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Frequency response of an automobile with semi-independent suspension system

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Abstract
Suspension system of an automobile not only supports the body of the vehicle, engine and passengers but also absorbs shocks arising from the roughness of the road. Most of the present day cars are provided with independent suspension for the front wheels and conventional suspension for the rear wheels. Such a suspension system is referred in this paper as semi-independent suspension system. When the automobile is moving, the roughness of the road keeps giving excitations to the suspension system through tyres. The frequency of excitation is directly proportional to the velocity of the vehicle and inversely proportional to the distance between two undulations of the road. The knowledge of natural frequency is important because the designer would prefer to keep the lowest natural frequency much higher than the frequency of excitation. In this paper, an attempt is made to study the frequency response of the suspension system.

1. Introduction

The suspension system is one of the most important systems of an automobile. Its main purpose is not only to support the engine, its components, passengers, but also to isolate them from shocks arising due to roughness of the road. It has been a practice from the beginning to have a frame called *chassis* which is being supported through springs and dampers by the front and rear axles. This type of suspension system is called *Conventional Suspension System*. There is yet another type of suspension called *Independent Suspension System*, in which the axle of a wheel is hinged to the body and is held in position by springs and dampers which are placed in between axle and the body. There is no separate chassis and the body of the vehicle itself acts as chassis. Many of the present day cars use independent suspension for the front wheels and conventional suspension for the rear wheels. In thispaper, such a system is referred as *Semi-independent Suspension System*.

The study of suspension systems has been a subject of interest for many researchers. A suspension system may be modelled as a quarter car model or a half car model or a full car model. The quarter car model or half car model yield the results very quickly but they are not accurate because they do not represent the system in a realistic way because the roll and/or pitch motions cannot be taken into account by these models. Full car model considers the entire vehicle as it is. The results can be considered to be accurate and realistic. However, the analysis becomes more complex.

Hedrick [1] considered a quarter car model with hydraulic actuator acting under the effect of coulomb friction. An absorber based nonlinear controller and adaptive nonlinear controller are proposed. Employing two sensors, one for displacement and other for velocity measurements, Majjad [2] considered a quarter car model and estimated the nonlinear damping parameters. Gobbi and Mastin [3],Wei Gao et al. [8] studied dynamic behaviour of passively suspended vehicles running on rough roads. The road profile is considered to give random inputs to the suspension system. Rajalingam and Rakheja [4] studied the dynamic behaviour of quarter car model under nonlinear suspension damper. Ahmed Faheem [5] studied the dynamic behaviour using quarter car model and half car model for different excitations given by the road. Jacquelien et al. [6] used electrical analogy in conjunction with quarter car model and studied the suspension system model and studied the control scheme of the suspension system. Wei Gao et al. [7] also studied the dynamic characteristics considering the mass, damping and tyre stiffness as random variables. Kamalakannan et al. [9] tried adaptive control by varying damping properties according to the road conditions. Sawant et al. [10] developed an experimental procedure for determining the suspension parameters using a quarter car model. Thite [11] refined the quarter car model to include the effect of series stiffness. State space equations are employed to calculate the natural frequency and model damping ratios. Gadhia et al. [12] analysed quarter car model for rear suspension using ADAMS software. Wei Gao et al. [13] investigated dynamic response of cars due to road roughness treating it as random excitation. Lin [14] performed a time domain direct identification for vehicle mass, damping and stiffness.

HusiynoAkcay [15] studied multi objective control of half car suspension system. It is observed that when the tyre damping coefficients are precisely estimated, the road holding quality of the suspension system can be improved to some extent. Li-Xing Gao [16] considered a half car model in conjunction with pseudo-excitation for the road conditions and studied the dynamic response of the vehicle. Thite et al. [17] used a frequency domain method for estimating suspension system parameters. Roberto Barbosa [18] studied the frequency response of half car model due to pavement roughness. Roberto Barbosa [19] also investigated vibrations of vehicles subjected to a long wave measured pavement irregularity.

