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Calculations on Cracking Strength in Whole Process to Elastic-Plastic Materials---The Genetic Elements and Clone Technology in Mechanics and Engineering Fields

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Abstract
For the decreasing experiments on fatigue, damage and fracture, the author bases on the principles of similar to the genetic genes in the life sciences, adopts the conventional material constants, discovers new constants to some materials of Masing's and elastic-plastic ones which show inherent properties; uses the theoretical approach which the mathematical derivations, the mechanics analysis and the calculations and analysis of the computer; proposes some new computing models from short to long crack which are the calculable mathematic models on crack problem as the threshold size, the transitional size and the critical ones; and sets up new computing formulas on the strength in different stages; provides the calculating criterions which are defined as the crack size and the stress intensity factor at each stage and in the whole process; and puts forward two kinds of the assessment methods in each stage and in whole process. In addition, to supplement again the comprehensive figure of the material behaviours; to give yet a detailed calculating example applied two of kinks for a safety assessment. This works may be there are practical significances for make linking and communication among the modern the material subject, fracture mechanics and damage mechanics; for to promote engineering applications.

1. Introduction

As is well-known, the traditional materials mechanics and the structure mechanics that is the calculable subjects and had made valuable contributions for every industrial engineering designs and calculations, but they cannot solve and calculate the strength and life problems in engineering materials and structures existed cracks. The fracture mechanics and the damage mechanics, which are just based on the local problems as researched objects for the local defects inside materials, to research the driving forces, the crack propagating (damage) rates and the life predictions in the crack growth process. But nowadays, latter these disciplines are all mainly dependent on tests.

The author thinks that in the mechanics and the engineering fields where exists such a scientific law as similar to genetic elements and cloning technology in the life sciences and has used the theoretical approach for similar principles, proposed some calculation models [1-2], recently sequentially discovered some new scientific laws to the Masing's and the elastic-plastic materials, and provides some new calculable models for the crack growth driving force, the calculating criterions and the assessment methods of the strength problems in the whole process which are from short to long crack. This is to try to make the modern fatigue, the damage mechanics and the fracture mechanics gradually become such calculable disciplines as the traditional materials mechanics. That way, it may be there are practical significances for decreasing experiments, for stinting manpower and funds, for promoting and developing engineering and applying it to relevant disciplines

2. A New Comprehensive Figure on Materials Behaviours

About problems among branch disciplines on fatigue-damage-fracture; about problems among the traditional material mechanics and the modern mechanics for communications and connecting their relations with each other, we must study and find out their correlations between the equations, even the relations between variables, between the material constants, and between the curves. This is because all the significant factors are to be researched and described for materials behaviours at each stage even in the whole process and are also all to have a lot of significations for the engineering calculations and designs. Therefore, we should research and find an effective tool used for analyzing the problems above mentioned. Here, the author provides the "Comprehensive figure of materials behaviors" as Figure 1 [3-4] (or the bidirectional combined coordinate system and simplified schematic curves in the whole process, or combined cross figure) that both is a principle figure of materials behaviors under monotonous loading, and is one under fatigue loading. It is also a comprehensive figure of multidisciplinary. Here in two problems to present as below:

2.1. Explanations on Their Geometrical and Physical Meanings for the Compositions of Coordinate System

In figure 1, it was being provided by the present author; at this time it has been corrected and complemented, that is, diagrammatically shown for the damage growth process or crack propagation process of materials behavior at each stage and in the whole course.

For the coordinate system, it is to consist of six abscissa axes O' I", O I', O_1 I, O_2 Ⅱ, O_3 Ⅲ, O_4 IV and a bidirectional ordinate axis O'_1 O_4. For the area between the axes O' I" and O I', it was an area applied as by the traditional material mechanics. Currently, it can also be applied for the micro-damage area by the very high cycle fatigue. Between the axes O I' and O_2 Ⅱ, it is calculating area applied for the micro-damage mechanics and the micro-fracture mechanics. For the areas among the O_2 Ⅱ, the O_3 Ⅲ and O_4 IV where they are calculatied and applied by the macro-damage mechanics and the macro-fracture mechanic. But for between the axes O_1 I and O_2 Ⅱ, it is calculated and applied in areas both for the micro-damage mechanics and for the macro-damage mechanics, or both for the micro-fracture mechanics and for the macro-fracture mechanics.

