Calculations and Assessment for Cracking Strength to Linear Elastic Materials in Whole Process---The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

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Citation

Abstract
The author bases on the principles of similar to the genetic genes in the life sciences, discovers some new constants shown material properties from a short crack to long crack, and proposes some new computing models which are the calculable length of a crack, the threshold size and the critical ones on crack problem to some metallic materials; which are to use the theoretical approach, to adopt the conventional material constants, to derive the new mathematical models and the stress factor of called crack strength, to provide simple assessment criterions on the crack strength and the calculating methods in each stage. In addition, supplements again the comprehensive figure of the material behaviours; gives yet a detailed calculating example for a safety assessment.

1. Introduction

The author thinks that in the mechanics and the engineering fields where it exists such a scientific law as similar to genetic elements and cloning technology in the life sciences, and had used the theoretical approach for similar principles, proposed some calculation models [1-6], recently sequentially discovered some new scientific laws to the Masing’s and the elastic-plastic materials, and provides some new calculable models for the crack growth driving force, the calculating criterions and the assessment methods about the strength problems in the whole process which are from short to long crack growth. This is to try to make the modern fatigue, the damage mechanics and the fracture mechanics gradually become such calculable disciplines as the traditional material mechanics and structural mechanics. That way, it may be there are practical significances for decreasing experiments to stint manpower and funds for promoting and developing engineering and applying it to relevant disciplines.
2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the “Comprehensive figure of materials behaviors” as Figure 1 (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

![Figure 1](image-url)  
*Figure 1. Comprehensive figure of material behaviours (Or called calculating figure of material behaviours or bidirectional combined coordinate system and simplified schematic curves in the whole process).*
2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes $O'$ $1''$, $O'1'$, $O_1''$, $O_1'$, $O_2''$, $O_2'$ and a bidirectional ordinate axis $O_1$ $O_2$. For the area between the axes $O'$ $1''$ and $O_1'$, it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes $O1'$ and $O_2''$, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the $O_2''$, the $O_1'''$ and $O_1'IV$, which are calculated and applied by the macro-damage mechanics and the macro-fracture mechanics. But for between the axes $O_1$ and $O_2''$, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

On the abscissa axes $O'$ $1''$, it are represented with parameters the stress $\sigma$ and the strain $\varepsilon$ as variables. On the abscissa axis $O1'$ there are the fatigue limits $\sigma_{1}^{s}$ at point “a” ($\sigma_{1}^{s} = 0$) and “b” ($\sigma_{1}^{s} \neq 0$) that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axes $O_1$ there are points “A” and “D” that just are the locations placed at threshold values as some materials. On the abscissa axes $O_1$ and $O_2''$, that they could all represented as variables with the short crack $a_2$ and the long crack $a_2$ (or damage $D_1$ and $D_2$). And here there are material constants of two that they are defined as the critical factor $K_1$ of crack-stress-intensity and the critical factor $K_2$ of the damage-stress-intensity at the first stage, where that are just two parameters corresponded to the transitional size $a_2$ of crack or the transitional value of damage $D_2$, they are just placed at point at the point B ($\sigma_{1}^{s} = 0$) and at point B, ($\sigma_{1}^{s} \neq 0$) corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors; but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level.

On the abscissa axis $O_3$ $III$, it is represented as variable with the stress intensity factor $\Delta K_1$ (or $\Delta \delta$) of long crack; it is also a boundary of the sizes as the residual strength between some elastic-plastic materials and brittle materials. On this axis $O_3$ $III$ there are the variables and the critical points at $D_1$ and $D_1c$, $A_1$ and $A_1c$, $C_1$. On abscissa $O_1$ $IV$, the point $A_1$ is corresponding to the fatigue strength coefficient $\sigma_f'$, the critical stress intensity factor values $K_{ic}(K_{2ic})$ and the critical values $D_{ic}$ and $a_{2c}$ for the mean stress $\sigma_{m} = 0$; the point $D_2$ is corresponding to the $\sigma_{n} \neq 0$; the point $C_2$ corresponding to the fatigue ductility coefficient $\epsilon_f'$ and critical crack tip open displacement value $\delta_f$; the point $F$ corresponding to a very high cycle fatigue strength coefficient $\sigma_{vf}$. In addition on the same $O_4$ $IV$, there are yet another critical values $J_f(\Delta I_f)$, etc. in the long crack propagation process.

For the ordinate axis, upward direction along the ordinate axis is represented as crack growth rate $da_i/dN_i$ or damage growth rate $dD_i/dN_i$ at each stage and in the whole process; the downward direction is represented as life $N_{ai},N_{aj}$ at each stage and in the whole lifetime $\Sigma N$.

