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Calculations and Assessment for Cracking Strength to Linear Elastic Materials in Whole Process---The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

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Abstract
The author bases on the principles of similar to the genetic genes in the life sciences, discovers some new constants shown material properties from a short crack to long crack, and proposes some new computing models which are the calculable length of a crack, the threshold size and the critical ones on crack problem to some metallic materials; which are to use the theoretical approach, to adopt the conventional material constants, to derive the new mathematical models and the stress factor of called crack strength, to provide simple assessment criterions on the crack strength and the calculating methods in each stage. In addition, supplements again the comprehensive figure of the material behaviours; gives yet a detailed calculating example for a safety assessment.

1. Introduction

The author thinks that in the mechanics and the engineering fields where it exists such a scientific law as similar to genetic elements and cloning technology in the life sciences, and had used the theoretical approach for similar principles, proposed some calculation models [1-6], recently sequentially discovered some new scientific laws to the Masing's and the elastic-plastic materials, and provides some new calculable models for the crack growth driving force, the calculating criterions and the assessment methods about the strength problems in the whole process which are from short to long crack growth. This is to try to make the modern fatigue, the damage mechanics and the fracture mechanics gradually become such calculable disciplines as the traditional material mechanics and structural mechanics. That way, it may be there are practical significances for decreasing experiments to stint manpower and funds for promoting and developing engineering and applying it to relevant disciplines.

2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the "Comprehensive figure of materials behaviors" as Figure 1 (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

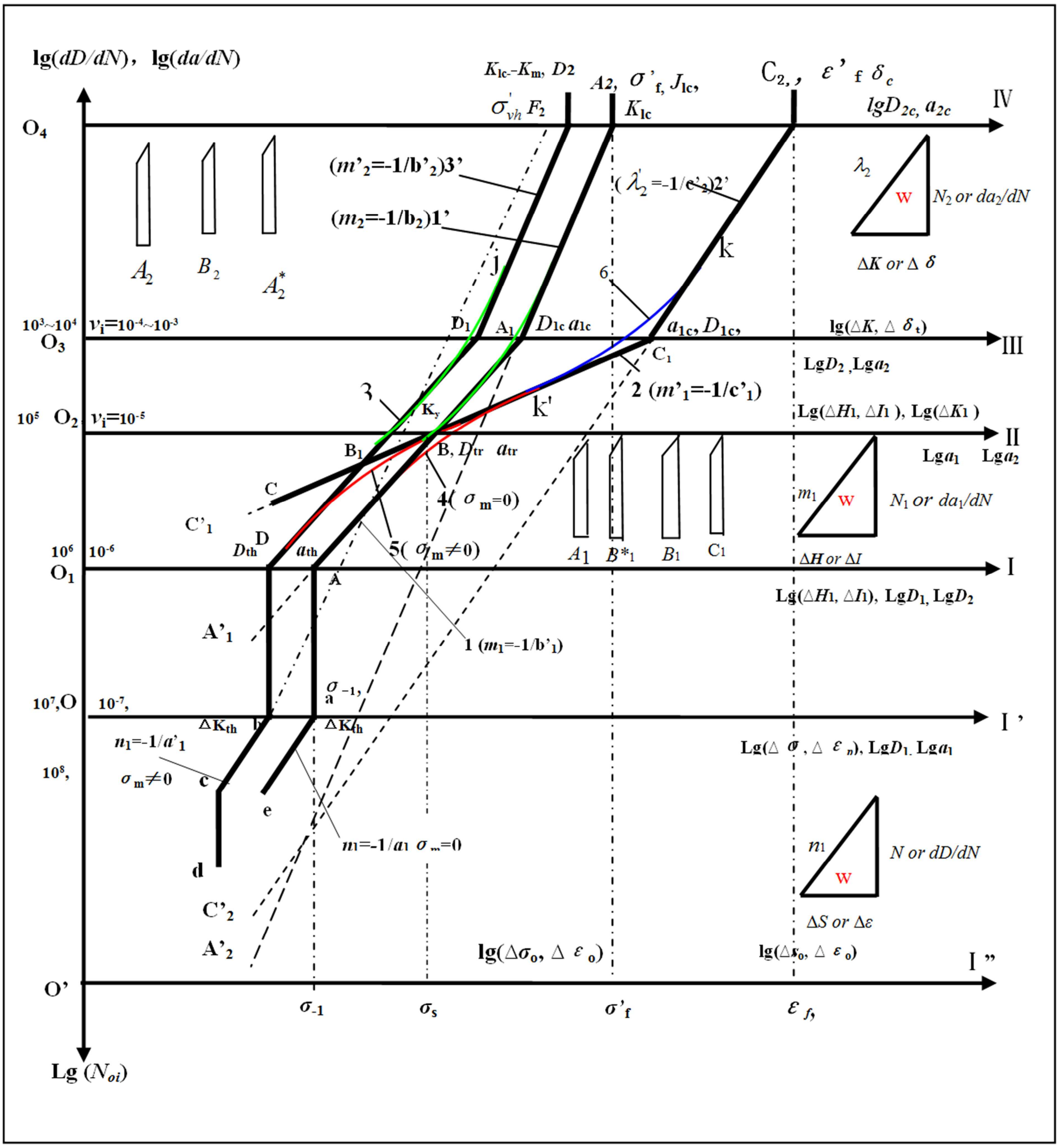

Figure 1. *Comprehensive figure of material behaviours (Or called calculating figure of material behaviours or bidirectional combined coordinate system and simplified schematic curves in the whole process).*

