that the Abraham density force arising in regions where the photon is propagating ought to be taken into account. In this case various known thought experiments and experimental evidence are consistent with each other completely.

2. Notions and Definitions

Above all things, it is worthwhile to note that our notion about the momentum of light is taken from mechanics where this notion was introduced several centuries ago for a body of mass M moving at speed v as a production of Mv. In accordance with Newton the momentum characterizes the "quantity of motion". A reason of a change of the momentum p is a force f and in accordance with the second Newton law $dp/dt=f$. In a closed system (one that does not exchange any matter with the outside and is not acted on by outside forces) the total momentum is constant. This fact, known as the law of conservation of momentum, is implied by Newton's laws of motion. [11, 12]. A magnitude of the momentum of a light pulse propagating in vacuum is generally accepted and is equal E_{pulse}/c, where E_{pulse} is the energy of the light pulse. A photon is considered as the light pulse where $E_{\text{pulse}}=h\upsilon$, υ is the frequency of the photon

If a continuous plane light wave is considered, its momentum is equal to infinity. In this case the momentum density given by $g=[E\Box H]/c^2=\varepsilon_0E^2/c$ is considered where E is the strength of the electrical field. The field is changed harmonically as $E_0\text{Sin}\omega t$ for the light wave. Then the momentum density averaged over period of oscillation is given by $<g>=\varepsilon_0E_0^2/(2c)= W_0/c$ where $W_0=\varepsilon_0E_0^2/2$ is the energy (electrical plus magnetic) density of the light wave averaged over period of oscillation, E_0 is the amplitude of the alternate electrical field . The momentum flux density (MFD) of the continuous plane light wave in free space is given by $(W_0c)/c=W_0$ [J/m^3=N/m^2]. We will call this MFD by the electromagnetic one because no material objects take part in its production. A mechanical pressure P applied to a body transmits to the body the mechanical MFD equal P [N/m^2=J/m^3].

There are optically induced forces (OIF) produced by the light propagating in an optical medium. As a result, the light interacts with matter (an exchange of the momentums between the light and matter takes place) and, therefore a propagation of light is accompanied by a change of the mechanical MFD in a general case. The law of the conservation of the momentums and the third Newton law are valid at this interaction. As a result, each OIF changes the mechanical MFD of matter. In turn, a counterpart of the MFD that arises in accordance with the third Newton law changes the electromagnetic MFD of light. Thus, each interaction is accompanied a redistribution between mechanical and electromagnetic MFDs. A sum of these MFDs is not changed. Usually, relations between electromagnetic and mechanical MFDs before interaction are known. The mechanical momentum of any light wave in free space is equal zero. In this case, having known a distribution of OIF in space and time, a behavior of the mechanical and electromagnetic MFDs in space and time can be calculated.

3. Contradiction between Experiments

The MFD of the block in accordance with Eq. (1) is changed from 0 to $W_0(1\text{-}1/n)\tau$ in time interval $0\ldots\tau$ where τ is duration of the light pulse. In this case the pressure on the front face of the block when the photon is entering the block is given by

$$P = W_0(1-1/n) \tag{2}$$

A generally accepted interpretation of this result is the following. The MFD of the light pulse in free space is equal W_0. The mechanical MFD that is transferred to the block is given by Eq. (2). Then MFD of the light pulse inside the block is equal to W_0/n. As is noted by authors of [7], "only the conservation of momentum and the uniform motion of the center of mass-energy are used, and it is difficult to see how any components of our derivation could seriously be open to question". Indeed, Eq.(1) derived for the photon where duration of the wave train τ is negligible small as compared with duration T of propagating of the light pulse through the block is not open to question. However, a conclusion that the MFD of the photon inside the block is equal to W_0/n is open to question when forces arising in regions of the block where the photon is propagating are taken into account. These forces are not taken into account because it was supposed that the photon enters the block instantly.

There is another thought experiment [13] where no assumption about kinds and physical origin of OIF is also made. A continuous light wave is reflected in serial from two parallel reflectors of a plane optical resonator located in free space. Block used in the Balazs thought experiment is imbedded in the resonator. It is shown that a net force applied to the block is equal zero. The pressure on the front face of the block produced by a travelling continuous light wave which MFD is equal W_0 is given by

$$P_M = W_0(1-n)\,. \tag{3}$$

The pressure on the back face is equal $-P_{\text{M}}$. In this case the block is expanded by the pressures only. There is the following result of the real experiment performed by Jones *et al* [5]. The radiation pressure exerted on the mirror immersed in a dielectric medium is proportional to the refractive index of the medium. An accuracy of this effect was 0.05%. Without a doubt, there is a steady-state in the experiment and no light pulses take part in the experiment. Thus, the momentum of a continuous light wave in matter increases by n times as compared with that in free space. Unambiguous results obtained from thought and real experiments are a reliable basis for the verification of any theory.

