Calculations on Cracking Strength in Whole Process to Elastic-Plastic Materials---The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

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Citation

Abstract
For the decreasing experiments on fatigue, damage and fracture, the author bases on the principles of similar to the genetic genes in the life sciences, adopts the conventional material constants, discovers new constants to some materials of Masing’s and elastic-plastic ones which show inherent properties; uses the theoretical approach which the mathematical derivations, the mechanics analysis and the calculations and analysis of the computer; proposes some new computing models from short to long crack which are the calculable mathematic models on crack problem as the threshold size, the transitional size and the critical ones; and sets up new computing formulas on the strength in different stages; provides the calculating criterions which are defined as the crack size and the stress intensity factor at each stage and in the whole process; and puts forward two kinds of the assessment methods in each stage and in whole process. In addition, to supplement again the comprehensive figure of the material behaviours; to give yet a detailed calculating example applied two of kinks for a safety assessment. This works may be there are practical significances for make linking and communication among the modern the material subject, fracture mechanics and damage mechanics; for to promote engineering applications.

1. Introduction

As is well-known, the traditional materials mechanics and the structure mechanics that is the calculable subjects and had made valuable contributions for every industrial engineering designs and calculations, but they cannot solve and calculate the strength and life problems in engineering materials and structures existed cracks. The fracture mechanics and the damage mechanics, which are just based on the local problems as researched objects for the local defects inside materials, to research the driving forces, the crack propagating (damage) rates and the life predictions in the crack growth process. But nowadays, latter these disciplines are all mainly dependent on tests.

The author thinks that in the mechanics and the engineering fields where exists such a scientific law as similar to genetic elements and cloning technology in the life sciences and has used the theoretical approach for similar principles, proposed some calculation
models [1-2], recently sequentially discovered some new scientific laws to the Masing’s and the elastic-plastic materials, and provides some new calculable models for the crack growth driving force, the calculating criterions and the assessment methods of the strength problems in the whole process which are from short to long crack. This is to try to make the modern fatigue, the damage mechanics and the fracture mechanics gradually become such calculable disciplines as the traditional materials mechanics. That way, it may be there are practical significances for decreasing experiments, for stinting manpower and funds, for promoting and developing engineering and applying it to relevant disciplines

2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the “Comprehensive figure of materials behaviors” as Figure 1 [3-4] (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes $O'1''$, $O'1'$, $O'I$, $O'I''$, $O'III$, $O'IV$ and a bidirectional ordinate axis $O'1$. For the area between the axes $O'1''$ and $O'1'$, it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes $O'I$ and $O'I''$, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the $O'II$, the $O'III$ and $O'IV$ where they are calculated and applied by the macro-damage mechanics and the macro-fracture mechanic. But for between the axes $O'I$ and $O'I''$, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

On the abscissa axis $O' I''$, it is represented with parameters the stress $\sigma$ and the strain $\epsilon$ as variables. On the abscissa axis $O'I$ there are the fatigue limits $\sigma_n$ at point “a” ($\sigma_n = 0$) and “b” ($\sigma_n \neq 0$) that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axis $O'I''$ there are points “A” and “D” that just are the locations placed at threshold values as some materials. On the abscissa axes $O'I$ and $O'I''$ that they could all represented as variables with the stress intensity factor range $\Delta K$ of short crack, and the strain intensity factor $\Delta l$, and the stress intensity factor range $K$ of long crack. On the other hand, they both could yet represented as variables with the short crack $a_i$ and the long crack $a_c$, (or damage $D_i$ and $D_c$). And here there are material constants of two that they are defined as the critical factor $K_v$ of crack-stress-intensity and the critical factor $K_c$, of the damage-stress-intensity at the first stage, where that are just two parameters corresponded to the transitional size $a_w$ of crack or the transitional value of damage $D_w$, they are just placed at point at the point $B$ ($\sigma_n = 0$) and at point $B_1$ ($\sigma_n \neq 0$) corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors, but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level.

On the abscissa axis $O'III$, it is represented as variable with the stress intensity factor range $\Delta K$, (or $\Delta \delta$) of long crack; it is a boundary of the sizes as the residual strength between some elastic-plastic materials and brittle materials. On this axes $O'III$ there are the critical points at $D_i(D_{ic})$, $A_i(A_{ic})$ and $C_1$. On abscissa $O'IV$, the point $A_t$ is corresponding to the fatigue strength coefficient $\sigma_{ct}$, the critical stress intensity factor values $K_{ic}(K_{2c})$ and the critical values $D'_{ct}$ and $a_{ct}$ for the mean stress $\sigma_n = 0$ ; the point $D_t$ is corresponding to the $\sigma_n \neq 0$ ; the point $C_2$ corresponding to the fatigue ductility coefficient $\epsilon_{ct}$ and critical crack tip open displacement value $\delta_{ct}$ ; the point $F$ corresponding to a very high cycle fatigue strength coefficient $\sigma_{ct}$. In addition on the same $O'IV$, there are yet another critical values $J'_{ct}(J_{ct})$, etc. in the long crack propagation process.

For an ordinate axis, an upward direction along the ordinate axis is represented as crack growth rate $da / dN$ or...
damage growth rate \( \frac{dD}{dN} \) in each stage and the whole process. But a downward direction is represented as life in each stage and the whole lifetime \( \Sigma N \).

\[
\text{damage growth rate } \frac{dD}{dN}
\]

**Figure 1.** Comprehensive figure of material behaviors 1 (Or called calculating figure of material behaviors or bidirectional combined coordinate system and simplified schematic curves in the whole process).

In the area between axes \( O' I' \) and \( O_2 I \), it is the fatigue history from un-crack to micro-crack initiation. In the area between axes \( O_1 I \) and \( O_2 II \), it is the fatigue history relative to life \( N_{\text{mic-mac}} \) from micro-crack growth to macro-crack forming. Consequently, the distance \( O_2 - O \) on ordinate axis is as the history relating to life \( N_{\text{mac}} \) from grains size to...
micro-crack initiation until macro-crack forming; the distance \( O_1 \cdot O \) is as the history relating to the lifetime life \( \sum N \) from micro-crack initiation until fracture.

