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Strength Calculations on Damage in Whole Process to Elastic-Plastic Materials---The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

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Abstract
The author bases on the principles of similar to the genetic genes in the life sciences, uses the theoretical approach, adopts the conventional material constants, discovers some new constants for some materials of Masing's and elastic-plastic ones which show inherent properties from micro to macro damage; and sets up new computing models to the damage strength in different stages; proposes the calculating criterions which are defined as the damage value and the damage factor; and provides two kinds of the assessment methods for them in whole process. In addition, to supplement the comprehensive figure of the material behaviours; to give yet a detailed calculating example for a safety assessment. This works may be there are practical significances for the decreasing experiments and for make linking and communication among the modern material subject, fracture mechanics and the damage mechanics.

1. Introduction

It is well known that for the traditional materials mechanics, which is a calculable subject and has made valuable contributions for every industrial engineering designs and calculations; but it cannot explain sudden fracture accidents on some structures for which working stresses are less than the original design permitting ones. In that the materials mechanics is only considered for those materials and the structures as the continuous and homogeneous medium to research and calculate, so that in their calculating models mainly there are the stresses and the strains and the relating material constants. Therefore, it can be generally only represented and calculated for those relations between the stresses and the strains about the strength problems. In practice in engineering materials and structures, usually there are various micro- or macro-flows (cracks), so it cannot solve and calculate once existed cracks. But in the fracture mechanics and the damage mechanics, which are just based on the local problems as researched objects for the local defects inside materials, to research the driving forces, the crack propagating (damage) rates and the life predictions in the crack growth process; in their calculating models to contain such calculable parameters as the crack size a or as the damage variable D. Consequently, these subjects can just solve and calculate problems mentioned above. But, nowadays, these latter disciplines are all mainly dependent on tests, and due to the micro-crack and the macro-crack behaviors there are obviously differences, who want to establish their calculating models about the strength, the crack propagation rate and the life predictions of the whole process from the micro-crack to macro-crack growth, which are more difficult and more complicated problems. To pay the manpower and money for experiments are more huge.

The author thinks that in the mechanics and the engineering fields there exists such a scientific law as similar to genetic elements and cloning technology in the life sciences and has used the theoretical approach for similar principles, proposed some calculation models [1-10], recently sequentially discovered some new scientific laws to the Masing's and the elastic-plastic materials, and provides some new calculable models for the crack growth driving force, the calculating criterions and the assessment methods of the strength problems on the damage in the whole process which are from midro to macro damage. This is to try to make the modern fatigue, the damage mechanics and the fracture mechanics gradually become such calculable disciplines as the traditional materials mechanics. That way, it may be there are practical significances for decreasing experiments to stint manpower and funds for promoting and developing engineering and applying it to relevant disciplines.

2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the "Comprehensive figure of materials behaviors" as Figure 1 (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) [10] that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes O' I", O I', O_1 I, O_2 II, O_3 III, O_4 IV and a bidirectional ordinate axis $O'_1 O_4$. For the area between the axes O' I" and O_1 I, it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes O I' and O_2 II, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the O_2 II, the O_3 III and O_4 IV where they are calculatied and applied by the macro-damage mechanics and the macro-fracture mechanic. But for between the axes O_1 I and O_2 II, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

On the abscissa axes O' I" and O_1 I, they are represented with parameters the stress σ and the strain ε as variables. On the abscissa axes O I' there are the fatigue limit σ_{-1} at point "a" $(\sigma_m = 0)$ and "b" $(\sigma_m \neq 0)$ that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axes O_1 I there are points "A" and "D" that just are the locations placed at threshold values to another materials. On the abscissa axes O_1 I and O_2 II that they could all represented as variables with the stress intensity factor range ΔH_1 of short crack, and the strain intensity factor ΔI, and the stress intensity factor range ΔK_1 of long crack. On the other hand, they both are yet represented as variables with the short crack a_1 and the long crack a_2 (or damage D_1 and D_2). And here there are materal constants of two that they are defined as the critical factor K_y of crack-stress-intensity and the critical factor K'_y of the damage-stress-intensity at the first stage, where that are just the transition parameters corresponded to the critical crack size $a_{tr}(=a_{1c})$ or the critical value of damage $D_{tr}(=D_{c1})$, they are just placed at point at the point B $(\sigma_m = 0)$ and at point B$_1$ $(\sigma_m \neq 0)$ corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors; but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level.