Attempts are being made to analyse the four wheeler with fully independent suspension system. Libin Li [20] performed computer simulation studies through multi body model, identifying twenty degrees of freedom. Pater Gaspar [21] considered full car model and proposed a method for identifying suspension parameters taking into account nonlinear nature of the components. Anil Shirahatt et al. [22] attempted to maximize the comfort level considering a full car model. Genetic algorithms have been employed to perform optimization to arrive at optimum values of suspension parameters. Hajkurami et al. [23] studied the frequency response of a full car model as a system of seven degrees of freedom. Ikbal Eski [24] obtained neural network base control system for full car model. Guidaa et al. [25] proposed a method of identifying parameter of a full car model. The analysis has been developed for designing an active suspension system. Balaraju and Venkatachalam analysed the dynamic behaviour of an automobile using full car model for both, fully conventional suspension systems [26] and fully independent suspension systems [27].

In this paper, an attempt is made to study the frequency response of the suspension system.

2. Formulation

Figure 1 shows a schematic arrangement of the semi-independent suspension system. It is to be observed that on the front side there is no axle, hence, there are only two masses, m_1 and m_2 which may represent the masses of the front tyres. It is also to be observed that on the rear side there is a mass m_3, which may be viewed as a mass taking into account the mass of the axle and also masses of the rear tyres. The mass of the main body may be represented by m.

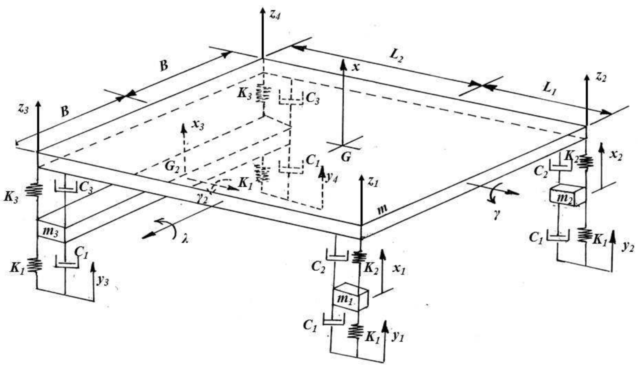


Figure 1. *Semi-independent Suspension System.*

G and G_2 are the centres of mass of the main body and the rear axle, respectively. The point G is located by the distances B, L_1 and L_2 as shown in the figure. (K_1,C_1),(K_2,C_2) and (K_3, C_3) may represent the stiffness and damping properties of a tyre, front shock absorber and rear shock absorber, respectively. Coordinates $x_1$and $x_2$may be used to represent the vertical motions of the masses m_1 and m_2, respectively. The rear axle can move up and down and can also exhibit roll motion. Coordinate $x_3$may represent the vertical motion of G_2, and γ_2 may represent the roll motion. The main body can exhibit roll and pitch motions, γ and λ , apart from the vertical motion x of the centre of mass G. In total, the motion of the entire system may be described by seven coordinates, x_1 , x_2 , x_3 , x , γ_2 , γ and λ .Thus the system is possessing seven degrees of freedom. It is also to be observed in the figure that the displacements induced by the road profile are indicated by y_i , i = 1 to 4.

The equations of motion may be derived as

$$m\ddot{x}+2(K_2+K_3)x+2(K_2L_1-K_3L_2)\lambda-K_2x_1-K_2x_2-2K_3x_3$$
$$+2(C_2+C_3)\dot{x}+2(C_2L_1-C_3L_2)\dot{\lambda}-C_2\dot{x}_1-C_2\dot{x}_2-2C_3\dot{x}_3=0 \qquad (1a)$$

$$I_r\ddot{\gamma}+2(K_2+K_3)B^2\gamma+K_2Bx_1-K_2Bx_2-2K_3B^2\gamma_2$$
$$+2(C_2+C_3)B^2\dot{\gamma}+C_2B\dot{x}_1-C_2B\dot{x}_2-2C_3B^2\dot{\gamma}_2=0 \qquad (1b)$$

