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# Damage Growth Rate Calculations Realized in Whole Process with Two Kinks of Methods

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## Abstract
Based on author's the new viewpoints who he thinks there are the genetic principles and clone technologies in the mechanics and the engineering fields; In view of micro-damage and macro-damage behaviors there are distinctly different; To use the theoretical approach, and adopt the simple stress-parameter, or the strain-parameter-method and the multiplication-method of two-parameters, to establish numerous new calculation models in whole damage growth process for elastic-plastic steels, which are the micro damage and macro growth driving forces, the damage growth rate equations for different stages, the damage-growth-rate-linking-equation in whole process; For the transitional point damage value and the damage growth rate at transitional point from micro damage to macro damage growth respectively to put forward a lot of expressions, to provide the concrete and detailed calculation process, the steps and the methods; For new discovering and there are functional relations of some key materials parameters, respectively to give the new calculable formulas, the new definitions, the new physical and geometrical meanings for them. Thereby make linking and communication for among the modern damage mechanics, the fracture mechanics and the traditional material mechanics; to realize calculations for the damage growth rate and lifetime prediction in whole process.

## 1. Introduction

In view of complexity of elastic-plastic material properties contained crack, to consider the micro and macro crack of behaviors are obviously different under different loading conditions, so to research the problems of the driving forces under so many factors and conditions, to establish so many damage growth rate models in the whole process, which are all very complicated problems.

As everyone knows for the traditional material mechanics, that is a calculable subject, and it has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the damage growth rate problems for some structures when it is pre-existing flaws and concentrated stress under repeated loading. In that it has no to contain such calculable parameters as crack variable $a$ or as the damage variable $D$ in its calculating models. But in the fracture mechanics and the damage mechanics, due to there are these variables, they can all calculate above problems. But nowadays latter these disciplines are all subjects mainly depended on tests. So that, for above elastic-plastic materials and structures of contained defects, if want to solve the damage growth rate calculations for the whole process from the micro damage to macro damage growth, that are more difficult, to pay the manpower and money for experiments are more huge.

Author thinks, in the mechanics and the engineering fields, where there are also such the scientific laws as similar to genetic elements and clone technologies in life science. Author was using the theoretical approach as above the similar principles, proposed some calculation models [1-7], recently sequentially discovers some new scientific laws, adopts the simple stress-parameter or the strain-parameter-method, and the multiplication-method of two parameters, provides some new calculable models for the damage growth driving force, and for the damage growth rate in whole process. Try to make the damage mechanics, the fracture mechanics, step by step become such calculable disciplines as the traditional material mechanics. Thereby, to realize the calculations in whole process for the damage growth rate based on conventional materials, it may be having practical significances for decreasing experiments, to stint man powers, power resources and funds, for promoting extensive applying to the aerospace, the transportations, the petrochemical engineering, the constructional and the machinery engineering, and for developing to relevant disciplines.

## 2. About Some Viewpoints of Existing "Genetic Gene" Principles and "Clone Technologies" in the Mechanics and Engineering Fields

As is well-known, in the traditional material mechanics, on describing material behaviors and its strength problems, its main calculating parameters are the stress $\sigma$, the strain $\varepsilon$ and relevant material constants, e.g. yield stress $\sigma_s(\sigma_y)$, elasticity modulus $E$ and reduction of area $\psi$, etc. In the damage mechanics it is based on the damage parameter $D$ as its variable, to adopt the fatigue strength coefficient $\sigma'_f$ and the fatigue ductility coefficient $\varepsilon'_f$ etc. as its material constants. In the fracture mechanics, describing material behaviors on the strength and the life prediction problems, it is based on the crack size $a$ as its variable, to use the fracture toughness $K_{1c}$ and the critical crack tip open displacement $\delta_c$ as its material constants.

Author thinks the gene and clone technologies in life science, for which their traits consist in: it has both him-self genetic properties, and has transferable and recombination properties. In fact, in fracture mechanics, in the stress intensity factor $K_1 = \sigma\sqrt{\pi a}$, in the crack tip open displacement $\delta_t$ and in their critical value $K_{1c} = \sigma\sqrt{\pi a_c}$ and the $\delta_c$, which are all including the parameters $\sigma$, $\varepsilon$, $\pi$ and their material constants $\sigma_s, \varepsilon_s$ and fracture stress $\sigma_f$ etc. Author thinks for the stress $\sigma$, the strain $\varepsilon$ and its relevant material constants $\sigma_s$ and $E$ etc in the material mechanics, for which can be considered as genetic elements; for the $D$ and $\sigma'_f$ etc in the damage mechanics can also be considered as genetic elements; and for the crack size $a$ in the fracture mechanics can also be considered as genetic elements. If can make a linking among the material mechanics, the damage mechanics and the fracture mechanics, and provide respectively some conversion methods, then they can also be converted each other for their relations between the variables, between the material constants, and between the dimensional units in the equations. For example, we can consider as gene for the stress $\sigma$ and its $\sigma_s, E, \psi$, to make them combination with the variable $D_1$ are together transferred into micro-damage-mechanics, and combination with the variable $D_2$ are transferred into macro-damage-mechanics; In the same way, we can also consider as gene for the stress $\sigma$ and its $\sigma_s, E, \psi$, to make them combination with the variable $a_1$ are together transferred into micro-fracture-mechanics, and combination with the variable $a_2$ are transferred into macro-fracture-mechanics. Then we are able by these stress $\sigma$, $\varepsilon$, $\sigma_f^{'}, \varepsilon_f^{'}$, etc, to renew establishing their the driving force models, the damage growth rate and its life equations or the crack growth rate and its life equations. Even we can also adopt the variable $D$ or $a$ to describe material behaviors in whole process.

Above the properties of those parameters and material constants, even though as compared to those ones the life science, due to they are in different disciplines. But, for which have both own inheritable properties (similar to genetic elements), and have the transferable, and the recombination properties, for these---on the epistemology and methodology, in practice they are all very similar.

Based on the cognitions and concepts mentioned above, author makes a linking among the material mechanics, the damage mechanics and the fracture mechanics, for relationship between their parameters are analyzed, for their equations are derived, for their dimensional units are converted each other, then for new made models are calculated, checked and validated again and again, finally, to provide the equations (1-49) in following text. Thereby try to make communications for among the traditional material mechanics, the damage mechanics and the fracture mechanics, then to make such new calculable mathematical models as those equations inside the material mechanics, which are the new driving force ones and the damage growth rate calculation equations. Author thinks if we can realize the goals, it will all be having practical significance for the engineering designs, the computational analysis for safe operation and assessment of machineries and structures.

## 3. Comprehensive Figure of Materials Behaviors

Among branch disciplines on fatigue-damage-fracture, among the traditional material mechanics and the modern mechanics, for communications and connecting their relations each other, it must study and find their correlations between the equations, even to be the relations between variables, between the material constants, between the curves.

Because which are all the significant factors to research and to describe for material behaviors at each stage even in whole process, and are also all to have significant significations for the engineering calculations and designs. Therefore it should research and find an effective tool used for analyzing problems above mentioned. Here author provides the "Comprehensive figure of materials behaviors" as Figure1 (or called the calculating figure of materials behaviors, or called bidirectional combined coordinate system and simplified schematic curves in the whole process, or called combined cross figure)[1-3]. Here below in two problems to present:

(1) The physical and geometrical meanings about the consisting, the partitions, the directions of the coordinate system and the critical points on coordinate axis.

In the figure 1, it was being provided by present author, at this time it has been corrected and complemented, that is diagrammatically shown for the damage growth process or crack propagation process of material behavior at each stage and in whole course.

For the coordinate system, it is to consist of six abscissa axes $O'$ I", $O$ I', $O_1$ I, $O_2$ II, $O_3$ III, $O_4$ IV and a bidirectional ordinate axis $O'_1\ O_4$.

For the area between the axes $0'$ I" and $0_1$ I, it was an area applied as by the traditional material mechanics, currently it can also be applied as micro-damage area by the very-high cycle fatigue. Among the axes $O$ I', $O_1$ I and $O_2$ II, they are calculating areas applied by the micro-damage mechanics and the micro-fracture mechanics. Between the axes $0_3$ III and $O_4$ IV, it is calculating area applied by the macro-damage mechanics and the macro-fracture mechanics. Between the axes $O_2$ II and $O_3$ III, it is both calculating area applied for the micro-fracture mechanics and macro-fracture mechanics, or it is both calculating area applied for the micro-fracture mechanics and macro fracture mechanics.