On the abscissa axis O' I", it is represented with parameters the stress σ and the strain ε as variables. On the abscissa axis O I' there are the fatigue limits σ_{-1} at point "a" ($\sigma_m = 0$) and "b" ($\sigma_m \neq 0$) that they just are the locations placed at threshold values for crack (damage) growth to some materials; on the abscissa axis O_1 I there are points "A" and "D" that just are the locations placed at threshold values as some materials. On the abscissa axes O_1 I and O_2 Ⅱ that they could all represented as variables with the stress intensity factor range ΔH_1 of short crack, and the strain intensity factor ΔI, and the stress intensity factor range ΔK_1 of long crack. On the other hand, they both could yet represented as variables with the short crack a_1 and the long crack a_2 (or damage D_1 and D_2). And here there are material constants of two that they are defined as the critical factor K_y of crack-stress-intensity and the critical factor K'_y of the damage-stress-intensity at the first stage, where that are just two parameters corresponded to the transitional size a_{tr} of crack or the transitional value of damage D_{tr}, they are just placed at point at the point B ($\sigma_m = 0$) and at point B_1 ($\sigma_m \neq 0$) corresponded to yield stress, that are also the boundary between short crack and long crack growth behaviors. but for some brittle materials would be happened to fracture to this point when their stresses are loaded to this level.

On the abscissa axis O3 Ⅲ, it is represented as variable with the stress intensity factor ΔK_1 (or $\Delta\delta_t$) of long crack; it is a boundary of the sizes as the residual strength between some elastic-plastic materials and brittle materials. On this axes O3 Ⅲ there are the critical points at $D_1(D_{1c})$, $A_1(A_{1c})$ and C_1. On abscissa O_4 IV, the point A_2 is corresponding to the fatigue strength coefficient σ'_f, the critical stress intensity factor values $K_{1c}(K_{2fc})$ and the critical values D'_{2c} and a_{2c} for the mean stress $\sigma_m = 0$; the point D_2 is corresponding to the $\sigma_m \neq 0$; the point C_2 corresponding to the fatigue ductility coefficient ε'_f and critical crack tip open displacement value δ_c; the point F corresponding to a very high cycle fatigue strength coefficient σ'_{vhf}. In addition on the same O_4 IV, there are yet another critical values $J'_{1c}(J_{1c})$, etc. in the long crack propagation process.

For an ordinate axis, an upward direction along the ordinate axis is represented as crack growth rate da/dN or damage growth rate dD/dN in each stage and the whole process. But a downward direction is represented as life N_{oi}, N_{oj} in each stage and the whole lifetime ΣN.