In the area between axes $O'$ $1''$ and $O_1$, it is the fatigue history from un-crack to micro-crack initiation. In the area between axes $O_1'IV$ and $O_2$, it is the fatigue history relative to life $N_{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_3-O_4$ on ordinate axis is as the history relating to life $N_{mic}$ from grains size to micro-crack initiation until macro-crack forming; the distance $O_4-O_4$ is as the history relating to the lifetime life $\Sigma N$ from micro-crack initiation until fracture.

At the crack forming stage, in the partial coordinate system made up of the upward ordinate axes $O_1$ $O_2$ and the abscissa axes $O_1'IV$, $O_1$ and $O_2'II$ is represented for the relationship between the crack growth rate $dD_1/dN_1$ (or the short crack growth rate $da_i/dN_i$) and the crack-stress factor range $\Delta H_1$ (or the damage strain factor range $\Delta L_1$). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis $O_1$ $O_2'$ and abscissa $O_2'$ $II$, $O_2'''$ and $O_4$ $IV$ at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range $\Delta K$, $J$-integral range $\Delta J$ and crack tip displacement range $\Delta \delta_1$ ($da_2/dN_2 - \Delta K$, $\Delta J$ and $\Delta \delta_1$). Inversely, the coordinate systems made up of the downward ordinate axis $O_1$ $O_1'$ and the abscissa axes $O_3$ $IV$, $O_3'''$, $O_2''$, $O_1$, and $O_1'$ are represented respectively as the relationship between the $\Delta H \sim \Delta K - range$ and each stage life $N_{ai},N_{aj}$ and the lifetime $\Sigma N$ (or between the $\Delta \varepsilon_{f}, \Delta \delta_{f} - range$ and the life $\Sigma N$).
2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve $ABA_1$ is represented as the varying laws as the behaviours of the elastic materials under high cycle loading in the macro-crack-forming stage (the first stage): positive direction $ABA_1$ represented as the relations between $da_1 / dN_1 - \Delta H$; inverted $A_1BA$, between the $\Delta H_1 - N_{a_1}$. The curve $CBC_1$ is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction $CBC_1$ is represented as the relations between $da_1 / dN_1 - \Delta I_1$; inverted $C_1BC$, the relations between the $\Delta \varepsilon_p - N_{a_1}$.

The curve $A_1A_2$ in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction $A_1A_2$ showed as $da_2 / dN_2 - \Delta K$ ($\Delta J$); inverted $A_2A_1$, between the $\Delta K_2, \Delta J - N_{a_2}$. The $C_1C_2$ is showed as: the positive, relation between the $da_2 / dN_2 - \Delta \delta$ under low-cycle loading, inverted $C_2C_1$, between $\Delta \delta_1 (\Delta J) - N_{a_2}$. By the way, the curves 'Dbcd', $\left(\sigma_\sigma = 0\right)$ and the 'Aae' $\left(\sigma_\sigma = 0\right)$ are represented as the laws under the very high cycle fatigue.

It should yet point that the curve $AA_1A_2$ ($1-1'$) is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, $da / dN \leq 10^{-6}$); the curve $DD_1D_2$ (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, $da / dN \leq 10^{-6}$). The curve $CC_1C_2$ (2-2') is depicted as the rate curve under low cycle loading. The curve $eaaBA_1A_2$ is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ($\sigma_\sigma = 0, da / dN < 10^{-7}$), the curves $dbcdDD_1D_2$ and $dcbf_2$ are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ($\sigma_\sigma = 0, da / dN < 10^{-7}$). Inversely, the curve $A_2A_1A$ is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, $N \leq 10^6$), the curve $D_2D_1D_3$ as the lifetime curve under unsymmetrical cycle loading ($N \leq 10^6$). The curve $C_1C_2C$ is depicted as the lifetime curve under low cycle loading ($N \leq 10^5$). On the other hand, the curve $A_2A_1Baae$ is as the lifetime one in whole process included very high cycle fatigue ($\sigma_\sigma = 0, N > 10^5$), the curves $D_1D_2Dbcd$ and $F_1bcd$ are all depicted as the lifetime ones in whole process ($\sigma_\sigma = 0, N > 10^5$).

It should also be explained that the comprehensive figure 1 of the materials behaviours may be as a complement for a fundamental research of the material subject; that is a tool to design and calculate for various kinds of structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.

3. Strength Calculations on a Crack Under Monotonic Loading

Here for the variable $a$ describing the crack growth process, it is defined as follows:
1) From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage, that is corresponded to the variable $a_1$ of the short crack, it is represented as the curve $AA_1$ in figure 2;

2) From the macro-crack propagation to the fracture process is defined in the long crack growth stage, or defined in the second stage. The variable $a_2$ of this stage is called as the long crack one, that it is corresponding curve $A_1A_2$ in figure 2;
3) From a micro-crack initiation to long crack growth until full fracture of a material, to adopt variable $a$ in the whole process, it is corresponding curve $AA_1A_2$ in figure 2.