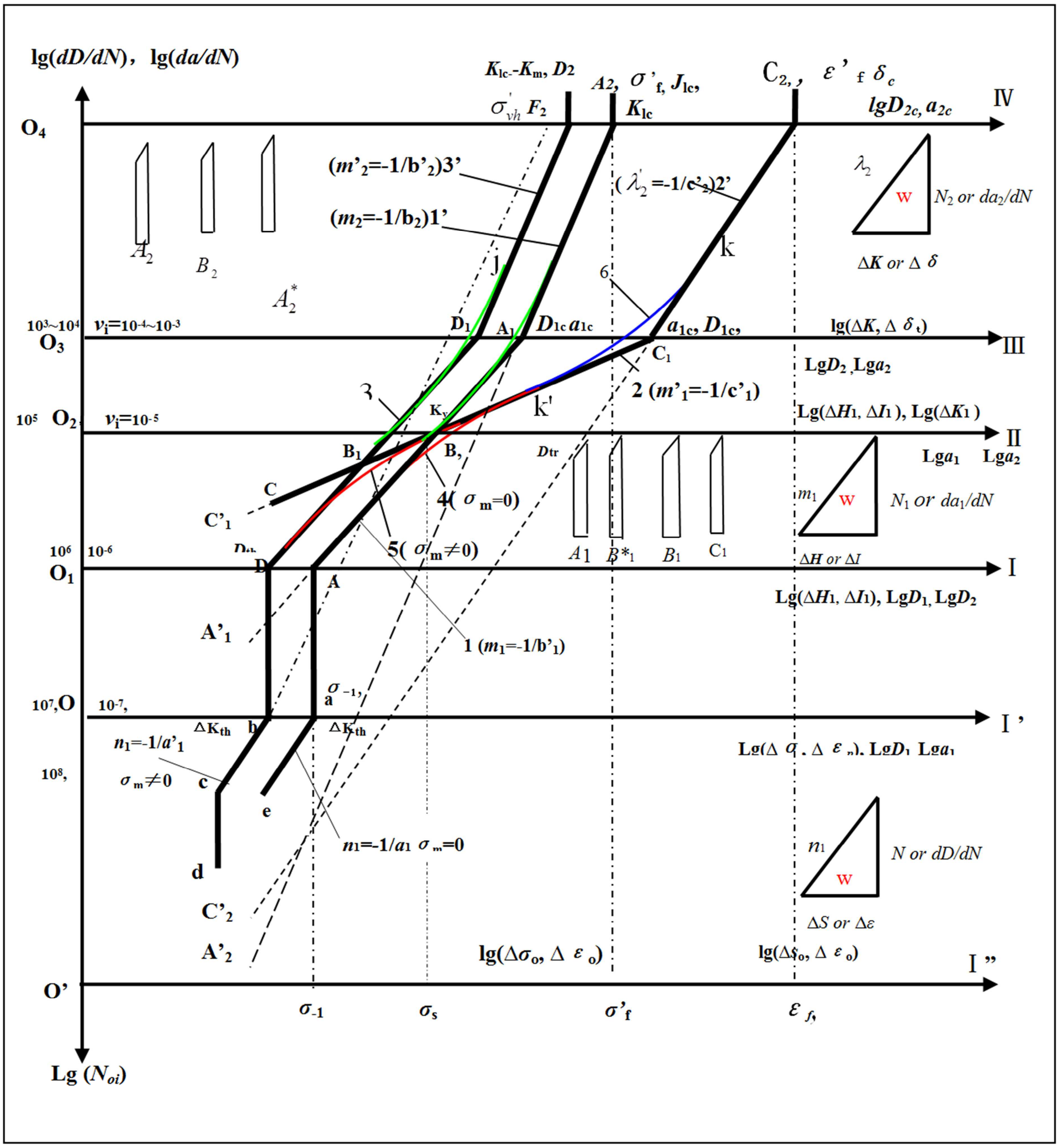

Figure 1. *Comprehensive figure of material behaviors (Or called calculating figure of material behaviors or bidirectional combined coordinate system and simplified schematic curves in the whole process).*

On the abscissa axes O3 Ⅲ, it is represented as variable with the stress intensity factor ΔK_1 (or $\Delta\delta_t$) of long crack; it is a boundary between the first stage and the second stage for some elastic-plastic materials. On this axes O3 Ⅲ there are the critical points at D1, A1, and C1 (D1c, A1c). On abscissa O_4 IV, the point A_2 is corresponding to the fatigue strength coefficient σ'_f, the critical stress intensity factor values $K_{1c}(K_{2fc})$ and the critical values D'_{2c} and a_{2c} for the mean stress $\sigma_m = 0$; the point D_2 is corresponding to the $\sigma_m \neq 0$; the point C_2 corresponding to the fatigue ductility coefficient ε'_f and critical crack tip open displacement value δ_c; the point F corresponding to a very high cycle fatigue strength coefficient σ'_{vhf}. In addition on the same O_4 IV, there are yet another critical values $J'_{1c}(J_{1c})$, etc. in the long crack propagation process.

For an ordinate axis, an upward direction along the ordinate axis is represented as crack growth rate da/dN or damage growth rate dD/dN in each stage and the whole process. But a downward direction is represented as life N_{oi}, N_{oj} in each stage and the whole lifetime ΣN.

In the area between axes O'I" and O_2 Ⅱ, it is the fatigue history from un-crack to micro-crack initiation. In the area between axes O_1 I' and O_2 II, it is the fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_2 - O^{'}$ on ordinate axis is as the history relating to life N_{mac} from grains size to micro-crack initiation until macro-crack forming; the distance O_4 - $O^{'}$ is as the history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

In the crack forming stage, the partial coordinate system made up of the upward and the ordinate axes O O_4 and the abscissa axes O I', O_1 I and O_2 II is represented as the relationship between the crack growth rate dD_1 / dN_1 (or the short crack growth rate da_1 / dN_1) and the crack-stressfactor range ΔH_1 (or the damage strain factor range ΔI_1). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis O_2 O_4 and abscissa O_2 II, O_3 III and O_4 IV at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range ΔK, J -integral range ΔJ and crack tip displacement range $\Delta\delta_t$ (da_2 / dN_2 - ΔK, ΔJ and $\Delta\delta_t$). Inversely, the coordinate systems made up of the downward ordinate axis O_4 O_1 and the abscissa axes O_4 IV, O_3 III, O_2 II, O_1 I, and O I' are represented respectively as the relationship between the ΔH -, ΔK - range and each stage life N_{oi}, N_{oj} and the lifetime $\sum N$ (or between the $\Delta\varepsilon_p$ -, $\Delta\delta_t$ - range and the life $\sum N$).

2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve ABA_1 is represented as the varying laws as the behaviours of the elastic materials or some elastic-plastic ones under high cycle loading in the macro-crack-forming stage (the first stage): positive direction ABA_1 represented as the relations between dD_1 / dN_1 (or da_1 / dN_1)- ΔH ; inverted A_1BA , between the $\Delta H_1 - N_{oi}$. The curve CBC_1 is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction CBC_1 is represented as the relations between da_1 / dN_1 - ΔI_1; inverted C_1BC , the relations between the $\Delta\varepsilon_p - N_{oi}$.