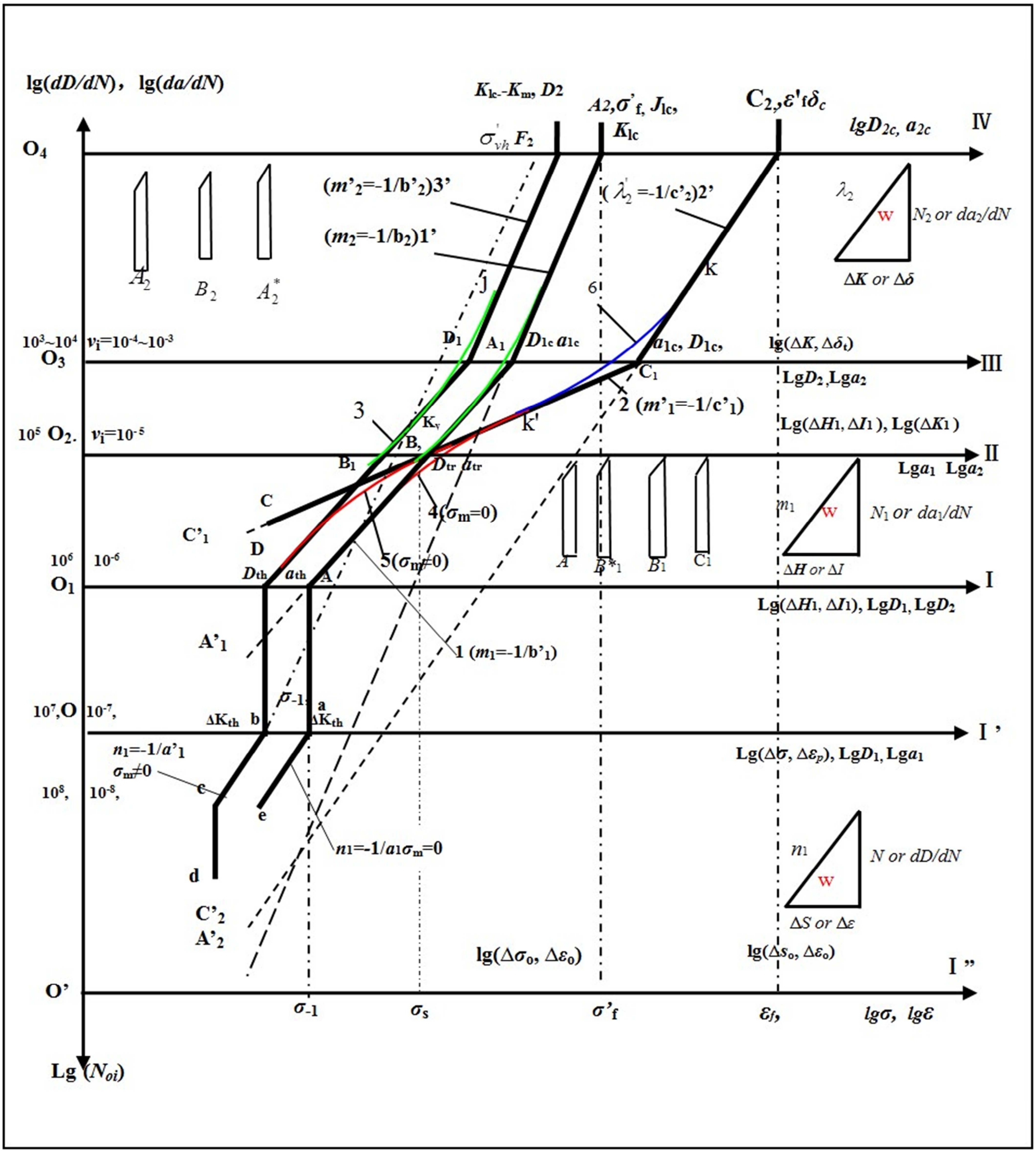

Figure 1. Comprehensive figure of material behaviors 1 (Or called calculating figure of material behaviors or bidirectional combined coordinate system and simplified schematic curves in the whole process).

In the area between axes O' I" and O_2 II, it is the fatigue history from un-crack to micro-crack initiation. In the area between axes O_1 I' and O_2 II, it is the fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_2 - O'$ on ordinate axis is as the history relating to life N_{mac} from grains size to micro-crack initiation until macro-crack forming; the distance O_4-O' is as the history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