In the crack forming stage, the partial coordinate system made up of the upward and the ordinate axes \( O \), \( O_1 \) and the abscissa axes \( O_1 \), \( O_1 \) II and \( O_2 \) II is represented as the relationship between the crack growth rate \( d\sigma_1 / dN \) (or the short crack growth rate \( d\sigma_1 / dN \)) and the crack-stress factor range \( \Delta H_1 \) (or the damage strain factor range \( \Delta H \)). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis \( O_2 \), \( O_2 \) I and abscissa \( O_2 \) II, \( O_3 \) I and \( O_3 \) IV at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range \( \Delta K \), J-integral range \( \Delta J \) and crack tip displacement range \( \Delta \delta \) ( \( d\sigma_1 / dN \)- \( \Delta K \), \( \Delta J \) and \( \Delta \delta \)). Inversely, the coordinate systems made up of the downward ordinate axis \( O_4 \), \( O \) and the abscissa axes \( O_4 \), \( O_1 \) II, \( O_1 \) III, \( O_2 \) II, \( O_2 \) I and \( O \) I are represented respectively as the relationship between the \( \Delta H_1 \), \( \Delta K \)-range and each stage life \( N_1 \), \( N_2 \) and the lifetime \( \Sigma N \) (or between the \( \Delta \varepsilon_p \), \( \Delta \delta \)-range and the life \( \Sigma N \)).

2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve \( ABA_1 \) is represented as the varying laws as the behaviours of the elastic materials or some elastic-plastic ones under high cycle loading in the macro-crack-forming stage (the first stage); positive direction \( ABA_1 \) represented as the relations between \( d\sigma_1 / dN \) (or \( d\sigma_1 / dN \)) \( \Delta H_1 \); inverted \( ABA_2 \), between the \( \Delta H_1 \), \( N_1 \) and the curve \( CBC_1 \) is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction \( CBC_1 \) is represented as the relations between \( d\sigma_1 / dN \) \( \Delta H_1 \); inverted \( CBC_2 \), the relations between the \( \Delta \varepsilon_p \), \( N_1 \).

The curve \( A_1A_2 \) in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction \( A_1A_2 \) showed as \( d\sigma_1 / dN \)- \( \Delta K \) ( \( \Delta J \)); inverted \( A_1A_2 \), between the \( \Delta K_2 \), \( \Delta J \)- \( \Delta \delta \). The \( C_1C_2 \) is showed as: the positive, relation between the \( d\sigma_1 / dN \)- \( \Delta \delta \) under low-cycle loading, inverted \( C_1C_2 \), between \( \Delta \delta \) \( \Delta J \)- \( \Delta \delta \). By the way, the curves ' \( Dbcd \)' ( \( \sigma \) = 0) and the ' \( Aae \)' ( \( \sigma \) = 0) are represented as the laws under the very high cycle fatigue.

It should yet point that the curve \( A_1A_2 (1-1') \) is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, \( d\sigma_1 / dN \leq 10^{-4} \)); the curve \( DD_2D \) (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, \( d\sigma_1 / dN \leq 10^{-4} \)). The curve \( CC_1C_2 (2-2') \) is depicted as the rate curve under low cycle loading. The curve \( caA_1A_1 \) is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ( \( \sigma \) = 0, \( d\sigma_1 / dN < 10^{-7} \)), the curves \( dcbDD_2D \) and \( dcbF_2 \) are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ( \( \sigma \) = 0, \( d\sigma_1 / dN < 10^{-7} \)). Inversely, the curve \( A_1A_1A \) is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, \( N \leq 10^6 \)), the curve \( D_2D_2D_2D \), as the lifetime curve under unsymmetrical cycle loading ( \( N \leq 10^6 \)). The curve \( C_1C_1C \) is depicted as the lifetime curve under low cycle loading ( \( N \leq 10^6 \)). On the other hand, the curve \( A_1A_1BAae \) is as the lifetime one in whole process included very high cycle fatigue ( \( \sigma \) = 0, \( N > 10^6 \)), the curves \( D_2D_2D_2D_2D \) and \( F_2bcd \) are all depicted as the lifetime ones in whole process ( \( \sigma \) = 0, \( N > 10^7 \)).

It should also be explained that the comprehensive figure 1 of the materials behaviours may be as a complement for the Fundamental knowledge of a material subject; that is a tool to design and calculate for various kinds of structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.


Here the variables for describing the crack growth process that are defined as follows:

1) From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage, the changing process of variable \( a \) corresponded to the short crack, it is represented with the curves \( aAB \) or \( AB \) in figure 1;

2) From the macro-crack propagation to the fracture process is defined in the crack growth stage, or defined in the second stage. The changing process corresponded in the long crack variable \( a \) in this stage, it is expressed in curve \( BA: A_1A_2 \);

3) From a short to long crack growth until fracture for a material, to adopt the \( a \) as the variable in the whole process, it is corresponded to curve “ \( aAA1: A_2 \)”.

3.1. About Threshold Size on Crack

In the figure 1 it is seen, for the general steels, they are always shown with various characters in the each stages. The author discovers, which they all have the threshold sizes \( a_0 \) of the crack growth as in table 1, and are only depended on the constant \( b_0 \) shown a material property. It should point, the locations of the threshold sizes \( a_0 \) of the cracks, some
materials are near at the point A where it is at the intersection one between the straight line AB and the abscissa axis O1 in figure 1; and other ones, near at the point a where it is at the intersection point between the straight line “Aa” and the abscissa axis O ’I. And the threshold size \( a_{th} \) can be calculable parameter as following formula [5],

\[
a_{th} = \left( \frac{1}{\pi \cdot \text{mm}} \right)^{0.5} (\text{mm})
\]

(1)

Or

\[
a_{th} = \left( \frac{1}{\pi \cdot \text{mm}} \right)^{0.5} (\text{mm})
\]

(2)

Where, the range of the \( a_{th} \)-length calculated by equations (1-2) is the 0.21 – 0.275 (mm) in table 1. Under the condition shown linear-elastic behaviour for some materials, by the crack size \( a_{th} \) is combined with the induced stress \( \sigma \) under the monotonic loading, here it can make a model of the driving force of a crack growth, that is as following form,

\[
H_{th} = \sigma \cdot a_{th}^{1/n} = \sigma \cdot n_{th} \sqrt{a_{th}} (\text{MPa} \cdot \text{m}^{1/n})
\]

(3)