On the abscissa axes $0'$ I", $0'$ I" and $0_1$ I, they are shown with stress $\sigma$ and the strain $\varepsilon$ parameters as variable, and on the abscissa axes $O$ I' there is a fatigue limit $\sigma_{-1}$ at point b. On the abscissa axes $0_2$ II, it is represented with the short crack stress intensity factor range $\Delta H$ or the short crack strain intensity factor $\Delta I$ as variable, and here there is to be been defined as the yield threshold stress intensity factor $\Delta K_y$ and the yield damage threshold values $\Delta K'_y$ at point B. On the abscissa axes $O_3$ III, it is shown with long crack stress intensity factor $\Delta K$ range (or $\Delta\delta_t$ ) as variable, that it is also a boundary between short crack and long crack growth behaviors (or between micro damage growth and macro damage growth behaviors), and it is also a boundary of transfer values ( $a_{tr} or D_{tr}$ ) between the first stage and the second stage. On abscissa $0_4$ IV, the point $A_2$ is corresponding to the fatigue strength coefficient $\sigma'_f$ and the critical stress intensity factor values $K_{1c}(K_{2fc})$; the point $C_2$ corresponding to the fatigue ductility coefficient $\varepsilon'_f$ and critical crack tip open displacement value $\delta_c$; the point $F$ corresponding to very-high cycle fatigue strength coefficient $\sigma'_{vhf}$ . On same the abscissa axes $0_4$ IV, there are also the critical values $K'_{1c}(K_{1c}), \delta'_c(\delta_c), J'_{1c}(J_{1c})$ ,etc. to fracture in long crack propagation process.

For ordinate axis, upward direction along the ordinate axis is represented as crack growth rate $da/dN$ or damage growth rate $dD/dN$ in each stage and the whole process. But downward direction, it is represented as life $N_{oi}, N_{oj}$ in each stage and lifetime $\Sigma N$ .

In area between axis $0'$ I" and $0_2$ II it is the fatigue history from un-crack to micro-crack initiation. In area between axes $O_2$ II and $O_3$ III, it is the fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_3 - O'$ on ordinate axis is as the history relating to life $N_{mac}$ from grains size to micro-crack initiation until macro-crack forming; the distance $O_4$ - $O'$ is as the history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

In crack forming stage, the partial coordinate system made up with the upward the ordinate axis $0\ 0_4$ and the abscissa axes $0$ I', $0_1$ I and $0_2$ II is represented to be as relationship between the crack growth rate $dD_1/dN_1$ ( or the short crack growth rate $da_1/dN_1$) and the crack stress factor range $\Delta H_1$ (or damage strain factor range $\Delta I_1$ ). In macro-crack growth stage, the partial coordinate system made up with the ordinate axis $0_3\ 0_4$ and abscissa $0_3$ III ( $0_4$ IV) at same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range $\Delta K$ , $J$ -integral range $\Delta J$ and crack tip displacement range $\Delta\delta_t$ ( $da_2/dN_2$ - $\Delta K$ , $\Delta J$ and $\Delta\delta_t$ ). Inversely the coordinate systems made up with downward ordinate axis $O_4\ O_1$ and abscissa axes $O_4$ IV, $O_3$ III, $O_2$ II, $O_1$ I , and $O$ I' are represented respectively as the relationship between the $\Delta H$-, $\Delta K$- range and the each stage life $N_{oi}, N_{oj}$ and the lifetime $\sum N$ (or between the $\Delta\varepsilon_p$-, $\Delta\delta_t$- range and the life $\sum N$ ).

(2) The physical and geometrical meanings for the directions, the partitions and the linking about relatived curves.

The curve $ABA_1$ shows the varying law as elastic material behaviors or as elastic-plastic material ones under high cycle loading in macro-crack-forming stage (the first stage): the positive direction $ABA_1$ shows the relation between $dD_1/dN_1$ (or $da_1/dN_1$)- $\Delta H$; the inverted $A_1BA$ , between the $\Delta H_1 - N_{oi}$. The curve $CBC_1$ shows the varying law of elastic-plastic material behaviors, it is under low-cycle loading at macro-crack forming stage: the positive direction $CBC_1$ shows the relation between $da_1/dN_1$- $\Delta I_1$; inverted $C_1BC$, it shows the relation between the $\Delta\varepsilon_p - N_{oi}$.

And the curve $A_1A_2$ in crack growth stage (the second stage) is showed as under high cycle loading: the positive direction $A_1A_2$, shows the $da_2/dN_2$- $\Delta K$ ($\Delta J$); the inverted $A_2A_1$, shows between the $\Delta K_2$, $\Delta J - N_{oj}$. The $C_1C_2$ shows: the positive direction relation between the $da_2/dN_2$- $\Delta\delta_t$ under low-cycle loading, the inverted $C_2C_1$, between $\Delta\delta_t$ ($\Delta J$)- $N_{oj}$. By the way, the curve abcd is the very-high cycle fatigue one corresponded to stress below fatigue limit.

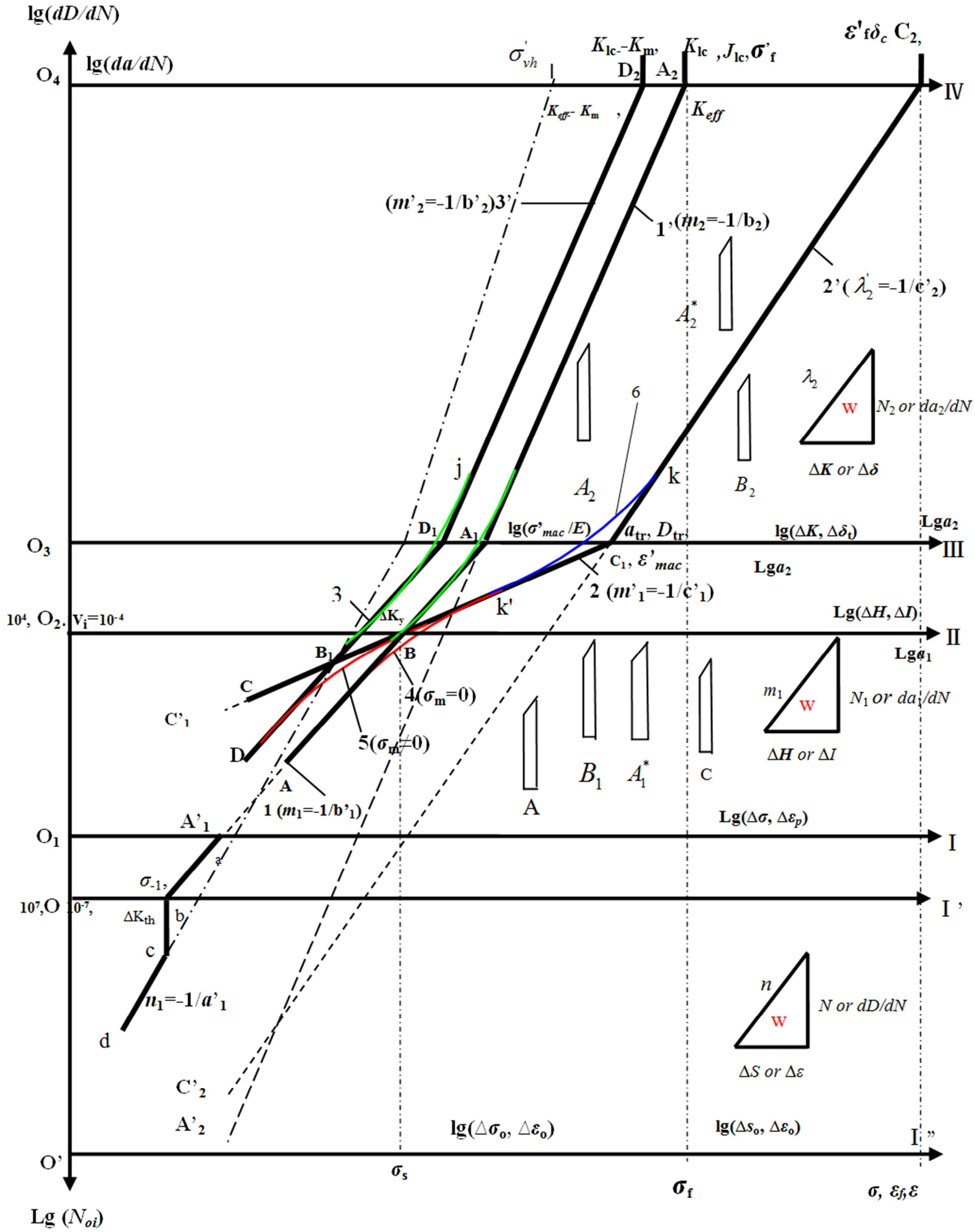


*Figure 1.* *Comprehensive figure of material behaviors (or called calculating figure of material behaviors, or called bidirectional combined coordinate system*

*and simplified schematic curves in the whole process) [1-3].*

It should point that the curve $AA_1A_2$ ( 1-1') is depicted as the damage (crack) growth rate curve in whole process under symmetrical and high cycle loading (i.e. zero mean stress); the curve $DD_1D_2$ (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress). The curves $dcbaBA_1A_2$ and $dcaB_1FG$ are depicted as the damage (crack) growth rate curve in whole process under very high cycle loading. The curve $CC_1C_2$ (2-2') is depicted as the rate curve under low cycle loading. Inverse, the curve $A_2A_1A$ is depicted as the lifetime curve under symmetrical cycle loading, the curve $D_2D_1D$, as the lifetime curve under unsymmetrical cycle loading. The curve $C_2C_1C$ is depicted as the lifetime curve under low cycle loading. The curves $A_2A_1Babcd$ and $GFB_1acd$ are depicted as the lifetime ones in whole process. And should yet point that the calculating figure 1 of materials behaviors may be a complement as a basis that it is to design and calculate for different structures and materials under different loading conditions, and it is also a tool and bridge, that is to communicate and link the traditional material mechanics and the modern mechanics.