$$I_p\ddot{\lambda}+2(K_2L_1-K_3L_2)x+2(K_2L_1^2+K_3L_2^2)\lambda-K_2L_1x_1-K_2L_1x_2+2K_3L_2x_3$$
$$+2(C_2L_1-C_3L_2)\dot{x}+2(C_2L_1^2+C_3L_2^2)\dot{\lambda}-C_2L_1\dot{x}_1-C_2L_1\dot{x}_2+2C_3L_2\dot{x}_3=0 \qquad (1c)$$

$$m_1\ddot{x}_1-K_2x+K_2B\gamma-K_2L_1\lambda+(K_1+K_2)x_1$$
$$-C_2\dot{x}+C_2B\dot{\gamma}-C_2L_1\dot{\lambda}+(C_1+C_2)\dot{x}_1=K_1y_1+C_1\dot{y}_1 \qquad (1d)$$

$$m_2\ddot{x}_2-K_2x-K_2B\gamma-K_2L_1\lambda+(K_1+K_2)x_2$$
$$-C_2\dot{x}-C_2B\dot{\gamma}-C_2L_1\dot{\lambda}+(C_1+C_2)\dot{x}_2=K_1y_2+C_1\dot{y}_2 \qquad (1e)$$

$$m_3\ddot{x}_3-2K_3x+2K_3L_2\lambda+2(K_1+K_3)x_3$$
$$-2C_3\dot{x}+2C_3L_2\dot{\lambda}+2(C_1+C_3)\dot{x}_3=K_1(y_3+y_4)+C_1(\dot{y}_3+\dot{y}_4) \qquad (1f)$$

$$I_2\ddot{\gamma}_2-2K_3B^2\gamma+2(K_1+K_3)B^2\gamma_2$$
$$-2C_3B^2\gamma+2(C_1+C_3)B^2\ddot{\gamma}_2=K_1B(y_4-y_3)+C_1B(\dot{y}_4-\dot{y}_3) \qquad (1g)$$

y_i may be regarded as excitation caused to the suspension system by the roughness of the road.

3. Analysis of the Suspension System

Based on a practical road vehicle *Santro Xing*, numerical values are assigned to various parameters involved, as follows.

m_1= 40 *kg* m_2= 40 *kg* L_1 = 1 *m* L_2 = 2.5 *m*

m= 1000 *kg* I_r= 500 $kg.m^2$ I_p = 1000 $kg.m^2$

m_3 = 100 *kg* I_2 = 20 $kg.m^2$ *B*= 0.75 *m*

K_1= $2{\times}10^5 N/m$ K_2 = K_3= $0.5{\times}10^5 N/m$

C_1= 1000 *N.s/m* C_2= C_3= 1000 *N.s/m*

Keeping the excitations as zeroes and also damping as zero, the undamped free vibration study was performed and the natural frequencies (NF's) are obtained as,