In the formula (2-3), \( m_{th} = -1/b_{th} \), the \( m_{th} \) is also only a constant shown a property, it is same with the \( b_{th} \) also a sole one. Here it should explain that the symbol of the constant \( b_{th} \) is a negative value; and the \( m_{th} \) is a positive one. The \( H_{th} \) is defined as the stress intensity factor of short crack. In an ordinary way, the \( \sigma \cdot a_{th}^{1/n} \) may be: the \( \sigma \cdot a_{th}^{1/n} \leq \sigma \cdot n_{th} \sqrt{a_{th}} \) or \( \sigma \cdot a_{th}^{1/n} \geq \sigma \cdot n_{th} \sqrt{a_{th}} \), then, the strength criterions for them are as below,

\[
H_{th} = \sigma \cdot a_{th}^{1/n} = \sigma \cdot n_{th} \sqrt{a_{th}} \leq H_{th} (\text{MPa} \cdot \text{m}^{1/n})
\]

(4)

Or

\[
H_{th} = \sigma \cdot a_{th}^{1/n} = \sigma \cdot n_{th} \sqrt{a_{th}} \geq H_{th} (\text{MPa} \cdot \text{m}^{1/n})
\]

(5)

Where the \( H_{th} \) is defined as the threshold factor of a crack. If the \( H_{th} < H_{th} \), the crack in a material does not grow; and the \( H_{th} > H_{th} \), the crack is sure to grow.

**Table 1.** Data calculated threshold sizes \( a_{th} \) for crack growth.

<table>
<thead>
<tr>
<th>Materials [6-7]</th>
<th>Heat- treatment</th>
<th>( \sigma_{p} ) MPa</th>
<th>( \sigma_{u} ) MPa</th>
<th>( b_{th} )</th>
<th>( m_{th} )</th>
<th>( a_{th}, \text{mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>①30CrMnSiA [17]</td>
<td>Hardening and Tempering</td>
<td>1177</td>
<td>1104.5</td>
<td>-0.0859</td>
<td>11.64</td>
<td>0.251</td>
</tr>
<tr>
<td>②LC4CS</td>
<td>CS</td>
<td>613.9</td>
<td>570.8</td>
<td>-0.0727</td>
<td>13.76</td>
<td>0.262</td>
</tr>
<tr>
<td>2024-T3 Aluminum</td>
<td>469</td>
<td>379</td>
<td>-0.124</td>
<td>8.06</td>
<td>0.218</td>
<td></td>
</tr>
<tr>
<td>③QT800-2</td>
<td>Normalization</td>
<td>913</td>
<td>584.3</td>
<td>-0.083</td>
<td>12.05</td>
<td>0.253</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering</td>
<td>1241</td>
<td>1172</td>
<td>-0.076</td>
<td>13.16</td>
<td>0.259</td>
</tr>
<tr>
<td>④40Cr</td>
<td>in oil-quenching 850°C, temper 560°C, air cooling</td>
<td>940</td>
<td>805</td>
<td>0.12</td>
<td>8.33</td>
<td>0.222</td>
</tr>
<tr>
<td>1005-1009</td>
<td>Hot rolled sheet</td>
<td>345</td>
<td>262</td>
<td>-0.109</td>
<td>9.174</td>
<td>0.231</td>
</tr>
<tr>
<td>1005-1009</td>
<td>Cold-draw sheet</td>
<td>414</td>
<td>400</td>
<td>-0.073</td>
<td>13.7</td>
<td>0.262</td>
</tr>
<tr>
<td>Steel: 1020</td>
<td>Hot rolled sheet</td>
<td>441</td>
<td>262</td>
<td>-0.12</td>
<td>8.33</td>
<td>0.222</td>
</tr>
<tr>
<td>BQC-100</td>
<td>Hot rolled sheet</td>
<td>931</td>
<td>883</td>
<td>-0.07</td>
<td>14.3</td>
<td>0.264</td>
</tr>
<tr>
<td>9262</td>
<td>anneal</td>
<td>924</td>
<td>455</td>
<td>-0.071</td>
<td>14.1</td>
<td>0.263</td>
</tr>
</tbody>
</table>

### 3.2. Calculation on Crack Strength in the First Stage

The material property had discovered by Masing, that is as well known [8-9]. For some called Masing’s materials, elastic-plastic and happened strain hardening ones, which are corresponded to the curve 1(\( AB \)) or aAB in the first stage between abscissa axis O’ and the O: II in figure 1, here can set up a criterion of the crack strength for it, that is as below form,

\[
H_{i} = \sigma \cdot a_{i}^{1/n} \leq [H_{i}] = H_{ic}/n_{i} (\text{MPa} \cdot \text{m}^{1/n})
\]

(6)

\[
H_{ic} = \sigma_{p} \times n_{th} \sqrt{a_{th}} (\text{MPa} \cdot \text{m}^{1/n})
\]

(7)

The \( H_{ic} \) in (6) is defined as a critical value of the stress intensity factor which is corresponded to the yield stress, so it must be the only a constant showing a material property. And the \( H_{ic} \) is also the critical value corresponded to the stress factor \( K_{i} \), and the transitional size \( a_{th} \) mentioned following text, their locations are near at points B on abscissa axis O2-II (in Fig.1) where it is on the boundary between the short crack and the long crack or between the first stage and the second stage.

The behabirous of crack for a material, it is changed with the loading ways and the stress levels, the crack size grewed before the yield stress \( \sigma_{p} \) under the monotonous loading can be caculated by following formula,

\[
a_{i} = \frac{\sigma_{p}^{2}}{\sigma_{tr} \times \pi} \nu \times (\text{mm})
\]

(8)

Where the \( \sigma_{tr} = \sigma_{p} \) is a stress value of proportional limit approximating elastic limit \( \sigma_{p} \), it can also approximatively be took in \((0.955 \sim 0.975)\sigma_{p} \) of the yield stress, as the data is to lack. The \( \nu \) is a conversion coefficient of the unit, \( \nu = 1\text{mm} \).

### 3.3. Calculation on Crack Strength in the Second Stage

1. The critical stress factor \( K_{i} \) and the critical size \( a_{th} \) at the transitional point
As is well known, the mathematic model to describe a crack in fracture mechanics [10-12], that is to adopt these “genes” $\sigma$ and $\pi$ and crack variable $a$, thereby to make the stress intensity factor $K_i = \sigma \times \sqrt{\pi a_i} (MPa \cdot \sqrt{m})$ that is as following form, 

$$K_i = \sigma \times \sqrt{\pi a_i} (MPa \cdot \sqrt{m}) \quad (9)$$

As explanation mentioned above, in the area between the abscissa axis O1-I and the O2-II in figure 1, where that stress factor $H_1$ of crack is equivalent to the $K_i$, but their dimensions and units are differences at this same point, they are corresponded to the variable $a_i$ from the threshold the $a_{th}$ to the $a_c$ in the section ($a_{th} \leq a_i \leq a_c = a_{max}$), it can set up two kinds of the mathematic models called the stress factors. In addition to above equations (6-8) can be applied; in theory below the mathematic models (9-11) are still suitable in the area.