4. Resolution of a Contradiction between Experiments

The first attempts to resolve the contradiction was undertaken by Jones [14]. He considered the photon as a "wave train" or a "light pulse" and believed that "besides the backwards impulse on the refracting interface there is also a forward bodily impulse communicated to the medium, while the wave is travelling…". However, Jones was not able to explain a physical nature of a rise of the forward bodily impulse. He writes "We are not able to specify the details of how this body impulse is created, but merely point out that it is demanded by the simple considerations of mechanics". Following Jones, we believe that the additional pressure P_A produced by the leading edge of the light pulse should be equal to a difference between pressures given by Eqs. (2) and (3). In this case

$$P_A = W_0(n - 1/n) \tag{4}$$

The additional pressure produced by the trailing edge of the pulse should be equal to $-P_A$. Let us assume that $\tau<T$, where τ is duration of the wave train, T is time of propagation of the wave train through the block. In this case a process of propagation of a light pulse through the block in the Balazs thought experiment looks like as follows. When only the leading edge of the pulse is propagating inside the block, there are two pressures applied to the block. Negative pressure given by Eq. (3) is applied to the front face of the block. Positive pressure given by Eq. (4) is applied to the region where the leading edge is propagating. Time instants when these pressures are terminated are identical and are equal $t=\tau$. As a result, a total pressure on the block is given by $W_0(1-n)+W_0(n-1/n)=W_0(1-1/n)$. This is in accordance with Eq.(2). Thus, the pressure applied to the block obtained from the Balazs thought experiment can be obtained on an assumption that additional pressure in accordance with Eq. (4) takes place in the region where the leading edge of the pulse is propagating. Unlike the notion of the Balazs thought experiment that the pressure on the front face of the block is positive and is given by Eq.(2), there is the negative pressure given by Eq.(3). Additional pressure P_A given by Eq. (4) that is applied to the region where the leading edge is propagating rather than to the front face of the block should be taken into account to obtain P in accordance with Eq. (2).

When the trailing edge enters the block, the negative pressure in accordance with Eq. (3) disappears. In the same time the negative additional pressure $-P_M$ in accordance with Eq.(4) in the region where the trailing edge is propagating arises. A sum of pressures produced by the leading and trailing edges of the pulse is equal to zero and the center of mass of the block moves uniformly.

The leading edge of the pulse produces the positive mechanical MFD $W_0(n-1/n)$ in regions where it is propagating. The trailing edge of the pulse produces the negative mechanical MFD $-W_0(n-1/n)$ in regions where it is propagating. As a result, the mechanical MFD is different from zero in the region between the leading and trailing edges of the pulse. The electromagnetic component of MFD of the pulse decreases by $-W_0(n-1/n)$ and becomes equal W_0/n. Both the electromagnetic and mechanical components are propagating together at light speed in the block c/n. Thus, unlike a general accepted interpretation of the Balazs thought experiment, the mechanical component accompanies the electromagnetic one and, as will be shown, also produces pressure on the reflector like the electromagnetic component does.

A process of propagation of the pulse in the block can be imagined in two ways. First, two pressures $W_0(n-1/n)$ and $-W_0(n-1/n)$ in the regions where leading and trailing edges are propagating are connected with the pulse which MFD is equal nW_0. Second, the mechanical component of MFD $W_0(n-1/n)$ accompanies the pulse which MFD is equal W_0/n. A penetration of the mechanical component in new regions of the block is accompanied by a rise of the pressure that provides a rise of the mechanical component in the new regions. In our opinion, the first way is preferable because well-known from mechanics notions about momentums and pressures are used. A notion about the mechanical component of MFD of the light pulse is used in the second way. Properties of this component are required to study to use it in practical applications.

When the leading edge leaves the block, positive pressure P_M in accordance with Eq. (3) arises on the back face of the block. In the same time positive pressure in the region where the leading edge is propagating disappears but the negative additional pressure produced by the trailing edge of the pulse remains. As a result, a sum of pressures is negative and is equal $-W_0(1-1/n)$. This sum provides a negative acceleration to the center of mass of the block. The center of mass stops when the trailing edge leaves the back face of the block. This picture is in a full compliance with results of the Balazs thought experiment. As is seen, no notion about the Abraham or Minkowski momentums has been used.