# 4. Damage Growth Rate Calculations in Whole Process for Elastic-Plastic Steels

For some elastic-plastic steels of pre-existed flaw, about some driving force and life's calculation equations for crack growth processes, for which some models have been proposed in reference[1-6]. Inside this paper, from micro damage to macro damage, it uses a called as "the single stress or strain parameters method" and the multiplication-method of "two-parameters" [7-8] for the crack propagating rates puts up the whole process calculations, that are by means of the stress $\sigma$ and the strain $\varepsilon$ as "genetic element" in first stage or by the stress intensity factor $K_1$ and the crack tip open displacement range $\delta_t$ as "genetic element" in the second stage, to establish various calculable models for the driving force and the damage growth rate, thereby achieve the calculations of damage growth rate in whole process under low cycle fatigue loading.

## 4.1. The Calculations for Micro Damage Growth Process

### 4.1.1. The Single Stress or Strain Parameter Method

Under the work stress is more than yield stress $\sigma > \sigma_s (= \sigma_y)$ or low cycle fatigue condition, the micro damage growth rate equation corresponded to positive direction curve $CC_1$ in fig.1, here to adopt the strain range $\Delta\varepsilon_p$ to express that is as following form

$$dD_1 / dN_1 = B_1 (\Delta I)^{m'_1} (mm/cycle) \tag{1}$$

Here

$$I'_1 = (\varepsilon_p)^{m_1'} \cdot D_1, (\%^{m_1'} \cdot damage-nuit-namber) \tag{2}$$

$$\Delta I'_1 = (\Delta\varepsilon_p)^{m_1'} \cdot D_1, (\%^{m_1'} \cdot damage-nuit-namber) \tag{3}$$

Here the damage variable $D_1$ (or below $D_2$ and $D$ ) is a non-dimensional value, it is equivalent to short crack $a_1$ discussed as reference[4-5]. Here must put up conversion for dimensions and units, and must be defined in 1mm (1 millimeter) of crack length equivalent to one of damage-unite (1 damage unit), in 1m (1 meter) equivalent to 1000 of damage-unit (1000 damage units). The $I'_1$ is defined as the damage strain factor, that is driving force of damage growth under monotonous loading; $\Delta I'_1$ is defined as the damage strain factor range, that is driving force of damage growth under fatigue loading, their units are "$\%^{m_1'} \cdot damage-nuit-namber$", in practice it is also a non-dimensional value. $\varepsilon'_f$ is a fatigue ductility factor, $m'_1$ is fatigue ductility exponent, $m'_1 = -1/c_1'$, $c_1'$ just is also a fatigue ductility exponent under low cycle fatigue. The $B_1'$ is defined as the calculable comprehensive material constants, its physical meaning is a concept of power, is a maximal increment value to give out energy for damage growth in one cycle before failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1) that is a projection of corresponding to curve 2 on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent $m'_1$ of the below formula (4). So the $B_1'$ is a calculable comprehensive material constant as follow,

$$B_1' = 2[2\varepsilon'_f]^{-m'_1} \times (v_{eff}')^{-1}, (\%)^{m_1'} \times damage-unit-number / cycle \tag{4}$$

Here

$$v'_{eff} = \frac{\ln(D_{1fc} / D_0)}{N_{1fc} - N_{01}} = \frac{[\ln(D_{1f}/D_0) - \ln D_1/D_{01})]}{N_{1fc} - N_{01}}, (damage-unit-number/\text{cycle}) \tag{5}$$

or

$$v'_{eff} = \frac{D_{1fc}\ln(1/1\text{-}\psi)}{N_{1fc} - N_{01}}, (damage-unit-number/\text{cycle}) \quad (6)$$

The $v'_{eff}$ in eqn (4-6) is defined as an effective damage rate correction factor in first stage, its physical meaning is the effective damage rate to cause whole failure of specimen material in a cycle, its unit is the $damage-unit-number/cycle$. $\psi$ is a reduction of area. $D_0$ is the pre-micro-damage value which has no effect on fatigue damage under prior cycle loading [9]. $D_{01}$ is an initial damage value, $D_{1fc}$ is a critical damage value before failure, $N_{01}$ is initial life in first stage, $N_{01} = 0$ ; $N_{1fc}$ is failure life, $N_{1fc} = 1$. Such, its final expansion equation for eqn. (1) is as following form,

$$dD_1/dN_1 = 2[2\varepsilon'_f]^{-m'_1} \times (\Delta\varepsilon_p)^{m'_1} \cdot D_1/v_{eff}(damage-unit-number/cycle) \quad (7)$$

If the materials occur strain hardening, and want via the stress $\sigma$ to express it, due to plastic strain occur cyclic hysteresis loop effect, then the damage growth rate equation corresponded to positive direction curve $CC_1$ in Fig1 should be

$$dD_1/dN_1 = A_1(\Delta H'_1/2)^{m_1}, (damage-unit-number/cycle), (\sigma > \sigma_s) \quad (8)$$

Where

$$H'_1 = \sigma \cdot D_1^{1/m_1} \quad (9)$$

$$\Delta H'_1 = \Delta\sigma \cdot D_1^{1/m_1} \quad (10)$$

$H'_1$ is defined as the damage stress factor, the $\Delta H'_1/2$ is the damage stress factor amplitude. Same, that $H'_1$ is a driving force of damage evolving under monotonous loading, and the $\Delta H'_1$ is a driving force of under fatigue loading. Its physical and geometrical meanings of the $A'_1$ are similar to above the $B'_1$. $A'_1$ is also calculable comprehensive material constant, for $\sigma_m = 0$, it is as below

$$A'_1 = 2(2\sigma'_f)^{-m_1}(v_{eff})^{-1}, (\sigma_m = 0) \quad (11)$$

But if $\sigma_m \neq 0$, here for the eqn (8) to adopt the correctional method for mean stress by in reference [10] as follow

$$A'_1 = 2[2\sigma'_f(1 - \sigma_m/\sigma_f)]^{-m_1}(v_{eff})^{-1}, (\sigma_m \neq 0) \quad (12)$$

Or

$$A'_1 = 2K'^{-m_1}[2\varepsilon'_f(1 - \sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1} \quad (\sigma_m \neq 0) \quad (13)$$

Where the $\sigma'_f$ is a fatigue strength coefficient, $K'$ is a cyclic strength coefficient. $m_1 = -1/b'_1$, $m_1$ and $b'_1$ are the fatigue strength exponent. $m_1 = -1/c'_1 \times n'$, $n' = b'_1/c'_1$, $n'$ is a strain hardening exponent. So that final expansion equation for (8) is as below form,

$$dD_1/dN_1 = 2(2\sigma'_f)^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot D_1/v_{eff}, (damage-unit-number/cycle), (\sigma > \sigma_s, \sigma_m = 0) \quad (14)$$

$$dD_1/dN_1 = 2[2\sigma'_f(1 - \sigma_m/\sigma_f)]^{-m_1}(0.5\Delta\sigma)^{m_1} \cdot D_1/v_{eff}, (damage-unit-number/cycle), (\sigma_m \neq 0) \quad (15)$$

If to take formula (13) to replace $A'_1$ into eqn. (8), its final damage rate expansion equation is as below forming

$$dD_1/dN_1 = 2K'^{-m_1}[2\varepsilon'_f(1 - \sigma_m/\sigma_f)]^{1/c'}(0.5\Delta\sigma)^{m_1} \cdot D_1/v_{eff}, (damage-unit-number/cycle), (\sigma_m \neq 0) \quad (16)$$

Here, for $\sigma >> \sigma_s$, influence of mean stress in eqn (15-16) can be ignored.

### 4.1.2. The Two Parameter Multiplication Method

Same, under $\sigma > \sigma_s$ condition, if to adopt the multiplication method of two parameters to express the micro-damage rate equation corresponded to positive direction curve $CC_1$, it is as following

$$dD_1/dN_1 = A'^{*}_1(0.25\Delta Q'_1)^{\frac{m_1 m'_1}{m_1 + m'_1}}, (damage-unit-number/cycle) \quad (17\text{-}1)$$

or

$$dD_1 / dN_1 = A'^{*}_1 (0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times a_1,(damage-unit-number/cycle) \quad (17\text{-}2)$$

Where the $Q'_1$ is defined as the damage $Q'_1$ -factor of two-parameter, the $\Delta Q'_1$ is defined the damage $Q'_1$ -factor range of two-parameter.

$$Q'_1 = (\varepsilon\cdot\sigma)\ D_1^{1/\frac{m_1 m'_1}{m_1+m'_1}} \quad (18)$$

$$\Delta Q'_1 = (\Delta\varepsilon\cdot\Delta\sigma)\ D_1^{1/\frac{m_1 m'_1}{m_1+m'_1}} \quad (19)$$

$$A'^{*}_1 = 2[4(\sigma'_f\,\varepsilon'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1}, (\text{MPa}^{\frac{m_1 m'_1}{m_1+m'_1}} damage-unit-number/cycle),\ (\sigma_m = 0) \quad (20)$$

$$A'^{*}_1 = 2[4(\sigma'_f\,\varepsilon'_f)(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (v_{eff})^{-1}, (\text{MPa}^{\frac{m_1 m'_1}{m_1+m'_1}} damage-unit-number/cycle)(\sigma_m \neq 0) \quad (21)$$

