The life predicting calculations in whole process realized from micro to macro damage with conventional materials constants

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Citation

Abstract
Based on researching and analyzing data by references, to study their compositions of various traditional mathematical models and modern models, to use the theoretical approach, especially to be the similar to “genetic elements” and “clone technology” in life science, by means of the conventional and the modern material constants, thereby have made up the calculating figure1 of material behaviors. For elastic-plastic steels containing pre-micro or pre-macro-flaws, to adopt the multiplication method of two parameters \( \sigma \cdot \varepsilon \) in micro damage process, the multiplication method of two parameters \( K_1 \cdot \delta \) in macro damage process, along with the program calculations of computer, so that to discover and establish lot of new calculation models from micro to macro and even all damage growth process, which are the equations of the driving forces and the various life predicting calculations. In addition, to propose yet many calculating expressions and calculating methods under different loading conditions, to define their physical and geometrical meanings for key parameters, to explain in detail the conversion methods between the damage variable \( D \) and the crack variable \( a \), between corresponding the dimensions-units and ones, which are relevant parameters in the damage mechanics and the fracture mechanics. Also to expound concretely the calculation methods and the processes for transition damage value \( D_T \) between two stages from the micro to macro damage growth process. The purpose is to try to make the modern fatigue-damage discipline nowadays depends on tests become gradually calculable subject as the conventional material mechanics, such that will be having practical significances for saving testing manpower and funds, for promoting applying and development relevant disciplines.

1. Introduction

As everyone knows for the conventional material mechanics, that is a calculable subject, and has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the life problems for some structures when it is pre-existing flaws and undergone fatigue damage under repeated loading. In that it has no to contain such calculable parameters as the damage variable \( D \) or as crack variable \( a \) in their calculating models. The damage mechanics and the fracture mechanics due to there are these variables, they can all calculate above problems. But nowadays latter these disciplines are all subjects mainly depended on tests.

Author thinks, in the mechanics and the engineering fields, in which are also to exist
such a scientific law as similar to genetic elements and clone technology in life science. Author has done some of works used the theoretical approach as above the similar principles [1-7], now continue applying those ones and to provide some calculable models for the damage growth driving force, and for the life predicting calculations. To try for the damage mechanics, step by step become calculable disciplines as the material mechanics. That way, may be having practical significances for decrease experiments, stint man powers and funds, for promoting engineering applying and developing for relevant disciplines.

2. About the Viewpoints of Existing Similar Genetic Gene and Clone Technology in the Mechanics and Engineering Fields

As is well-known in the conventional material mechanics, on describing material behaviors and its strength problems, its main calculating parameters are the stress $\sigma$, the strain $\varepsilon$ and relevant material constants, e.g. yield stress $\sigma_y$, reduction of area $\psi$ and elasticity modulus $E$, etc. In the damage mechanics it is based on the damage parameter $D$ as its variable, to adopt the fatigue strength coefficient $\sigma_f$ and the fatigue ductility coefficient $\varepsilon_f$, etc. as its material constants. And in the fracture mechanics, describing material behaviors on the strength and the life prediction problems, it is based on the crack size $a$ as its variable, to use the fracture toughness $K_i$ and the critical crack tip open displacement $\delta_{ct}$ as its material constants.

Author thinks that the gene and clone technology in life science, for which their traits consist in: both have them-self genetic properties, and have transferable and recombination properties. In fact, in fracture mechanics, in the stress intensity factor $K_1 = \sigma \sqrt{a}$, in the crack tip open displacement $\delta_{ct}$ and in their critical value $K_{cr} = \sigma_y \sqrt{a}$, and the $\delta_{ct}$, which are all including the parameter $\sigma$, $\varepsilon$, $\pi$ and their material constants $\sigma_y$, $\varepsilon_y$ and $\sigma_f$, etc. Author think for the stress $\sigma$, the strain $\varepsilon$ and its relevant material constants $\sigma_y$ and $\varepsilon_y$ etc in the material mechanics, for which can be considered as genetic elements; for the $D$ and $\sigma_f$, etc in the damage mechanics can also be considered as genetic elements; and for the crack size $a$ in the fracture mechanics can also be considered as genetic elements. If can make a link among the material mechanics, the damage mechanics and the fracture mechanics, and provide respectively some conversion methods, then they can be converted each other for their relations between the variables, between the material constants, and between the dimensional units in the equations. For example, we can consider as gene for the stress $\sigma$ and its $\sigma_y, E, \psi$, to make them combination with the variable $D_i$ are transferred into micro-damage-mechanics, and combination with the variable $D_g$ are transferred into macro-damage-mechanics; In the same way, we can also consider as gene for the stress $\sigma$ and its $\sigma_y, E, \psi$, to make them combination with the variable $a_i$ are transferred into micro-fracture-mechanics, and combination with the variable $a_g$ are transferred into macro-fracture-mechanics. Then we are able by these stress $\sigma$, $\varepsilon$, $\sigma_f$, $\varepsilon_f$, etc, to establish their the new driving force models, the damage growth rate and its life equations or the new crack growth rate and its life equations. Even we can also adopt the variable $D$ or $a$ to describe material behaviors in overall process.

Above the properties of those parameters and material constants, even though as compared to those ones the life science, due to they are in different disciplines. But, for which both have own inheritable properties (similar to genetic elements), and have the transferable, and the recombination properties, for these---on the epistemology and methodology, in practice they are all very similar.

Based on the cognitions and concepts mentioned above, author makes a link among the material mechanics, the damage mechanics and the fracture mechanics, for relationship between their parameters are analyzed, for their equations are derived, for their dimensional units are converted each other, then for new made models are calculated, checked and validated again and again, finally, to provide the equations (1-54) in following text. Thereby try to make communications for among the conventional material mechanics, the damage mechanics and the fracture mechanics, then to make such new calculable mathematical models as those equations inside the material mechanics, which are the new driving force ones and the life calculation equations. Author thinks if can realize the goals, it will all have practical significances for the engineering designs, the computational analysis for safe operation and assessment of machineries and structures.

3. Calculating Figure of Materials Behaviors

Among branch disciplines on fatigue-damage-fracture, among the conventional material mechanics and the modern mechanics, for communications and connecting their relations each other, it must study and find their correlations between the equations, even to be the relations between variables, between the material constants, between the curves. Because which are all the significant factors to research and to describe for material behaviors at each stage even in whole process, and are also all to have significant significations for the engineering calculations and designs. Therefore it should research and find an effective tool used for analyzing problems above mentioned. Here author provides the “calculating figure of materials behaviours” as Figure1 (or called bidirectional combined coordinate system and
simplified schematic curves in the whole process, or called combined cross figure).

Figure 1. Calculating figure of material behaviors (Bidirectional combined coordinate system and simplified schematic curves in the whole process).

In the figure 1, it had been provided by present author [1-2]. At this time it has been corrected and complemented, that is shown diagrammatically for the damage growth process or crack propagation process of material behavior at each stage and in whole course. It is to consist of six abscissa axes $O'$ I', $O'$ II', $O'$ III', $O'$ IV and two bidirectional ordinate axis $O'$ I and $O'$ II'. Between the axes $O'$ I' and
It was an area applied by the conventional material mechanics, currently it can be applied as micro-damage area by the very-high cycle fatigue. Among the axes \( O_{\Gamma} \), \( O_{I} \) and \( O_{2} \), they are calculation areas applied by the micro-damage mechanics and the macro-fracture mechanics. Between the axes \( O_{\Gamma} \), \( O_{I} \) and \( O_{2} \), it is calculation area applied by the macro-damage mechanics and the macro-fracture mechanics. Between the axes \( O_{\Gamma} \) and \( O_{I} \), it is both calculation area applied for the micro-damage mechanics and macro-fracture mechanics, or it is both calculation area applied for the micro-damage mechanics and macro damage mechanics.