The curve A_1A_2 in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction A_1A_2 showed as da_2 / dN_2 - ΔK (ΔJ); inverted A_2A_1, between the ΔK_2, $\Delta J - N_{oj}$. The C_1C_2 is showed as: the positive, relation between the da_2 / dN_2 - $\Delta\delta_t$ under low-cycle loading, inverted C_2C_1, between $\Delta\delta_t$ (ΔJ)- N_{oj}. By the way, the curves $'Dbcd', (\sigma_m = 0)$ and the $'Aae'$ ($\sigma_m = 0$) are represented as the laws under the very high cycle fatigue.

It should yet point that the curve AA_1A_2 (1-1') is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, $da / dN \le 10^{-6}$); the curve DD_1D_2 (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, ($da / dN \le 10^{-6}$). The curve CC_1C_2 (2-2') is depicted as the rate curve under low cycle loading. The curve $eaABA_1A_2$ is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ($\sigma_m = 0, da / dN < 10^{-7}$), the curves $dcbDD_1D_2$ and $dcbF_2$ are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ($\sigma_m \ne 0, da / dN < 10^{-7}$). Inversely, the curve A_2A_1A is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, $N \le 10^6$), the curve D_2D_1D, as the lifetime curve under unsymmetrical cycle loading $(N \le 10^6)$. The curve C_2C_1C is depicted as the lifetime curve under low cycle loading $(N \le 10^5)$. On the other hand, the curve A_2A_1BAae is as the lifetime one in whole process included very high cycle fatigue ($\sigma_m = 0, N > 10^7$), the curves D_2D_1Dbcd and F_2bcd are all depicted as the lifetime ones in whole process $(\sigma_m \ne 0, N > 10^7)$.

It should also be explained that the comprehensive figure 1 of the materials behaviours may be a complement as a fundamental research; that is a tool to design and calculate for different structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.

3. Strengh Calculations on Damage Under Monotonic Loading

Here the damage variables D for describing the damage growth process that are defined as follows:

1) From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage. If applying the concept of the damage mechanics, it is defined in the micro-damage stage, and it adopted variable D1 called as the micro-damage variable, corresponded to the variable a1 of a short crack that it is corresponding curve AA1 in figure 1;
2) From the macro-crack propagation to the fracture process is defined in the crack growth stage, or defined in the second stage, and here it is also applying the concept of the damage mechanics, it is defined in the macro-damage stage. The damage variable D2 of this stage is called in the macro damage variable, corresponds to the variable a2 of the long crack that it is corresponding curve A1 A2 in figure 1;
3) From micro-damages to full failure of a material, to

adopt the parameter D as the variable in the whole process, it corresponds to the crack variable a in the whole process from short crack to long crack growth until full fracture that it is corresponding curve AA1 A2 in figure 1.

3.1. About Threshold Value on Damage

Under the monotonous loading, the behabirous of damage for a material, it is changed with the loading ways and the stress levels, its damage value shown before the yield stress σ_s can be caculated by following formula,

$$D_1 = \left(\frac{\sigma}{\sigma_{pr}}\right)^{m_1} \times D_{1c}, \quad (1)$$

Where the $\sigma_{pr} \approx \sigma_e$ is a stress value of proportional limit (approximating elastic limit, it can also approximatively be took for definite ratio by the yield stress, for example $\sigma_{pr} = (0.95 \sim 0.97)\sigma_s$, if the data is to lack. The m_1 is only a shown the constant of a property, it is same with the b_1 also a sole one $m_1 = -1/b_1$. Here it should explain that the operative symbol of the constant b1 is a negative value; and the m1 is a positive one.

In the figure 1 it is seen, for the general steels, they are always shown with various characters in the each stages. The author discovers they all have the threshold values D_{th} of the damage in table 1, and only depended on the exponent b_1.

It should point the location of the threshold value D_{th} of damage is at the point A where it is at the intersection one between the straight line AA1 and the abscissa axis O_1I in figure 1. And the threshld D_{th} can be calculable one with as following formula, it should be

$$D_{th} = \left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} \quad (damage-units) \quad (2)$$

Or

$$D_{th} = \left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5-(1/m_1)}} \quad (damage-units) \quad (3)$$

The range of the D_{th} in equation (2-3) is in 0.21~0.275 (damage-units), it is equivalent to in length 0.21~0.275 (mm) of short crack [11].

At the stage happened elastic behaviour for elastic-materials, when the damage value is combined by the D_{th} with the stress σ induced under the monotonic loading, so that to form a model of the driving force that is as below,

$$H'_1 = \sigma \cdot D_{th}^{1/m_1} = \sigma^{m_1} \cdot D_{th} \; [MPa \cdot (damage-units)^{1/m_1}] \quad (4)$$

In the above formula, $m_1 = -1/b_1$; the H'_1 is defined as the stress intensity factor of micro damage. Because the variable D_1 is a dimensionless value, it is equivalent to the length a_1 of short crack, here it must defined in "1mm-length of crack " equivalent to "1-unit-damage-value", in "1m-length of crack" equivalent to "1000-damage units". In an ordinary way, the $\sigma \cdot D_1^{1/m_1}$ may be: the $\sigma \cdot \sqrt[m_1]{D} \le \sigma \cdot \sqrt[m_1]{D_{th}}$ or $\sigma \cdot \sqrt[m_1]{D} \ge \sigma \cdot \sqrt[m_1]{D_{th}}$, for the strength criterions of them are as below,

$$H'_1 = \sigma \cdot \sqrt[m_1]{D_{th}} \le H'_{th} [MPa \cdot (damage-units)^{1/m_1}], \quad (5)$$