Same, the $Q'_1$ in (18) is driving force of micro-damage under monotonic loading, and the $\Delta Q'_1$ in (19) is driving force under fatigue loading. It should be point that, the parameter $A'^{*}_1$ in (17) is also a calculable comprehensive material constant. Its physical and geometrical meaning of the $A'^{*}_1$ is similar to above the $A'_1$. And its slope of the micro trapezium bevel edge just is corresponding to the exponent $m_1 m'/(m_1 + m'_1)$ of the formula (20-21). By the way, here is also to adopt those material constants $\sigma'_f, b'_1, \varepsilon'_f, c'_1$ as "genes" in the fatigue damage subject [3]. Therefore, for the eqn (17), its final expansion equation corresponded to curve 2' $(CC_1)$ (Fig 1.) is as below form:

$$dD_1 / dN_1 = 2(4\sigma'_f\,\varepsilon'_f)^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times D_1/v_{eff}, (damage-unit-number/cycle), (\sigma_m = 0) \quad (22)$$

$$dD_1 / dN_1 = 2[4(\sigma'_f\,\varepsilon'_f)(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}} \times D_1/v_{eff}, (damage-unit-number/cycle), (\sigma_m \neq 0) \quad (23)$$

Where, influence of mean stress in eqn (23) can also be ignored. But it must point that the total strain range $\Delta\varepsilon$ in eqn (22-23) should be calculated by Masing law as following eqn.[11]

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \quad (24)$$

### 4.2. The Calculations for Mocro Damage Growth Process

#### 4.2.1. The Single Stress Parameter Method

Under $\sigma > \sigma_s$ condition, due to the material behavior comes into the macro damage growth stage, the exponent in macro damage growth rate $dD_2 / dN_2$ equation also shows change from $m'_1$ to $\lambda_2$; and due to it occurs cyclic hysteresis loop effect, its damage rate model corresponded to positive direction curve $C_1C_2$ in figure 1 is as below form

$$dD_2 / dN_2 = B'_2\left[y_2(a/b)\Delta\delta'_t/2\right]^{\lambda_2}, (damage-unit-number/cycle) \quad (25)$$

Where

$$\delta'_t = 0.5\pi\times\sigma_s\times D_2(\sigma/\sigma_s + 1)/E, \quad (26)$$

$$\Delta\delta'_t = 0.5\pi\times\sigma_s\times D_2(\Delta\sigma/2\sigma_s + 1)/E, \quad (27)$$

Where $\delta'_t$ is the damage crack tip open displacement, it is equivalent to the crack tip open displacement $\delta_t$ in [12], but both units are different, here is a dimensionless quantity. $\Delta\delta'_t$ is a damage-crack tip open displacement range. The $y_2(a/b)$ is a correction factor [13] related to long crack form and structure size. Here should note the $B'_2$ is also a calculable comprehensive material constant,

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{2eff}/E)\right]^{\lambda_2} \times v_{pv}, (\sigma_m = 0) \tag{28-1}$$

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{2eff}/E)\right]^{\lambda_2} \times v_{pv}, (\sigma_m \neq 0) \tag{28-2}$$

$$v'_{pv} = \frac{(D_{2pv} - D_{02})}{N_{2eff} - N_{02}} \approx 3\times10^{-5} \sim 3\times10^{-4} = v^*(damage-unit-number/cycle) \tag{29}$$

Where $\lambda_2$ is defined to be a ductility exponent in macro-damage growth process, $\lambda_2 = -1/c_2^{'}$, $c_2^{'}$ is a fatigue ductility exponent under low cycle in second stage. the $v'_{pv}$ is defined to be the virtual rate, it is an equivalent rate caused in precrack, its dimension is similar to the $v^*$-value in reference [14], but both units are different, where is the "$m/Cycle$", here its unit is $damage-unit-number/cycle$.

So that, the conclusive expansion equations is derived from above mentioned eqn. (25) as follow

For $\sigma_m = 0$,

$$dD_2/dN_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{2eff}/E)\right]^{\lambda_2} v_{pv} \times \left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2} \tag{30}$$
$$(damage-unit-number/cycle)$$

For $\sigma_m \neq 0$, it should be

$$dD_2/dN_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{2eff}/E)\right]^{\lambda_2} v_{pv} \times \left[y_2(a/b)\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2} \tag{31}$$
$$(damage-unit-number/cycle)$$

Where, influence to mean stress can usually ignored in the eqn (31). $D_{2eff}$ is an effective damage value, it can calculate from effective damage crack tip opening displacement $\delta'_{2eff}$

$$D_{2eff} = \frac{E\times\delta'_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s+1)}, (damage-unit-number) \tag{32}$$

And

$$\delta'_{2eff} = (0.25\sim0.4)\delta'_c, (damage-unit-number) \tag{33}$$

Here the $\delta'_c$ is critical damage crack tip displacement, it is equivalent the critical crack tip displacement $\delta_c$ in fracture mechanics, both is only on the unit to be different. So the $D_{2eff}$ in (30-31) can be converted and calculated out by $\delta_c$-value in "1mm" value equivalent to "1 damage-unit" by means of equations (32-33). It must be point that the life units in eqns (25,30-31) are all cyclic number.

### 4.2.2. The Two Parameter Multiplication Method

In the multiplication method of two parameters to calculate the damage growth rate in second stage, it can yet use two kinds of methods: the $Q'_2$-factor method and the $\sigma$-stress method.

1) $Q'_2$-factor method

To use $Q'_2$-factor method calculating the macro damage growth rate, here its effective models corresponded to positive direction curve $C_1C_2$ in figure 1 is as below form

$$dD_2/N_2 = B^{*}_2 \times (0.25y_2(a/b)\Delta Q'_2)^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}}, (damage-unit-number) \tag{34}$$

Where

$$Q'_2 = y_2(a/b)K_1\delta_t, (MPa\cdot\sqrt{1000 damage-unit}\cdot damage-unit) \tag{35}$$

$$\Delta Q'_2 = y_2(a/b)(\Delta K_2\cdot\Delta\delta_t), (MPa\cdot\sqrt{1000 damage-unit}\cdot damage-unit) \tag{36}$$

$$K'_2 = K'_1 = \sigma\sqrt{\pi D_2}, (MPa\cdot\sqrt{1000 damage-unit}) \tag{37}$$

The $Q'_2$-factor and $\Delta Q'_2$ are defined as macro damage driving force, which are respectively under monotonous and repeated loading, their units are all "$(MPa\cdot\sqrt{1000 damage-unit}\cdot damage-unit$". The $K'_1$ is defined as damage stress intensity factor, its unit is also "$MPa\cdot\sqrt{1000 damage-unit}$, it is equivalent to stress intensity factor $K_1$. $B'^*_2$ is also calculable comprehensive material constant, on exponent as compared with above eqn (20) and (21) that is not different.

$$B'^*_2 = 2[4(K'_{2fc}\delta'_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \times v'_{pv}, (\sigma_m = 0), (MPa^{\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \cdot damage - unit - number / cycle \tag{38-1}$$

$$B'^*_2 = 2[4K'_{2fc}\delta'_{2fc}(1 - K'_{2m}/K'_{2fc})]^{-\frac{m_2\lambda'_2}{m_2+\lambda'_2}} \times v'_{pv}, (\sigma_m \neq 0), \tag{38-2}$$

Note, where $m_2$ is an linear elastic exponent in macro damage growth process, $m_2 = -1/b'_2$; And $\lambda_2$ is a ductility exponent, $\lambda_2 = -1/c'_2$.

So the conclusive damage growth rate expanded equation corresponded to positive direction curve $C_1C_2$ in fig.1 should be

For $\sigma_m = 0$

$$dD_2/N_2 = 2[4K'_{2eff}\delta'_{2eff}]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v'_{pv} \times [0.25 y_2(a/b)\Delta K'_2 \cdot \Delta\delta'_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \ (\sigma_m = 0) \tag{39-1}$$

For $\sigma_m \neq 0$,

$$dD_2/N_2 = 2[4K'_{2eff}\delta'_{2eff}(1 - K'_{2m}/K'_{2fc})]^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v'_{pv} \times [0.25 y_2(a/b)\Delta K'_2 \cdot \Delta\delta'_t]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \quad (\sigma_m \neq 0) \tag{39-2}$$

In reference [15-16] refer to the effective stress intensity factor in fracture mechanics, same, here there is also an effective value $K'_{2eff}$ to propose as follow,

$$K'_{2eff} \approx (0.25 - 0.4)K'_{2fc} \tag{40-1}$$

$$K'_{2fc} = \sigma'_f\sqrt{\pi D_{fc}} \tag{40-2}$$

$$K'_{2eff} = \sigma'_f\sqrt{\pi D_{2eff}} \tag{40-3}$$

Where $K'_{2fc}$, $K'_{2m}$ and $K'_{2eff}$ are respectively the critical damage stress intensity factor, the mean and the effective damage stress intensity factor under fatigue loading in the second stage; The $\delta'_{2eff}$ is the effective damage crack tip open displacement. The $D_{fc}$ in (40-2) is the critical damage value, the $D_{2eff}$ in (40-3) is an effective damage value, for which is obtained and calculated from eqns (32-33), (37) and (40) under fatigue loading, and it should take less value.