In the crack forming stage, the partial coordinate system made up of the upward and the ordinate axes $O\ O_4$ and the abscissa axes O I', O_1 I and O_2 II is represented as the relationship between the crack growth rate dD_1/dN_1 (or the short crack growth rate da_1/dN_1) and the crack-stressfactor range ΔH_1 (or the damage strain factor range ΔI_1). In the macro-crack growth stage, the partial coordinate system made up with the ordinate axis $O_2\ O_4$ and abscissa O_2 II, O_3 III and O_4 IV at the same direction is represented to be the relationship between the macro-crack growth rate and the stress intensity factor range ΔK, J-integral range ΔJ and crack tip displacement range $\Delta\delta_t$ (da_2/dN_2 - ΔK, ΔJ and $\Delta\delta_t$). Inversely, the coordinate systems made up of the downward ordinate axis $O_4\ O_1$ and the abscissa axes O_4 IV, O_3 III, O_2 II, O_1 I, and O I' are represented respectively as the relationship between the ΔH -, ΔK - range and each stage life N_{oi}, N_{oj} and the lifetime $\sum N$ (or between the $\Delta\varepsilon_p$ -, $\Delta\delta_t$ - range and the life $\sum N$).

2.2. Explanations on the Physical and Geometrical Meanings of Relevant Curves

The curve ABA_1 is represented as the varying laws as the behaviours of the elastic materials or some elastic-plastic ones under high cycle loading in the macro-crack-forming stage (the first stage): positive direction ABA_1 represented as the relations between dD_1/dN_1 (or da_1/dN_1)- ΔH; inverted A_1BA, between the $\Delta H_1 - N_{oi}$. The curve CBC_1 is represented as the varying laws of the behaviours of the elastic-plastic materials or some plastic ones under low-cycle loading at the macro-crack forming stage: positive direction CBC_1 is represented as the relations between da_1/dN_1 - ΔI_1; inverted C_1BC, the relations between the $\Delta\varepsilon_p - N_{oi}$.

The curve A_1A_2 in the crack growth stage (the second stage) is showed as under high cycle loading: positive direction A_1A_2 showed as da_2/dN_2 - ΔK (ΔJ); inverted A_2A_1, between the $\Delta K_2, \Delta J - N_{oj}$. The C_1C_2 is showed as: the positive, relation between the da_2/dN_2 - $\Delta\delta_t$ under low-cycle loading, inverted C_2C_1, between $\Delta\delta_t$ (ΔJ)- N_{oj}. By the way, the curves $'Dbcd', (\sigma_m = 0)$ and the $'Aae'$ ($\sigma_m = 0$) are represented as the laws under the very high cycle fatigue.

It should yet point that the curve AA_1A_2 (1-1') is depicted as the rate curve of damage (crack) growth in whole process under symmetrical and high cycle loading (i.e. zero mean stress, $da/dN \le 10^{-6}$); the curve DD_1D_2 (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress, ($da/dN \le 10^{-6}$). The curve CC_1C_2 (2-2') is depicted as the rate curve under low cycle loading. The curve $eaABA_1A_2$ is depicted as the damage (crack) growth rate curve in whole process under very high cycle loading ($\sigma_m = 0, da/dN < 10^{-7}$), the curves $dcbDD_1D_2$ and $dcbF_2$ are depicted as ones of the damage (crack) growth rates in whole process under very high cycle loading ($\sigma_m \ne 0, da/dN < 10^{-7}$). Inversely, the curve A_2A_1A is depicted as the lifetime curve under symmetrical cycle loading (i.e. zero mean stress, $N \le 10^6$), the curve D_2D_1D, as the lifetime curve under unsymmetrical cycle loading ($N \le 10^6$). The curve C_2C_1C is depicted as the lifetime curve under low cycle loading ($N \le 10^5$). On the other hand, the curve A_2A_1BAae is as the lifetime one in whole process included very high cycle fatigue ($\sigma_m = 0, N > 10^7$), the curves D_2D_1Dbcd and F_2bcd are all depicted as the lifetime ones in whole process ($\sigma_m \ne 0, N > 10^7$).

It should also be explained that the comprehensive figure 1 of the materials behaviours may be as a complement for the

Fundamental knowledge of a material subject; that is a tool to design and calculate for various kinds of structures and materials under different loading conditions, and it is also a bridge to communicate and link the traditional material mechanics and the modern mechanics.