Or

$$H'_1 = \sigma \cdot \sqrt[m_1]{D_{th}} \ge H'_{th} [MPa \cdot (damage-units)^{1/m_1}], \quad (6)$$

Where the H'_{th} is defined as the threshold factor of damage. It should point the location of the threshold D_{th} is at the point A where it is at the intersection one between the straight line AA1 and the the abscissa axis O1I in figure 1. If to take the yield stress σ_s to replace the σ in the equaton (4), it is come as following form

$$H'_{th-y} = \sigma_s \cdot \sqrt[m_1]{D_{th}} \;, [MPa \cdot (damage-units)^{1/m_1}], \quad (7)$$

Then the H'_{th-y} is defined as the threshold factor corresponded to the yield stress, so that the H'_{th-y} must be the only a constant showing a material property; And the damage of a material is sure to grow if the $H'_1 \ge H'_{th-y}$.

Table 1. *Data of threshold values on damage.*

Materials [12-13]	Heat- treatment	σ_b, MPa	σ_s, MPa	b_1	m'_1	D_{th}, damage − units
①30CrMnSiA	Hardeningand Tempering	1177	1104.5	-0.0859	11.64	0.251
①LC4CS	CS	613.9	570.8	-0.0727	13.76	0.262
2024-T3 Aluminum		469	379	-0.124	8.06	0.218
②QT800-2	Normalization	913	584.3	-0.083	12.05	0.253
4340	Quenching and tempering	1241	1172	-0.076	13.16	0.259
④40Cr	in oil-quench 850°C, temper 560°C, air cooling	940	805	0.12	8.33	0.222
1005-1009	Hot rolled sheet	345	262	-0.109	9.174	0.231
1005-1009	Cold-draw sheet	414	400	-0.073	13.7	0.262
Steel: 1020	Hot rolled sheet	441	262	-0.12	8.33	0.222
RQC-100	Hot rolled sheet	931	883	-0.07	14.3	0.264
9262	anneal	924	455	-0.071	14.1	0.263

3.2. Strength Calculation on Damage at the First Stage

The material property had discovered by Masing, that as is well known [14]. For some Masing's, elastic-plastic and happened strain hardening materials, which are corresponded to the curve 1(AB) in the first stage between abscissa axis O1 I and the O2 II in figure 1. If the stress inside a material is gradually loaded to the yield stress (at point B on abscissa axis O2 II) or over this level to the A1 (at point A1 on abscissa axis O3 III), then its damage will also increasingly grow, here can set up a criterion of the damage strength for it, that is as below form,

$$H'_1 = \sigma \cdot D_1^{1/m_1} \le [D_1] = H'_{1c} \ / n_1, (MPa \cdot damage - units^{1/m_1}) \quad (8)$$

$$H'_{1c} = \sigma_s \times \sqrt[m_1]{D_{1c}}, (MPa \cdot damage - units^{1/m_1}) \quad (9)$$

The H'_{1c} in (9) is defined as a critical value of the stress intensity factor on damage, the H'_{1c} is a value corresponded to the critical value K_y and the transition value Dtr of damage, also is the boundary between the short crack and the long crack. Their locations are at points B on abscissa axis O2-II (in Fig. 1). But for some cast iron, low strength steels and brittle materials which could be happened to fracture when their stresses are loaded to this level.

As is well know the mathematic model to describe a long crack in fracture mechanics, that is to adopt these "genes" σ and π and crack variable a , thereby to make the stress intensity factor $K_1 = \sigma\sqrt{\pi a}$ [15-17]; Here can take the variable D of macro damage in the name of macro damage mechanics to displace the crack size a inside the K_1 , then it can still derive the equation of driving force for the describing behavior of macro damage, that is as following form,

$$K'_1 = \sigma \times \sqrt{\pi D_1}, (MPa \cdot \sqrt{1000\text{-}damage\text{-}units}) \quad (10)$$

Where the K'_1 is a stress intensity factor of the macro damage, it is equivalent to H'_1 , but their dimensions and units are differences at this same point. Here is sure to explain, the area between the abscissa axis O1-I and the O2-II in Fig. 1, where it is corresponded to the D -values from the threshold D_{th} to D_{1c} （$D_{th} \le D_1 \le D_{tr} = D_{\mathrm{mac}} = D_{1c}$）, in the area it can set up two kinds of the mathematic models called the stress factors. In addition to above equations (8-9) can be applied; in theory below the mathematic models (10-14) are still suitable in the first stage.

For some Masing's materials, their damage values D_{tr} at transition points between the elastic and the plastic strain, it is also be calculable, and can be calculated by means of the following equation,

$$D_{tr} = \left(\sigma_s^{(1-n')\ /n'} \times \frac{E \times \pi^{1/2 \times n'}}{K^{1/n'}} \right)^{\frac{2m_1 n_1}{2n_1 - m_1}}, \ (damage - units) \quad (11)$$

Where K is a strength coefficient under monotonic loading, the n' is a exponent happened strain hardening. Then the model of driving force at this point should be as follow

$$K'_y = \sigma_s \cdot \sqrt{\pi D_{tr}}, (MPa \cdot \sqrt{damage - unit - number}) \quad (12)$$

Where the K'_y is defined as the stress factor of damage that is relevant to the damage value D_{tr} at transition point, just is corresponding to size $a_{mac} (\approx a_{tr})$ of forming macro crack, it is the very at point B related yield stress σ_s on abscissa axis O2-II in fig. 1, also a the critical value D_{1c} in the first stage, where it is on boundary between the first stage and the second stage.