2) $\sigma$ -stress method

For the $\Delta Q'_2$ and $B'^*_2$ in eqn (34), if adopt stress calculations, it should all be expressed by the stress $\sigma$, it is as following forms

$$Q'_2 = 0.5\, y_2(a/b)\sigma \cdot \sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s + 1)]/E \tag{41}$$

$$\Delta Q'_2 = 0.5\, y_2(a/b)\Delta\sigma \cdot \sigma_s(\sqrt{\pi D_2})^3(\Delta\sigma/2\sigma_s + 1)]/E \tag{42}$$

For $\sigma = 0$

$$B^*_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s + 1)}{E}(\sqrt{\pi a_{2f}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v_{pv}, \{[MPa(\sqrt{m})^3]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \cdot mm/cycle\}, \tag{43}$$

$$B^*_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s + 1)}{E}(\sqrt{\pi D_{2f}})^3](1 - \sigma_m/\sigma'_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v'_{pv}, \\ \{[MPa(\sqrt{1000 - damage - unit})^3]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \cdot damage - unit/cycle\}, \tag{44}$$

Therefore the damage growth rate equation of corresponded to positive direction curve $C_1C_2$ in fig.1, its conclusive expansion equation is as below form,

For $\sigma_m = 0$,

$$dD_2/N_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s + 1)}{E}(\sqrt{\pi D_{2eff}})^3]\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}} \times v'_{pv} \times \left([y_2(a/b)0.5\,\Delta\sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\Delta\sigma/2\sigma_s + 1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}, \\ (damage - unit - number/cycle) \tag{45}$$

For $\sigma_m \neq 0$,

$$dD_2/N_2 = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v'_{pv} \times\left\{[y_2(a/b)0.5\,\Delta\sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\Delta\sigma/2\sigma_s+1)]/E\right\}^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \quad (46)$$

## 4.3. Calculations for the Damage Growth Rate in Whole Process

### 4.3.1. The Single Parameter Method

Due to micro damage behaviors and macro damage one there are distinctly different, for availing to the damage growth rate calculation in whole process, author proposes an research result and method: that is how calculating problem for the transition damage value $D_{tr}$ from micro-damage to macro damage growth process: It can be derived to set up equal expression at transition point between both damage growth rate equations. The calculating model is as follow:

$$(dD_1/dN_1)_{D_{01}->D_{tr}} <= dD_{tr}/dN_{tr} =< (dD_2/dN_2)_{D_{tr}->D_{eff}} \quad (47)$$

Here the equation (47) is defined as the damage-growth-rate-linking-equation in whole process, the $dD_{tr}/dN_{tr}$ in (47) is defined as the damage growth rate at transition point.

For $\sigma_m \neq 0$, to select the driving force equations (10) and (27), to select the formula (13) and (28-2) for relative comprehensive material constant $A_1'$ and $B'_2$, for above related parameters are substituted into eqn (47), then to derive its expanded damage-growth-rate-linking-equation for eqn (47) corresponded to positive curve $CC_1C_2$ is as following form (48),

$$\frac{dD_1}{dN_1} = \left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'}\times(v_f\times D_{tr})^{-1}\times(\Delta\sigma/2)^{m_1}\times D_1\right\}_{D_{01}->D_{tr}} <= \frac{dD_{tr}}{dN_{tr}} =<$$

$$\frac{dD_2}{dN_2} = \left\{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{eff}/E)\right]^{\lambda_2}\times v'_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D}{E}\right]^{\lambda_2}\right\}_{D_{tr}->D_{eff}} \quad (48)$$

$$(damage-unit-number/cycle),(\sigma\neq 0)$$

### 4.3.2. The Multiplication Method of two Parameters

For the multiplication method of two parameters, if in $\sigma_m \neq 0$ as example, its expanded damage-growth-rate-linking-equation corresponded to positive curve $CC_1C_2$ is as following form (49),

$$\frac{dD_1}{dN_1} = \left\{2[4\sigma'_f\,\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v'_{eff})^{-1}\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}D_1\right\}_{D_{01}->D_{tr}} <= \frac{dD_{tr}}{dN_{tr}}$$

$$= \frac{dD_2}{dN_2} =< \left\{\begin{array}{l} 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi D_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times \\ \times\left([0.5\,\sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}} \end{array}\right\}_{D_{tr}->D_{eff}}, damage-unit-number/cycle,(\sigma\neq 0) \quad (49)$$

It must point that the calculations for the damage growth rate in whole process should be according to different stress level and loading conditions, to select appropriate calculable equation. And here have to explain that its meaning of the eqns (47-49) are to make a linking relation-formula for the damage growth rate between the first stage and the second stage, in which before the transition-point value $D_{tr}$, its damage growth rate should be calculated by the micro damage growth rate equation; and after the transition-point value $D_{tr}$ it should be calculated by the macro damage growth rate equation. About calculation tools methods, it can be calculated by means of computer for different damage growth values [17-18].

## 5. Calculating Example

### 5.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR, its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, fatigue limit $\sigma_{-1} = 267.2MPa$, reduction of area is $\psi = 0.51$, modulus of elasticity $E = 200000MPa$; Cyclic strength coefficient $K' = 1165\,MPa$, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma'_f = 947.1MPa$, fatigue strength exponent $b'_1 = -0.111$, $m_1 = 9.009$; Fatigue ductility coefficient $\varepsilon'_f = 0.464$, fatigue ductility exponent $c'_1 = -0.5395$, $m'_1 = 1.8536$. Threshold value $\Delta K_{th} = 8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7\text{MPa}\sqrt{\text{m}}$, critical damage stress intensity factor $K_{1c}(K_{2c})$. Its working stress $\sigma_{\max} = 450MPa$, $\sigma_{\min} = 0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b) = 1$. Other computing data are all included in table 1.

***Table 1.*** *Computing data.*

| $K_{1c}, MPa\sqrt{m}$ | $K_{eff}, MPa\sqrt{m}$ | $K_{th}, MPa\sqrt{m}$ | $v_{pv}$ | $m_2$ | $\delta_c, mm$ | $\lambda_2$ | $y_2(a/b)$ | $a_{th}, mm$ |
|---|---|---|---|---|---|---|---|---|
| 92.7 | 28.23 | 8.6 | $2\times10^{-4}$ | 3.91 | 0.18 | 2.9 | 1.0 | 0.07 |

### 5.2. Required Calculation Data

Try to calculate respectively by calculating methods of two kinds as follow requested different data and depicting their curves:

(1) To calculate damage value $D_{tr}$ at the transitional point between two stages;
(2) To calculate the damage growth rate $da_{tr}/dN_{tr}$ at transitional point;
(3) To calculate the damage growth rate $dD/dN$ from micro damage value $D_{01} = 0.02 - damage - unit$ growth to macro-damage value $D = 2damage - unit$;
(4) To calculate the macro damage growth rate $dD/dN$ from $D_{1eff} = 0.2damage - unit$ to effective damage value $D_{2\text{eff}} = 5damage - unit$;
(5) Calculating for damage growth rate $dD/dN$ in the whole process;
(6) To depict the curves of the damage growth rate $dD/dN$, and convert to become the curves of the crack growth rate $da/dN$ in whole process.

### 5.3. Calculating Processes and Methods

#### 5.3.1. Calculations for Relevant Parameters

The concrete calculation methods and processes are as follows:

1) Conversions for Calculating data

***Table 2.*** *Computing data after conversions.*

| $K'_{1c}, MPa\sqrt{1000damage - nuit}$ | $K'_{eff}, MPa\sqrt{1000damage - nuit}$ | $\delta_c$ damage-unit | $\lambda_2$ | $y_2(a/b)$ | $D_{eff}$ damage − unit |
|---|---|---|---|---|---|
| 92.7 | 28.23 | 0.18 | 2.9 | 1.0 | 2 |

2) Calculations for stress range and mean stress:
Stress range calculation:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450(MPa);$$

Mean stress calculation:

$$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (450 - 0)/2 = 225MPa.$$

3) Calculation for effective damage value $D_{eff}$

For the effective damage value $D_{eff}$ in first stage and the second stage, both can be calculated respectively, and can take smaller one of both. According to formulas (32), calculation for effective damage $D_{2eff}$ in second stage is as follow,

$$D_{2eff} = \frac{E \times \delta'_{eff}}{\pi\sigma_s(\sigma_f/\sigma_s + 1)} = \frac{200000 \times 0.25 \times 0.18}{\pi 361(947.1/361 + 1)} = 2.1(damage - unit),$$

Take $D_{eff} = 2.0mm$, here for $D_{1eff}$ in first stage to take same value by the second stage,

$$D_{1eff} = D_{2eff} = 2damage - unit.$$

4) According to formulas (6), to calculate correction coefficient $v'_{eff}$ in first stage:

$$v'_{eff} = D_{eff}\ln[1/(1-\psi)] = 2\times\ln[1/(1-0.51)] = 1.43, (\text{damage - nuit}/cycle).$$