2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes O' I”, O I’, O_1 I, O_2 Ⅱ, O_3 Ⅲ, O_4 IV and a bidirectional ordinate axis $O'_1\ O_4$. For the area between the axes O' I” and O_1 I’, it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes O I’ and O_2 Ⅱ, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the O_2 Ⅱ, the O_3 Ⅲ and O_4 IV, which are calculated and applied by the macro-damage mechanics and the macro-fracture mechanics. But for between the axes O_1 I and O_2 Ⅱ, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

On the abscissa axes O' I”, it are represented with parameters the stress σ and the strain ε as variables. On the abscissa axis O I’ there are the fatigue limits σ_{-1} at point “a” $(\sigma_m = 0)$ and “b” $(\sigma_m \neq 0)$ that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axes O_1 I there are points “A” and “D” that just are the locations placed at threshold values as some materials. On the abscissa axes O_1 I and O_2 Ⅱ that they could all represented as variables with the stress intensity factor range ΔH_1 of short crack, and the strain intensity factor ΔI, and the stress intensity factor range ΔK_1 of long crack. On the other hand, they both could yet represented as variables with the short crack a_1 and the long crack a_2 (or damage D_1 and D_2). And here there are material constants of two that they are defined as the critical factor K_y of crack-stress-intensity and the critical factor K'_y of the damage-stress-intensity at the first stage, where that are just two parameters corresponded to the transitional size a_{tr} of crack or the transitional value of damage D_{tr}, they are just placed at point at the point B $(\sigma_m = 0)$ and at point B_1 $(\sigma_m \neq 0)$ corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors; but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level.

On the abscissa axis O3 Ⅲ, it is represented as variable with the stress intensity factor ΔK_1 (or $\Delta\delta_t$) of long crack; it is also a boundary of the sizes as the residual strength between some elastic-plastic materials and brittle materials. On this axis O3 Ⅲ there are the variables and the critical points at D_1 and D_{1c}, A_1 and A_{1c}, C_1. On abscissa O_4 IV, the point A_2 is corresponding to the fatigue strength coefficient σ'_f, the critical stress intensity factor values $K_{1c}(K_{2fc})$ and the critical values D'_{2c} and a_{2c} for the mean stress $\sigma_m = 0$; the point D_2 is corresponding to the $\sigma_m \neq 0$; the point C_2 corresponding to the fatigue ductility coefficient ε'_f and critical crack tip open displacement value δ_c; the point F corresponding to a very high cycle fatigue strength coefficient σ'_{vhf}. In addition on the same O_4 IV, there are yet another critical values $J'_{1c}(J_{1c})$, etc. in the long crack propagation process.

For the ordinate axis, upward direction along the ordinate axis is represented as crack growth rate da/dN or damage growth rate dD/dN at each stage and in the whole process; the downward direction is represented as life N_{oi}, N_{oj} at each stage and in the whole lifetime ΣN.

In the area between axes O' I” and O_2 Ⅱ, it is the fatigue history from un-crack to micro-crack initiation. In the area between axes O_1 I’ and O_2 Ⅱ, it is the fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_2 - O'$ on ordinate axis is as the history relating to life N_{mac} from grains size to micro-crack initiation until macro-crack forming; the distance O_4 - O' is as the history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

At the crack forming stage, in the partial coordinate system made up of the upward ordinate axes $O\ O_4$ and the abscissa axes O I’, O_1 I and O_2 Ⅱ is represented for the relationship between the crack growth rate dD_1/dN_1 (or the short crack growth rate da_1/dN_1) and the crack-stress factor range ΔH_1 (or the damage strain factor range ΔI_1). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis $O_2\ O_4$ and abscissa O_2 Ⅱ, O_3 Ⅲ and O_4 IV at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range ΔK, J-integral range ΔJ and crack tip displacement range $\Delta\delta_t$ (da_2/dN_2 - ΔK, ΔJ and $\Delta\delta_t$). Inversely, the coordinate systems made up of the downward ordinate axis $O_4\ O_1$ and the abscissa axes O_4 IV, O_3 Ⅲ, O_2 Ⅱ, O_1 I, and O I’ are represented respectively as the relationship between the ΔH -, ΔK - range and each stage life N_{oi}, N_{oj} and the lifetime $\sum N$ (or between the $\Delta\varepsilon_p$ -, $\Delta\delta_t$ - range and the life $\sum N$).