A replacement Δz of the center of mass of the block that takes place in accordance with the Balazs thought experiment after transmission of the photon through the block can be explained alternatively as a majority of replacements of mass ΔM located between the leading and trailing edges of the pulse. The leading edge of the pulse provides a motion of the region where the edge is propagating. This motion continues until the trailing edge propagating through the region stops it. Thus, the region is moving for time τ and travels distance Δz_1. This is valid for all region of the block. As a result the block travels as an indivisible whole distance Δz_1 for time $\tau+T$. It is worthwhile to pay attention that whole block travels distance Δz_1 for very small time measured by picoseconds if its length L<10 cm. Replacement Δz_1 can be determined from the following proportion. The momentum density $\tau W_0(1-1/n)$ corresponds to replacement $\Delta z=(n-1)Lh\upsilon(Mc^2)$ where M and L are the mass and length of the block, respectively [7]. Then mechanical momentum density $\tau W_0(n-1/n)$ provides replacement $\Delta z_1=\Delta z(n-1/n)/(1-1/n)=(n+1)\Delta z$.

In the same time there is negative replacement $\Delta z_2=-n\Delta z$ due to momentum density $\tau W_0(1-n)$ transferred to the block through the front face and momentum density $-\tau W_0(1-n)$ transferred to the block through the back face with time delay T. As is seen, $\Delta z=\Delta z_1+\Delta z_2$. Behaviors of the block due to these replacements are perfectly different. The whole block is replaced by Δz_1 in time interval T. The replacement of the block at Δz_2 consists of negative replacement Δz_3 of the region near the front face and positive replacement Δz_4 of the region near the back face. As a result of transient processes a sum of $\Delta z_3+\Delta z_4$ gives Δz_2. The time constant of these processes is determined by the sound speed in matter that is smaller than the light speed by five orders of magnitude. These processes are terminated when the light pulse has propagated at great distance above 10^5L.

As is seen, a transmission of the momentum to the block differs essentially from the simplest view accepted in the interpretation of the Balazs thought experiment where it is supposed that the momentum of the pulse simply decreases to n times at entering the block and recovers its value at exiting the block. In reality, it is a complex procedure where the propagation is accompanied by various pressures arising in various regions of space in various time instants. These pressures produce various mechanical momentums of different signs in different regions of the block. The light pulse transmits to the block mechanical momentums of different signs and leaves the block at light speed.

Replacements Δz_1 and Δz_2 are not taken into account in the general accepted interpretation of the Balazs thought experiment. Only their sum is considered. Because of this there is the contradiction between the Balazs and Jones experiments.

Alternatively, we can imagine the mechanical component of MFD as a small mass $m=(L/c)W_0(n-1/n)/(c/n)=(E_{\mathrm{pulse}}/c^2)(n^2-1)$ that is distributed between edges of the pulse and is propagating together with the pulse at speed c/n, and provides the mechanical momentum given by Eq.(4).

Thus, a joint consideration of all forces enables one to match contradictory results of the Balazs and Jones experiments. No notion about the Abraham or Minkowski momentum of light is required to interpret a behavior of the block derived from the Balazs thought experiment as well as the pressure produced on a reflector in the Jones experiments. The simplest description of propagation of light in matter can be presented as follows. When a continuous light wave enters the block, there is a steady-state and the MFD of the wave increases by n times. When a steady-state is absent and the energy flux density of the light changes in time, additional pressures in accordance with Eq.(4) ought to be taken into account at propagation of light in matter. These pressures change the mechanical momentum of the matter and the counterpart of these pressures change the momentum of light. As is seen, only well-known notions and relations are used.

Let us next analyze an origin of pressure on the leading and trailing edges of a light pulse given by Eq. (4). The density force on the leading edge of the pulse where the energy density W is changed from 0 to W_0 is given by $f_A=dP_A/dz = (n-1/n)dW/dz$. Since $z=tc/n$, we have $f_A=\frac{(n^2-1)}{c}\frac{dW}{dt}$. There is no reflection at the entrance of the light pulse into the block. In this case the energy flux density <S> inside the block and free space is identical and is given by <S>=Wc, where symbol < > means an average over period of oscillation. Since $\langle S\rangle=\langle[E\times H]\rangle$, we have

$$f_A=\frac{(n^2-1)}{c^2}\frac{d\langle[E\times H]\rangle}{dt}. \tag{5}$$

Eq.(5) determines the Abraham density force [15-19]. Thus, coordination of results of unambiguous thought and real experiments leads to the need to recognize an existence of the Abraham force which existence is discussed for a long time. Unlike the Lorentz force considered by Mansuripur [8], it is the Abraham force that is responsible for a rise of pressures in the regions where edges of a light pulse are propagating.