For some Masing’s materials, their transitional sizes $a_{tc}$ at transition points between the elastic and the plastic strain, for which are also can be calculated and can be means by of the following equation,

$$a_{tc} = \left(\sigma_{\pi}^{12} \times \frac{E \times \pi^{1/2}}{K^{1/2}}\right)^{2m_{tc}} \times 2\pi a_{tc} (mm) \quad (10)$$

Here, the $K$ is a strength coefficient under monotonic loading, the $n$ is an exponent happened strain hardening. Then the model of driving force at this point should be as follow

$$K_y = \sigma_y \times \sqrt{\pi a_{yc}} (MPa \cdot \sqrt{m}) \quad (11)$$

Where the $K_y$ is called as the critical stress factor that is relevant to the crack size $a_{yc}$ at transitional point, just is corresponding to size $a_{max} (= a_c)$ of forming macro crack, this factor $K_y$ should theoretically be equivalent to above mentioned the $H_1$, although the dimensions and units between them are differences.

2. The critical stress factor $K_{ic}$ and the critical size $a_{ic}$ during long crack growth

$$K = \sigma \times \sqrt{\pi a_{ic}} = 584.3MPa \sqrt{\pi 2.944(mm)} = 1777MPa\sqrt{mm} = 56.2MPa\sqrt{m}$$

On the other hand, the practical calculable critical factor $K_{ic}$ on fracture mechanics subject should also be,

$$K_{ic} = \sigma \times \sqrt{\pi a_{ic}} = 584.3MPa \sqrt{\pi 2.944(mm)} = 1777MPa\sqrt{mm} = 56.2MPa\sqrt{m}$$

So, the calculating results are completely consistent. It can see from the table 5, this data is close to experiment one ($K_{ic} = 47.6MPa\sqrt{m}$). Therefore, its unit of the $K$ to be “MPa” which was called as strength coefficient on material subject is actually the very critical stress intensity factor on fracture mechanics, and the unit of which should be all

Over the abscissa axis O2-II, the crack over the transitional point size $a_{ic}$ is to adopt the $a_c$ as the variable. When the crack growth gets to the macro crack, where it is corresponding to the curve $BA_A$, in figure 1. But, for some cast irons, steels of the low toughness and brittle materials, which their behaviours are depicted in curve $BA$ between the abscissa axis OII and the OiiII. When their stresses are loaded to this level, or gotten to the critical values $a_{ic}$ of macro crack, that may be happened to fracture, then its strength criterion should be used as following form,

$$K_i = \frac{y(a/b) \cdot \sigma \cdot \sqrt{\pi a_{ic}}}{K_{ic}} \leq \frac{K_i}{n_i \pi} (MPa\sqrt{m}) \quad (12)$$

$$K_{ic} = \sigma \times \sqrt{\pi a_{ic}} (MPa \cdot \sqrt{m}) \quad (13)$$

Where the $y(a/b)$ [13-14] is a correcting factor related with the shape and the size of a crack; the $K_{ic}$ is a the critical factor called during the long crack growth, that is corresponded to the critical size $a_{ic}$ on abscissa axis OiiIII in fig. 1, also a the critical value in the second stage. It should point, because the yield stresses $\sigma_y$ in the formula (13) is the constant of uniquenesses for a material, the critical size of crack $a_{ic}$ can also be applied as an important parameter showed its property. In practice, the critical value $a_{ic}$ could be calculated by means of below formula:

$$a_{ic} = \frac{K^2}{\sigma \times \pi} (mm) \quad (14)$$

The author researches to discover that the strength coefficient $K$ in the material and the fatigue subjects is virtually the very critical stress intensity factor $K_{ic}$ in fracture mechanics under monotonous loading, if they take same calculating parameter and unit. For instance, for the QT800-2 in table 2, its $\sigma = 584.32MPa$ , $a_{ic} = 2.944mm = 2.944 \times 10^{-3}m$, then if to adopt the calculating model in fracture mechanics to calculate the strength coefficient $K$, that is as below,

"MPa\sqrt{mm}".

Here has to point the above equations, the data error in the calculations is bigger for those materials happened strain softening.

In the table 2, here are listed to the critical sizes $a_{ic}$ of crack for 13 kinds of materials.
3. The critical stress factor $K_{2c}$ and the critical size $a_{2c}$ at the momentary fracture

For the behaviours of another materials could be over the abscissa axis OI\(\text{III}\) in figure, they get to own critical values $a_{2c}$ of long crack which are usually later than the brittle materials above mentioned, their life are also longer. So the abscissa axis OI\(\text{III}\) is a boundary that can be the residual intemnsity sizes between different materials in crack growth process. In this case that strength criterion (12-14) on crack can still be suted for calculations.

When the crack growth over the abscissa axis OI\(\text{III}\) to the OI\(\text{IV}\) in figure 1, the strength criterion of crack at later time in the second stage should be calculated by following form,

$$K_2 = y(a/b)\sigma \cdot \sqrt{\pi a_2} \leq [K] = \frac{K_{2c}}{n}(\text{MPa} \sqrt{\text{m}})$$

$$K_{2c} = \sigma_f \cdot \sqrt{\pi a_{2c}} (\text{MPa} \sqrt{\text{m}})$$

Where the $K_2$ is also defined as the stress factor of crack in the second stage, the $K_{2c}$ is called as a critical factor at the momentary fracture, that it is the very the critical stress intensity factor $K_i$ in fracture mechanics. The $\sigma_f$ is a fracture stress, the $a_{2c}$ is a critical crack size at the momentary fracture where it is at the crossing point $A_2$ of the abscissa axis O4-IV and the straight line $A_1 A_2$ in Fig. 1.

It should yet explain because the $K_{2c}$ is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by mens of the fracture stress $\sigma_f$. So that the critical size $a_{2c}$ at the momentary fracture under corresponding to true stress $\sigma_f$ should also be the only data. In theory, it must be to exist as following functional relationship,

$$a_{2c} = \frac{K_{2c}^2}{\sigma_f^2 \sqrt{\pi}} (\text{mm})$$

(17)

In the table 3, to list the critical sizes $a_{2c}$ at the momentary fracture in second stage to some materials.