Here it need yet explain, this factor K'_y should theoretically be equivalent to above mentioned the H'_{1c} in first stage, although the dimensions and units between them are differences. Therefore the strength criterion of its damage should be calculated as following form,

$$K'_1 = y(a/b) \cdot \sigma \cdot \sqrt{\pi D_1} \le [K] = K'_{1c} / n_1, (MPa\sqrt{1000 damage - units}) \quad (13)$$

$$K'_{1c} = \sigma_s \cdot \sqrt{\pi D_{1c}} \quad (MPa\sqrt{1000 damage - units}) \quad (14)$$

Where the $y(a/b)$ [18-19] is a correcting factor related with the shape and the size of a crack; the K'_{1c} is a the critical value of damage corresponded to the D_{1c} mentioned above. It shoud point, because the yield stresses σ_s is the constant of uniquenesses for a material, the critical value of damage D_{1c} can also be applied as an important parameter showed its property. In practice, the critical value D'_{1c} could be calculated by means of below formula:

$$D'_{1c} = \frac{K^2}{\sigma_s^2 \times \pi}, (damage - units) \quad (15\text{-}1)$$

Here has to point the above equations, the data error in the calculations is bigger for those materials happened strain softening.

In the table 2, here are listed to the critical values D_{1c} of damage for 13 kinds of materials.

Table 2. *The critical values* D_{1c} *of damage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	D_{1c},
Hot rolled sheet 1005-1009	345	262	531	1.31
Steel: 1005-1009 Cold-draw sheet	414	400	524	0.546
RQC-100, Hot rolled sheet	931	883	1172	0.561
4340, quench and tempering	1241	1172	1579	0.578
Aluminum 2024-T3	469	379	455	0.46
①30CrMnSiA, Hardening and tempering	1177	1104.5	1475.76	0.568
①LC4CS, Heat treatment-CS	613.9	570.8	775.05	0.587
③40Cr	940	805	1592	1.25
③60Si2Mn, quench, medium-temperature tempering	1504.8	1369	1721	0.503

Note: σ_b is a strength limit; σ_s is an yield limit;
①---The Masing's materials; ③-Cyclic softening.

3.3. Strength Calculation on Damage at the Second Stage

When the damage growth gets to the macro damage stage where it is corresponding to the curve BA_1A_2 in figure 1, for the behaviour of some materials corresponded curve BA_1 between the abscissa axis OII and the O_3III (or over the O_3III), they form the critical values D_{1c} of macro damage are usually later than those brittle materials, their life are also longer, so the transition points between two stages in damage process are on the abscissa axis O_3III that just is as the boundary of them. In this case that strength criterion (12-13) on damage in first stage can still be sutied for calculations in the second stage.

By the way, when a structure is being calculated in design, if the damage grows to the stage of macro damage, then the damage value D_2 in the equation (12) can also be calculated for the predictions by following formula

$$D_2 = \frac{\sigma^2 \times \pi}{\sigma_y^2}, (damage-units) \tag{15-2}$$

When the damage growth over the abscissa axis O_3III to the O4IV in figure 1, the strength criterion of damage at later time in the second staege should be as following form [20-22],

$$K'_2 = y(a/b)\sigma \cdot \sqrt{\pi D} \le [K] = K_{2c}/n, (MPa\sqrt{1000 damage-units}) \tag{16}$$

$$K'_{2c} = \sigma_f \cdot \sqrt{\pi D_{2c}}, (MPa\sqrt{1000 damage-unit-number}) \tag{17}$$

Where the K'_2 is defined as the stress factor of damage in the second, the K'_{2c} is a critical factor of damage that it is equivalent to the critical stress intensity factor K_{1c} in fracture mechanics. The σ_f is a fracture stress, the D_{2c} is a critical damage value where it is at the crossing point A2 of the abscissa axis O4-IV and the straight line 1 (A_1A_2) in Fig. 1.

It should yet explain because the K'_{2c} is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by mens of the fracture stress σ_f (in the table 2). So that the critical value of damage D_{2c} under corresponding to true stress σ_f should also be the only data. In theory, it must be to exist as following functional relationship,

$$D_{2c} = \frac{K^2}{\sigma_f^2 \times \pi}, (damage-units) \tag{18}$$

In the table 3 to include the critical values D_{2c} of some materials.

Table 3. *The critical values* D_{2c} *of damage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	σ_f, MPa	D_{2c}, damage − units
Hot rolled sheet 1005-1009	345	262	531	848	0.125
Steel: 1005-1009 Cold-draw sheet	414	400	524	841	0.124
RQC-100, Hot rolled sheet	931	883	1172	1330	0.247
4340, quench and tempering	1241	1172	1579	1655	0.280
Aluminum 2024-T3	469	379	455	558	0.212
①30CrMnSiA, Hardening and tempering	1177	1104.5	1475.76	1795.1	0.215
①LC4CS, Heat treatment-CS	613.9	570.8	775.05	710.62	0.379
②QT800-2, normalizing	913	584.3	1777	946.8	1.121
③40Cr	940	805	1592	1305	0.474
③60Si2Mn, quench, medium-temperature tempering	1504.8	1369	1721	2172.4	0.20

Note: σ_b is a strength limit; σ_s is an yield limit;
①---The Masing's materials; ②---The cycle-harden materials; ③-Cyclic softening materials.

3.4. Strength Calculations on Damage in Whole Process

Due to the behaviors shown by the materials are different at each stage, their dimensions and units in computing models about strength problem are also differences. But the author researches finding recently, on account of the critical values at each stage are all inherent constants, they are the only depended on the material properties, therefore here to exist necessary critical values of damage at each stage. So that the critical values of damage as the material constants in whole process have been proposed as following forms:

$$D_{wc} = D_{th} + D_{1c} \quad (19)$$

Or

$$D_{wc} = D_{tr} + D_{1c} \quad (20)$$

It shoud point that the data in table 4 is the material constants for six kinds of materials, which they are under the monotonous loading. The data in table 5 is called as the critical value of damage which is calculated with calculable formulas.