On the abscissa axes \( O_{\Gamma} \), \( O_{2} \), and \( O_{I} \), they are shown with stress \( \sigma \) and strain \( \varepsilon \) parameters as variable, and on the abscissa axes \( O_{\Gamma} \) there is a fatigue limit \( \sigma_{f} \) at point b.

On the abscissa axes \( O_{\Gamma} \), it is represented with the short crack stress intensity factor range \( \Delta K \) or the short crack strain intensity fact \( \Delta \varepsilon \) as variable, and here there are the threshold stress intensity factor \( \Delta K_{\text{th}} \) and the damage threshold values \( \Delta K_{\text{r}} \). On the abscissa axes \( O_{\Gamma} \), it is shown with long crack stress intensity factor \( \Delta K \) range (or \( \Delta \varepsilon \)) as variable, that it is also a boundary between short crack and long crack growth behaviors (or between micro damage and macro damage growth), and it is also a boundary of transfer values ( \( a_{o}, \tau_{D_{o}} \) ) between the first stage and the second stage. On abscissa \( O_{\Gamma} \), the point \( A_{o} \) is corresponding to fatigue strength coefficient \( \sigma_{f} \) and critical values \( K_{c} \), etc; the point \( C_{\Delta} \) corresponding to fatigue ductility coefficient \( \varepsilon_{f} \) and critical value \( \delta_{\varepsilon} \); the point \( F \) corresponding to very-high cycle fatigue strength coefficient \( \sigma_{f}^{(2)} \), i.e. on same the abscissa axes \( O_{\Gamma} \), there are also the critical values \( K_{c}^{(2)}(K_{c}^{(2)})_{o} \), \( \delta_{\varepsilon}^{(2)} \), \( J_{c}^{(2)}(J_{c}^{(2)})_{o} \) to fracture in long crack growth process.

Upward direction along the ordinate axis is represented as crack growth rate \( d\alpha/dN \) or damage growth rate \( dD/dN \) in each stage and the whole process. But downward direction, it is represented as life \( N_{\text{life}}, N_{\text{N}} \) in each stage and lifetime \( N \).

In area between axis \( O_{\Gamma} \) and \( O_{2} \), it is a fatigue history from un-crack to micro-crack initiation. In area between axes \( O_{\Gamma} \) and \( O_{I} \), it is a fatigue history relative to life \( N_{\text{life}} \) from micro-crack growth to macro-crack forming. Consequently, the distance \( O_{\Gamma} - O_{\Gamma} \) on ordinate axis is as a history relating to life \( N_{\text{life}} \) from grains size to micro-crack initiation until macro-crack forming; the distance \( O_{\Gamma} - O_{\Gamma} \) is as a history relating to the lifetime life \( N \) from micro-crack initiation until fracture.

In crack forming stage, the partial coordinate system made up with the upward the ordinate axis \( O_{\Gamma} \) and the abscissa axes \( O_{\Gamma} \), \( O_{I} \) and \( O_{2} \) is represented to be as relationship between the damage evolving rate \( dD_{2}/dN_{2} \) (or the short crack growth rate \( da_{2}/dN_{2} \)) and the damage stress factor amplitude \( \Delta H_{1}/2 \) (or damage strain factor amplitude \( \Delta \varepsilon_{1}/2 \)). In macro-crack growth stage, the partial coordinate system made up with the ordinate axis \( O_{\Gamma} \) and abscissa \( O_{\Gamma} \) at same direction is represented to be the relationship between macro-crack growth rate and the stress intensity factor amplitude \( \Delta K/2 \), \( \int \Delta K/2 \) and crack tip displacement amplitude \( \Delta \delta_{2}/2 \). Inversely the coordinate systems made up with downward ordinate axis \( O_{\Gamma} \) and abscissa axes \( O_{\Gamma} \), \( O_{\Gamma} \), \( O_{2} \), \( O_{I} \) and \( O_{\Gamma} \) are represented respectively as the relationship between the \( \Delta H/2 \), \( \Delta K/2 \)-amplitude and the each stage life \( N_{\text{life}}, N_{\text{N}} \), the lifetime \( \Sigma N \) even (or between the \( \Delta \varepsilon_{2}/2 \), \( \Delta \delta_{2}/2 \)-amplitude and the life \( \Sigma N \)).
And should yet point that the calculating figure 1 of materials behaviors may be a complement as a basis that it is to design and calculate for different structures and materials under different loading conditions, and it is also a tool and bridge, that is to communicate and link the conventional material mechanics and the modern mechanics.

4. The Life Prediction Calculations for Elastic-Plastic Materials Containing Pre-Flaws

If under repeating load, for some elastic-plastic materials containing pre-flaws, its life predicting calculations for its micro-damage growth process can be calculated by means of the method in reference [8-9], that is to do damage calculations by multiplication $\varepsilon \cdot \sigma$ with stress and strain parameters; For its macro-damage growth process, this paper also adopts similar to above method, that is a method by the multiplication parameters $K', \delta_{2}'$ with the damage stress intensity factor $K'$ and the damage crack tip open displacement $\delta_{2}'$, thereby achieve severally for their life’s calculations in each stage.

4.1. The Life Prediction Calculations in Micro Damage Process

The life curves of micro damage growth in the first stage are just described with curves 1 ($\sigma < \sigma_c, \sigma_m = 0$), 2 ($\sigma > \sigma_c$) and 3 ($\sigma < \sigma_c, \sigma_m \neq 0$) at reversed direction coordinate system in fig.1, here for them are only described with two parameters Multiplication method in micro-damage process.

$$A'_1 = \frac{2}{4} [\sigma', \varepsilon'_f] m'_m, \frac{m}{m_1} \times (v'_{eff})^{-1}, (MPa^{m/m_1} \text{damage-nuit-number/cycle}), (\sigma_m = 0)$$

$$A'_1 = \frac{2}{4} [\sigma', \varepsilon'_f] (1-\sigma_m/\sigma' f)] m'_m, \frac{m}{m_1} \times (v'_{eff})^{-1}, (MPa^{m/m_1} \text{damage-nuit-number/cycle}), (\sigma_m \neq 0)$$

Here the eqn (2) is a driving force of micro-damage under monotonic loading, and the eqn (3) is driving force under fatigue loading. $\Delta \sigma = \Delta \sigma / E$. The parameter $A'_1$ is a comprehensive material constant, and $A'_1$ is a calculable one of having function relation with other parameters $\sigma'_f, b'_f, \psi$ in [2]. Author research and think, its physical meaning of the $A'_1$ is a concept of power, that is to give out an energy to resist outside force, it just is a maximal increment value to give out energy in one cycle, before the specimen material makes failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1 ($\sigma_m = 0$) or 3 $\sigma_m \neq 0$ on the y-axis, also is an intercept between $O_1 - O_1$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent $m'_m / (m_1 + m'_1)$ of the formula (4-5). The parameters $m_1$ and $m'_1$ are respectively material constants under high cycle or low cycle fatigue. The $m_1 = -1/b'_1$, $b'_1$ is the fatigue strength exponent under high cycle fatigue; the $m'_1 = -1/c'_1$, $c'_1$ is the fatigue ductility exponent under low cycle fatigue. It should be point that, the $A'_1$ in eqn (4) is corresponding reversed curves 1, its mean stress $\sigma_m = 0$, the $A'_1$ in eqn (5) is corresponding curves 3, its mean stress is $\sigma_m \neq 0$, here for the eqn (5) to adopt the correctional method for mean stress $\sigma_m \neq 0$ in reference [10].