3. Calculations on Cracking Strengh for Elastic Plastic Materials Under Monotonic Loading

Here the variables for describing the crack growth process that are defined as follows:

1) From micro-crack initiation to macro-crack forming process, it is defined in the crack forming stage or defined in the first stage, the changing process of variable a_1 corresponded to the short crack, it is represented with the curves aAB or AB in figure 1;
2) From the macro-crack propagation to the fracture process is defined in the crack growth stage, or defined in the second stage. The changing process corresponded in the long crack variable a_2 in this stage, it is expressed in curve BA_1A_2;
3) From a short to long crack growth until fracture for a material, to adopt the a as the variable in the whole process, it is corresponded to curve "$aAA_1\ A_2$".

3.1. About Threshold Size on Crack

In the figure 1 it is seen, for the general steels, they are always shown with various characters in the each stages. The author discovers, which they all have the threshold sizes a_{th} of the crack growth as in table 1, and are only depended on the constant b_1 shown a material property. It should point, the locations of the threshold sizes a_{th} of the cracks, some materials are near at the point A where it is at the intersection one between the straight line AB and the abscissa axis O_1I in figure 1; and other ones, near at the point *a* where it is at the intersectional point between the straight line "Aa" and the abscissa axis O I'. And the threshold size a_{th} can be calculable parameter as following formula [5],

$$a_{th} = \left(\frac{1}{(\pi \cdot mm)^{0.5}}\right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} \ (mm) \tag{1}$$

Or

$$a_{th} = \left(\frac{1}{(\pi \cdot mm)^{0.5}}\right)^{\frac{1}{0.5-(1/m_1)}}, \ (mm) \tag{2}$$

Where, the range of the a_{th}-length calculated by equations (1-2) is the 0.21~0.275 (mm) in table 1. Under the condition shown linear-elastic behaviour for some materials, by the crack size a_{th} is combined with the induced stress σ under the monotonic loading, here it can make a model of the driving force of a crack growth, that is as following form,

$$H_1 = \sigma \cdot a_{th}^{1/m_1} = \sigma \cdot \sqrt[m_1]{a_{th}} \ [MPa \cdot (m)^{1/m_1}] \tag{3}$$

In the formula (2-3), $m_1 = -1/b_1$, the m_1 is also only a constant shown a property, it is same with the b_1 also a sole one. Here it should explain that the symbol of the constant b_1 is a negative value; and the m_1 is a positive one. the H_1 is defined as the stress intensity factor of short crack. In an ordinary way, the $\sigma \cdot a_1^{1/m_1}$ may be: the $\sigma \cdot a_1^{1/m_1} \le \sigma \cdot \sqrt[m_1]{a_{th}}$ or $\sigma \cdot a_1^{1/m_1} \ge \sigma \cdot \sqrt[m_1]{a_{th}}$, then, the strength criterions for them are as below,

$$H_1 = \sigma \cdot a_1^{1/m_1} = \sigma \cdot \sqrt[m_1]{a_{th}} \le H_{th}[MPa \cdot (m)^{1/m_1}], \tag{4}$$

Or

$$H_1 = \sigma \cdot a_1^{1/m_1} = \sigma \cdot \sqrt[m_1]{a_{th}} \ge H_{th}[MPa \cdot (m)^{1/m_1}], \tag{5}$$

Where the H_{th} is defined as the threshold factor of a crack. If the $H_1 < H_{th}$, the crack in a material does not grow; and the $H_1 \ge H_{th}$, the crack is sure to grow.