It is worthwhile to note that we have considered momentums, pressures and forces averaged over period of oscillations of a light wave. It is justified in optics where period of oscillations is extremely small and oscillation in time of these parameters is not interesting. Because of this we do not consider the Abraham force exited in an optical medium in a region where a plane part of a light pulse is propagating. This force in any point of optical medium averaged over time is equal to zero. Besides, this force at any time instant averaged over wavelength where the light pulse is located is also equal to zero. Nevertheless, ought to bear in mind that this force exists and oscillates at very great frequency.

5. What is the Magnitude of the Photon Momentum in Matter

A device that enables one to determine a magnitude of the momentum of light in matter is a reflector that has been used in the Jones experiments [4, 5]. Let us consider one more thought experiment where experimental evidence is available for a particular case. A light of the energy density W_0 is propagating in free space and enters the block without reflection at t=0 where it is propagating in an optical medium and is reflecting from the ideal reflector located on the back face of the block. There is no antireflection λ/4 film at the back face of the block.

Let us first consider a propagation of a continuous light wave in the block. The pressures applied to the front face of the block and total pressure applied to the system block + reflector are given by $2W_0(1\text{-}n)$ and $2W_0$, respectively. The pressure on the reflector P_r is determined from relation $2W_0=P_r+2W_0(1\text{-}n)$ and, therefore, $P_r=2nW_0$. The pressure applied to the reflector is positive and is greater by n times than the pressure applied to the reflector by the same wave propagating in free space. This agrees with results of Jones real experiments [4, 5] and thought experiment [13]. The Abraham forces are absent in this case and, therefore, there is no mechanical component of the MFD propagating together with the light pulse because the light pulse is absent.

Let us next consider a propagation of a light pulse in the block. Let $\tau >> T$ and the leading and trailing edges be propagating in free space. The leading edge is propagating after reflection in the medium in backward direction. The trailing edge is propagating in the forward direction and has not entered the block yet. In this case the leading edge produces after reflection the negative mechanical MFD located between it and the reflector. This looks like as if the positive mechanical component is reflected from the reflector. The mechanical component at the reflection produces pressure $2W_0(n\text{-}1/n)$. The electromagnetic component produces pressure $2W_0/n$. The total pressure on the reflector produced by the total MFD is equal $2W_0n$.

Let us next assume that $\tau<T$ and $\tau_e<<\tau$ where τ_e is duration of edges of the pulse. When the pulse is reflecting from the reflector, MFD of the pulse changes its direction and pressure $P_R=2W_0n$ is applied to the reflector in time interval τ. The additional pressures at leading and trailing edges of the pulse in matter are both negative at reflection of the pulse and their sum is equal to $-2W_0(n\text{-}1/n)$. But they are applied to the matter rather than to the reflector.

Thus, a magnitude of the photon momentum in an optical medium increases by n times as compared with that in free space. The momentum consists of the electromagnetic and mechanical components that propagate together with the photon and produce on a reflector the same pressure as if the momentum is indivisible. In the same time the mechanical component provides a replacement of the transparent block through which it propagates in the direction of propagation of the photon in accordance with the conclusion of the Balazs thought experiment.

Alternatively, a propagation of the photon in matter can be imaged as a propagation of the light pulse which momentum is increased by n times as compared with that in free space. Besides, a propagation of the photon is accompanied by two pressures. One pressure is propagating together with the leading edge of the photon. The pressure is equal $W_0(n\text{-}1/n)$ and is applied to the medium in the region where the leading edge is propagating. The counterpart of this pressure is applied to the leading edge. Another pressure is propagating together with the trailing edge. As a result, the pressures applied to the medium produce the mechanical momentum between the edges. The pressures applied to the edges change the momentum of the pulse and converts a part of its electromagnetic component into its mechanical one.

6. Conclusion

The total momentum of a photon in matter is greater by n times than that in free space. The Abraham density forces arising in regions of the matter where leading and trailing edges of the photon are propagating were not taken into account in the generally accepted interpretation of the Balazs thought experiment. Accounting for these forces enables one to match the contradictory results of experiments without introducing notions about different kinds of the momentum of light in matter. It is shown that these forces produce the mechanical momentum distributed between leading and trailing edges of the photon. The mechanical momentum is propagating and reflecting together with the photon. When a photon is propagating in matter, material objects located between the leading and trailing edges of the photon are moving in the direction of propagation. As a result, all material objects through which the photon has propagated are replaced although a net force applied by the photon to the matter when the photon is propagating in matter is equal zero.

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