Where

$$v'_{eff} = \ln(D_f / D_m)N_{1,10} - N_{1,1} = \ln(D_f / D_m) - \ln D_f / D_m)N_{1,1} - N_{1,1} \text{damage-unit-number/cycle) (6)}$$

(1) Under work stress $\sigma < \sigma_c$, condition (or high cycle loading)

In fig.1, under work stress $\sigma < \sigma_c$, condition, the life prediction equation corresponding reversed curves 1 and 3 can be calculated as following form

$$N_i = \int_{D_m}^{D_f} \frac{dD}{A'_1 \times (\Delta Q_1')^{m/m_1} D_i} \text{ (cycle)}$$

or

$$N_i = \int_{D_m}^{D_f} \frac{dD}{A'_1 \times (\Delta \varepsilon_0 \cdot \Delta \sigma)^{m/m_1} D_i} \text{ (cycle)}$$

Where the damage variable $D_i$ (or below $D_2$ and $D$) is a non-dimensional value, it is equivalent to short crack $a_i$. discused as reference [1-2]. Here must put up conversion for dimensions and units, and must be defined in 1mm (1 millimeter) of crack length equivalent to one-damage-unit, in 1m-crack-length (1 meter) equivalent to 1000-damage-unit (1000 damage-unit). The $Q_1'$ is defined as the $Q_1'$-factor of two-parameter, the $\Delta Q_1'$ is defined the $Q_1'$-factor range of two-parameter. Here,

$$Q_1' = (\varepsilon \cdot \sigma) D_i^{m/m_1}$$

$$\Delta Q_1' = (\Delta \varepsilon \cdot \Delta \sigma) D_i^{m/m_1}$$

In fig.1, under work stress $\sigma < \sigma_c$, condition, the life prediction equation corresponding reversed curves 1 and 3 can be calculated as following form

$$N_i = \int_{D_m}^{D_f} \frac{dD}{A'_1 \times (\Delta Q_1')^{m/m_1} D_i} \text{ (cycle)}$$

or

$$N_i = \int_{D_m}^{D_f} \frac{dD}{A'_1 \times (\Delta \varepsilon_0 \cdot \Delta \sigma)^{m/m_1} D_i} \text{ (cycle)}$$

Where the damage variable $D_i$ (or below $D_2$ and $D$) is a non-dimensional value, it is equivalent to  short crack $a_i$ discussed as reference [1-2]. Here must put up conversion for dimensions and units, and must be defined in 1mm (1 millimeter) of crack length equivalent to one-damage-unit, in 1m-crack-length (1 meter) equivalent to 1000-damage-unit (1000 damage-unit). The $Q_1'$ is defined as the $Q_1'$-factor of two-parameter, the $\Delta Q_1'$ is defined the $Q_1'$-factor range of two-parameter. Here,
or

\[ v_{\text{eff}} = [D_o \ln(1/1-\psi)]/N_{ij} - N_{ii} \text{(damage-unit/cycle)} \] (7)

Where the \( v_{\text{eff}} \) in eqn (6-7) is defined as an effective damage history correction factor in first stage, its physical meaning is the damage rate of whole failure to cause specimen material in a cycle, its unit is the damage - unit - number / cycle. \( \psi \) is a reduction of area.

\( D_o \) is pre-micro-damage value which has no effect on fatigue damage under prior cycle loading [11]. \( D_{ii} \) is an initial damage value, \( D_j \) is a critical damage value before failure, \( N_{ii} \) is initial life in first stage, \( N_{ij} = 0 \); \( N_{ij} \) is failure life, \( N_{ij} = 1 \). By the way, here is also to adopt those material constants \( \sigma'_i, b_i, \varepsilon'_i, c_i \) as "genes" in the fatigue damage subject. The \( D_o \) in eqn (1) is a transitional damage value transited from micro to macro damage, \( D_m = D_{mac} \cdot D_{mac} \) is a macro damage value corresponded to forming macro-crack size \( a_{mac} \). \( D_{ii} \) is a medial damage value between initial damage value and transitional damage value corresponding mental life \( N_{ii} \).

So, for the eqn (1), its final expansion equation corresponded reversed to curves 1 (\( \sigma i A \)) is as below form:

\[ N_{ii} = \frac{\ln D_{ii} - \ln D_i}{2(4\sigma'_i \times \varepsilon'_i) \times (v_{\text{eff}})^{-1} \times (\Delta \sigma \times \Delta \varepsilon)^{m_i}} \] (8)

And its final expansion equation corresponded reversed to curves 3 (\( D_i D \)) should be:

\[ N_{ii} = \frac{\ln D_{ii} - \ln D_i}{2(4\sigma'_i \times \varepsilon'_i (1-S_{\text{med}} / \sigma'_i)) \times (v_{\text{eff}})^{-1} \times (\Delta \sigma \times \Delta \varepsilon)^{m_i}} \] (9)

(2) Under work stress \( \sigma > \sigma_i \) condition (or low cycle loading)

When under \( \sigma > \sigma_i \) condition, due to plastic strain occurring cyclic hysteresis loop effect, its life equation corresponded to reversed direction curve \( C_i C \) is as following

\[ N_i = \int_{D_i}^{D_f} \frac{dD_i}{A_i \times (\Delta Q_i / 2)} \] (10-1)

Here influence of mean stress in eqn (12) can be ignored. But it must point that the total strain range \( \Delta \varepsilon \) in eqn (11-12) should be calculated by Masing law as following eqn. [12]

\[ \Delta \varepsilon = \frac{\Delta \sigma}{E} + 2\left(\frac{\Delta \sigma}{2K_2}\right)^{\frac{1}{n}} \] (13)

4.2. The Life Prediction Calculations in Macro Damage Growth Process (or Called the Second Stage)

In Fig.1, the residual life curves of macro damage in the second stage are just described with curves 1’ (\( \sigma < \sigma_i, \sigma_m = 0 \)) , 2’ (\( \sigma > \sigma_i \)) and 3’ (\( \sigma < \sigma_i, \sigma_m \neq 0 \)) at reversed direction coordinate system. For the driving force and the life calculating problems in macro-damage process, here to adopt the two parameters \( K_1 \delta_i \) -method and the stress \( \sigma \) method are described and calculated for them.

1) Under work stress \( \sigma < \sigma_i \) condition

For its life prediction equation corresponded reversed curves \( A_i A_i \) and \( D_i D_i \) should be as below
\[
N_2 = \int_{D_2}^{D_2'} \frac{dD_2}{A'_{2}(\Delta Q_2)} \text{(cycle)}
\]

Or
\[
N_2 = \int_{D_2}^{D_2'} \frac{dD_2}{A'_{2}(\Delta Q_2)} \text{(cycle)}
\]

Where
\[
A'_{2} = 2(4\left(\frac{K_{2,fc}'}{m_{2} + m_{2}'}\right) - \frac{m_{2}/m_{2}'}{m_{2} + m_{2}'}) \times \nu'_{\rho}, (\text{MPa} \cdot \text{m}^{2} / \text{cycle}), (\sigma_{\rho} = 0)
\]

And the \( A'_{2} \) corresponded to curve \( D_{1}D_{2} \), it should be as following form
\[
A'_{2} = 2(4\left(\frac{K_{2,fc}'}{m_{2} + m_{2}'}\right) - \frac{m_{2}/m_{2}'}{m_{2} + m_{2}'}) \times \nu'_{\rho}, (\text{MPa} \cdot \text{m}^{2} / \text{cycle}), (\sigma_{\rho} = 0)
\]