Table 1. *Data calculated threshold sizes a_{th} for crack growth.*

Materials [6-7]	Heat- treatment	σ_bMPa	σ_sMPa	b_1	m_1'	a_{th},mm
①30CrMnSiA [17]	Hardeningand Tempering	1177	1104.5	-0.0859	11.64	0.251
①LC4CS	CS	613.9	570.8	-0.0727	13.76	0.262
2024-T3 Aluminum		469	379	-0.124	8.06	0.218
②QT800-2	Normalization	913	584.3	-0.083	12.05	0.253
4340	Quenching and tempering	1241	1172	-0.076	13.16	0.259
④40Cr	in oil-quench 850°C, temper 560°C, air cooling	940	805	0.12	8.33	0.222
1005-1009	Hot rolled sheet	345	262	-0.109	9.174	0.231
1005-1009	Cold-draw sheet	414	400	-0.073	13.7	0.262
Steel: 1020	Hot rolled sheet	441	262	-0.12	8.33	0.222
RQC-100	Hot rolled sheet	931	883	-0.07	14.3	0.264
9262	anneal	924	455	-0.071	14.1	0.263

3.2. Calculation on Crack Strength in the First Stage

The material property had discovered by Masing, that is as well known [8-9]. For some called Masing's materials, elastic-plastic and happened strain hardening ones, which are corresponded to the curve 1($\overline{AB}$) or aAB in the first stage between abscissa axis O I' and the O2 II in figure 1, here can set up a criterion of the crack strength for it, that is as below form,

$$H_1 = \sigma \cdot a_1^{1/m_1} \le [H_1] = H_{1c} / n_1, (MPa \cdot m^{1/m_1}) \tag{6}$$

$$H_{1c} = \sigma_s \times \sqrt[m_1]{a_{tr}}, (MPa \cdot m^{1/m_1}) \tag{7}$$

The H_{1c} in (6) is defined as a critical value of the stress intensity factor which is corresponded to the yield stress, so it must be the only a constant showing a material property. And the H_{1c} is also the critical value corresponded to the stress factor K_y and the transitional size a_{tr} mentioned following text, their locations are near at points B on abscissa axis O2-II (in Fig. 1) where it is on the boundary between the short crack and the long crack or between the first stage and the second stage.

The behabirous of crack for a material, it is changed with the loading ways and the stress levels, the crack size growed before the yield stress σ_s under the monotonous loading can be caculated by following formula,

$$a_1 = \frac{\sigma^2}{\sigma_{pr}^2 \times \pi} v, (mm) \tag{8}$$

Where the $\sigma_{pr} \approx \sigma_e$ is a stress value of proportional limit (approximating elastic limit σ_e), it can also approximatively be took in $(0.955 \sim 0.975)\sigma_y$ of the yield stress, as the data is to lack. The v is a conversion coefficient of the unit, $v = 1mm$.

3.3. Calculation on Crack Strength in the Second Stage

1. The critical stress factor K_y and the critical size a_{tr} at the transitional point

As is well known, the mathematic model to describe a crack in fracture mechanics [10-12], that is to adopt these “genes” σ and π and crack variable a, thereby to make the stress intensity factor $K_1 = \sigma\sqrt{\pi a}$, that is as following form,

$$K_1 = \sigma \times \sqrt{\pi a_1}, (MPa \cdot \sqrt{m}) \tag{9}$$

As explanation mentioned above, in the area between the abscissa axis O1-I and the O2-II in figure 1, where that stress factor H_1 of crack is equivalent to the K_1, but their dimensions and units are differences at this same point, they are corresponded to the variable a_1 from the threshold the a_{th} to the a_{tr} in the section（$a_{th} \le a_1 \le a_{tr} = a_{\text{mac}}$）, it can set up two kinds of the mathematic models called the stress factors. In addition to above equations (6-8) can be applied; in theory below the mathematic models (9-11) are still suitable in the area.