Table 4. *Data of material's performances.*

Materials [12-13]	Heat- treatment	σ_b, MPa	σ_s, MPa	K, MPa	n_1	E MPa	σ_f, MPa	b_1	$m_1^{'}$
①30CrMnSiA	Hardeningand Tempering	1177	1104.5	1475.76	0.063	203005	1795.1	-0.0859	11.64
①LC4CS	CS	613.9	570.8	775.05	0.063	72571.8	710.62	-0.0727	13.76
2024-T3 Aluminum		469	379	455	0.032	70329	558	-0.124	8.06
②QT800-2	Normalization	913	584.3	1777	0.2034	160500	946.8	-0.083	12.05
4340	Quenching and tempering	1241	1172	1579	0.066		1655	-0.076	13.16
40Cr	in oil-quench 850°C, temper 560°C, air cooling	940	805	1592	0.173		1305	0.12	8.33

①---The Masing's materials; ②---The strain harding material;
③---it is a material happened the strain softening under cyclic loading.

Table 5. *Calculated data with calculable formulas.*

Materials	Heat- treatment	D_{th} / D_{tr} ⑤	K'_{1c} ⑥	D_{1c} ⑤	K'_{2c} ⑥	D_{2c} ⑤	K'_{wc} ⑥	D_{wc} ⑤	K_c ④
①30CrMnSiA	Hardeningand Tempering	0.251/0.291	46.64	0.568	46.64	0.212	92.05/94.2	0.837/0.877	98.9
① LC4CS,	CS	0.262	24.51	0.587	24.51	0.3786	36.7	0.849	38.5
2024-T3	Aluminum	0.218	14.4	0.459	14.4	0.212	25.8	0.68	31
②QT800-2	Normalization	0.253	56.2	2.944	56.2	1.121	58.56	4.318	47.6
4340	Quenching and tempering	0.259	49.94	0.578	49.94	0.29	60	1.127	50
40Cr ③	in oil-quench 850°C, temper 560°C, air cooling	0.222	50.36	1.245	50.36	0.475	58.64	1.941	154

①---The Masing's materials; ②---The cycle-hardening material;
③---it is a material happened the strain softening under cyclic loading; ④ K_c is the experiment data of critical stress intensity factor, $MPa\sqrt{m}$;
⑤—Units are the "damage-units"; ⑥--- Units are the " $MPa\sqrt{1000\text{-damage-units}}$ ".

It can seen from the above table 5, for the critical factors K'_{1c} and the K'_{2c} of damage, $K'_{1c} = K'_{2c}$, because corresponding to point of the K'_{1c} -value just is the one of the K'_{2c} - value, where they are the same at point A2 on abscissa axis O4 IV; but for their critical values of damages, $D_{2c} \neq D_{1c}$. So, when to take the value for the $[K]$ it must only be caculated by the K'_{1c}/n or K'_{2c}/n with the safe factor n .

Here can be compared in calculated data with the experiment ones K_c , and can be seen out: (1) the calculating value for the nodular cast iron QT800-2 which is materials happened strain harding, the K'_{wc} of it close to the experimental data K_c ; the K'_{wc} for the steel 4340 by quenching and tempering is also close to the experimental K_c ; the calculating values K_{Wc} for Masing's materials 30CrMnSiA and LC4CS, between the K_{Wc} and the K_c , both is approximating; the K'_{wc} for the aluminum 2024-T3 is also close to the experimental K_c . (2) But for the steel 40Cr of shown strain softening, both error between the calculated K'_{wc} and the experimental data K_c is bigger.

The calculating criterions about the damage strength in whole process there are two kinds of ways: 1) The assessment method of the damage factor; 2) The assessment method of the damage value. It should explain, if to apply the assessment method of the damage value, it must use the variables D_1, D_2 and D in whole process, and it has to adopt their critical values shown different performances which relevant material conatants $D_{th}, D_{tr} D_{1c}, D_{2c}$ and D at each stage.

For instance, for a material, it can adopt the following ways:

1. The assessment method of the damage factor

(1) The condition 1

Calculating for the stress factor of micro damage is

If the $H'_1 = \sigma \times \sqrt[m_1]{D_1} \le H_{th}$,（$MPa\sqrt[m_1]{1000 damage-units}$）,

Then the damage for the material will not grow.

If the $H'_1 = \sigma \times \sqrt[m_1]{D_1} \ge H_{th}$,（$MPa\sqrt[m_1]{1000 damage-units}$）,

The damage will be growth.

(2) The condition 2

Calculating for the stress intensity factor in the first stage is as below,

1) $H'_1 = \sigma \times \sqrt[m_1]{D_1} \le H'_{tr}$,

Here $H'_{tr} = \sigma_s \times \sqrt[m_1]{D_{tr-y}}$;

2) $H'_1 = \sigma \times \sqrt[m_1]{D_1} \le [H_1] = H'_{1c}/n$,
（$MPa\sqrt[m_1]{1000 damage-units}$）,

Here $H'_{1c} = \sigma_s \times \sqrt[m_1]{D_{1c}}$,

Here, for the materials of happened strain harding, $n=3$; and for Masing materials, $n=1.6$.

(3) Condition 3

Calculating for the stress intensity factor K'_2 macro damage in the second stage, that is as following formula,

1) $K'_2 = y(a/b) \times \sigma_s \times \sqrt{\pi D_2} \le [K'_2] = K'_{1c}/n$

The $[K'_2] = K_{1c}/n = \sigma_s\sqrt{\pi D_{1c}}/n$

Here, for the materials of happened strain harding, $n=3$; and for Masing materials, $n=1.6$.