Here the \( K_{2,fc} \) is a critical damage stress intensity factor, \( K_{2,fc} \) is mean damage stress intensity factor; \( \delta_{2,fc} \) is a critical damage crack tip open displacement, for which they are all respectively equivalent to the \( K_{fc}, K_{\rho}, \delta_{fc} \) in fracture mechanics. It should be point that their physical and geometrical meanings for the \( A'_{2} \) are similar with those concept in micro damage process mentioned above. But should explain that unit of \( A'_{2} \) is \( \text{MPa} \cdot \text{m}^{2} / \text{cycle} \). The \( \nu''_{\rho} \) is defined to be the virtual damage rate, its physical meaning with the parameter \( \nu''_{\rho} \) is similar in reference [14], but the dimension and unit are different, because where the \( \nu'' = 3 \times 10^{-6} \sim 3 \times 10^{-7} (\text{m/cycle}) \), here its unit is converted to \( \text{MPa} \cdot \text{m}^{2} / \text{cycle} \).

\[
N_{2,eff} = \left[ \frac{4(m_{2} + m_{2}')}{4m_{2} + 4m_{2}' - 6m_{2}m_{2}'} \times \left(\frac{2m_{2} + 2m_{2}' - 3m_{2}m_{2}'}{2(m_{2} + m_{2}')}\right) \times \nu''_{\rho} \times (\nu''(a/b) \Delta K_{2,fc}' \Delta \delta_{2,fc}') \right] \text{(Cycle)}(\sigma_{\rho} = 0)
\]

In reference [15-16] refer to the effective stress intensity factor in fracture mechanics. Same, here there are also two effective values \( K'_{2,eff} \) and \( \delta'_{2,eff} \) corresponding to the critical \( K'_{2,fc} \) and \( \delta'_{2,fc} \) to propose as follow:
\[
K'_{2,eff} = K'_{th} K'_{2,fc}, \text{ or } K'_{2,eff} = (0.25 - 0.4)K'_{2,fc}
\]
\[
\delta'_{2,eff} = (0.25 - 0.4)\delta'_{2,fc}
\]

Where the \( D_{2,eff} \) in (24) is an effective damage value, it is obtained and calculated from eqns (21), (23) and (25-1), (25-2), and to take less value. The \( K'_{th} \) is a damage threshold stress intensity factor value, that is one equivalent to threshold stress intensity factor of long crack. So the effective life expanded equation corresponding reversed direction curve \( D_{1}D_{2} \) should be
\[
Q'_{2} = K'_{2,fc} \varphi_{2}'(\text{MPa} \cdot \text{damage-unit-number} \text{orMPa} ), \text{(16)}
\]
\[
\Delta Q_{2} = (\Delta K_{2,fc} \Delta \delta_{2,fc}),(\text{MPa} \cdot \text{damage-unit-number}) \text{orMPa} \text{(17)}
\]
Due to word stress is still \( \sigma / \sigma \ll 1 \) (\( \sigma \leq 0.5 \sigma \_1 \)), the macro damage residual life equation of corresponded reversed direction curve \( A_1 A_1 \) in fig.1 is as following form

\[
N_{2\text{eff}} = \frac{4(m_2 + m_1^*)}{4m_1 + 4m_2 - 6m_1m_2} \left( \frac{D_{2\text{eff}}}{m_2m_1^*} \right)^{\frac{2m_2 + 2m_1 - 3m_2m_1^*}{2(m_2 + m_1^* + 2m_1m_2)} - D_{02}} \left( \frac{m_2^*}{m_2 + m_1^* + 2m_1m_2} \right)^{\text{(Cycle)}, \ (\sigma_m \neq 0)}
\]

(26)

Its medial life \( N_{2\text{mf}} \) in second stage is

\[
N_{2\text{mf}} = \frac{4(m_2 + m_1^*)}{4m_1 + 4m_2 - 6m_1m_2} \left( \frac{D_{2\text{mf}}}{m_2m_1^*} \right)^{\frac{2m_2 + 2m_1 - 3m_2m_1^*}{2(m_2 + m_1^* + 2m_1m_2)} - D_{02}} \left( \frac{m_2^*}{m_2 + m_1^* + 2m_1m_2} \right)^{\text{(Cycle)}, \ (\sigma_m \neq 0)}
\]

(27)

2) \( \sigma \)-stress method

Due to word stress is still \( \sigma / \sigma \ll 1 \) (\( \sigma \leq 0.5 \sigma \_1 \)), the macro damage residual life equation of corresponded reversed direction curve \( A_1 A_1 \) in fig.1 is as following form

\[
Q'O'_2 = \frac{\sigma^3}{E\sigma} \left( \sqrt{\bar{D}} \right)^3 \left( \text{MPa} \cdot \text{damage} - \text{unit} - \text{number} \right)
\]

(29)

\[
A'_2 = 2 \left( \frac{\sigma^3}{E\sigma} \left( \sqrt{\bar{D}} \right)^3 \right)^{\frac{m_2^*}{m_2 + m_1^*}} \times v_{pp} \cdot \left( \text{MPa} \cdot \text{damage} - \text{unit} - \text{number} / \text{cycle} \right) (\sigma_m = 0)
\]

(30)

\[
A'_2 = 2 \left( \frac{\sigma^3}{E\sigma} \left( \sqrt{\bar{D}} \right)^3 \right)^{\frac{m_2^*}{m_2 + m_1^*}} \times v_{pp} \cdot \left( \text{MPa} \cdot \text{damage} - \text{unit} - \text{number} / \text{cycle} \right) (\sigma_m \neq 0)
\]

(31)

So for \( \sigma_m = 0 \), its final expansion equation corresponded to reversed direction curve \( A_2 A_2 \) is as below form,

\[
N_{2g} = \frac{4(m_2 + m_1^*)}{4m_1 + 4m_2 - 6m_1m_2} \left( \frac{D_{2g}}{m_2m_1^*} \right)^{\frac{2m_2 + 2m_1 - 3m_2m_1^*}{2(m_2 + m_1^* + 2m_1m_2)} - D_{02}} \left( \frac{m_2^*}{m_2 + m_1^* + 2m_1m_2} \right)^{\text{Cycle}}, \ (\sigma_m \neq 0)
\]

(32)

For \( \sigma_m \neq 0 \), the life equation corresponded to reversed direction curve \( D_2D_1 \) is following
\[ N_{2\text{eff}} = \frac{4(m_2 + m'_2)}{m_2 + 4m'_2 - 6m_2} \left( \frac{2m_2 + 2m'_2 - 3m_2m'_2}{2(m_2 + m'_2 - 3m_2m'_2)} \right) - \frac{D_{02}}{D_{\text{eff}}} \] \[ \text{(34)} \]

(2) Under work stress \( \sigma > \sigma'_w \) condition

1) \( \sigma - \) factor method

Under \( \sigma > \sigma'_w \) condition, due to the materials occur plastic strain, the exponent of its equation also to show change from \( m_2 \) to \( \lambda'_2 \); and due to occur cyclic hysteresis loop effect, its effective life calculable models corresponded to reversed curve \( C_2C_1 \) figure 1 is below form

\[ B'_2 = 2[4K'_2, \delta'_2] \left( \frac{m_2}{m_2 + \delta'_2} \right) \times \sigma'_p \cdot (\text{MPa})^{m_2 + \delta'_2} \cdot \text{damage - unit - number / cycle} (\sigma_w = 0) \] \[ \text{(35)} \]

Where \( m_2 \) is an linear elastic exponent in long crack growth process, \( m_2 = -1/b_2 \). But, \( \lambda'_2 \) is a ductility exponent, \( \lambda'_2 = -1/\lambda'_2 \).

\[ D_{2\text{eff}} = \frac{E \times \delta'_2}{\pi \sigma'_w (\sigma'_w / \sigma'_w + 1)} \cdot \text{(damage - unit - number)} \] \[ \text{(36)} \]

\[ \text{For } \sigma_w = 0 \]

\[ N_{2\text{eff}} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2 \lambda_2} \left( \frac{2m_2 + 2\lambda_2 - 3m_2 \lambda_2}{2(m_2 + \lambda_2 - 3m_2 \lambda_2)} \right) - \frac{D_{02}}{D_{\text{eff}}} \] \[ \text{(37)} \]

\[ \text{For } \sigma_w \neq 0 \]

\[ N_{2\text{eff}} = \frac{4(m_2 + \lambda_2)}{4m_2 + 4\lambda_2 - 6m_2 \lambda_2} \left( \frac{2m_2 + 2\lambda_2 - 3m_2 \lambda_2}{2(m_2 + \lambda_2 - 3m_2 \lambda_2)} \right) - \frac{D_{02}}{D_{\text{eff}}} \] \[ \text{(38)} \]