5) By eqn (29), to select virtual rate $v'_{pv}$ in second stage, here take:

$$v_{pv} = \frac{D_{2eff} - D_{02}}{N_{2fc} - N_{02}} \approx 2.0\times10^{-4}(damage-unit/cycle),\ \ (N_{2fc}=1,\ \ N_{02}=0).$$

Here by means of two kinds of methods to calculate respectively as below:

### 5.3.2. The Single Stress Parameter Method

(1) To calculate damage value $D_{tr}$ at the transitional point between two stages

a) By the damage growth-rate-linking formulas (47, 48-1), to select relevant equation for micro damage growth rate calculating:

At first, calculation for comprehensive material constant $A'_1$ by eqn (13)

$$A_1 = 2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'}\times(D_{1eff}\times v_f)^{-1} = 2\times1165^{-9.01}\times[2\times0.464(1-225/947.1)]^{1/-0.5395}(2\times0.713)^{-1}$$
$$= 6.28\times10^{-28},(MPa^{m_1}\sqrt{damage-unit})^{-m_1}\times damage-unit/cycle$$

Here select the rate equation (8, 15), and for damage growth rate in first stage to simplify calculations as follow form,

$$dD_1/dN_1 = A'_1\times(\Delta\sigma/2)^{m_1}\times D_1 = 3.193\times10^{-28}\times(450/2)^{9.01}\times D_1 = 6.28\times10^{-28}\times1.56\times10^{21}\times D_1 = 9.8\times10^{-7}\times D_1$$

Still by the rate-link-formulas (48-1), calculating for macro damage growth rate in second stage:

Calculation for comprehensive material constant $B'_2$ by eqn (28)

$$\mathrm{B}'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{eff}/E)\right]^{-\lambda_2}\times v'_{pv} = 2\left[2(3.1416\times361(947.1/361+1)(1-225/947.1)\times2/200000)\right]^{-2.9}$$
$$\times2\times10^{-4} = 9.1988,\ (\mathrm{damage-unit})^{-\lambda_2}\times\mathrm{damge-unit/cycle}$$

Select the rate equation (31), to calculate the damage growth rate in second stage, and to simplify calculation equation as follow form,

$$dD_2/dN_2 = B'_2\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2} = 9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)D_2}{E}\right]^{2.9} = 9.1988\times1.6698\times10^{-7}D_2^{2.9}$$
$$= 1.5384\times10^{-6}D_2^{2.9}(damage-unit/cycle)$$

2) Calculation for damage value $D_{tr}$ at transitional point:

According to the equations (47) and (48-1), to do calculation for damage value $D_{tr}$ at the transitional point; then, to take brief damage growth rate-linking-calculating-formulas as follow form,

$$6.28\times10^{-28}\times1.56\times10^{21}\times D_{tr} = 9.1988\times1.6698\times10^{-7}\times D_{tr}^{2.9}$$

$$D_{tr} = (0.638)^{\frac{1}{1.9}} = (0.638)^{0.5263} = 0.789(damage-unit)$$

So to obtain the transitional point damage value=0.789 $damage-unit$, it is equivalent to crack size $0.789mm$.

(2) To calculate the damage growth rate at transitional point $D_{tr}$

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = 9.8\times10^{-7}D_1 = 9.8\times10^{-7}\times0.789 = 7.74\times10^{-7}(damage-unit/cycle)$$

$$dD_2/dN_2 = dD_{tr}/dN_{tr} = 1.5384\times10^{-6}D_{tr}^{2.9} = 1.5384\times10^{-6}\times(0.789)^{2.9} = 7.74\times10^{-7}(damage-unit/cycle)$$

Here it can be seen, the damage growth rate at the transition point is same, it is all equal $7.74\times10^{-7}(damage-unit/cycle)$, that is equivalent to crack growth rate $7.74\times10^{-7}mm/cycle$.

(3) Calculations for the damage growth rates $dD/dN$ in whole process

a) To select eqn (48-1), the $dD/dN$ from micro-damage value $D_{01}=0.02damage-unit$ to transitional point $D_{tr}=0.789damage-unit$, again to long-crack $D_{eff}=5damage-unit$ is as follow,

$$\frac{dD_1}{dN_1}=\left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'}\times(v_f\times a_{tr})^{-1}\times(\Delta\sigma/2)^{m_1}\times D_1\right\}_{D_{01}->D_{tr}}<=\frac{dD_{tr}}{dN_{tr}}=<$$

$$\frac{dD_2}{dN_2}=\left\{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)a_{eff}/E)\right]^{-\lambda_2}\times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2}\right\}_{D_{tr}->D_{eff}},(mm/cycle),(\sigma\neq 0)$$

1) To put into related data to the micro damage rate equation in the first stage

$$dD_1/dN_1=A_1\times(\Delta\sigma/2)^{m_1}\times D_1=3.193\times10^{-28}\times(450/2)^{9.01}\times D_1=6.28\times10^{-28}\times1.56\times10^{21}\times D_1=9.8\times10^{-7}\times D_1$$

2) To put into related data to the macro damage rate equation in the second stage

$$dD_2/dN_2=B_2\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)D_2}{E}\right]^{\lambda_2}=9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)D_2}{E}\right]^{2.9}=9.1988\times1.6698\times10^{-7}D_2^{2.9}$$
$$=1.5384\times10^{-6}D_2^{2.9}(damage-unit/cycle)$$

3) To put into relevant data to the damage-rate-linking-equation, and to do calculations,

$$\frac{dD_1}{dN_1}=\left\{3.193\times10^{-28}\times(450/2)^{9.01}\times D_1\right\}_{D_{01}->D_{tr}}<=\frac{dD_{tr}}{dN_{tr}}$$
$$=<\frac{dD_2}{dN_2}=\left\{9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)D_2}{E}\right]^{2.9}\right\}_{D_{tr}->D_{eff}},(damage-unit-number/cycle),(\sigma\neq 0)$$

$$\frac{dD_1}{dN_1}=\left\{3.193\times10^{-28}\times(450/2)^{9.01}\times D_1\right\}_{D_{01}->D_{tr}}<=\frac{dD_{tr}}{dN_{tr}}$$
$$=<\frac{dD_2}{dN_2}=\left\{9.1988\times\left[\frac{0.5\pi361(450/(2\times361)+1)D_2}{E}\right]^{2.9}\right\}_{D_{tr}->D_{eff}},(damage-nuit-number/cycle),(\sigma\neq 0)$$

$$\frac{dD_1}{dN_1}=\left\{6.28\times10^{-28}\times1.56\times10^{21}\times D_1\right\}_{D_{01}->D_{tr}}<=\frac{dD_{tr}}{dN_{tr}}$$
$$=<\frac{dD_2}{dN_2}=\left\{9.1988\times1.6698\times10^{-7}D_2^{2.9}\right\}_{D_{tr}->D_{eff}},(damage-unit-number/cycle),(\sigma\neq 0)$$

4) For above formulas, it can derive simplified damage-rate-linking-equation in whole process as follow form,

$$\frac{dD_1}{dN_1}=\left\{9.8\times10^{-7}\times D_1\right\}_{D_{01}->D_{tr}}<=\frac{dD_{tr}}{dN_{tr}}=<\frac{dD_2}{dN_2}=\left\{1.5384\times10^{-6}D_2^{2.9}\right\}_{D_{tr}->D_{eff}},(damage-unit-number/cycle),(\sigma\neq 0)$$

According to above the simplified damage rate-linking-equation, by means of a computer to do the damage growth rate computing from micro damage $D_{01}=0.02damage-unit$ to transitional point $D_{tr}=0.789damage-unit$, again to macro-damage $D_{eff}=5damage-unit$. For the damage growth rate data corresponded to different damage value, Then are all converted to become different crack sizes, and are included in table 2~4.

5) To depict the damage growth rate curves in the whole process

By the data in tables 2-4, the damage growth rate curves for two stages and whole process are depicted respectively in figure 2 and 3.