2) $K'_2 = y(a/b) \times \sigma_f \times \sqrt{\pi D_2} \le [K'_2] = K'_{2c}/n$.

The $[K'_2] = K_{2c}/n = \sigma_f\sqrt{\pi D_{2c}}/n$.

Here, for the materials of happened strain harding, $n=3$; and for Masing materials, $n=1.6$.

For example, it can see the D_{2c} of the QT800-2 and 4340 in the table 5.

(4) Condition 4

Calculating for the stress intensity factor K'_w in the whole process, that is as following formula,

$$K'_w = y(a/b) \times \sigma \times \sqrt{\pi D_w} \le [K'_w] = K'_{wc}/n \qquad (21)$$

Here the K'_w is fefined as the stress factor K'_w of damage in whole process, the D_w is a total value of damage, that can be calculated with following formulas,

$$D_w = D_{th} + D_1 \qquad (22)$$

$$D_w = D_{tr} + D_1 \qquad (23)$$

For the materials of happened strain harding and Masing's ones, that $[K'_w]$ is

$$[K'_w] = K_{wc}/n = \sigma_f\sqrt{\pi D_{wc}}/n,\ n=3 .$$

For example, where it can see the D_{wc} of the steels 30CrMnSiA and LC4CS in the table 5, total value D_{wc} of the critical damage can be calculated by following formulas, $D_{wc} = D_{th} + D_{1c}$ or $D_{wc} = D_{tr} + D_{1c}$; $n=3$

If it can be corresponded under all conditions mentioned above for a material, then, it is placed under a safe state in a definite time; Otherwise, that isn't the safe.

2. The assessment method for the damage value

(1) The cases 1

If the damage values are as following cases

It may be: $D_1 \le D_{th}$,（$damage-units$）, or $D_1 \ge D_{th}$,（$damage-units$）.

(2) The condition 2

1) The damage value in the first stage is as below,

$$D_1 \le [D] = D_{tr-y}/n,（damage-units）,$$

Here, for Masing's material, $n=1.6$; for the materials of happened strain hardening, $n=3$.

2) Calculating to apply the damage value in the second stage, that is

$$D_2 \le [D],（damage-units）, [D_2] = D_{1c}/n$$

Here for the materials of happened strain hardening, $n=3$; and for Masing's ones, $n=1.6$.

(3) The condition 3

To apply the assessment method of the total damage value in the whole process, that is

$$D_w \le [D] = D_{wc}/n ,$$

Here, $D_w = D_1 + D_2$;

$$D_{wc} = D_{th} + D_{1c} \text{ or } D_{wc} = D_{tr} + D_{1c};\ n=3 .$$

If it can be corresponded under all conditions mentioned above, then it is subjected to a safe state in a definite time; Otherwise, it isn't a safe case.

4. Calculating Example

The steel 30CrMnSiA is a Masing's material, its strength limit $\sigma_b = 1177\,MPa$, yield limit $\sigma_s = 1104.5MPa$, $E = 203005MPa$, $n' = 0.063$, $b'_1 = -0.0859$; Working stress $\sigma_{\max} = 1000\,MPa$, supposing proportional limit $\sigma_p = 0.95\sigma_s = 1050\,MPa$, $y(a/b)=1$. If it is being calculated in design, to try to calculate respectively following data:

(1). Calculate the threseld value D_{th} of damage, the critical value D_{1c} , the D_{2c} and the total D_{wc} of damage for the material;

(2). Calculate the factors $H'_{th}, K'_{1c}, K'_{2c}$ and K'_{wc} of damage stress intensity corresponded to each critical damage values mentioned above, respectivaly;

(3). Use two kind of assessment methods to make evaluating for the security of the material.

The processes and steps of calculations are as below.

1. Calculating for each threshold value D_{th} , critical ones D_{1c} , D_{2c} and D_{wc} of damage, and make assessment for the

material's safety.

(1) According to the formulas (15-1), (1), (2), and (11), for the critical value of damage D_{1c}, the damage value D_1, the threshold D_{th} and the transition D_{tr} are calculated respectively as below:

1) The critical value D_{1c} is as below,

$$D_{1c} = \frac{K^2}{\sigma_s^2 \times \pi} = \frac{1475.8^2}{1104.5^2 \times \pi} = 0.568(damage-units);$$

2) The value D_1 of damage is,

$$D_1 = \left(\frac{\sigma}{\sigma_p}\right)^{m_1} D_{1c} = \left(\frac{1000}{1050}\right)^{11.64} \times 0.568 = 0.322(damage-units);$$

3) The threshold value D_{th} of damage is,

$$D_{th} = \left(\frac{1}{\pi^{0.5}}\right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+(-0.0859)}} = 0.251(damage-units);$$

4) The transitional value between two stages is as below,

$$D_{tr} = \left(\sigma_s^{(1-n')/n'} \times \frac{E \times \pi^{1/2\times n'}}{K'^{1/n'}}\right)^{\frac{2m_1n_1}{2n_1-m_1}} = \left(\sigma_s^{(1-0.063)/0.063} \times \frac{203005 \times \pi^{1/2\times 0.063}}{1475.76^{1/0.063}}\right)^{\frac{2\times 11.64\times 0.063}{2\times 0.063-11.64}} = 0.291(damage-unit);$$

So that $D_1 = 0.322 > D_{th} = 0.251(damage-unite)$, the damage must grow for the material.

Now the $D_1 = 0.322 > [D_1](damage-unit)$ and $D_1 > D_{tr} = 0.291(damage-unit)$,

Therfore, data in the first stage is already subjected under a case of unsafety.