3.1. About the Driving Force and Threshold Size on Crack Growth

In the figure 2, it can be seen that differences with the loading ways and the stress levels, for the general steels, their behaviours were always shown differences in the each stages, but they are all to exist the threshold values $a_1$, of the crack, only depended on the exponents $b_1$ related to the material character in table 1.
It should point, the locations of the threshold sizes $a_{th}$ of the cracks, some materials are near at the point $A$ where it is at the intersection one between the straight line AB and the abscissa axis $O_1I$ in figure 1; and other ones, near at the point $a$ where it is at the intersectional point between the straight line “Aa” and the abscissa axis $O_1A$. And the threshold size $a_{th}$ can be calculable parameter with as following formula, it should be [10]

$$a_{th} = \left( \frac{1}{(\pi \times 1 \text{mm})^{0.5}} \right)^{1/0.564} (\text{mm})$$  \hspace{1cm} (1)

Or

$$a_{th} = \left( \frac{1}{(\pi \times 1 \text{mm})^{0.5}} \right)^{1/0.564} (\text{mm})$$  \hspace{1cm} (2)

The range of the threshold size $a_{th}$ is the 0.21~0.275 (mm). For linear elastic materials, to make the $a_{th}$ is combined with the stress $\sigma$, so that it can make a model of the driving force that is as below.

$$H_1 = \sigma \cdot a_{1/m} \leq \sigma_{m} \cdot a_{th} = H_{th}[\text{MPa} \cdot (m)^{1/m}]$$  \hspace{1cm} (3)

In the formulas (2-3), $m_i = -1/b_i$. The $H_1$ is defined as the stress intensity factor of short crack [10]. In an ordinary way, the $\sigma \cdot a_{1/m}$ may be: the $\sigma \cdot a_{1/m} < \sigma_{m} \cdot a_{th} = H_{th}$ or $\sigma \cdot a_{1/m} \geq \sigma_{m} \cdot a_{th} = H_{th}$, then the strength criterions for them are as below,

$$H_1 = \sigma \cdot a_{1/m} \leq H_{th}[\text{MPa} \cdot (m)^{1/m}] \text{ or } [\text{MPa} \cdot (mm)^{1/m}]$$  \hspace{1cm} (4)

Or

$$H_1 = \sigma \cdot a_{1/m} \geq H_{th}[\text{MPa} \cdot (m)^{1/m}] \text{ or } [\text{MPa} \cdot (mm)^{1/m}]$$  \hspace{1cm} (5)

Where the $H_{th}$ is defined as the threshold factor of the short crack. If the $H_1 < H_{th}$, the crack in a material does not grow; but, the $H_1 \geq H_{th}$, the crack is must to be grow.

3.2. Strength Calculation on Crack at the First Stage

When a short crack gradually grow to the long crack where it is corresponding to the curve 1($AB$) or the “aAB” between the abscissa axes $O_1I$ and the $O_2II$ in figure 2. Here it can set up the strength criterion for it in the first stage, which is as below form,

$$H_1 = \sigma \cdot a_{1/m} \leq [H] = H_{lc} / n_i[\text{MPa} \cdot m^{1/m}]$$  \hspace{1cm} (6)

$$H_{lc} = \sigma \times \sqrt{\pi a_{th}}[\text{MPa} \cdot m^{1/m}]$$  \hspace{1cm} (7)

Where the $H_{lc}$ in (7) is defined as a critical value of the stress intensity factor in first stage, it is a value corresponded to the critical value $K_{cr}$ and the transitional size $a_{th}$ of a crack, also are the constant values on the boundary between the short crack and the long crack. Their locations are respectively at points B on abscissa axis $O_2II$ in (Fig 2).

It should yet explain, the crack $a_{th}$ in the eq. (6) mentioned above may be calculated to take the size of preexisted a flaw in a component, or it can also applied into predicating calculations by a designer for a design. If the designing stress is less than the elastic limit $\sigma_{cr}$, the calculating of the crack length can be adopted as following calculable formula,

$$a_{th} = \frac{\sigma_{cr}^2}{\sigma_{cr}^2 \times \pi} \times \nu (\text{mm})$$  \hspace{1cm} (8)

Here the $\sigma_{cr} = \sigma_{cr}$ is a stress value of proportional limit (approximating to the elastic limit), it can also approximatively be took for definite ratio by the yield stress, for example $\sigma_{cr} = (0.96 \sim 0.97)\sigma_{cr}$, if the data is to lack.

The $\nu$ is a conversion coefficient of the unit, $\nu = 1 mm$. 

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Table 1. Threshold sizes of the crack shown the material character.