By the way, when a structure is being calculated in design, if the crack grows to the stage of long crack, then the crack sizes $a_i$ and $a_s$ in above equations (9, 12, 15) can also be calculated for the predictions by following formula

$$a_2 = \frac{\sigma_i^2 \pi}{\sigma_f^2} (\text{mm})$$

(18)

*Table 2. The critical sizes $a_{2c}$ during long crack growth of some materials in second stage.*

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\sigma_{cr}, \text{MPa}$</th>
<th>$\sigma_f, \text{MPa}$</th>
<th>$K_{2c}, \text{MPa}$</th>
<th>$a_{2c}, \text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot rolled sheet 1005-1009</td>
<td>345</td>
<td>262</td>
<td>531</td>
<td>1.31</td>
</tr>
<tr>
<td>Steel: 1005-1009 Cold-draw sheet</td>
<td>414</td>
<td>400</td>
<td>524</td>
<td>0.546</td>
</tr>
<tr>
<td>RQC-100, Hot rolled sheet</td>
<td>931</td>
<td>883</td>
<td>1172</td>
<td>0.651</td>
</tr>
<tr>
<td>4340, quench and tempering</td>
<td>1241</td>
<td>1172</td>
<td>1579</td>
<td>0.578</td>
</tr>
<tr>
<td>Aluminum 2024-T3</td>
<td>469</td>
<td>379</td>
<td>455</td>
<td>0.46</td>
</tr>
<tr>
<td>30CrMnSiA, (1) Hardening and tempering</td>
<td>1177</td>
<td>1104.5</td>
<td>1475.76</td>
<td>0.568</td>
</tr>
<tr>
<td>LC4CS, (1) Heat treatment-CS</td>
<td>613.9</td>
<td>570.8</td>
<td>775.05</td>
<td>0.587</td>
</tr>
<tr>
<td>40Cr(3)</td>
<td>940</td>
<td>805</td>
<td>1592</td>
<td>1.25</td>
</tr>
<tr>
<td>60Si2Mn, quench, medium-temperature tempering (3)</td>
<td>1504.8</td>
<td>1369</td>
<td>1721</td>
<td>0.503</td>
</tr>
<tr>
<td>QT800-2, (2) normalizing</td>
<td>913</td>
<td>584.3</td>
<td>1777</td>
<td>2.944</td>
</tr>
</tbody>
</table>

Note: $\sigma_{cr}$ is a strength limit; $\sigma_f$ is an yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen; (1)—The Masing’s materials; (2)—The cycle-harden materials; (3)—Cyclic softening materials.

*Table 3. The critical sizes $a_{2c}$ at the momentary fracture in second stage.*

<table>
<thead>
<tr>
<th>Materials</th>
<th>$\sigma_{cr}, \text{MPa}$</th>
<th>$\sigma_f, \text{MPa}$</th>
<th>$K_{2c}, \text{MPa}$</th>
<th>$\sigma_f, \text{MPa}$</th>
<th>$a_{2c}, \text{mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hot rolled sheet 1005-1009</td>
<td>345</td>
<td>262</td>
<td>531</td>
<td>848</td>
<td>0.125</td>
</tr>
<tr>
<td>Steel: 1005-1009 Cold-draw sheet</td>
<td>414</td>
<td>400</td>
<td>524</td>
<td>841</td>
<td>0.124</td>
</tr>
<tr>
<td>RQC-100, Hot rolled sheet</td>
<td>931</td>
<td>883</td>
<td>1172</td>
<td>1330</td>
<td>0.247</td>
</tr>
<tr>
<td>4340, quench and tempering</td>
<td>1241</td>
<td>1172</td>
<td>1579</td>
<td>1655</td>
<td>0.280</td>
</tr>
<tr>
<td>Aluminum 2024-T3</td>
<td>469</td>
<td>379</td>
<td>455</td>
<td>558</td>
<td>0.212</td>
</tr>
<tr>
<td>30CrMnSiA, (1) Hardening and tempering</td>
<td>1177</td>
<td>1104.5</td>
<td>1475.76</td>
<td>1795.1</td>
<td>0.215</td>
</tr>
<tr>
<td>LC4CS, (1) Heat treatment-CS</td>
<td>613.9</td>
<td>570.8</td>
<td>775.05</td>
<td>710.62</td>
<td>0.379</td>
</tr>
<tr>
<td>QT8000-2, (2) normalizing</td>
<td>913</td>
<td>584.3</td>
<td>1777</td>
<td>946.8</td>
<td>1.121</td>
</tr>
<tr>
<td>40Cr(3)</td>
<td>940</td>
<td>805</td>
<td>1592</td>
<td>1305</td>
<td>0.474</td>
</tr>
<tr>
<td>60Si2Mn, quench, medium-temperature tempering (3)</td>
<td>1504.8</td>
<td>1369</td>
<td>1721</td>
<td>2172.4</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Note: $\sigma_{cr}$ is a strength limit; $\sigma_f$ is an yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen; (1)—The Masing’s materials; (2)—The cycle-harden materials; (3)—Cyclic softening materials.
3.4. Calculations on Cracking Strength in Whole Process

Due to the behaviors shown by the materials are different at each stage, their dimensions and units in computing models about strength problem are also differences. But the author recently researches finding, on account of the critical sizes in each stage are all inherent constants, they are the only depended on the material properties, therefore which exist necessary the critical sizes of crack in each stage. So that the total critical sizes of crack, which is calculated with calculable formulas, as the material constants in whole process should be applied as the important parameter for the assessment, here it proposes as follow:

\[ a_{w} = a_{t} + a_{c} \]  \hspace{1cm} (19)

Or

\[ a_{w} = a_{t} + a_{c} \]  \hspace{1cm} (20)

It should point that the data in table 4 is the material constants for six kinds of materials, which they are under the monotonous loading. And the data in table 5 is called as the critical size of crack, which is calculated with calculable formulas.