For some Masing’s materials, their transitional sizes a_{tr} at transition points between the elastic and the plastic strain, for which are also be calculable, and can be calculated by means of the following equation,

$$a_{tr} = \left(\sigma_s^{(1-n')/n'} \times \frac{E \times \pi^{1/2\times n'}}{K^{1/n'}} \right)^{\frac{2m_1 n_1}{2n_1 - m_1}}, \ (mm) \tag{10}$$

Here, the K is a strength coefficient under monotonic loading, the n is an exponent happened strain hardening. Then the model of driving force at this point should be as follow

$$K_y = \sigma_s \cdot \sqrt{\pi a_{tr}}, (MPa \cdot \sqrt{m}) \tag{11}$$

Where the K_y is called as the critical stress factor that is relevant to the crack size a_{tr} at transitional point, just is corresponding to size $a_{mac} (\approx a_{tr})$ of forming macro crack, this factor K_y should theoretically be equivalent to above mentioned the H_{1c}, although the dimensions and units between them are differences.

2. The critical stress factor K_{1c} and the critical size a_{1c} during long crack growth

Over the abscissa axis O2-II, the crack over the transitional point size a_{tr} is to adopt the a_2 as the variable. When the crack growth gets to the macro crack, where it is corresponding to the curve BA_1A_2 in figure 1. But, for some cast irons, steels of the low toughness and brittle materials, which their behaviours are depicted in curve BA_1 between the abscissa axis OII and the O3III. When their stresses are loaded to this level, or gotten to the critical values a_{1c} of macro crack, that may be happened to fracture, then its strength criterion should be used as following form,

$$K_1 = y(a/b) \cdot \sigma \cdot \sqrt{\pi a_2} \le [K] = K_{1c} / n_1, (MPa\sqrt{m}) \tag{12}$$

$$K_{1c} = \sigma_s \cdot \sqrt{\pi a_{1c}} \quad (MPa\sqrt{m}) \tag{13}$$

Where the $y(a/b)$ [13-14] is a correcting factor related with the shape and the size of a crack; the K_{1c} is a the critical factor called during the long crack growth, that is corresponded to the critical size a_{1c} on abscissa axis O3-III in fig. 1, also a the critical value in the second stage. It shoud point, because the yield stresses σ_s in the formula (13) is the constant of uniquenesses for a material, the critical size of crack a_{1c} can also be applied as an important parameter showed its property. In practice, the critical value a_{1c} could be calculated by means of below formula:

$$a_{1c} = \frac{K^2}{\sigma_s^2 \times \pi}, (mm) \tag{14}$$

The author researches to discover that the strength coefficient K in the material and the fatigue subjects is virtually the very the critical stress intensity factor K_{Ic} in fracture mechanics under monotonous loading, if they take same calculating parameter and unit. For instance, for the QT800-2 in table 2, its $\sigma_s = 584.32MPa$, $a_{1c} = 2.944mm = 2.944 \times 10^{-3} m$, then if to adopt the calculating model in fracture mechanics to calculate the strength coefficient K, that is as below,

$$K = \sigma_s \sqrt{\pi a_{1c}} = 584.3MPa\sqrt{\pi 2.944(mm)} = 1777MPa\sqrt{mm} = 56.2MPa\sqrt{m};$$

On the other hand, the practical calculable critical factor K_{Ic} on fracture mechanics subject should also be,

$$K_{Ic} = \sigma_s \sqrt{\pi a_{1c}} = 584.32MPa\sqrt{\pi 2.944(mm)} = 1777MPa\sqrt{mm} = 56.2MPa\sqrt{m}.$$

So, the calculating results are completely consistent. It can see from the table 5, this data is close to experiment one ($K_{Ic} = 47.6MPa\sqrt{m}$). Therefore, its unit of the K to be "MPa" which was called as strength coefficient on material subject is actually the very critical stress intensity factor on fracture mechanics, and the unit of both should be all "$MPa\sqrt{mm}$".

Here has to point the above equations, the data error in the calculations is bigger for those materials happened strain softening.

In the table 2, here are listed to the critical sizes a_{1c} of crack for 13 kinds of materials.