2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve ABA_1 is represented as the varying laws as the behaviours of the elastic materials or some elastic-plastic ones under high cycle loading in the macro-crack-forming stage (the first stage): positive direction ABA_1 represented as the relations between da_1/dN_1 - ΔH ; inverted A_1BA, between the $\Delta H_1 - N_{oi}$. The curve CBC_1 is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction CBC_1 is represented as the relations between da_1/dN_1 - ΔI_1 ; inverted C_1BC, the relations between the $\Delta\varepsilon_p - N_{oi}$.

The curve A_1A_2 in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction A_1A_2 showed as da_2/dN_2 - ΔK (ΔJ); inverted A_2A_1, between the $\Delta K_2, \Delta J - N_{oj}$. The C_1C_2 is showed as: the positive, relation between the da_2/dN_2 - $\Delta\delta_t$ under low-cycle loading, inverted C_2C_1, between $\Delta\delta_t$ (ΔJ)- N_{oj}. By the way, the curves $'Dbcd', (\sigma_m = 0)$ and the $'Aae'$ ($\sigma_m = 0$) are represented as the laws under the very high cycle fatigue.

It should yet point that the curve AA_1A_2 (1-1') is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, $da/dN \le 10^{-6}$); the curve DD_1D_2 (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, $(da/dN \le 10^{-6}$). The curve CC_1C_2 (2-2') is depicted as the rate curve under low cycle loading. The curve $eaABA_1A_2$ is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ($\sigma_m = 0, da/dN < 10^{-7}$), the curves $dcbDD_1D_2$ and $dcbF_2$ are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ($\sigma_m \ne 0, da/dN < 10^{-7}$). Inversely, the curve A_2A_1A is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, $N \le 10^6$), the curve D_2D_1D, as the lifetime curve under unsymmetrical cycle loading $(N \le 10^6)$. The curve C_2C_1C is depicted as the lifetime curve under low cycle loading $(N \le 10^5)$. On the other hand, the curve A_2A_1BAae is as the lifetime one in whole process included very high cycle fatigue ($\sigma_m = 0, N > 10^7$), the curves D_2D_1Dbcd and F_2bcd are all depicted as the lifetime ones in whole process $(\sigma_m \ne 0, N > 10^7)$.

It should also be explained that the comprehensive figure 1 of the materials behaviours may be as a complement for a fundamental research of the material subject; that is a tool to design and calculate for various kinds of structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.

3. Strength Calculations on a Crack Under Monotonic Loading

Here for the variable a describing the crack growth process, it is defined as follows:

1) From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage, that is corresponded to the variable a_1 of the short crack, it is represented as the curve AA_1 in figure 2;

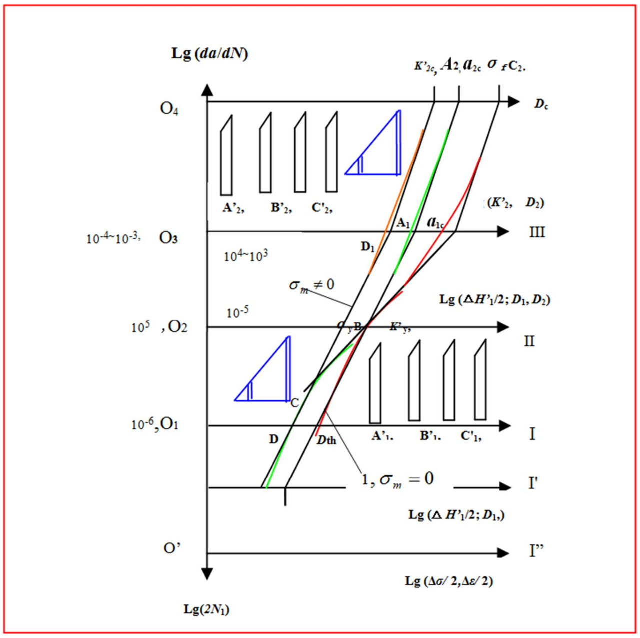

Figure 2. The figure of the crack behaviours in whole process.

2) From the macro-crack propagation to the fracture process is defined in the long crack growth stage, or defined in the second stage. The variable a_2 of this stage is called as the long crack one, that it is corresponding curve A_1A_2 in figure 2;
3) From a micro-crack initiation to long crack growth until full fracture of a material, to adopt variable a in the whole process, it is corresponding curve AA_1 A_2 in figure 2.

3.1. About the Driving Force and Threshold Size on Crack Growth

In the figure 2, it can be seen that differences with the loading ways and the stress levels, for the general steels, their behaviours were always shown differences in the each stages, but they are all to exist the threshold values a_{th} of the crack, only depended on the exponents b_1 related to the material character in table 1.