**Table 4. Data of material’s performances.**

<table>
<thead>
<tr>
<th>Materials [15-16]</th>
<th>Heat-treatment</th>
<th>( \sigma_{e} ) MPa</th>
<th>( \sigma_{f} ) MPa</th>
<th>( K_{c} ) MPa m</th>
<th>( n_{i} )</th>
<th>( E ) MPa</th>
<th>( \sigma_{f} ) MPa</th>
<th>( b_{i} )</th>
<th>( m_{i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 30CrMnSiA</td>
<td>Hardening and Tempering CS</td>
<td>1177</td>
<td>1104.5</td>
<td>1475.76</td>
<td>0.063</td>
<td>203005</td>
<td>1795.1</td>
<td>-0.0859</td>
<td>11.64</td>
</tr>
<tr>
<td>(2) 2024-T3 Aluminum</td>
<td>Normalization</td>
<td>469</td>
<td>379</td>
<td>455</td>
<td>0.032</td>
<td>70329</td>
<td>558</td>
<td>-0.124</td>
<td>8.06</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>913</td>
<td>584.3</td>
<td>1777</td>
<td>0.2034</td>
<td>160500</td>
<td>946.8</td>
<td>-0.083</td>
<td>12.05</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>1241</td>
<td>1172</td>
<td>1579</td>
<td>0.066</td>
<td>1655</td>
<td>0.076</td>
<td>13.16</td>
<td></td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>940</td>
<td>805</td>
<td>1592</td>
<td>0.173</td>
<td>1305</td>
<td>0.12</td>
<td>8.33</td>
<td></td>
</tr>
</tbody>
</table>

(1)---The Masing’s materials; (2)---The strain hardening material; (3)---it is a material happened the strain softening during cyclic loading.

**Table 5. Calculated data by means of the calculable formulas.**

<table>
<thead>
<tr>
<th>Materials [15-16]</th>
<th>Heat-treatment</th>
<th>( \sigma_{e} ) MPa</th>
<th>( \sigma_{f} ) MPa</th>
<th>( K_{c} ) MPa m</th>
<th>( n_{i} )</th>
<th>( K_{Wc} )</th>
<th>( \sigma_{f} ) MPa</th>
<th>( b_{i} )</th>
<th>( m_{i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) 30CrMnSiA</td>
<td>Hardening and Tempering CS</td>
<td>0.251</td>
<td>46.64</td>
<td>0.568</td>
<td>46.64</td>
<td>0.212</td>
<td>92.05</td>
<td>0.837</td>
<td>98.9</td>
</tr>
<tr>
<td>(1) 2024-T3 Aluminum</td>
<td>Normalization</td>
<td>0.262</td>
<td>24.51</td>
<td>0.58</td>
<td>24.71</td>
<td>0.3786</td>
<td>36.7</td>
<td>0.849</td>
<td>38.5</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>0.218</td>
<td>14.4</td>
<td>0.459</td>
<td>14.4</td>
<td>0.212</td>
<td>25.8</td>
<td>0.68</td>
<td>31</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>0.253</td>
<td>56.2</td>
<td>2.944</td>
<td>56.2</td>
<td>0.121</td>
<td>58.56</td>
<td>4.318</td>
<td>47.6</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>0.259</td>
<td>49.94</td>
<td>0.578</td>
<td>49.94</td>
<td>0.29</td>
<td>60</td>
<td>1.127</td>
<td>50</td>
</tr>
<tr>
<td>4340</td>
<td>Quenching and tempering in oil-quench 850°C, temper 560°C, air cooling</td>
<td>0.222</td>
<td>50.36</td>
<td>1.245</td>
<td>50.36</td>
<td>0.475</td>
<td>58.64</td>
<td>1.941</td>
<td>154</td>
</tr>
</tbody>
</table>

(1)---The Masing’s materials; (2)---The cycle harden material; (3)---it is a material happened the strain softening under cyclic loading;

(4) \( K_{c} \) is the experiment data of critical stress intensity factor \( (\text{MPa}\sqrt{\text{m}}) \); \( K_{Wc} \); \( \sigma_{f} \); \( D_{f} \); \( K_{i} \)---Units are the “\( \text{mm} \)”; \( \sigma_{f} \); (5)---Units are the “\( \text{MPa} \)".

It can be seen from the above table 5, for the critical factors \( K_{c} \) and the \( K_{2c} \) of the crack, \( K_{c} = K_{2c} \), because corresponding to end point of the \( K_{c} \) -value just is the starting point of the \( K_{2c} \)-value, where they are the same at point A2 on abscissa axis O4 IV; but for their critical sizes of sizes, \( a_{2c} \neq a_{c} \). So, when to take the value for the \( \{K\} \) it should only be calculated by the \( K_{r} / n \) or \( K_{w} / n \) with the safe factor \( n \).

Here can be compared in calculated data with the experiment ones \( K_{c} \), and can be seen out: 1) the calculated value for the nodular cast iron QT800-2 which is materials happened strain hardening, the calculated \( K_{2c} \) of it closes to the experimental data \( K_{c} \); the \( K_{2c} \) for the steel 4340 by quenching and tempering is also close to the experimental \( K_{c} \); the calculated values \( K_{Wc} \) for Masing’s materials 30CrMnSiA and LC4CS, between the calculated \( K_{Wc} \) and the \( K_{c} \), both is approximating; But for the steel 40Cr of shown strain softening, both error between the calculated and the experimental data is bigger.

The calculating criterions about the crack strength in whole process there are two kinds of ways: 1) The assessment method for the stress intensity factor; 2) The assessment method for the crack size. It should explain, if to apply the assessment method for the crack size, it must use the variable \( a \) in whole process and those relevant material constants, it has to adopt their critical sizes shown different performances at each stage.

For instance, for a material there are following cases: 1. The assessment method for the stress intensity factor

\[ H_{i} = \sigma \sqrt[2]{a_{i}} \leq H_{a} \quad (\text{MPa} \sqrt{\text{m}}) \]

Or

\[ H_{i} = \sigma \sqrt[2]{a_{i}} \geq H_{a} \quad (\text{MPa} \sqrt{\text{m}}) \]

(2) The condition 2

If to use the assessment method of the stress intensity factor in the first stage, it is as below,
\[ H_i = \sigma \times \sqrt{a} \leq [H_i] = H_{iy}, \quad n \times (MPa\sqrt{m}) \]

\[ H_{ik} = \sigma \times \sqrt{a_n} \]

Here, for masing’s material, \( n = 1.6 \); for the materials of happened strain hardening, \( n = 3 \).