<table>
<thead>
<tr>
<th>Materials [7-9]</th>
<th>Heat treatment</th>
<th>$\sigma_{th}$, MPa</th>
<th>$\sigma_{y}$, MPa</th>
<th>$b$</th>
<th>$a_{th}$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHW35</td>
<td>Normalizing 920°C, temper 620°C</td>
<td>670</td>
<td>538</td>
<td>-0.0719</td>
<td>0.2626</td>
</tr>
<tr>
<td>QT450-10</td>
<td>As cast condition</td>
<td>498.1</td>
<td>393.5</td>
<td>-0.1027</td>
<td>0.237</td>
</tr>
<tr>
<td>QT800-2</td>
<td>Normalizing</td>
<td>913.0</td>
<td>584.32</td>
<td>-0.0830</td>
<td>0.253</td>
</tr>
<tr>
<td>ZG35</td>
<td>Normalizing</td>
<td>572.3</td>
<td>366.27</td>
<td>-0.0988</td>
<td>0.240</td>
</tr>
<tr>
<td>60Si2Mn</td>
<td>Quench and medium-temperature tempering</td>
<td>1504.8</td>
<td>1369.4</td>
<td>-0.1130</td>
<td>0.228</td>
</tr>
<tr>
<td>45</td>
<td>Normalizing 850°C</td>
<td>576–624</td>
<td>377</td>
<td>-0.123</td>
<td>0.219</td>
</tr>
<tr>
<td>40Cr</td>
<td>Oil quenching 850°C, temper 560°C</td>
<td>845–940</td>
<td>670</td>
<td>-0.120</td>
<td>0.222</td>
</tr>
<tr>
<td>16MnL</td>
<td>Hot rolling</td>
<td>570</td>
<td>307</td>
<td>-0.1066</td>
<td>0.233</td>
</tr>
<tr>
<td>20</td>
<td>Hot rolling</td>
<td>432</td>
<td>307</td>
<td>-0.12</td>
<td>0.222</td>
</tr>
<tr>
<td>40CrNiMoA</td>
<td>Oil quenching 850°C, temper 580°C</td>
<td>1167</td>
<td>-0.061</td>
<td>0.271</td>
<td></td>
</tr>
<tr>
<td>BHW35</td>
<td>Normalizing 920°C, temper 620°C</td>
<td>670</td>
<td>538</td>
<td>-0.0719</td>
<td>0.262</td>
</tr>
<tr>
<td>30Cr2MoV</td>
<td>Normalizing 940°C, oil cooling 840°C, furnace cooling 700°C</td>
<td>719</td>
<td>-0.0731</td>
<td>0.261</td>
<td></td>
</tr>
<tr>
<td>30CrMnSiNi2A</td>
<td>Heat 900°C, isothermy 245°C, air cooling, temper 270°C</td>
<td>1655</td>
<td>1334</td>
<td>-0.1026</td>
<td>0.237</td>
</tr>
<tr>
<td>2A12CZ</td>
<td>Natural aging (CZ)</td>
<td>545</td>
<td>-0.0638</td>
<td>0.269</td>
<td></td>
</tr>
<tr>
<td>2A50 CS</td>
<td>Artificial aging (CS)</td>
<td>513</td>
<td>-0.0845</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>Ti6Al4V (TC4)</td>
<td>Air cooling 800°C</td>
<td>989</td>
<td>-0.07</td>
<td>0.264</td>
<td></td>
</tr>
</tbody>
</table>
### 3.3. Strength Calculation on Crack at the Second Stage

As is well known, the mathematical model to describe a crack in fracture mechanics that it is to adopt these "genes" \( \sigma \) and \( \pi \) and crack variable \( a \), thereby to make the stress intensity factor; Here it can make the model of driving force for the describing behavior of it as following form [11-13].

\[
K_i = \sigma \sqrt{\pi a_i} (\text{MPa} \cdot \sqrt{\text{m}})
\]

Here is sure to explain, the area between the abscissa axis O-1 and the O-2 in fig. 2, the crack size \( a \) from the threshold \( a_\text{th} \) to \( a_\text{c} \) \( (a_\text{th} \leq a \leq a_\text{c} = a_\text{max}) \), there are the mathematic models of the stress intensity factors of two kinds, which are all suited in the section. In addition to above equations (6-7) can be applied, in theory another mathematic models (9-13) are still suitable in the area.

Where the \( K_\text{c} \) is also called as a stress intensity factor of short crack that it is equivalent to \( H_\text{c} \), but their dimensions and units are differences at this same point, then the model of driving force corresponded at that critical point B should be as follow

\[
K_y = \sigma_y \sqrt{\pi a_y} (\text{MPa} \cdot \sqrt{\text{m}})
\]

\[
a_y = \left( \frac{E \times \pi a_y^2}{K y \times \text{mm}} \right)^{2/n} (\text{mm})
\]

Where the \( K_y \) is defined as the critical stress factor that is corresponding to a crack size \( a_y \) of the transitional point, and just is to that size \( a_\text{max} \) as \( a_y \), forming macro crack, is the very at point B to the yield stress \( \sigma_y \) on absissa axis O-II in fig. 2. Here it need yet explain, this factor \( K \), should theoretically be equivalent to above mentioned the \( H_\text{c} \) in first stage, although the dimensions and units between them are different. In addition, the \( K \) is a strength coefficient under monotonic loading, its unit is the \( \text{"MPa} \times \sqrt{\text{mm}} \) \. The \( n \) is an exponent happened strain hardening.