2) \( \sigma - \) stress method

Due to word stress \( \sigma > \sigma'_s \), if adopt stress to express it, the \( B'_2 \) in eqn (35) should be

For \( \sigma = 0 \)

\[ B'_2 = 2 \left[ \frac{\sigma'_s \cdot \sigma'_w}{E} \left( \sqrt{\pi D_{2f}} \right)^2 \right] \left( \frac{m_2}{m_2 + \delta'_2} \right) \times \sigma'_p \cdot (\text{MPa})^{m_2 + \delta'_2} \cdot \text{damage - unit - number / cycle}, \] \[ \text{(39)} \]

For \( \sigma \neq 0 \)

\[ B'_2 = 2 \left[ \frac{\sigma'_s \cdot \sigma'_w}{E} \left( \sqrt{\pi D_{2f}} \right)^2 \right] \left( \frac{m_2}{m_2 + \delta'_2} \right) \times \sigma'_p \cdot (\text{MPa})^{m_2 + \delta'_2} \cdot \text{damage - unit - number / cycle}, \] \[ \text{(40)} \]
\[ B'_2 = 2\left[\frac{\sigma_{\mu} \cdot \sigma_\theta / \sigma_\mu + 1}{E} \left(\sqrt{D_{2f}}\right)^{m_{2f} z_{2f}} [1 - \sigma_m / \sigma_\mu]\right] \frac{m_{2f} z_{2f}}{m_{2f} + z_{2f}} \times \frac{m_{2f} z_{2f}}{m_{2f} + z_{2f}} \cdot \text{damage - unit - number / cycle}, \]  

Therefore the residual life equation of corresponded to reversed direction curve \( D_2 D_1 \) in fig.1, its final expansion equation is as below form.

For \( \sigma_m = 0 \),

\[ N_{2\text{eff}} = \frac{4(m + \lambda)}{4m + 4\lambda_z - 6m_z \lambda_z} \left\{ D_{2\text{eff}} \left[\frac{2m_2 + 2d_2 - 3m_2 d_{2f}}{2(m_2 + d_2)}\right] - D_1 \left[\frac{2m_2 + 2d_2 - 3m_2 d_{2f}}{2(m_2 + d_2)}\right]\right\} \times \frac{1}{\left[\frac{0.5 \sigma \cdot [\sqrt{D_{2\text{eff}}}] (\sigma / \sigma_p + 1) / E}\right]^{m_{2f} z_{2f}}}, \text{(cycle)}, \]  

For \( \sigma_m \neq 0 \),

\[ N_{2\text{eff}} = \frac{4(m + \lambda)}{4m + 4\lambda_z - 6m_z \lambda_z} \left\{ D_{2\text{eff}} \left[\frac{2m_2 + 2d_2 - 3m_2 d_{2f}}{2(m_2 + d_2)}\right] - D_1 \left[\frac{2m_2 + 2d_2 - 3m_2 d_{2f}}{2(m_2 + d_2)}\right]\right\} \times \frac{1}{\left[\frac{0.5 \sigma \cdot [\sqrt{D_{2\text{eff}}}] (\sigma / \sigma_p + 1) / E}\right]^{m_{2f} z_{2f}}}, \text{(cycle)}, \]  

Here, influence to mean stress in eqn (45) usually can be ignored, and it must be point that the units in the life equations are all cyclic number.

### 4.3. The Life Prediction Calculations for Whole Process Damage Growth

For availing to life calculation in whole process, author proposes it need to take the damage value \( D_n \) of transition point between two stages from micro to macro damage growth process, and the damage value \( D_n \) at transition point can be derived to make equal between the damage rate equations by two stages, for instance [17],

\[ dN_1 / dN = dN_2 / dN = dD_1 / dD_2 \]  

(1) Under work stress \( \sigma < \sigma_p \).

For \( \sigma < \sigma_p, \sigma_m = 0 \), its expanded rate link equation for eqn (46) corresponding to positive curve \( AA_2 \) is as following form

\[ \frac{dD_{\text{eff}}}{dN} = \left\{ \frac{\sigma_\theta}{E \sigma_\mu} \left(\sqrt{D_{2\text{eff}}}\right)^{m_{2f} z_{2f}} \times y_2(a \cdot b / \Delta \sigma) \right\}, \text{(\( \sigma_m = 0 \))damage - unit - number / cycle} \]  

For \( \sigma < \sigma_p, \sigma_m \neq 0 \), its expanded rate link equation for eqn (45) corresponding to positive curve \( DD_1 D_2 \) is as following form

\[ \frac{dD_{\text{eff}}}{dN} = \left\{ \frac{\sigma_\theta}{E \sigma_\mu} \left(\sqrt{D_{2\text{eff}}}\right)^{m_{2f} z_{2f}} \times y_2(a \cdot b / \Delta \sigma) \right\}, \text{damage - unit - number / cycle} \]  

(\( \sigma \neq 0 \))
And the life equations in whole process corresponding to reversed direction curves \( A_{2A} \) and \( D_{2D} \) should be as below

\[
\sum N = N_1 + N_2 = \int_{D_{2D}} dD \frac{dD}{D_{B}} \sum_{m_{1}^{m_{2}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}}} + \int_{D_{2D}} dD \frac{dD}{D_{B}} \sum_{m_{1}^{m_{2}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}}} + D_{2D} \left[ y_2(a/b) \Delta \sigma \cdot \Delta E \right] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}},
\]  

(49)

Its expanded equation corresponding to reversed direction curves \( A_{2A} \) is as following form

\[
\sum N = N_1 + N_2 = \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 2[4(\sigma' / \sigma') (1 - \sigma_m / \sigma_f)] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}} \right] 

+ \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 2 \left( \frac{\sigma_m^2}{E2\sigma_r} \right) \left( \sqrt{\sigma_{m/2 \sigma_{eff}}} \right)^2 \left( 1 - \sigma_m / \sigma_f \right) \times y_m \left( a \times b \Delta \sigma \cdot \Delta E \right) \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \right] \left( \sigma_m = 0 \right)
\]

(50)

But for \( \sigma_m \neq 0 \), its expanded equation corresponding to reversed direction curves \( D_{2D} \) should be

\[
\sum N = N_1 + N_2 = \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 2[4(\sigma' / \sigma') (1 - \sigma_m / \sigma_f)] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}} \right] 

+ \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 2 \left( \frac{\sigma_m^2}{E2\sigma_r} \right) \left( \sqrt{\sigma_{m/2 \sigma_{eff}}} \right)^2 \left( 1 - \sigma_m / \sigma_f \right) \times y_m \left( a \times b \Delta \sigma \cdot \Delta E \right) \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \right] \left( \sigma_m \neq 0 \right)
\]

(51)

(2) Under work stress \( \sigma > \sigma_f \)

Under work stress \( \sigma > \sigma_f \), its expanded rate link equation for eqn (46) corresponding to positive curve \( CC_1C_2 \) is as following form

\[
\frac{dD_{nu}}{dN_1} = \left[ 2[4(\sigma' / \sigma') (1 - \sigma_m / \sigma_f)] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}} \right] \left( \sigma_m = 0 \right) 

+ \frac{dD_{nu}}{dN_1} \left[ 0.25 \Delta \sigma \times \Delta E \times \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \right] \left( \sigma_m \neq 0 \right)
\]