### 5.3.3. The Two Parameter Multiplication Method

(1) To calculate damage value $D_{tr}$ at the transitional point between two stages

1) By the damage growth-rate-linking formulas (47, 49-1), to select relevant equation for micro damage growth rate calculating:

At first, calculation for comprehensive material constant $A_1^*$ in first stage by eqn (21)

$$A_1^* = 2[4\sigma'_f\,\varepsilon'_f\,(1-\sigma_m/\sigma'_f)]^{-\frac{m_1 m'_1}{m_1+m'_1}} \times (a_{eff}\times v_f)^{-1} = 2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}} \times (2\times0.713)^{-1}$$

$$= 2.216\times10^{-5}(MPa^{\frac{m_1 m'_1}{m_1+m'_1}} mm/cycle)$$

Here select the micro damage growth rate equation (23) in first stage to simplify calculations as follow form,

$$dD_1/dN_1 = A_1^*\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1 m'_1}{m_1+m'_1}}\times D_1 = 2.216\times10^{-5}\times(0.25\times\Delta\varepsilon\cdot\Delta\sigma)^{\frac{m_1 m'_1}{m_1+m'_1}}\times D_{tr} =$$

$$2.216\times10^{-5}\times(0.25\times450\times2.553\times10^{-3})^{\frac{9.009\times1.8536}{9.009+1.8536}}\times D_{tr} = 3.22\times10^{-6}D_{tr},(damage-unit-number/cycle),(\sigma_m\neq0)$$

2) Select the macro damage growth rate equation (46) in second stage, to simplify calculations as follow form,

Same, first calculation for comprehensive material constant $B_2^*$ in second stage by eqn (44)

$$B_2^* = 2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times v_{pv} =$$

$$= 2\{[\frac{947.1\times361(947.1/361+1)}{200000}(\sqrt{\pi\times2})^3](1-225/947.1)\}^{-\frac{3.91\times2.9}{3.91+2.9}}\times2\times10^{-4}$$

$$= 2\{[6.1945\times\pi^{1.5}2^{1.5}]0.7624\}^{-1.665}\times2\times10^{-4} = 2\{74.381\}^{-1.665}\times2\times10^{-4} = 3.0625\times10^{-7},(MPa^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\cdot mm/cycle)$$

Then by the macro damage growth rate equation (46) in second stage to simplify calculations,

$$dD_2/N_2 = B_2^*\times[(\Delta\sigma/2)\cdot0.5\sigma_s(\sqrt{\pi a_{tr}})^3(\Delta\sigma/2\sigma_s+1)/E]^{\frac{m_2\lambda_2}{m_2+\lambda_2}} =$$

$$3.0625\times10^{-7}\times[0.5(450/2)\cdot361(\sqrt{\pi D_{tr}})^3(450/2\times361+1)/E]^{\frac{3.91\times2.9}{3.91+2.9}} = 2.6695\times10^{-6}\times a_{tr}^{2.4975}$$

3) Calculation for transitional value $D_{tr}$ at transitional point

According to the damage growth rate-linking-equations (47) and (49), to use the similar method and the steps mentioned above, to obtain simplified calculation:

$$3.22\times10^{-6}D_{tr} = 2.6695\times10^{-6}\times D_{tr}^{2.4975};$$

$$D_{tr} = 1.2062^{0.6678} = 1.133(damage-unit);$$

So the transitional point damage value $D_{tr} = 1.133(damage-unit)$ between two stages.

(2) Calculations for the damage growth rate at transitional point $D_{tr}$

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = 3.22\times10^{-6}\times D_{tr} = 3.22\times10^{-6}\times1.133 = 3.648\times10^{-6}(damage-unit/cycle)$$

$$dD_2/dN_2 = dD_{tr}/dN_{tr} = 2.6695\times10^{-6}D_{tr}^{2.4975} = 2.6695\times10^{-6}\times1.133^{2.4975} = 3.646\times10^{-6}(damage-unit/cycle)$$

Thus it can be seen, the damage growth rate at the transition point $D_{tr} = 1.113(damage-unit)$ is corresponding, that is $3.646\times10^{-6}$ $damage-unit/cycle$.

(3) Calculations for the damage growth rates in whole process by crack growth sizes

Select the damage growth rate equation (49), the calculations for the damage growth rates $dD/dN$ in whole process from micro crack $D_{01} = 0.02damage-unit$ to transitional point $D_{tr} = 1.113damage-unit$, again to macro damage value $D_{eff} = 5damage-unit$ are as follow,

1) According to the damage growth rate-linking-equations (49-1),

$$\frac{dD_1}{dN_1}=\left\{2[4\sigma'_f\,\varepsilon'_f\,(1-\sigma_m/\sigma'_f)]^{-\frac{m_1m'_1}{m_1+m'_1}}\times(v'_{eff})^{-1}\times(0.25\Delta\sigma\times\Delta\varepsilon)^{\frac{m_1m'_1}{m_1+m'_1}}D\right\}_{D_{01}->D_{tr}} <= \frac{dD_{\mathrm{tr}}}{dN_{\mathrm{tr}}}$$

$$=\frac{dD_2}{dN_2}=<\left\{\begin{array}{l}2\{[\frac{\sigma_{fc}\cdot\sigma_s(\sigma_{fc}/\sigma_s+1)}{E}(\sqrt{\pi a_{2eff}})^3](1-\sigma_m/\sigma_{fc})\}^{-\frac{m_2\lambda_2}{m_2+\lambda_2}}\times\\ \times\left([0.5\ \sigma\cdot\sigma_s(\sqrt{\pi D_2})^3(\sigma/\sigma_s+1)]/E\right)^{\frac{m_2\lambda_2}{m_2+\lambda_2}}\end{array}\right\}_{D_{tr}->D_{eff}},damage-unit/cycle,(\sigma\neq 0)$$

2) In equation (49-1), to input the above relevant data:

$$\frac{dD_1}{dN_1}=\left\{2[4(947.1\times0.464)(1-225/947.1)]^{-\frac{9.009\times1.8536}{9.009+1.8536}}\times(2\times0.7133)^{-1}\times(0.25\times\Delta\varepsilon\cdot450)^{\frac{m_1m'_1}{m_1+m'_1}}D\right\}_{0.02->1.133} <= \frac{dD_{\mathrm{tr}}}{dN}$$

$$=\frac{dD_2}{dN_2}=<\left\{\begin{array}{l}2\{[\frac{947.1\times361(947.1/361+1)}{200000}(\sqrt{\pi\times2})^3](1-225/947.1)\}^{-\frac{3.91\times2.9}{3.91+2.9}}\times2\times10^{-4}\\ [0.5(450/2)\times361(\sqrt{\pi D_2})^3(450/2\times361+1)/200000]^{\frac{3.91\times2.9}{3.91+2.9}}\end{array}\right\}_{1.133->5},damage-unit/cycle$$

3) From above calculations, it can derive simplified the damage growth rate-linking-equations in whole process corresponded to different damage value, in the end to get the simplified rate-linking-equation as follow form,

$$\frac{dD_1}{dN_1}=\left\{3.22\times10^{-6}\times D_1\right\}_{D_{01}->D_{tr}} <= \frac{dD_{\mathrm{tr}}}{dN_{\mathrm{tr}}} =< \frac{dD_2}{dN_2}=\left\{2.6695\times10^{-6}D_2^{2.4975}\right\}_{D_{tr}->D_{eff}},damage-unit/cycle,(\sigma\neq 0)$$

According to above the simplified rate-linking-equation, by means of a computer to do computing from micro damage $D_{01}=0.02damage-unit$ to transitional point $D_{tr}=1.113damage-unit$, again to macro damage $D_{eff}=5damage-unit$.

The result data calculated with the multiplication method of two parameters, then to convert to become crack sizes are included in tables 2-4.

(4) To depict the damage (crack) growth rate curves in the whole process

By the data in tables 2-4, the damage (crack) growth rate curves for two stages and whole process are depicted respectively in figure 2 and 3.

Here comparisons for crack growth rate data of calculating results by two kinds of methods are also included in table 3, 4 and 5.

***Table 3.*** *Comparisons for damage growth rate data in two stages by two kinds of methods.*

| **Data point of number** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|
| Crack size (mm) | 0.02 | 0.04 | 0.1 | 0.2 | 0.4 |
| Rate by single-parameter in first stage | $1.96\times10^{-8}$ | $3.92\times10^{-8}$ | $9.8\times10^{-8}$ | $1.96\times10^{-7}$ | $3.92\times10^{-7}$ |
| Rate by two-parameter in first stage | $6.44\times10^{-8}$ | $1.29\times10^{-7}$ | $3.22\times10^{-7}$ | $6.44\times10^{-7}$ | $1.29\times10^{-6}$ |
| Ratio | 3.25/1 | 3.25/1 | 3.25/1 | 3.25/1 | 3.25/1 |
| Rate by single-parameter in second stage | Invalid section | | | $1.446\times10^{-8}$ | $1.079\times10^{-7}$ |
| Rate by two-parameter in second stage | Invalid section | | | $4.79\times10^{-8}$ | $2.71\times10^{-7}$ |

***Table 4.*** *Comparisons for damage growth rate data in two stages by two kinds of methods.*

| **Data point of number** | **5** | **6** | **7** | **Transition point** | **8-Transition point** |
|---|---|---|---|---|---|
| Crack size (mm) | 0.5 | 0.6 | 0.7 | 0.789 | 1.133 |
| Rate by single-parameter in first stage | $4.95\times10^{-7}$ | $5.88\times10^{-7}$ | $6.869\times10^{-7}$ | $7.732\times10^{-7}$ | $1.11\times10^{-6}$ |
| Rate by two-parameter in first stage | $1.61\times10^{-6}$ | $1.93\times10^{-6}$ | $2.25\times10^{-6}$ | $2.54\times10^{-6}$ | $3.65\times10^{-6}$ |
| Ratio | 3.25/1 | 3.25/1 | 3.25/1 | 3.25/1 | 3.25/1 |

| Data point of number | 5 | 6 | 7 | Transition point | 8-Transition point |
|---|---|---|---|---|---|
| Rate by single-parameter in second stage | $2.06\times10^{-7}$ | $3.497\times10^{-7}$ | $5.468\times10^{-7}$ | $7.732\times10^{-7}$ | $2.21\times10^{-6}$ |
| Rate by two-parameter in second stage | $4.73\times10^{-7}$ | $7.45\times10^{-7}$ | $1.1\times10^{-6}$ | $1.48\times10^{-6}$ | $3.65\times10^{-6}$ |
| Ratio | 3.23/1 | 2.13/1 | 2.01/1 | 1.91/1 | 1.65/1 |