Over the abscissa axis O2-II, the crack over the transitional point size \( a_\text{c} \) is to adopt the \( a_\text{c} \) as the variable. During a crack growth gets to the size of long crack, which it is depicted as corresponding to the curve \( BA_1A_2 \) in figure 2, then its strength criterion should be calculated as following form,

\[
K_\text{lc} = \sigma_\text{lc} \sqrt{\pi a_\text{lc}} (\text{MPa} \cdot \sqrt{\text{m}})
\]

Where the \( y(a / b) \) \[14-15\] is a correcting factor related with the shape and the size of a crack, the \( K_\text{lc} \) is a the critical factor called during the long crack growth, it is corresponded to the critical size \( a_\text{lc} \) on absissa axis O-II in fig. 1, also a the critical value in the second stage. The \([H]\) is defined as the permitted value; the \("n") is a safety factor; and the \( a_\text{lc} \) is a critical size corresponded to the yield stress in the first stage. It should point, because the yield stresses \( \sigma_y \) is the constant of uniquenesses for a material, the critical size of crack \( a_\text{lc} \) can also be applied as an important parameter showed its property. In practice, the critical value \( a_\text{lc} \) could be calculated by means of below formula:

\[
a_\text{lc} = \frac{K_\text{lc}^2}{\sigma_y^2 \times \pi} (\text{mm})
\]

But, for some cast irons, steels of the low toughness and brittle materials, which their behaviours are depicted as curve \( BA_1 \) between the absissa axis OII and the OIII. When their stresses are loaded to this level, or gotten to the critical values \( a_\text{lc} \) of long crack, that may be happened to fracture.

Here has to point the calculating equations mentioned above are only suitable for some brittle materials and strain hardening ones. The calculating error is larger for the materials to happened strain softening.

In the table 2, here are listed to the critical sizes \( a_\text{lc} \) of crack for 13 kinds of materials.

### Table 2. The critical sizes \( a_\text{lc} \) of crack in first stage.

<table>
<thead>
<tr>
<th>Materials</th>
<th>( \sigma_y \text{MPa} )</th>
<th>( \sigma_\text{lc} \text{MPa} )</th>
<th>( K \text{MPa} )</th>
<th>( a_\text{lc} \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot rolled sheet 1005-1009</td>
<td>345</td>
<td>262</td>
<td>531</td>
<td>1.31</td>
</tr>
<tr>
<td>Steel: 1005-1009 Cold-draw sheet</td>
<td>414</td>
<td>400</td>
<td>524</td>
<td>0.546</td>
</tr>
<tr>
<td>RQC-100, Hot rolled sheet</td>
<td>931</td>
<td>883</td>
<td>1172</td>
<td>0.561</td>
</tr>
<tr>
<td>4340, quench and tempering</td>
<td>1241</td>
<td>1172</td>
<td>1579</td>
<td>0.578</td>
</tr>
<tr>
<td>Aluminum 2024-T3</td>
<td>469</td>
<td>379</td>
<td>455</td>
<td>0.46</td>
</tr>
<tr>
<td>30CrMnSiA, (1) Hardening and tempering</td>
<td>1177</td>
<td>1104.5</td>
<td>1475.76</td>
<td>0.568</td>
</tr>
<tr>
<td>LC4CS, (1) Heat treatment-CS</td>
<td>613.9</td>
<td>570.8</td>
<td>775.05</td>
<td>0.587</td>
</tr>
<tr>
<td>40Cr (3)</td>
<td>940</td>
<td>805</td>
<td>1592</td>
<td>1.25</td>
</tr>
<tr>
<td>60Si2Mn, quench, medium-temperature tempering (3)</td>
<td>1504.8</td>
<td>1369</td>
<td>1721</td>
<td>0.503</td>
</tr>
<tr>
<td>QT800-2, (2) normalizing</td>
<td>913</td>
<td>584.3</td>
<td>1777</td>
<td>2.94</td>
</tr>
<tr>
<td>QT800-2, (B) (2) normalizing</td>
<td>748.4</td>
<td>456.5</td>
<td>1440</td>
<td>3.167</td>
</tr>
<tr>
<td>QT600-2, (A) (2) normalizing</td>
<td>677</td>
<td>521.3</td>
<td>1622</td>
<td>3.08</td>
</tr>
<tr>
<td>ZG35 (2) normalizing</td>
<td>572.3</td>
<td>366.3</td>
<td>1218</td>
<td>3.51</td>
</tr>
</tbody>
</table>