(52)

The life equations in whole process corresponding to reversed direction curve \( C_2C_1 \) should be as following

\[
\sum N = N_1 + N_2 = \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 0.5 \times \sigma_m \left( \sigma_m / \sigma_f + 1 \right) \right] \left( \sigma_m \neq 0 \right)
\]

\[
\frac{dD_{nu}}{dN_2} = \left[ 2[4(\sigma' / \sigma') (1 - \sigma_m / \sigma_f)] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}} \right] \left( \sigma_m = 0 \right)

+ \frac{dD_{nu}}{dN_2} \left[ 0.5 \times \sigma_m \left( \sigma_m / \sigma_f + 1 \right) \right] \left( \sigma_m \neq 0 \right)
\]

(53)

And the expanded life prediction expression in whole process, corresponding reversed curve \( C_2C_1 \) should be as following

\[
\sum N = N_1 + N_2 = \int_{D_{2D}} dD \frac{dD}{D_{B}} \left[ 0.5 \times \sigma_m \left( \sigma_m / \sigma_f + 1 \right) \right] \left( \sigma_m \neq 0 \right)
\]

\[
\frac{dD_{nu}}{dN_2} = \left[ 2[4(\sigma' / \sigma') (1 - \sigma_m / \sigma_f)] \frac{m_{1}^{m_{2}}}{m_{1}^{m_{2}}} \times (\Delta \sigma \times \Delta E)^{m_{1}^{m_{2}}} \right] \left( \sigma_m = 0 \right)

+ \frac{dD_{nu}}{dN_2} \left[ 0.5 \times \sigma_m \left( \sigma_m / \sigma_f + 1 \right) \right] \left( \sigma_m \neq 0 \right)
\]
\[ \sum N = \frac{dD}{N_{ref}} + \int \frac{dD}{2\left[\frac{\sigma_f}{\sigma_m} - \left(1 - \frac{\sigma_m}{\sigma_f}\right)\right]^{\frac{m-1}{m}}\times(v_{eff})^{-1}\times(0.25\Delta\sigma\times\Delta\varepsilon)} \]

\[ D = \int_{\sigma_f}^{\sigma_m} \left[\frac{\sigma_f}{\sigma_m} - \left(1 - \frac{\sigma_m}{\sigma_f}\right)\right]^{\frac{m-1}{m}}\times(v_{eff})^{-1}\times(0.25\Delta\sigma\times\Delta\varepsilon) \]

It should point that the calculations for rate and life in whole process should be according to different stress level, to select appropriate calculable equation. Here must explain that whole process should be done in two stages. But the life calculations for two stages can be added together. About calculation tool, it can be calculated by means of computer doing computing by different damage value [18].

### 5. Computing Example

#### 5.1. Contents of Example Calculations

A pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material \(\sigma_s = 573\text{MPa}\), yield limit \(\sigma_y = 361\text{MPa}\), fatigue limit \(\sigma_f = 267.2\text{MPa}\), reduction of area is \(\psi = 0.51\), modulus of elasticity \(E = 200000\text{MPa}\).


<table>
<thead>
<tr>
<th>Table 1. Computing data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_{1c}), MPa√(m)</td>
</tr>
<tr>
<td>92.7</td>
</tr>
</tbody>
</table>

#### 5.2. Required Calculating Data

Try to calculate respectively as following different data and depicting their curves:

1. To calculate the transitional point damage value \(D_{tr}\) between two stages;
2. To calculate the damage rate for transitional point \(D_{tr}\) between two stages;
3. To calculate the life \(N_{t}\) in first stage from micro damage value \(D_{t} = 0.02\text{damage – unit}\) growth to transitional point \(D_{tr}\);
4. To calculate the effective life \(N_{eff}\) in second stage from transitional point \(D_{tr}\) to effective damage value \(D_{eff} = 5\text{damage – unit}\);
5. To calculate the whole service life \(\Sigma N\)
6. Depicting life curve in whole process.

#### 5.3. Calculating Processes and Methods

The concrete calculation methods and processes are as follows

1. Conversions for dimensions and units


<table>
<thead>
<tr>
<th>Table 2. Computing data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K'_{1c}), MPa√(1000\text{damage – unit})</td>
</tr>
<tr>
<td>92.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Computing data</th>
</tr>
</thead>
<tbody>
<tr>
<td>(v_{pu})</td>
</tr>
<tr>
<td>2×10^{-4}</td>
</tr>
</tbody>
</table>
(2) Calculations for relevant parameters
To calculate related strain values by reference (31)

Total strain:
\[
\varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^\frac{1}{2} = \frac{450}{200000} + \left( \frac{450}{1165} \right)^\frac{1}{2}
\]
\[
= 2.25 \times 10^{-3} + 6.178 \times 10^{-3} = 8.4278 \times 10^{-3} \text{(m)} = 8428 \mu \varepsilon
\]
\[
\varepsilon_{\text{max}} = 8.4278 \times 10^{-3} \text{(m)} = 8428 \mu \varepsilon; \quad \varepsilon_{\text{min}} = 0
\]
\[
\varepsilon_{\text{n}} = (\varepsilon_{\text{max}} + \varepsilon_{\text{min}}) / 2 = (8.4278 \times 10^{-3} + 0) / 2 = 4.2148 \times 10^{-3}
\]
Stress range calculation:
\[
\Delta \sigma = \sigma_{\text{max}} - \sigma_{\text{min}} = 450 - 0 = 450 \text{MPa}
\]
Mean stress calculation:
\[
\sigma_{\text{m}} = (\sigma_{\text{max}} + \sigma_{\text{min}}) / 2 = (450 - 0) / 2 = 225 \text{MPa}
\]
Total strain range:
\[
\Delta \varepsilon = \frac{\Delta \sigma}{E} + 2 \left( \frac{\Delta \sigma}{2K} \right)^\frac{1}{2} = \frac{450}{200000} + 2 \left( \frac{450}{2 \times 1165} \right)^\frac{1}{2}
\]
\[
= 2.25 \times 10^{-3} + 3.034 \times 10^{-4} = 2.5534 \times 10^{-4} \text{(m)} = 2.553 \text{mm}
\]
Elastic strain range:
\[
\Delta \varepsilon_e = 2.25 \times 10^{-3} \text{(m)} = 2.25 \text{mm} ;
\]
\[
A_i^* = 2[4\sigma', \varepsilon', (1 - \sigma_m / \sigma')^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma')^{-1}] = 2[4(947.1 \times 0.464)(1 - 225/947.1)]^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (2 \times 0.713)^{-1}
\]
\[
= 2.216 \times 10^{-3} \text{(MPa/m}, \mu \varepsilon, \text{damage - number/cycle})
\]

2) Calculation for comprehensive material constant \(B_i^*\) in second stage by eqn (43)

\[
B_i^*_{\text{eff}} = 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}] = 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}]
\]
\[
= 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}] = 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}]
\]
\[
= 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}] = 2[\left( \sigma_{\text{m}} / \sigma_{\text{m}} \right)^{\frac{m_{\mu,m_1}}{m_{\mu,m_1}}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{-1}]
\]

3) Calculation for transitional point crack size \(a_i\)

According to the equations (46) and (52),
\[
dD_i / dN_i = dD_i / dN_i = dD_i / dN_i
\]

Then, calculation for the transitional damage value \(D_i\) between two stages, it can make equal between the rate expansion equations at left side and the rate one at right side as following
\[
A_i^* \times (0.25 \times \Delta \varepsilon \times \Delta \sigma)^{m_{\mu,m_1}} \times D_i = A_i^* \times (\Delta \sigma / 2) \times 0.5 \sigma_{\text{m}} \times (\sigma_{\text{m}} / \sigma_{\text{m}})^{m_{\mu,m_1}}
\]
Then make simplified calculation:
\[
3.22 \times 10^6 D_\sigma = 2.6695 \times 10^6 \times D_{\nu}^{2.4975};
\]
\[
D_\nu = 1.2062^{0.6678} = 1.133 (\text{damage unit});
\]
So to obtain the transitional point damage value \( D_\sigma = 1.133 (\text{damage unit}) \), that is equivalent to crack size \( a_\nu = 1.113 \text{mm} \) at this point.