***Table 1.** Threshold sizes of the crack shown the material character.*

Materials [7-9]	Heat treatment	σ_b, MPa	σ_s, MPa	b_1	a_{th}, mm
BHW35	Normalizing 920°C, temper 620°C	670	538	-0.0719	0.2626
QT450-10	As cast condition	498.1	393.5	-0.1027	0.237
QT800-2	Normalizing	913.0	584.32	-0.0830	0.253
ZG35	Normalizing	572.3	366.27	-0.0988	0.240
60Si2Mn	Quench and medium-temperature tempering	1504.8	1369.4	-0.1130	0.228
45	Normalizing 850°C	576~624	377	-0.123	0.219
40Cr	Oil quenching 850°C, temper 560°C	845~940		-0.120	0.222
16MnL	Hot rolling	570		-0.1066	0.233
20	Hot rolling	432	307	-0.12	0.222
40CrNiMoA	Oil quenching 850°C, temper 580°C	1167		-0.061	0.271
BHW35	Normalizing 920°C, temper 620°C	670	538	-0.0719	0.262
30Cr2MoV	Normalizing 940°C, oil cooling 840°C, furnace cooling 700°C	719		-0.0731	0.261
30CrMnSiNi2A	Heat 900°C, isothermy 245°C, air cooling, temper 270°C	1655	1334	-0.1026	0.237
2A12CZ	Natural aging (CZ)	545		-0.0638	0.269
2A50 CS	Artificial aging (CS)	513		-0.0845	0.252
Ti6Al4V (TC4)	Air cooling 800°C	989		-0.07	0.264

It should point, the locations of the threshold sizes a_{th} of the cracks, some materials are near at the point A where it is at the intersection one between the straight line AB and the abscissa axis O_1 I in figure 1; and other ones, near at the point *a* where it is at the intersectional point between the straight line "Aa" and the abscissa axis O I'. And the threshold size a_{th} can be calculable parameter with as following formula, it should be [10]

$$a_{th} = \left(\frac{1}{(\pi \times 1mm)^{0.5}}\right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} \quad (mm) \tag{1}$$

Or

$$a_{th} = \left(\frac{1}{(\pi \times 1mm)^{0.5}}\right)^{\frac{1}{0.5-(1/m_1)}} \quad (mm) \tag{2}$$

The range of the threshold size a_{th} is the 0.21~0.275 (mm). For linear elastic materials, to make the a_{th} is combined with the stress σ, so that it can make a model of the driving force that is as below.

$$H_1 = \sigma \cdot a_1^{1/m_1} = \sigma^{m_1} \cdot a_{th} = H_{th}[MPa \cdot (m)^{1/m_1}] \tag{3}$$

In the formulas (2-3), $m_1 = -1/b_1$. The H_1 is defined as the stress intensity factor of short crack [10]. In an ordinary way, the $\sigma \cdot a_1^{1/m_1}$ may be: the $\sigma \cdot a_1^{1/m_1} < \sigma^{m_1} \cdot a_{th} = H_{th}$ or $\sigma \cdot a_1^{1/m_1} \geq \sigma^{m_1} \cdot a_{th} = H_{th}$, then the strength criterions for them are as below,

$$H_1 = \sigma \cdot a_1^{1/m_1} \leq H_{th}[MPa \cdot (m)^{1/m_1}] \text{ or } [MPa \cdot (mm)^{1/m_1}] \tag{4}$$

Or

$$H_1 = \sigma \cdot a_1^{1/m_1} \geq H_{th}[MPa \cdot (m)^{1/m_1}] \text{ or } [MPa \cdot (mm)^{1/m_1}] \tag{5}$$

Where the H_{th} is defined as the threshold factor of the short crack. If the $H_1 < H_{th}$, the crack in a material does not grow; but, the $H_1 \geq H_{th}$, the crack is must be to grow.

3.2. Strength Calculation on Crack at the First Stage

When a short crack gradually grow to the long crack where it is corresponding to the curve 1(AB) or the "aAB" between the abscissa axes O I' and the O_2 II in figure 2. Here it can set up the strength criterion for it in the first stage, which is as below form,

$$H_1 = \sigma \cdot a_1^{1/m_1} \leq [H] = H_{1c} \,/\, n, (MPa \cdot m^{1/m_1}) \tag{6}$$

$$H_{1c} = \sigma_s \times \sqrt[m_1]{a_{tr}}, (MPa \cdot m^{1/m_1}) \tag{7}$$

Where the H_{1c} in (7) is defined as a critical value of the stress intensity factor in first stage, it is a value corresponded to the critical value K_y and the transitional size a_{tr} of a crack, also are the constant values on the boundary between the short crack and the long crack. Their locations are respectively at points B on abscissa axis O_2-II (in Fig. 2).

It should yet explain, the crack a_1 in the eq. (6) mentioned above may be calculated to take the size of preexisted a flaw in a component, or it can also applied into predicating calculations by a designer for a design. If the designing stress is less than the elastic limit σ_e, the calculating of the crack length can be adopted as following calculable formula,

$$a_1 = \frac{\sigma^2}{\sigma_{pr}^2 \times \pi} v, (mm) \tag{8}$$

Here the $\sigma_{pr} \approx \sigma_e$ is a stress value of proportional limit (approximating to the elastic limit), it can also approximatively be took for definite ratio by the yield stress, for example $\sigma_{pr} = (0.96 \sim 0.97)\sigma_s$, if the data is to lack.