Note: \( \sigma \) is strength limit; \( \sigma_y \) is yield limit; (A)-Bar \( \varphi = 30 \) ; (B)-Y-type test specimen; (1)—The Masing’s materials; (2)—The cycle-harden material (3)—Cyclic softening.
It could see from table 2, where the materials from the first to ninth kind are all the steels, their critical sizes of cracks are the range in 0.43~1.42mm to this stage. But the materials from the tenth to thirteenth, which are the nodular cast irons and a cast iron respectively, their critical sizes are the range in 2.94~3.51 mm. In practice as the cast irons are subject to brittle materials which get already the critical values of the fracture under the equivalent yield stress, then those materials will occur to the failures.

For the behaviours of another materials could be over the abscissa axis O3III in figure 1, while they get to own critical values \( a_{3c} \) of long crack, which are usually later than the brittle materials above mentioned, their life are also longer. So the abscissa axis O3III is a boundary that can be thought for the residual intensity sizes between different materials in crack growth process. In this case, that strength criterion (12-14) on crack mentioned above can still be suited for calculations.

When the crack growth over the abscissa axis O3III in figure 2, the strength criterion of crack at later time in the second stage should be as below form

\[
K_2 = y(a/b)\sigma - \sqrt{\pi a} = \frac{1}{n_c}(MPa\sqrt{m}) \quad (15)
\]

\[
K_{2c} = \sigma_f - \sqrt{\pi a_{2c}} (MPa\sqrt{m}) \quad (16)
\]

Where the \( K_2 \) is also the stress factor of crack in the second; the \( K_{2c} \) is a critical factor when it is momentary fracture to the crack, that it is equivalent to the critical stress intensity factor \( K_{3c} \) in fracture mechanics. The \( \sigma_f \) is a fracture stress, the \( a_{2c} \) is a critical crack value where it is at the crossing point A2 on the abscissa axis O3-IV and the straight line \( (A_1A_2) \) in fig. 2.

It should yet explain because the \( K_{2c} \) is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by means of the fracture stress \( \sigma_f \) (table 2). So that the critical size of crack \( a_{2c} \) under corresponding to the true stress \( \sigma_f \) should also be the only data. In theory, it must be there is as following functional relationship,

\[
a_{2c} = \frac{K^2}{\sigma_f^2 \times \pi} (mm) \quad (17)
\]

By the way, when a structure is calculated for a crack size predicting in design, the crack length \( a_2 \) in the equations (11, 14) can be used as following calculable formula,

\[
a_2 = y(a/b)\sigma^2 x \pi t \quad (18)
\]

Here \( t \) is a converting coefficient, 1-damage unit=1mm, \( t=1-mm \).

Recently the author researches to discover that the strength coefficient \( K \) on material subject is virtually the very the critical stress intensity factor \( K_{3c} \) on fracture mechanics under the monotonous loading, if their calculating parameters take all same units. For instance, for the hot rolled sheet 4340 in table 2-3, its \( K=1579MPa \), \( \sigma_f=1655MPa \), \( a_{2c}=0.29mm=2.9\times10^{-4}m \), then if to adopt the calculating model in fracture mechanics to calculate the strength coefficient \( K \), that is as below,

\[
K = \sigma_f - \sqrt{\pi a_{2c}} =1565MPa\sqrt{m} =1579.7(\text{MPa}\sqrt{\text{mm}}) = 49.95\text{MPa}\sqrt{\text{m}} .
\]

On the other hand, the practical calculable critical factor \( K_{3c} \) on fracture mechanics subject should also be,

\[
K_{3c} = \sigma_f - \sqrt{\pi a_{3c}} =1565MPa\sqrt{m} =1578\text{MPa}\sqrt{m} = 49.95\text{MPa}\sqrt{m} .
\]

So, the calculating results are completely consistent. Where its unit to be "\( \text{MPa}\sqrt{\text{m}} \)" of the \( K \) which was called as the strength coefficient in material subject that it is actually the very the critical stress intensity factor in fracture mechanics, and the units of both should be all "\( \text{MPa}\sqrt{\text{m}} \)" or "\( \text{MPa}\sqrt{\text{m}} \). Here it should be point that the experiment values of the \( K_{3c} \) are also 50~63 \( \text{MPa}\sqrt{\text{m}} \).

In the table 3 is listing the critical sizes \( a_{2c} \) of some materials.