The damage rate calculations for transitional point \( D_\nu \):
\[
dD_1 / dN_1 = dD_\sigma / dN_\nu = 3.22 \times 10^6 \times D_\nu = 3.22 \times 10^6 \times 1.133 = 3.648 \times 10^6 \text{ (damage – unit / cycle)}
\]
\[
dD_2 / dN_2 = dD_\nu / dN_\nu = 2.6695 \times 10^6 \times a_\nu^{2.4975} = 2.6695 \times 10^6 \times 1.133^{2.4975} = 3.646 \times 10^6 \text{ (damage – unit / cycle)}
\]
That is equivalent to crack growth rate \( 3.648 \times 10^6 \text{ (mm / cycle)} \).

Thus it can be seen, the crack growth rate at the transition point crack size \( a_\nu = 1.113 \text{ (mm)} \) is same, it is \( 3.646 \times 10^6 \text{ (mm / cycle)} \).

(3) Life prediction calculations in whole process
1) Select life predicting calculation equation (12), to calculate life \( N_1 \) in first stage from micro-damage \( D_{01} = 0.02 \text{ (damage – unit)} \) to transitional point \( D_\nu = 1.113 \text{ (damage – unit)} \) is as follow,
\[
N_1 = \frac{\ln D_\sigma - \ln D_{01}}{\ln 1.133 - \ln 0.02} = \frac{2[4\sigma'_f, \epsilon'_f (1 - \sigma'_u / \sigma'_f)^{m_{\text{m1}}} \times (D_{\text{off}} \times \nu_f)^{m_{\text{m2}}} \times (0.25 \times \Delta \epsilon \times \Delta \sigma)^{m_{\text{m3}}} \times D_\sigma = 2\left[\frac{\sigma_0 \cdot \sigma_\nu (\sigma_\nu / \sigma_0 + 1)}{E} (\sqrt{2D_{\text{off}}} / (\Delta \sigma / 2\sigma_\nu + 1)) \right] (1 - \sigma_\nu / \sigma_0)}{m_{\text{m1}} + m_{\text{m2}} + m_{\text{m3}}} \times \nu_f}
\]
\[
= \frac{2[4(947.1 \times 0.464)(1 - 225 / 947.1)]^{m_{\text{m1}}} \times (2 \times 0.7133)^{m_{\text{m2}}} \times (0.25 \times 2.553 \times 10^3 \times 450)^{m_{\text{m3}}} \times D_\sigma}{200000}
\]
Then make simplified calculation:
\[
3.22 \times 10^6 D_\sigma = 2.6695 \times 10^6 \times D_{\nu}^{2.4975};
\]
\[
D_\nu = 1.2062^{0.6678} = 1.133 (\text{damage – unit});
\]
So first stage life \( N_1 = 1253693 \text{ (Cycle)} \).

From above formula we can derive simplified life equation in first stage corresponded to different damage value as follow form
\[
N_1 = \frac{1}{3.22 \times 10^6 D_1}
\]

Select life predicting calculation equation (45), to calculate life \( N_2 \) in second stage from transitional point \( D_\nu = 1.113 \text{ (damage – unit)} \) to \( D_{\nu} = 5 \text{ (damage – unit)} \), It is as below,


From above formula we can also derive simplified life equation corresponding different damage value as follow form

\[ N_2 = \frac{1}{2.6695 \times 10^{10} D_{cr}^{2.9}} \]

Therefore, whole process life is

\[ \Sigma N = N_1 + N_2 = 1253693 + 185014 \times 1438707(\text{Cycle}) \]

If use integral equation (53) and (54) to calculate the service life in whole process, it is

\[ \Sigma N = \int_{0.02}^{1} \int_{0.02}^{D_{cr}^{2.9}} \frac{1}{A_1 \times \Omega^{m_1}} \, \frac{dD}{dD} \, dD = \frac{1}{1.133} \int_{0.02}^{1} \int_{0.02}^{1} \frac{1}{A_1 \times \Omega^{m_1}} \, \frac{dD}{dD} \, dD \]

\[ = \frac{1.133}{0.02} \times \frac{dD}{dD} \times \frac{dD}{dD} = 3.322 \times 10^{4} \times D_{cr}^{2.6695} \\
\times \frac{1}{0.5(450/2) \times 26695 \times 10^{-5} \times D_{cr}^{2.9}} = 1300000 + 185020 = 1485020(\text{Cycle}) \]

5.4. Calculating Results

5.4.1. Life data of Each Stage and Whole Process

Calculating results are that the first stage life is \( N_1 = 1300000 \) cycle from micro-crack size 0.02 mm to transition point size 1.113mm; the second stage life is 185020 cycle from transition point-crack size 1.113 mm growth to long crack length 5 mm; the service life in whole process is 1485020 cycle. This result is consistent by and large with expansion equations calculation results data from above equation.

The life data corresponded to different crack length in crack propagation course are all included in table 4, 5, 6.

<table>
<thead>
<tr>
<th>Data point of number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack size (mm)</td>
<td>0.02</td>
<td>0.04</td>
<td>0.1</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Data of the first stage</td>
<td>15527950</td>
<td>7763975</td>
<td>3105990</td>
<td>1552795</td>
<td>776398</td>
</tr>
<tr>
<td>Data of the second stage</td>
<td>Invalid section</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4. Crack growth life data in whole process
**Table 5. Crack growth life data in whole process**

<table>
<thead>
<tr>
<th>Data point of number</th>
<th>6</th>
<th>7</th>
<th>8 transition point</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack size (mm)</td>
<td>0.6</td>
<td>0.7</td>
<td>1.113</td>
<td>1.5</td>
</tr>
<tr>
<td>Data of the first stage</td>
<td>517598</td>
<td>443656</td>
<td>274103</td>
<td>207039</td>
</tr>
<tr>
<td>Data of the second stage</td>
<td>1341644</td>
<td>912931</td>
<td>274240</td>
<td>136076</td>
</tr>
</tbody>
</table>

**Table 6. Crack growth life data in whole process**

<table>
<thead>
<tr>
<th>Data point of number</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crack size (mm)</td>
<td>1.5</td>
<td>2.0</td>
<td>3.0</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Data of the first stage</td>
<td>207039</td>
<td>155280</td>
<td>Invalid section</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Data of the second stage</td>
<td>136076</td>
<td>66336</td>
<td>24097</td>
<td>11747</td>
<td>6728</td>
</tr>
</tbody>
</table>

5.4.2. To Depict the Life Curves in Each Stage and Whole Process

According to crack growth data for different crack size depicting the curves in whole process, which is showed in figures 2 and 3.

![Figure 2. Life curve in whole course (in decimal coordinate system)](image)

(A) 2-1—data curve in first stage obtained by two-parameters calculating method;
(B) 2-2—data curve in second stage obtained by two-parameters calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5mm is just at eighth point (crack size 1.113mm).

![Figure 3. Life curve in whole course (in logarithmic coordinate system)](image)

(A) 2-1—data curve in first stage obtained by two-parameters calculating method;
(B) 2-2—data curve in second stage obtained by two-parameters calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5mm is just at eighth point (crack size 1.113mm).