1.796	2.129	3.875	11.436	12.605	12.625	18.890

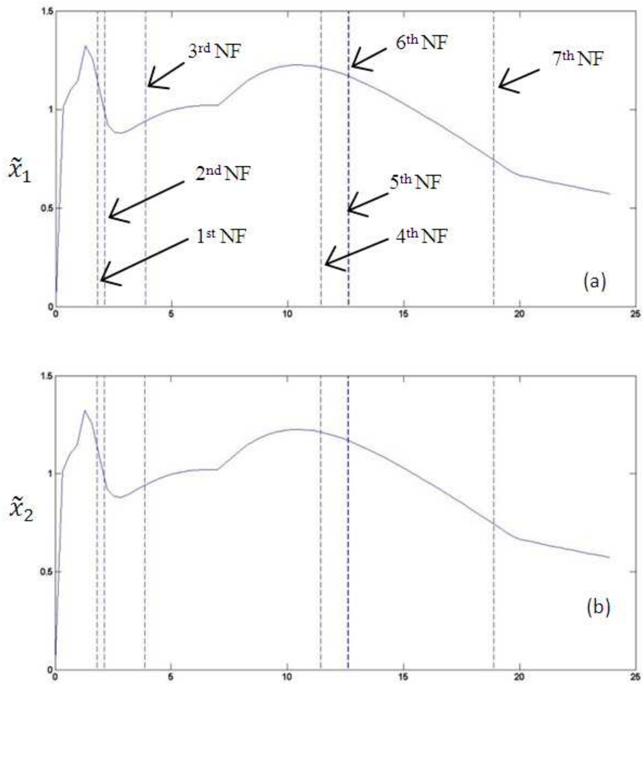
It is observed that first three frequencies are small and close to each other, the other four frequencies are very large, and there is a large gap between third and fourth frequencies. If the road profile creates an excitation whose frequency matches with any of the seven natural frequencies resonance may occur. The frequency of road excitation is directly proportional to the velocity of the vehicle and inversely proportional to the distance between the undulations existing on the road. Among these seven frequencies, the lowest one is very important because under normal circumstances one may at the most cross the lowest frequency. In this paper, it is intended to study the behaviour of the system under harmonic excitations, which may be simulated through the variables $\boldsymbol{y_i}$. Harmonic excitations are applied at all the four wheels as

$$y_i(t)= y_{m_i}\sin\omega t \quad , \qquad i = 1 \text{ to } 4 \qquad (2)$$

The equations of motion in (1) may be integrated using Runge Kutta method and the time histories of various variables may be observed. For the sake of convenience, the variables are expressed in non-dimensional form as

$$\tilde{x}_1=\frac{x_1}{y_{m_1}} \quad \tilde{x}_2=\frac{x_2}{y_{m_1}} \quad \tilde{x}_3=\frac{x_3}{y_{m_1}} \quad \tilde{x}=\frac{x}{y_{m_1}}$$
$$\tilde{\gamma}_2=\frac{\gamma_2 B}{y_{m_1}} \quad \tilde{\gamma}=\frac{\gamma B}{y_{m_1}} \quad \tilde{\lambda}=\frac{\lambda L_2}{y_{m_1}} \qquad (3)$$

Taking y_{m_i} = 0.05 *m*, *i* = 1 to 4, the equations of motion are integrated for different values of the exciting frequency ω in the range zero to 150 *rad/s* (0 to 24 *Hz*). This range of ω covers all the seven natural frequencies. The time histories of each of the variables are closely observed and the maximum values are noted. Expressing these maximum values in non-dimensional form as in Equation (3), they are plotted against the exciting frequency. Figure 2 shows the variation of the maximum values of various variables with frequency.