<table>
<thead>
<tr>
<th>Materials</th>
<th>( \sigma_f ), MPa</th>
<th>( \sigma_f ), MPa</th>
<th>( \sigma_f ), MPa</th>
<th>( \sigma_f ), MPa</th>
<th>( a_{2c}, \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot rolled sheet 1005-1009</td>
<td>345</td>
<td>262</td>
<td>531</td>
<td>848</td>
<td>0.125</td>
</tr>
<tr>
<td>Steel: 1005-1009 Cold-draw sheet</td>
<td>414</td>
<td>400</td>
<td>524</td>
<td>841</td>
<td>0.124</td>
</tr>
<tr>
<td>RQC-100, Hot rolled sheet</td>
<td>931</td>
<td>883</td>
<td>1172</td>
<td>1330</td>
<td>0.247</td>
</tr>
<tr>
<td>4340, quench and tempering</td>
<td>1241</td>
<td>1172</td>
<td>1579</td>
<td>1655</td>
<td>0.280</td>
</tr>
<tr>
<td>Aluminum 2024-T3</td>
<td>469</td>
<td>379</td>
<td>455</td>
<td>558</td>
<td>0.212</td>
</tr>
<tr>
<td>30CrMnSiA, (1) Hardening and tempering</td>
<td>1177</td>
<td>1104.5</td>
<td>1475.76</td>
<td>1795.1</td>
<td>0.215</td>
</tr>
<tr>
<td>LCACS, (1) Heat treatment-CS</td>
<td>613.9</td>
<td>570.8</td>
<td>775.05</td>
<td>710.62</td>
<td>0.379</td>
</tr>
</tbody>
</table>
4. Calculating Example

A test specimen made of nodular cast iron QT800-2, its strength limit $\sigma_s = 913 MPa$, yield limit $\sigma_y = 584.3 MPa$, $E = 160500$, its material constant $b_1 = -0.083$, $m_1 = 12.078$, the strain hardening exponent $n = 0.2034$, fracture stress $\sigma_f = 946.8 MPa$; If a designer needs to do predicting calculations for a crack strength, to suppose working stress $\sigma = 550 MPa$, the threshold factor $H_{th}$, the critical size $a_{cr}$ of crack, the critical factors $H_{kr}$ of the material.

The processes and steps of calculations are as below.

1. To calculate the crack length $a_1$ under work stress, the threshold size $a_{th}$, the critical size $a_{cr}$ and the $a_{2e}$ of crack for the material, respectively;

2. To calculate the stress factor $H_1$, the threshold factor $H_{th}$, the critical factors $H_{kr}$, $K_{kr}$ and $K_{2e}$ of the crack, respectively;

3. To use the assessment method of the stress factor to do an assessment for it.

The processes and steps of calculations are as below.

1) To calculate the crack length $a_1$ under work stress, the threshold size $a_{th}$, the critical size $a_{cr}$ and the $a_{2e}$ of crack, and to do an assessment for the material.

Here $m_1 = -1$, $b_1 = -1$, $-0.083 = 12.048$

1) According to the formula (1), the threshold size is,

$$a_{th} = \left( \frac{1}{(\pi 1 mm)^{0.2}} \right) = (0.564 mm)^{0.5} \times (0.083) = 0.253 (mm)$$

2) According to the formula (11), the transitional size is as below,

$$a_{tr} = \left( \frac{\sigma_{m}^{(1-a)}}{E} \times \pi \times \sigma_{m}^{1-a} \times K_{tr} \right) = (578.3)^{0.2034} \times 160500 \times \pi \times 0.2034 = 0.2875 (mm)$$

3) According to the formula (14), its critical size of the crack at the first stage is

$$a_{cr} = \frac{K^2}{\sigma \times \pi} = \frac{1777^2}{584.3^2 \times \pi} = 2.944 (mm)$$

4) By the formula (8), its crack size corresponded to the working stress 550 MPa should be as below,

$$a_1 = \frac{550^2}{\sigma \times \pi} = 677$$

So the crack in the material is necessarily to grow.

5) According to the formula (18), its crack length under work stress at the second stage is

$$a_2 = y(a/b) \frac{\sigma \times \pi}{\sigma} = 1.05 \times 550^2 \times \pi = 2.784 (mm)$$

Note: $\sigma$ is a strength limit; $\sigma_y$ is an yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen;

(1)---The Masing’s materials; (2)---The cycle-harden material (3)-Cyclic softening.

For the material, respectively;
\[ [H_i] = H_{ic} / n = 360.21 / n = 120.07 \, (MPa \cdot m^{1/m}) \]

So that \( H_i = 280 > [H_i] = 120.07 (MPa \sqrt{m}) \).

Therefore, the calculating result by the criterion in the first stage, that is not safe.

(3). To calculate the stress intensity factors \( K_1 \) and \( K_2 \), the critical values \( K_{ic} \) and \( K_{ic} \) for long crack in the second stage, respectively.