The v is a conversion coefficient of the unit, $v = 1mm$.

3.3. Strength Calculation on Crack at the Second Stage

As is well known, the mathematic model to describe a crack in fracture mechanics that it is to adopt these "genes" σ and π and crack variable a, thereby to make the stress intensity factor; Here it can make the model of driving force for the describing behavior of it as following form [11-13].

$$K_1 = \sigma\sqrt{\pi a}(MPa\cdot\sqrt{m}) \tag{9}$$

Here is sure to explain, the area between the abscissa axis O_1-I and the O_2-II in fig. 2, the crack size a_1 from the threshold a_{th} to a_{tr} （$a_{th} \le a_1 \le a_{tr} = a_{\mathrm{mac}}$）, there are the mathematic models of the stress factors of two kinds, which are all suited in the section. In addition to above equations (6-7) can be applied, in theory another mathematic models (9-13) are still suitable in the area.

Where the K_1 is also called as a stress intensity factor of short crack that it is equivalent to H_1, but their dimensions and units are differences at this same point, then the model of driving force corresponded at that critical point B should be as follow

$$K_y = \sigma_s\cdot\sqrt{\pi a_{tr}}, (MPa\cdot\sqrt{m}) \tag{10}$$

$$a_{tr} = \left(\sigma_s^{(1-n')\,/\,n'} \times \frac{E\times\pi^{1/2\times n'}}{K^{1/n'}}\right)^{\frac{2m_1n_1}{2n_1-m_1}}, (mm) \tag{11}$$

Where the K_y is defined as the critical stress factor that is corresponding to a crack size a_{tr} of the transitional point, and just is to that size $a_{mac}(\approx a_{tr})$ of forming macro crack, is the very at point B to the yield stress σ_s on abscissa axis O2-II in fig. 2. Here it need yet explain, this factor K_y should theoretically be equivalent to above mentioned the H_{1c} in first stage, although the dimensions and units between them are different. In addition, the K is a strength coefficient under monotonic loading, its unit is the "$MPa\sqrt{mm}$". The n is an exponent happened strain hardening.

Over the abscissa axis O2-II, the crack over the transitional point size a_{tr} is to adopt the a_2 as the variable. During a crack growth gets to the size of long crack, which it is depicted as corresponding to the curve BA_1A_2 in figure 2, then its strength criterion should be calculated as following form,

$$K_1 = y(a/b)\cdot\sigma\cdot\sqrt{\pi a_2} \le [K] = K_{1c}/n_1, (MPa\sqrt{m}) \tag{12}$$

$$K_{1c} = \sigma_s\cdot\sqrt{\pi a_{1c}} \quad (MPa\sqrt{m}) \tag{13}$$

Where the $y(a/b)$ [14-15] is a correcting factor related with the shape and the size of a crack, the K_{1c} is a the critical factor called during the long crack growth, it is corresponded to the critical size a_{1c} on abscissa axis O3-III in fig. 1, also a the critical value in the second stage. The $[H]$ is defined as the permitted value; the "*n*" is a safety factor; and the a_{1c} is a critical size corresponded to the yield stress in the first stage. It should point, because the yield stresses σ_s is the constant of uniquenesses for a material, the critical size of crack a_{1c} can also be applied as an important parameter showed its property. In practice, the critical value a_{1c} could be calculated by means of below formula:

$$a_{1c} = \frac{K^2}{\sigma_s^2\times\pi}, (mm) \tag{14}$$

But, for some cast irons, steels of the low toughness and brittle materials, which their behaviours are depicted as curve BA_1 between the abscissa axis OII and the O3III. When their stresses are loaded to this level, or gotten to the critical values a_{1c} of long crack, that may be happened to fracture.

Here has to point the calculating equations mentioned above are only suitable for some brittle materials and strain hardening ones. The calculating error is larger for the materials to happened strain softening.

In the table 2, here are listed to the critical sizes a_{1c} of crack for 13 kinds of materials.

Table 2. *The critical sizes a_{1c} of crack in first stage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	a_{1c}, mm
Hot rolled sheet 1005-1009	345	262	531	1.31
Steel: 1005-1009 Cold-draw sheet	414	400	524	0.546
RQC-100, Hot rolled sheet	931	883	1172	0.561
4340, quench and tempering	1241	1172	1579	0.578
Aluminum 2024-T3	469	379	455	0.46
30CrMnSiA, (1) Hardening and tempering	1177	1104.5	1475.76	0.568
LC4CS, (1) Heat treatment-CS	613.9	570.8	775.05	0.587
40Cr (3)	940	805	1592	1.25
60Si2Mn, quench, medium-temperature tempering (3)	1504.8	1369	1721	0.503
QT800-2, (2) normalizing	913	584.3	1777	2.94
QT600-2, (B), (2) normalizing	748.4	456.5	1440	3.167
QT600-2, (A) (2) normalizing	677	521.3	1622	3.08
ZG35 (2) normalizing	572.3	366.3	1218	3.51

Note: σ_b is strength limit; σ_s is yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen;
(1)---The Masing's materials; (2)---The cycle-harden material (3)-Cyclic softening.