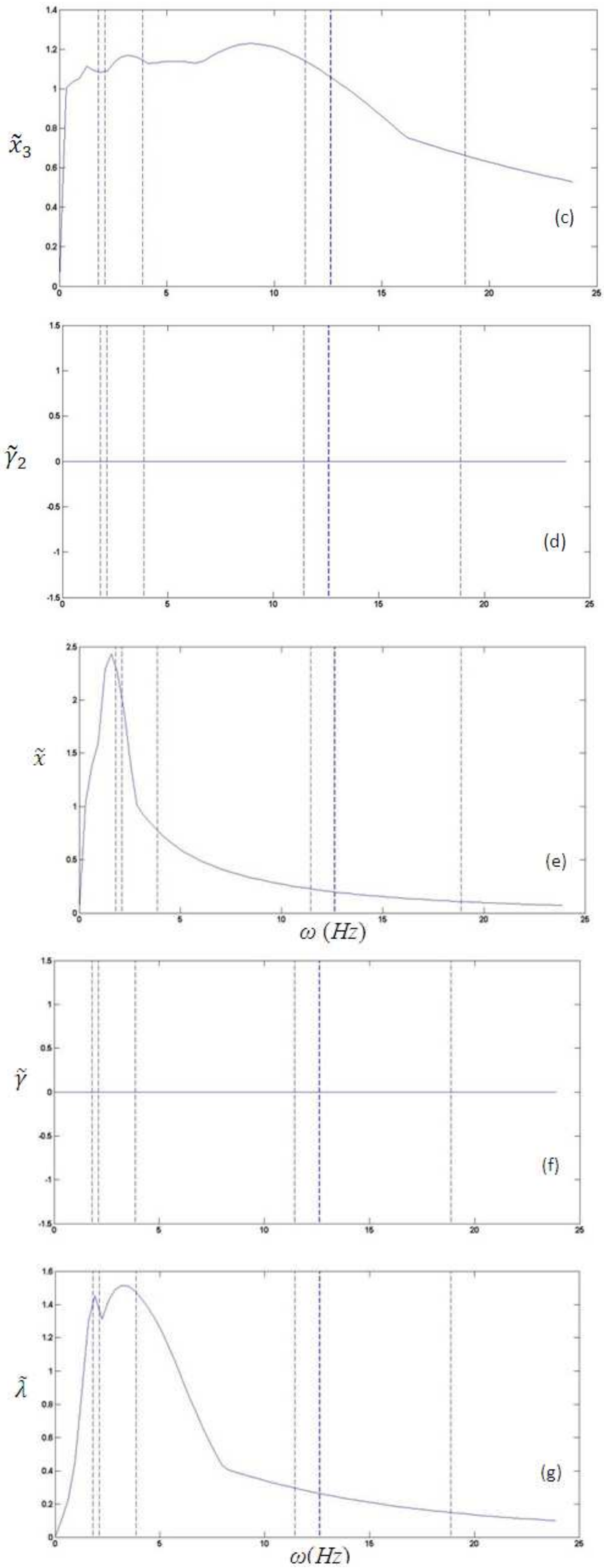


Figure2. *Semi-independent Suspension System.*

It is observed from Figures 2a and b that the responses of x_1 and x_2 are exactly the same. This is because the excitation given at the front two wheels moves the front two wheels in the same manner always. The peak values may be observed to occur at the frequency which is less than the first natural frequency. Similar trend may also be observed at the fourth natural frequency. Figure 2c shows the response of up and down motion x_3 of the rear axle. The peak amplitudes may be observed at frequencies less than the first, third and fourth natural frequencies. Figure 2d shows the response of the roll motion γ_2 of the rear axle. It is to be observed that the axle is not undergoing any kind of roll motion. This may be because the given excitation keeps moving the entire vehicle as a single unit. Figure 2e shows the response of x. It is to be observed that the resonance is prominent only at the first natural frequency. Here also the peak is occurring at a frequency slightly less than the first natural frequency. Figure 2f shows the response of roll motion γ , of the main body. As in Figure 2d, the main body is observed to have negligible roll motion. This may be for the same reason. Figure 2g shows the response of pitch motion λ , of the main body. Unlike the roll motion, the automobile can have pitch motion because the centre of mass G is not at the middle point of the longitudinal axis ($L_1 \neq L_2$). The resonance may be observed to be prominent at the first and the third natural frequencies.

4. Conclusions

The work presented in this paper and significant conclusions that may be drawn based on the present work may be summarized as follows.

(i)	Full car model for semi-independent suspension system is studied for its frequency response.
(ii)	For the purpose of study, the values of parameters are taken which correspond to a real practical automobile.
(iii)	The harmonic excitations are given to all the four wheels simultaneously.
(iv)	The frequency of excitation is varied and the behaviour of various variables are noted.
(v)	Frequency response plots are made for all the variables and various observations are made and physically interpreted.
(vi)	It is observed that resonance is occurring predominantly at the first natural frequency only.

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