6. Discussions

Author putted forward such a view point—there are a scientific law of similar to gene principle and cloning technique in the mechanics, aviation, machinery and civil engineering etc. fields. Because where there are common scientific laws as life science: 1) each unit cell combined in a genetic structure has all its own genetic (or inheritable) characters; 2) to have the clonable (or can copy), the transferable characters, and can be recombined together by inherent relationship; 3) the new combined structures there are also with new characters and functions. Those parameters inside from equations (1) to (54) in this paper, which there are all as "genetic character of unit cells", e.g. as the stress parameter $\sigma$, the strain $\varepsilon$ and their material
constants $\sigma, \psi, E$ etc subjected traditional materials mechanics; and these “genetic elements” all to maintain its original calculable properties and functions. But once they are transferred into in new areas as the damage mechanics or the fracture mechanics, they are been formed by new structure-equations. Then the new equations are also shown with new calculable functions as those equations in the traditional material mechanics. Author just is according to such thinking logics and methodologies, and by means of various relations among the material constants each other, e.g. $m_i = -1/b_i, \; m_i = -1/c_i, \; n = b_i/c_i, \; n = -1/c_i n$; $m_2 = -1/b_2, \; \lambda_2 = -1/c_2$ etc, and according to the cognition for their physical and geometrical significance for key parameters, thereby derives the above mentioned a lot of calculable models.

7. Conclusions

1) About the significations of figure1: The calculating figure of materials behaviors may be a calculation route diagram on fatigue-damage-fracture, it is a tool and bridge to communicate relations among the conventional material mechanics, the modern mechanics and the engineering materials disciplines.

2) About difference problem for material constants: True material constants must show the inherent characters of materials, such as the $\sigma_f$ and $E, \delta, \psi$ etc in the material mechanics; for instance the $\sigma_f$ and $\sigma_f'$; $\epsilon_f$ and $\epsilon_f'$; $b_i$ and $b_i'$; $c_i$ and $c_i'$ and so on in the fatigue damage mechanics; $K_i, \sigma, m_i, \lambda_i$ and so on in the fracture mechanics, which could all be checked and obtained from general handbooks. But some new material constants in the damage and fracture mechanics which are essentially functional formulas, for which can all be calculated by means of the relational expressions, e.g. eqns (4-5), (18-19), (31-32), (36-37), (42-43), (46-48), (52), etc. and must combine experiments to verify. Therefore for this kind of material constants can be defined as comprehensive materials constants.

3) About cognitions to the physical and mathematical meanings for key parameters $A_i$, $A'_i$, $A''_i$, $B'_i$: The parameters $A_i$ in the first stage and $A'_i$, $A''_i$, $B'_i$ in the second stage, they are all calculable comprehensive materials constants, are all the formulas with other parameters to have functional relations. Their physical meanings of the $A_i$ and $A'_i$, $B'_i$ are a concept of power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

4) About conversion regulations on variables, dimensions and units: Inside mathematical models to convert crack variable $a$ into damage variable $D$, it must define “1mm-crack-length” equivalent to ‘one-damage-unit’, “1m-crack-length” equivalent to ‘1000-damage-unit’, this is a key for making link between the damage mechanics and the fracture mechanics.

5) About the regulations and methods for whole process rate and life calculations: For damage transition value $D_n$ can be calculated from to make equal between the micro-damage rate and the macro-damage rate equation; For rate calculation, before the transition point $D_n$ it should be calculated by the micro damage rate equation, after the transition point $D_n$ it should be calculated by the macro damage rate equation. But the lifetime calculations can be added together by life cycle number of two stages.

6) Based on the traditional material mechanics is a calculable subject, in consideration of the traditional parameters there are “the hereditary characters”, In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions of the (1)--(5), then nowadays for the fatigue-, the damage-, and the fracture disciplines yet mainly depended on tests, thereby make them become calculable subjects, under the conditions which are via theoretical calculation is given as priority, via the experiments is verified as complementary, that will be possible.

Acknowledgments

At first, author sincerely thanks scientists David Broek, Miner, P. C. Paris, Coffin, Manson, Basquin, Y. Murakami, S. Ya. Yaliema, Morrow J D, etc, they have be included or no included in this paper reference, for they have all made out valuable contributions for the fatigue-damage-fracture subjects. Due to they hard research, make to discover the fatigue damage and crack behavioral law for materials, to form the modern fatigue-damage-fracture mechanics; due to they work like a horse, make to develop the fatigue-damage-fracture mechanics subjects, gain huge benefits for accident analysis, safety design and operation for which are mechanical equipments in engineering fields. Particularly should explain that author cannot have so many of discovery and provide above the calculable mathematical models and the combined figure 1, if have no their research results.

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Definitions and Nomenclature

$a_{ml}, a_{mw}$: initial and medial short crack at the first stage; $a_{ir}, a_{ime}$: transited crack size from short crack in first stage to long crack in second stage, $a_i = a_{ime} ; N_{0i}, N_m$: initial life and medial life in first stage; $N_{1f}$: failure life for cycle number $N_{1f} = 1$ in first stage; $N_i$: life in first stage; $m_i, b_i$: fatigue strength exponent under high cycle fatigue in first stage, $m_i = -1/b_i ; m_i', c_i$: fatigue ductility exponent under
low cycle fatigue in first stage, $m_1' = -1/c'_2$; $A'_1$ : comprehensive material constant in first stage; $\theta$ : reduction of area; $\sigma_f' $ : fatigue strength coefficient; $E_f' $ : fatigue ductility coefficient; $\sigma_m$ : mean stress; $\epsilon_m$ : mean strain; $Q_1, \Delta Q_1$ : short-crack stress-strain factor and stress-strain factor range in first stage; $Q_2', \Delta Q_2'$ : damage stress-strain factor and damage stress-strain factor range in first stage; $a_{ij}$ : initial crack size at the second stage; $a_{cr} :$ critical crack size to make fracture in one cycle; $m_2, b_2$ : fatigue strength exponent under high cycle fatigue in second stage, $m_2 = -1/b_2 ; A_2, c_2$ : fatigue ductility exponent under low cycle fatigue in second stage, $A_2 = -1/c_2 ; A'_2$ : comprehensive material constant at the second stage; $A_{2e}^\prime$ : effective comprehensive material constant at the second stage; $A_{2eff}^\prime$ : effective damage comprehensive material constant at the second stage; $\delta_{eff}$ : effective value of crack tip open displacement; $\delta_{cr}$ : critical crack tip open displacement; $K_m$ : mean stress intensity factor; $\delta_e$ : mean value of crack tip open displacement; $\delta_{cr} = \delta_{cr}^C$ : critical crack tip open displacement; $K_{2eff}$, $K_{2eff}$ : effective value of stress intensity factor; $\delta_{cr}$ : effective value of crack tip open displacement; $K_m$ : mean stress intensity factor; $\delta_e$ : mean value of crack tip open displacement; $\delta_e = \delta_e^2$ : critical crack tip open displacement; $K_m = K_{cr}$ : fracture toughness; $Q_2, \Delta Q_2$ : long crack stress-strain factor and stress-strain factor range in second stage; $Q_2', \Delta Q_2'$ : damage stress-strain factor and damage stress-strain factor range in second stage; $v_{eff}$ : virtual rate at the second stage; $a_{12}, a_{ij}, a_{ij}, a_{12}$ : initial, medial, effective and critical size of crack at the second stage. $a_{12} \leq a_{ij} \leq a_{eff} < a_{12}$ ; $D_{12}, D_{ij}, D_{ij}, D_{12}$ : initial, medial, effective and critical damage value in second stage, $D_{12} \leq D_{ij} \leq D_{eff} < D_{12}$ ; $N_{a_{ij}}, N_{a_{ij}}, N_{a_{ij}}, N_{a_{ij}}$ : initial life, medial life, effective and critical life in second stage, $N_{a_{ij}} \leq N_{a_{ij}} \leq N_{a_{eff}} < N_{a_{ij}}$ : life in second stage; $\sum N$ : life in whole process.

References