It could see from table 2, where the materials from the first to ninth kind are all the steels, their critical sizes of cracks are the range in 0.43~1.42mm to this stage. But the materials from the tenth to thirteenth, which are the nodular cast irons and a cast iron respectively, their critical sizes are the range in 2.94~3.51 mm. In practice as the cast irons are subject to brittle materials which get already the critical values of the fracture under the equivalent yield stress, then those materials will occur to the failures.

For the behaviours of another materials could be over the abscissa axis O3III in figure 1, while they get to own critical values a_{1c} of long crack, which are usually later than the brittle materials above mentioned, their life are also longer. So the abscissa axis O3III is a boundary that can be thought for the residual intensity sizes between different materials in crack growth process. In this case，that strength criterion (12-14) on crack mentioned above can still be suited for calculations.

When the crack growth over the abscissa axis O_3III in figure 2, the strength criterion of crack at later time in the second stage should be as below form

$$K_2 = y(a/b)\sigma \cdot \sqrt{\pi a} \le [K] = K_{2c}/n, (MPa\sqrt{m}) \tag{15}$$

$$K_{2c} = \sigma_f \cdot \sqrt{\pi a_{2c}}, (MPa\sqrt{m}) \tag{16}$$

Where the K_2 is also the stress factor of crack in the second; the K_{2c} is a critical factor when it is momentary fracture to the crack, that it is equivalent to the critical stress intensity factor K_{1c} in fracture mechanics. The σ_f is a fracture stress, the a_{2c} is a critical crack value where it is at the crossing point A_2 on the abscissa axis O_4-IV and the straight line 1 (A_1A_2) in fig. 2.

It should yet explain because the K_{2c} is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by means of the fracture stress σ_f (table 2). So that the critical size of crack a_{2c} under corresponding to the true stress σ_f should also be the only data. In theory, it must be there is as following functional relationship,

$$a_{2c} = \frac{K^2}{\sigma_f^2 \times \pi}, (mm) \tag{17}$$

By the way, when a structure is calculated for a crack size predicting in design, the crack length a_2 in the equations (11, 14) can be used as following calculable formula,

$$a_2 = y(a/b)\frac{\sigma^2 \times \pi}{\sigma_s^2} t \tag{18}$$

Here t is a converting coefficient, 1-damage unit=1mm, $t = 1\text{-}mm$.

Recently the author researches to discover that the strength coefficient K on material subject is virtually the very the critical stress intensity factor K_{1c} on fracture mechanics under the monotonous loading, if their calculating parameters take all same units. For instance, for the hot rolled sheet 4340 in table 2-3, its $K = 1579MPa$, $\sigma_f = 1655MPa$, $a_{2c} = 0.29mm = 2.9\times10^{-4} m$, then if to adopt the calculating model in fracture mechanics to calculate the strength coefficient K, that is as below,

$$K = \sigma_f\sqrt{\pi a_{2c}} = 1655MPa\sqrt{\pi 0.29(mm)} = 1579.7(MPa\sqrt{mm}) = 49.95MPa\sqrt{m}.$$

On the other hand, the practical calculable critical factor K_{1c} on fracture mechanics subject should also be,

$$K_{1c} = \sigma_f\sqrt{\pi a_{2c}} = 1655MPa\sqrt{\pi 0.29(mm)} = 1578MPa\sqrt{mm} = 49.95MPa\sqrt{m}.$$

So, the calculating results are completely consistent. Where its unit to be "MPa" of the K which was called as the strength coefficient in material subject that it is actually the very the critical stress intensity factor in fracture mechanics, and the units of both should be all "$MPa\sqrt{mm}$" or "$MPa\sqrt{m}$". Here it should be point that the experiment values of the K_{1c} are also 50~63 $MPa\sqrt{m}$.

In the table 3 is listing the critical sizes a_{2c} of some materials.