(2) According to the formulas (15-2), for the damage D_2 in the second stage are calculated as follow:

1) The damage value is,

$$D_2 = \frac{\sigma^2 \times \pi}{\sigma_y^2} = \frac{1000^2 \times \pi}{1104.5^2} = 2.575,(damage-units)$$

$$D_2 = 2.575 > [D] = D_{1c}/n = 0.568/1.6 = 0.366(damage-units)$$

So, the calculating result of this stage is also not safe.

(3) According to the formulas (19) and (20), calculations for total damage values in the whole process are as follow,

1) The total value of damage is,

$$D_w = D_1 + D_2 = 0.322 + 2.573 = 2.895(damage-units)$$

2) The critical values of damage are,

$$D_{wc} = D_{th} + D_{1c} = 0.251 + 0.586 = 0.837(damage-units),$$

$$D_{wc} = D_{tr} + D_{1c} = 0.291 + 0.586 = 0.877(damage-units).$$

Here it should take the larger value of both data.

3) The permited value is

$$[D_w] = D_{wc}/n = 0.877/3 = 0.292(damage-units);$$

The results from the two data can see as following,

Both the $D_w = 2.895 > [D] = 0.292(damage-units)$, and $D_w = 2.895 > D_{wc} = 0.877,(damage-units)$

So, the material for the structure is necessarily the result of fracture.

2. To calculate the factor's values K_2', K'_y and K'_{wc} of the stress intensity, make an assessment for the safety, respectively.

According to the formulas (11), (12), (13) and (14), for each factor values $K'_{tr}, H_1', K_2', K_w'$ and total critical K'_{wc} of damages in the whole process are calculated as below, respectively.

1) The factor value K_{tr}' corresponded to the yield stress σ_s in the first stage is as below

$$K'_{tr} = \sigma_s \times \sqrt{\pi D_{tr}} = 1104.5 \times \sqrt{\pi 2.91 \times 10^{-4}} = 33.4(MPa \cdot \sqrt{1000-damage-units})$$

2) The factor value K_1' of macro damage in the first stage is,

$$K'_1 = y(a/b)\sigma \times \sqrt{\pi D_1} = 1 \times 1000 \times \sqrt{\pi 3.22 \times 10^{-4}} = 31.81(MPa\sqrt{1000\text{-}damage-units})$$

3) The factor value K_2' of damage in the second stage should be,

$$K'_2 = y(a/b)\sigma \times \sqrt{\pi D_2} = 1 \times 1000 \times \sqrt{\pi 2.573 \times 10^{-3}} = 89.91(MPa\sqrt{1000\text{-}damage-units}),$$

And the $K_{1c} = \sigma_s \sqrt{\pi D_{1c}} = 1104.5\sqrt{\pi 5.68\times10^{-3}} = 46.66(MPa\sqrt{1000\text{-}damage-units})$

The permited value is,

$$[K'_2] = K'_{1c}/n = 46.66/1.6 = 29.16(MPa\sqrt{1000\text{-}damage-units}$$

So that the $K'_2 = 89.19$ not only over the $[K'_2] = 29.16$, but also over the $K'_{1c} = 46.66$, it must be unsafety.

The total value of damage factor in the whole process is as below,

$$K'_{w1} = y(a/b)\sigma\times\sqrt{\pi D} = 1\times1000\times\sqrt{\pi 2.895\times10^{-3}} = 95.37(MPa\sqrt{1000\text{-}damage-units})$$

And the total critical value of damage factor in the whole process is

$$K'_{wc} = \sigma_f\times\sqrt{\pi D_{wc}} = 1795.1\times\sqrt{\pi 8.77\times10^{-4}} = 94.2(MPa\sqrt{1000\text{-}damage-units}),$$

Here can see, this critical value of damage with the experimental data ($K_c = 98.9MPa\sqrt{m}$ of that stress intensity factor in table 5 is relatively approximating.

The permited value is,

$$[K'_w] = K'_{wc}/n = 94.2/3 = 31.4(MPa\sqrt{1000\text{-}damage-units}\ .$$

Now, $K_w = 95.37 > [K_w] = 31.4(MPa\sqrt{1000-damage-units})$

and $K'_w = 95.37MPa\sqrt{1000\text{-}damage-units} > K'_{wc} = 94.2(MPa\cdot\sqrt{1000-damage-units})$

So that the damage state for the material will certainly fracture.

Here it can find out, the results for two kinds of calculating and assessment that is coincident.

5. Conclusions

(1). The calculatung equation that it is a damage value D_{tr} at transition point between the elastic and plastic strain, it is suited to materials shown strain hardening and particularly Masing ones.

(2). As the yield stress $\sigma_s(\sigma_y)$ is the only the constant shown own inherent property, so that the new critical value D_{1c} depended on the σ_s, that is also the sole, and the D_{1c} is a calculable parameter. Similarly, because the fracture stress σ_f is the only the constant shown own inherent one, the new critical value D_{2c} depended on the σ_f that is also the sole.

(3). The critical D_{1c} and D_{2c} of damage are inherent constants shown the materials'characters; so the critical stress factors K_{1c} and K_{2c} based on D_{1c} and D_{2c} are also sole values, and are all calculable ones.

(4). In assessment for damage strength, there are two kinds of methods that are calculated by the damage factor and by the damage value, both is consistent, and the later way is more simpleness.

(5). For Masing's materials, the total critical D_{wc} of damage is also the only, and it is another new constant shown a material's propertiy; therefore its critical stress factor K_{wc} based on D_{wc} is also the sole value. Those D_{wc} and K_{wc} are also calculable; Their computing models can be used to calculate both for the safe assessment to some materials preexistted a flaw and for predicting damage in some structure designs; But for some steels as the 40Cr of shown strain softening, both error between the calculated and the experimental data is bigger.

(6). In those computing models are proposed by the author, if readers want to apply into engineering calculations, it must yet be verified by combined experiments, and it has to consider the influences for the shape and size to the crack and the structure.

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