(3) Condition 3
1) If to use the assessment method of the stress intensity factor \( K', \) at earlier time in the second stage, that is as following formula,

\[ K_i = y(a/b) \times \sigma \times \sqrt{a} \leq [K'_i] = K_{ik}/n \]

Here, for the materials of happened strain hardening, that the \([K'_i]\)

\[ [K'_i] = K_{ik}/n = \sigma \sqrt{a_n} / n , \quad n = 3 \]

For the Masing’s materials, it is

\[ [K'_i] = K_{ik}/n = \sigma \sqrt{a_n} / n , \quad n = 1.6 \]

2) If to calculate the stress intensity factor \( K_2 \) at the latter time in the second stage, that is as below,

\[ K_2 = y(a/b) \times \sigma \times \sqrt{a} \leq [K_2] = K_{ik}/n \]

Here, for the materials of happened strain hardening, that the \([K'_2]\)

\[ [K'_2] = K_{ik}/n = \sigma \sqrt{a_n} / n , \quad n = 3 \]

where it can see, that the \( a_n \) of the steels QT800-2 and 4340 in the table 5.

For the Masing’s materials, it is

\[ [K'_2] = K_{ik}/n = \sigma \sqrt{a_n} / n , \quad n = 1.6 \]

Where, it can see, that the \( a_n \) of the steels 30CrMnSiA and LC4CS in the table 5.

3) If to use the assessment method of the stress intensity factor \( K_{uc} \) in whole process, that is as follow,

\[ K_u = y(a/b) \times \sigma \times \sqrt{a} \leq [K_u] = K_{uc}/n \quad (21) \]

\[ [K_{uc}] = K_{uc}/n = \sigma \sqrt{a_n} / n , \quad n = 3 \quad (22) \]

Where, total size of a crack in whole process, \( a_n = a_i + a_2 \); total critical size \( a_{uc} \) of a crack, \( a_{uc} = a_i + a_2 \)
or \( a_{uc} = a_1 + a_2 \), here to take smaller one of both data; \( n = 3 \).

2. The assessment method for the crack size
1) The cases 1
If the crack size is

It may be: \( a_i \leq a_{uc}, (mm) \), then the short crack will grow;
or \( a_i \geq a_{uc}, (mm) \), then it will not grow.

(2) The condition 2
1) To apply the assessment method of the crack size in the first stage, it is as below,

\[ a_i \leq [a_i] = a_{uc}/n , (mm) \quad (23) \]

Here, for Masing’s material, \( n = 1.6 \); for the materials of happened strain hardening, \( n = 3.0 \).

2) To apply the assessment method of the crack sizes at earlier time in the second stage, that is

\[ a_2 \leq [a_2], (mm) \quad (24) \]

Here for Masing’s material, \( n = 1.6 \);
Here, for the materials of happened strain hardening, \( n = 3 \).

3) The condition 3
To apply the assessment method of the total crack size in the whole process, that is

\[ a \leq [a] = a_{uc}/n \quad (25) \]

Here, for Masing’s materials, \( a_{uc} = a_i + a_2 \) or
\( a_{uc} = a_i + a_2 \), to take smaller one of both data; \( n = 3 \).

If it can be corresponded under all conditions mentioned above, then is placed to a safe state in a definite time; Otherwise, it isn’t a safe case.

4. Calculating Example

The steel 30CrMnSiA is a Masing’s material, its strength limit \( \sigma = 1177MPa \), yield limit \( \sigma_y = 1104.5MPa \),
\[ E = 203005MPa \]
\[ K = 1475.76MPa, \sigma = 1795.1MPa, \quad n' = 0.063, b' = -0.0859 \]
Working stress \( \sigma_{max} = 960MPa \) under monotonous loading, and
\( \sigma_{pr} = 0.97 \sigma = 0.97 \times 1104.5 = 1071(MPa) \);
\[ y(a/b) = 1 \]. If it is being calculated by a designer in design, to try to calculate respectively following data:

1) Calculate the threshold size \( a_0 \) of a crack, the critical size \( a_1 \), the \( a_2 \), and the total \( a_{uc} \) for the material;
2) Calculate the factors \( H_{ik}, K_{ik}, K_{uc} \) of stress intensity corresponded to each critical crack size mentioned above, respectively;
3) Use two kind of assessment methods to make evaluating for the security of the material.

The processes and steps of calculations are as below.
1. To adopt the assessment method of crack size
1) Calculate each critical size \( a_0, a_1, a_2 \), and \( a_{uc} \) for the crack, and make an assessment for safety of the material
According to the formulas (8), (1), and (10), the crack size \( a_1 \), the threshold \( a_0 \) and the transitional \( a_{uc} \) are respectively calculated as below,
\[ \sigma_{pr} = 0.97 \sigma = 0.97 \times 1104.5 = 1071(MPa) \].
\[ a_1 = \frac{\sigma^2}{\sigma_{yr}^2 \times \pi}, \quad v = \frac{960^2}{1071^2 \times \pi}, \quad 1 \text{mm} = 0.256, (\text{mm}) \]

\[ a_{th} = \left( \frac{1}{(2 \text{mm})^{0.5 \times 0.55}} \right)^{1.5} = (0.564)^{0.5 \times 0.55} = (0.564)^{0.5 \times -0.0859} = 0.251(\text{mm}). \]

The result \( a_1 = 0.256(\text{mm}) > a_{th} = 0.251(\text{mm}) \), then it can predict that the micro crack must initiate on the material.

And \( a_w = \left( \sigma_{yr}^{(-0.063) \times 0.063} \times \frac{203005 \times \pi}{1475.76} \right)^{2/0.063} \times 2011.64 \times 0.063 \times 11.84 \times 0.568 = 0.291(\text{mm}); \)

\[ [a] = a_w / n = 0.291/1.6 = 0.182(\text{mm}). \]

The results \( a_1 = 0.256(\text{mm}) > a_{th} = 0.251(\text{mm}) \) and \( a_1 = 0.256 > [a] = 0.181(\text{mm}) \), so the crack must also grow in the material.