Table 3. *The critical sizes a_{2c} of crack in second stage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	σ_f, MPa	a_{2c}, mm
Hot rolled sheet 1005-1009	345	262	531	848	0.125
Steel: 1005-1009 Cold-draw sheet	414	400	524	841	0.124
RQC-100, Hot rolled sheet	931	883	1172	1330	0.247
4340, quench and tempering	1241	1172	1579	1655	0.280
Aluminum 2024-T3	469	379	455	558	0.212
30CrMnSiA, (1) Hardening and tempering	1177	1104.5	1475.76	1795.1	0.215
LC4CS, (1) Heat treatment-CS	613.9	570.8	775.05	710.62	0.379

Materials	σ_b, MPa	σ_s, MPa	K, MPa	σ_f, MPa	a_{2c}, mm
40Cr (3)	940	805	1592	1305	0.474
60Si2Mn, quench, medium-temperature tempering (3)	1504.8	1369	1721	2172.4	0.20
QT800-2, (2) normalizing	913	584.3	1777	946.8	1.121
QT600-2, (B), (2) normalizing	748.4	456.5	1440	856.5	0.90
QT600-2, (A) (2) normalizing	677	521.3	1622	888.8	1.06
ZG35 (2) normalizing	572.3	366.3	1218	809.4	0.721

Note: σ_b is a strength limit; σ_s is an yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen;
(1)---The Masing's materials; (2)---The cycle-harden material (3)-Cyclic softening.

4. Calculating Example

A test specimen made of nodular cast iron QT800-2, its strength limit $\sigma_b = 913MPa$, yield limit $\sigma_s = 584.3MPa$, $E = 160500$, its material constant $b_1 = -0.083$, $m_1 = 12.078$, the strength coefficient $K = 1777MPa$, the strain hardening exponent $n = 0.2034$, fracture stress $\sigma_f = 946.8MPa$; If a designer needs to do predicting calculations for a crack strength, to suppose working stress $\sigma_{\max} = 550MPa$, the proportional limit $\sigma_p = 0.97\sigma_s = 567MPa$, $y(a/b) = 1.0$, to try to calculate respectively following data:

(1). Calculate the length a of the crack, the threshold size a_{th}, the critical size a_{1c} and the a_{2c} of crack for the material, respectively;

(2). Calculate the stress factor H_1, the threshold factor H_{th}, the critical factors H_{1c}, K_{1c} and K_{2c} of the crack, respectively;

(3). To use the assessment method of the stress factor to do an assessment for it.

The processes and steps of calculations are as below.

(1). To calculate the crack length a_1 under work stress, the threshold size a_{th}, the transitional size a_{tr}, the critical a_{1c} and a_{2c} of crack, and to do an assessment for the material.

Here $m_1 = -1/b_1 = -1/-0.083 = 12.048$

1) According to the formula (1), the threshold size is,

$$a_{th} = \left(\frac{1}{(\pi 1mm)^{0.5}}\right)^{\frac{1}{0.5+b_1}} v = (0.564mm)^{\frac{1}{0.5+b_1}} = (0.564mm)^{\frac{1}{0.5+(-0.083)}} = 0.253(mm);$$

2) According to the formula (11), the transitional size is as below,

$$a_{tr} = \left(\sigma_s^{(1-n')/n'} \times \frac{E\times\pi^{1/2\times n'}}{K^{1/n'}}\right)^{\frac{2m_1n_1}{2n_1-m_1}} = \left(578.3^{(1-0.2034)/0.2034} \times \frac{160500\times\pi^{1/2\times0.2034}}{K^{1/n'}}\right)^{\frac{2\times12.048\times0.2034}{2\times0.2034-12.048}} = 0.2875,(mm)$$

3) According to the formula (14), its critical size of the crack at the first stage is

$$a_{1c} = \frac{K^2}{\sigma_s^2\times\pi} = \frac{1777^2}{584.3^2\times\pi} = 2.944(mm);$$

4) By the formula (8), its crack size corresponded to the working stress 550 MPa should be as below,

$$a_1 = \frac{\sigma^2}{\sigma_p^2\times\pi} = \frac{550^2}{567^2\times\pi} = 0.3(mm)$$

$a_1 = 0.3 > a_{th} = 0.253(mm)$; $a_1 = 0.3 > a_{tr} = 0.2875(mm)$

So the crack in the material is necessarily to grow.

5) According to the formula (18), its crack length under work stress at the second stage is

$$a_2 = y(a/b)\frac{\sigma^2\times\pi}{\sigma_s^2} = 1.0\frac{550^2\times\pi}{584.3^2} = 2.784(mm)$$

6) By the formula (17), its critical size at the second stage should be,

$$a_{2c} = \frac{K^2}{\sigma_f^2\times\pi}v = \frac{1777^2}{946.8^2\times\pi}1 = 1.1216(mm);$$

(2). To calculate the stress intensity factor H_1 and the critical value H_{1c} for the crack in the first stage, respectively.

1) According to the formula (6), its stress factor of crack is

$$H_1 = \sigma\times\sqrt[m_1]{a_1} = 550\times\sqrt[12.048]{3\times10^{-4}} = 280.52(MPa\sqrt[m_1]{m}).$$

2) The critical factor of the crack is as below,

$$H_{1c} = \sigma_s\cdot a_{1c}^{1/m_1} = 584.3\times\sqrt[12.048]{2.944\times10^{-3}} = 360.21,(MPa\cdot m^{1/m_1}).$$

3) Its permitting value should be,

$$[H_1] = H_{1c} / n = 360.21 / n = 120.07\ (MPa \cdot m^{1/m_1}).$$

So that $H_1 = 280 > [H_1] = 120.07(MPa \times \sqrt[m_1]{m})$.