***Table 5.*** *Comparisons for damage growth rate data in two stages by two kinds of methods.*

| Data point of number | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|
| Crack size (mm) | 1.5 | 2.0 | 3.0 | 4 | 5 |
| Rate by single-parameter in first stage | $1.47\times10^{-6}$ | $1.96\times10^{-6}$ | Invalid section | | |
| Rate by two-parameter in first stage | $4.83\times10^{-6}$ | $6.44\times10^{-6}$ | Invalid section | | |
| Ratio | 3.25/1 | 3.25/1 | | | |
| Rate by single-parameter in second stage | $4.986\times10^{-6}$ | $1.148\times10^{-5}$ | $3.72\times10^{-5}$ | $8.57\times10^{-5}$ | $1.64\times10^{-4}$ |
| Rate by two-parameter in second stage | $7.35\times10^{-6}$ | $1.51\times10^{-5}$ | $4.15\times10^{-5}$ | $8.51\times10^{-5}$ | $1.49\times10^{-4}$ |
| Ratio | 1.47/1 | 1.32/1 | 1.12/1 | 0.99/1 | 0.91/1 |

From tables 2~4, it is observed that the micro crack size from 0.02 mm to macro crack 2mm (as the first stage), comparison for result data calculated by the single parameter method and the two-parameter multiplication-method, both ratio is all 3.25/1; Comparison for their result data calculated from the crack size 0.5 mm to 5mm (as the second stage), both ratio is gradually to reduce from 2.3/1 to 0.91/1. These data are shown as so long history in whole crack growth process from micro damage to macro damage, for these calculating results data can all be accepted for two of mothers.

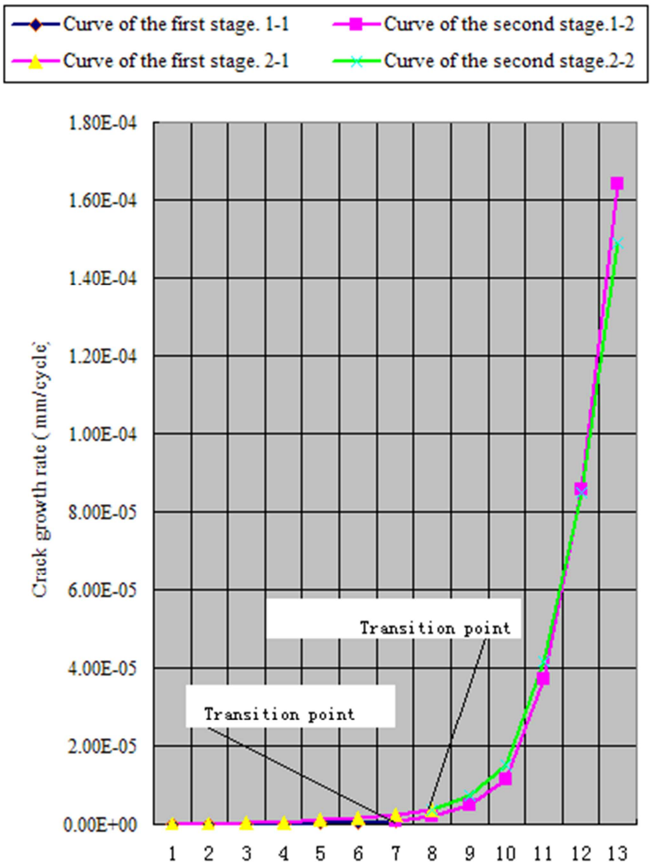


***Figure 2.*** *Comparison of life curves in whole process (in decimal coordinate system).*

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;
(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size $0.789mm$ ).
(D) 2-1---Curve in first stage depicted by two-parameter calculating data;
(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;
(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size $1.113mm$ ).

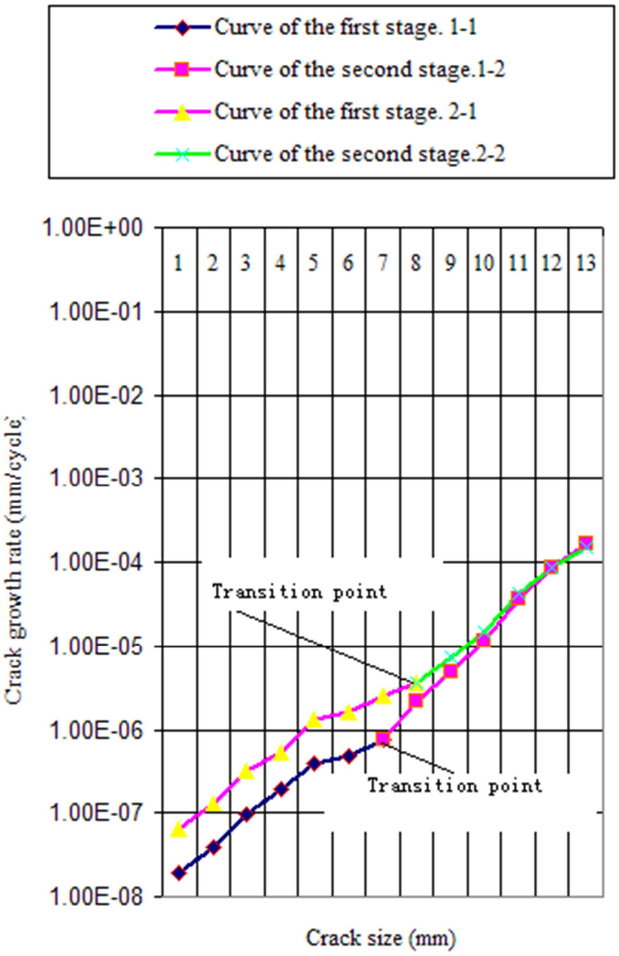


***Figure 3.*** *Comparison of damage growth rate curves in whole process (in logarithmic coordinate system).*

(A) 1-1---Curve in first stage depicted by single-parameter calculating data;
(B) 1-2--- Curve in second stage depicted by single-parameter calculating data;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size $0.789mm$ ).
(D) 2-1---Curve in first stage depicted by two-parameter calculating data;
(E) 2-2--- Curve in second stage depicted by two-parameter calculating data;
(F) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at eighth point (crack size $1.113mm$ ).

## 6. Conclusions

(1) About comparison for calculating methods of two kinds: Looked from the overall trend for the damage growth rate curves, the result data calculated by two methods is basically closer in whole process; especially both crack rate data in second stage is closer. For the single parameter method, its calculation model is simpler; for the two-parameter method, its calculation in whole process is more moderate, but its calculation models are more complex.

(2) About the theory basis of the whole process damage rate model：Although there are the micro fatigue damage behavior and the macro damage behavior to be obvious different, in the micro damage growth to the macro damage growth process it must exist a same damage value at the transition point, and the damage growth rate at this point must be equal. According to this reasoning, with the help of the same value location at transition point as the linking point, thereby to establish the fatigue damage-growth rate-linking-equation in whole process between the first and the second stage, this is just the theory basis of damage rate equation as whole process.

(3) About new cognition for some key material constants: True material constants must show their inherent characters of materials, such as the $\sigma_s$ and $E,\delta,\psi$ etc. in the material mechanics; and for instance the $\sigma_f$ and $\sigma'_f$ ; $\varepsilon_f$, and $\varepsilon'_f$ ; $b_1$ and $b'_1$ ; $c_1$ and $c'_1$ and so on in the fatigue damage mechanics; which could all be checked and obtained from general handbooks; But for some key new material constants $A_1$ , $A_1^*$ and $B_2, B_2^*$ about the damage growth rate equations in the damage mechanics, in practice there are functional relations with other parameters, they are all calculable parameters by means of the relational expressions (4,11-13), (20-21), (28), (38), etc. Therefore for this kind of key parameters

can be defined as the calculable comprehensive materials constants.

(4) About cognitions to the physical and geometrical meanings for key parameters: The parameters $A_1$ , $A_1^*$ in the first stage and $B_2, B_2^*$ in the second stage, their physical meanings are all a concept of power, just are a maximal increment value paying energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

(5) About the calculating methods for the damage-growth-rate-linking-equation in whole process: Calculation for damage growth transition-point value $D_{tr}$ between two stages, it can be calculated to set up equal expression at transition-point between the micro damage growth rate and the macro damage growth rate equation; before the transition point $D_{tr}$ , its damage growth rate should be calculated by the micro damage growth rate equation, after the transition point $D_{tr}$ it should be calculated by the macro damage growth rate equation.

(6) Total conclusion: Based on the traditional material mechanics is a calculable subject, in consideration of the conventional constants there are "the hereditary characters", In view of the relatedness and the transferability between related parameters among each disciplines; And based on above viewpoints and cognitions (1)~(5), then make the modern damage mechanics and the fracture mechanics disciplines become calculable subjects as the traditional material mechanics, that will be to exist possibility.

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