2) According to the formula (18), its growed size of the crack under work stress \( \sigma \) in the second stage is

\[ a_2 = \frac{\sigma^2 \times \pi}{\sigma_{yr}^2} (\text{mm}) = \frac{960^2 \times \pi}{1104.5^2} = 2.373\text{mm} \]

3) According to the formula (14), its critical size during long crack growth should be,

\[ a_{cr} = \frac{K^2}{\sigma_{yr}^2 \times \pi} = \frac{1475.8^2}{1104.5^2 \times \pi} = 0.568(\text{mm}) \]

\[ [a] = a_{cr} / n = 0.568 / 1.6 = 0.355. \]

The crack both \( a_1 = 2.373 > [a] = 0.355(\text{mm}) \) and \( a_2 = 2.373 > a_{cr} = 0.568(\text{mm}) \),

4) The total crack size in the whole process is:

\[ a_w = a_1 + a_2 = 0.256 + 2.373 = 2.629(\text{mm}) \]

According to the formula (19), the total critical size of crack should be as below,

\[ a_{wc} = a_{th} + a_{cr} = 0.251 + 0.568 = 0.819(\text{mm}) \cdot \]

\[ [a] = a_{wc} / n = 0.819 / 3 = 0.273(\text{mm}) \]

Now, the crack not only \( a_2 = 2.629 > [a_{wc}] = 0.273(\text{mm}) \), but also \( a_w = 2.629 > a_{wc} = 0.819(\text{mm}) \), so that the material of the structure must be fracture.

2. To adopt the assessment method of stress intensity factor.

1) According to the formulas (6) and (7), to calculate the stress factor \( H_1 \) and its critical value in the first stage as below,

\[ H_1 = \sigma \times \sqrt{a_1} = 960 \times \sqrt[3]{2.56 \times 10^{-3}} = 471.74(\text{MPa} \cdot \text{m}^{1/2}) \]

\[ H_1 = \sigma \times \sqrt{a_{wc}} = 1104.5 \times \sqrt[3]{5.68 \times 10^{-3}} = 581.2(\text{MPa} \cdot \text{m}^{1/2}) \]

Therefore, it is already insecurity in the first stage.

2) According to the formulas (13), (12) and (14), the factor values \( K_y, K_y \) and total critical \( K_{wc} \) of crack are calculated as below.

Calculate the values \( K_y, K_y \) and \( K_{wc} \) of stress intensity factors of the crack in the second stage, respectively;

a) The stress intensity under work stress is

\[ K_y = \frac{y(a/b) \sigma \times \sqrt{a_{cr}}}{1 \times 960 \times \sqrt{\pi} \times 2.373 \times 10^{-3}} = 82.89(\text{MPa} \cdot \text{m}^{1/2}) \]

b) The threshold values \( K_{cr} \) of the stress intensity at transitional point under yield stress is as below

\[ K_{cr} = \sigma \times \sqrt{a_{cr}} = 1104.5 \times \sqrt{2.51 \times 10^{-3}} = 31(\text{MPa} \cdot \text{m}^{1/2}) \]

\[ K_{cr} = \sigma \times \sqrt{a_{cr}} = 1104.5 \times \sqrt{2.51 \times 10^{-3}} = 31(\text{MPa} \cdot \text{m}^{1/2}). \]

\[ [K_{cr}] = K_{cr} / n = 31 / 1.6 = 19.4(\text{MPa} \cdot \text{m}^{1/2}) \]

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\[ K_{wc} = \sigma \times \sqrt{a_{wc}} = 1795.1 \times \sqrt{8.19 \times 10^{-3}} = 91.1(\text{MPa} \cdot \text{m}^{1/2}) \]

Here can see, this critical size of crack with the
experimental data \( (K_c = 98.9 \text{MPa}\sqrt{m}) \) of that stress intensity factor in table 5 is relatively approximating.

4) Its allowable value of stress intensity factor is,

\[
[K] = K_{nc} / n = 91.1/3 = 30.4(\text{MPa}\sqrt{m}),
\]

Now, \( K_n = 87.25 > [K] = 30.4(\text{MPa}\sqrt{m}) \).

So that the material must be fracture.

Here it can find out, the results for two kinds of calculating and assessment that is coincident.

5. Conclusions

(1) Due to the \( b_1 \) is the only constants for the materials behaviours shown inherent properties, so that the new threshold values \( a_{t1} \) depended on them, that are also the sole, and calculable one.

(2) The equation that crack size \( a_{t1} \) at transitional point between the elastic and plastic strain, that it is suited to calculate for the materials shown strain hardening and Masing’s ones.

(3) As the yield stress \( \sigma_c \) the is the only the constant shown own inherent property, so that the new critical value \( a_{t1} \) depended on the \( \sigma_c \), that is also the sole, and the \( a_{t1} \) is a calculable parameter. Similarly, because the fracture stress \( \sigma_f \) is the only the constant shown inherent only one, so that the new critical value \( a_{t2} \) depended on the \( \sigma_f \) that is also the sole.

(4) The critical \( a_{t1} \) and \( a_{t2} \) of a crack are inherent constants shown the materials’ characters; therefore the critical stress factors \( K_{t1} \) and \( K_{t2} \) based on \( a_{t1} \) and \( a_{t2} \) are also sole values, and are all calculable ones.

(5) Because corresponding to the factor at end point of the \( K_{t1} \) is the very one at starting point of the \( K_{t2} \), where they are the same point at point A2 on abscissa axis O4 IV; but for their critical values of cracks, \( a_{t1} \neq a_{t2} \). So, for some materials of the elastic-plastic and happened strain hardening when to take the value for the \( [K] \), it must only be calculated by the \( K_{t1} / n \) or \( K_{t2} / n \) in the safe factor \( n \).

(6) In assessment for crack strength, there are two kinds of methods that are calculated by the stress factor and by the crack size, both is consistent. For Masing’s materials, it must be calculated by \( K_{nc} / n = [K] \) or by \( a_{nc} / n = [a] \); for the materials happened strain hardening, it must be calculated by \( K_{nc} / n = [K] \) or \( a_{nc} / n = [a] \); by the way, the assessment way of the crack size is more simpleness.

(7) For Masing’s materials, the total critical \( a_{nc} \) of a crack is also the only, and it is another new constant shown a material’s property; therefore its critical stress factor \( K_{nc} \) based on \( a_{nc} \) is also the sole value, and also calculable; Their computing models can be used to calculate both for the safe assessment to some materials preexistent a flaw and for predicting crack in the structure designs; But for as the steel 40Cr shown strain softening, both error between the calculated and the experimental data is bigger.

(8) If to take same the yield stress and the calculating unit \( (\text{mm}) \) to calculate the strength coefficient \( K \) in fatigue subject, then the \( K \) is virtually the very the critical stress intensity factor \( K_c \) on fracture mechanics under monotonous loading.

(9) In those computing models are proposed by the author, if readers want to apply in engineering calculations, it must yet be verified to combine experiments, and it has to consider the influences for the shape and size to the crack and the structure.

References