Therefore, the calculating result by the criterion in the first stage, that is not safe.

(3). To calculate the stress intensity factors K_1 and K_2, the critical values K_{1c} and K_{2c} for long crack in the second stage, respectively.

1) According to the formulas (10) ~ (16), the factor K_1, the threshold value K_y corresponding the yield stress σ_s and the critical one of long crack are respectively as follow,

a) For the stress factor K_1 of the long crack is

$$K_1 = y(a/b)\sigma \times \sqrt{\pi a_1} = 1.0 \times 550 \times \sqrt{\pi 2.784 \times 10^{-3}} = 51.44(MPa\sqrt{m})$$

b) The threshold value K_y of the crack corresponding to the yield stress is as below

$$K_y = \sigma_s \times \sqrt{\pi a_{tr}} = 584.3 \times \sqrt{\pi 2.875 \times 10^{-4}} = 17.56(MPa\sqrt{m})$$

So $K_1 = 51.44 > K_y = 17.56(MPa\sqrt{m})$

Then, the crack must be to grow.

c) The critical factor of crack in this stage is

$$K_{1c} = \sigma_s \times \sqrt{\pi a_{1c}} = 584.3 \times \sqrt{\pi 2.994 \times 10^{-3}} = 56.64(MPa \cdot \sqrt{m}),$$

d) Its permitted value should be,

$$[K_1] = K_{1c} / n = 56.64/3 = 18.9(MPa \cdot \sqrt{m}),$$

e) On the other hand, the critical value of stress factor when the crack are been at momentary fracture is as follow,

$$K_{2c} = \sigma_f \times \sqrt{\pi a_{2c}} = 946.8 \times \sqrt{\pi 1.1216 \times 10^{-3}} = 56.2(MPa \cdot \sqrt{m}).$$

Here it can see out that the $K_{1c} = K_{2c} = 56.2(MPa\sqrt{m})$

Its permissible value of crack factor is

$$[K_2] = K_{2c} / n = 56.2/3 = 18.8(MPa\sqrt{m})$$

So that $K_2 = 51.43 > [K_2] = 18.8(MPa \cdot \sqrt{m})$,

Therefore, the data calculated by the criterion for the macro-crack and the result calculated for the design are not all safe to the material.

Here it can see from the above calculations, for the critical factors of a crack, the $K_{1c} = K_{2c}$, because corresponding to the point of the K_{1c}-value just is the one of the K_{2c}-value where they are at same point A_2 on abscissa axis O_4 IV; but for their critical sizes of cracks, $a_{1c} \neq a_{2c}$. So when to take the value for the $[K]$, it must only be calculated by the K_{1c}/n or K_{2c}/n with the safe factor "n".

5. Conclusions

(1). The crack length a at different stage can be predicted to calculate out by means of the conventional stress and the material constants $b_1, \sigma_s, \sigma_f, K$ and π.

(2). The new threshold size a_{th} of the short crack that can show own inherent property, that is depended on the sole material constant b_1, is a calculable one.

(3). For some materials of the brittle and happened strain hardening under monotonous loading, as the yield stresses $\sigma_s(\sigma_y)$ is the only the constant shown own inherent property, so that the new critical size a_{1c} of crack depended on the σ_s to be also the sole, and the a_{1c} is a calculable parameter. Similarly, because the fracture stresses σ_f is the only the constant shown own inherent one, so that the new critical size a_{2c} of crack depended on the σ_f to be also the sole and calculable.

(4). The critical sizes a_{1c} and a_{2c} of cracks are inherent constants shown the materials' characters; so the critical stress factors K_{1c} and K_{2c} based on a_{1c} and a_{2c} are also sole values, and are all calculable ones; Their computing models can be used to calculate both for the safe assessment to materials preexisted a flaw and for the predicting crack strength in design process; But it may be calculating error to be larger for the shown strain softening's ones.

(5). Because corresponding to the factor-value of the K_{1c} is the very one of the K_{2c} where they are the same at point A_2 on abscissa axis O_4 IV; but for their critical sizes of cracks, $a_{2c} \neq a_{1c}$. So for some materials of the brittle and happened strain hardening when to take the value for the $[K]$ it must only be calculated by the K_{1c}/n or K_{2c}/n with the safe factor n.

(6). The strength coefficient K on material subject is virtually the very the critical stress intensity factor K_{Ic} under monotonous loading on fracture mechanics, if to take the fracture stress σ'_f and the same unit (mm) to calculate for them K and K_{Ic}. But, the unit for the K is the "$MPa\sqrt{mm}$", not foregone that "MPa".

(7). In those computing models are proposed in the paper, if readers want to apply in engineering calculations, it must yet be checked to combine experiments, and it have to consider the influences for the shape and the size to a crack and a structure.

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