1) According to the formulas (10) ~ (16), the factor \( K_1 \), the threshold value \( K_{y} \) corresponding the yield stress \( \sigma_y \) and the critical one of long crack are respectively as follow,

- a) For the stress factor \( K_1 \) of the long crack is

\[ K_1 = \gamma(a/b)\sigma \times \sqrt{a} = 1.0 \times 550 \times \sqrt{a} = 51.44 (MPa \sqrt{m}) \]

- b) The threshold value \( K_y \) of the crack corresponding to the yield stress is as below

\[ K_y = \sigma_y \times \sqrt{a} = 584.3 \times \sqrt{a} \times 2.875 \times 10^{-4} = 17.56 (MPa \sqrt{m}) \]

So \( K_1 = 51.44 > K_y = 17.56 (MPa \sqrt{m}) \)

Then, the crack must be to grow.

- c) The critical factor of crack in this stage is

\[ K_{ic} = \sigma_y \times \sqrt{a_{ic}} = 584.3 \times \sqrt{a_{ic}} \times 2.994 \times 10^{-4} = 56.64 (MPa \sqrt{m}) \]

- d) Its permitted value should be,

\[ [K_1] = K_{ic} / n = 56.64 / 3 = 18.9 (MPa \sqrt{m}) \]

- e) On the other hand, the critical value of stress factor when the crack are been at momentary fracture is as follow,

\[ K_{ic} = \sigma_y \times \sqrt{a_{ic}} = 946.8 \times \sqrt{a_{ic}} = 56.2 (MPa \sqrt{m}) \]

Here it can see out that the \( K_{ic} = 56.2 (MPa \sqrt{m}) \)

Its permissible value of crack factor is

\[ [K_y] = K_{ic} / n = 56.2 / 3 = 18.8 (MPa \sqrt{m}) \]

So that \( K_y = 51.43 > [K_y] = 18.8 (MPa \sqrt{m}) \)

Therefore, the data calculated by the criterion for the macro-crack and the result calculated for the design are not all safe to the material.

Here it can see from the above calculations, for the critical factors of a crack, the \( K_{ic} = K_{ic} \), because corresponding to the point of the \( K_{ic} \)-value just is the one of the \( K_{ic} \)-value where they are at same point \( A_2 \) on abscissa axis \( O_i \) IV; but for their critical sizes of cracks, \( a_{ic} \neq a_{ic} \). So when to take the value for the \( [K] \), it must only be calculated by the \( K_{ic} / n \) or \( K_{ic} / n \) with the safe factor “n”.

5. Conclusions

(1). The crack length \( a \) at different stage can be predicted to calculate out by means of the conventional stress and the material constants \( b_i, \sigma_f, \sigma_y, K \) and \( \pi \).

(2). The new threshold size \( a_{ic} \) of the short crack that can show own inherent property, that is depended on the sole material constant \( b_i \), is a calculable one.

(3). For some materials of the brittle and happened strain hardening under monotonous loading, as the yield stresses \( \sigma_y(\sigma_f) \) is the only the constant shown own inherent property, so that the new critical size \( a_{ic} \) of crack depended on the \( \sigma_y \) to be also the sole, and the \( a_{ic} \) is a calculable parameter. Similarly, because the fracture stresses \( \sigma_y \) is the only the constant shown own inherent one, so that the new critical size \( a_{ic} \) of crack depended on the \( \sigma_y \) to be also the sole and calculable.

(4). The critical sizes \( a_{ic} \) and \( a_{ic} \) of cracks are inherent constants shown the materials’ characters; so the critical stress factors \( K_{ic} \) and \( K_{ic} \) based on \( a_{ic} \) and \( a_{ic} \) are also sole values, and are all calculable ones; Their computing models can be used to calculate both for the safe assessment to materials preexisting a flaw and for the predicting crack strength in design process; But it may be calculating error to be larger for the shown strain softening’s ones.

(5). Because corresponding to the factor-value of the \( K_{ic} \) is the very one of the \( K_{ic} \) where they are the same at point \( A_2 \) on abscissa axis \( O_i \) IV; but for their critical sizes of cracks, \( a_{ic} \neq a_{ic} \). So for some materials of the brittle and happened strain hardening when to take the value for the \( [K] \) it must only be calculated by the \( K_{ic} / n \) or \( K_{ic} / n \) with the safe factor \( n \).

(6). The strength coefficient \( K \) on material subject is virtually the very the critical stress intensity factor \( K_{ic} \) under monotonous loading on fracture mechanics, if to take the fracture stress \( \sigma_f \) and the same unit (mm) to calculate for them \( K \) and \( K_{ic} \). But, the unit for the \( K \) is the “MPa \sqrt{mm}”, not foregone that “MPa”.

(7). In those computing models are proposed in the paper, if readers want to apply in engineering calculations, it must yet be checked to combine experiments, and it have to consider the influences for the shape and the size to a crack and a structure.

Acknowledgments

The author thanks sincerity the Zhejiang Guangxin New Technology Application Academy of Electromechanical and Chemical Engineering gives to support and provides research
funds of 500 thousand yuan RMB.

References