***Table 2.** The critical sizes a_{1c} during long crack growth of some materials in second stage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	a_{1c}, mm
Hot rolled sheet 1005-1009	345	262	531	1.31
Steel: 1005-1009 Cold-draw sheet	414	400	524	0.546
RQC-100, Hot rolled sheet	931	883	1172	0.561
4340, quench and tempering	1241	1172	1579	0.578
Aluminum 2024-T3	469	379	455	0.46
30CrMnSiA, (1) Hardening and tempering	1177	1104.5	1475.76	0.568
LC4CS, (1) Heat treatment-CS	613.9	570.8	775.05	0.587
40Cr(3)	940	805	1592	1.25
60Si2Mn, quench, medium-temperature tempering (3)	1504.8	1369	1721	0.503
QT800-2, (2) normalizing	913	584.3	1777	2.944

Note: σ_b is a strength limit; σ_s is an yield limit;

(A)-Bar $\varphi = 30$; (B)-Y-type test specimen;

(1)---The Masing's materials; (2)---The cycle-harden materials; (3)-Cyclic softening.

3. The critical stress factor K_{2c} and the critical size a_{2c} at the momentary fracture

For the behaviours of another materials could be over the abscissa axis O3III in fagure, while they get to own critical values a_{1c} of long crack which are usually later than the brittle materials above mentioned, their life are also longer. So the abscissa axis O3III is a boundary that can be the residual intennsity sizes between different materials in crack growth process. In this case that strength criterion (12-14) on crack can still be sutied for calculations.

When the crack growth over the abscissa axis O3III to the O4IV in figure 1, the strength criterion of crack at later time in the second staege should be calculated by following form,

$$K_2 = \mathrm{y}(a/b)\sigma \cdot \sqrt{\pi a_2} \le [K] = K_{2c}/n, (MPa\sqrt{m}) \tag{15}$$

$$K_{2c} = \sigma_f \cdot \sqrt{\pi a_{2c}}, (MPa\sqrt{m}) \tag{16}$$

Where the K_2 is also defined as the stress factor of crack in the second stage, the K_{2c} is called as a critical factor at the momentary fracture, that it is the very the critical stress intensity factor K_{1c} in fracture mechanics. The σ_f is a fracture stress, the a_{2c} is a critical crack size at the momentary fracture where it is at the crossing point A2 of the abscissa axis O4-Ⅳ and the straight line 1 (A_1A_2) in Fig. 1.

It should yet explain because the K_{2c} is also a material constant, it must be the data of uniqueness to show a material performance, and it could be calculated out by mens of the fracture stress σ_f. So that the critical size a_{2c} at the momentary fracture under corresponding to true stress σ_f should also be the only data. In theory, it must be to exist as following functional relationship,

$$a_{2c} = \frac{K^2}{\sigma_f^2 \times \pi} v, (mm) \tag{17}$$

In the table 3, to list the critical sizes a_{2c} at the momentary fracture in second stage to some materials.

By the way, when a structure is being calculated in design, if the crack grows to the stage of long crack, then the crack sizes a_1 and a_2 in above equations (9, 12, 15) can also be calculated for the predictions by following formula

$$a_2 = \frac{\sigma^2 \times \pi}{\sigma_y^2}, (mm) \tag{18}$$

***Table 3.** The critical sizes a_{2c} at the momentary fracture in second stage.*

Materials	σ_b, MPa	σ_s, MPa	K, MPa	σ_f, MPa	a_{2c}, mm
Hot rolled sheet 1005-1009	345	262	531	848	0.125
Steel: 1005-1009 Cold-draw sheet	414	400	524	841	0.124
RQC-100, Hot rolled sheet	931	883	1172	1330	0.247
4340, quench and tempering	1241	1172	1579	1655	0.280
Aluminum 2024-T3	469	379	455	558	0.212
30CrMnSiA, (1) Hardening and tempering	1177	1104.5	1475.76	1795.1	0.215
LC4CS, (1) Heat treatment-CS	613.9	570.8	775.05	710.62	0.379
QT800-2, (2) normalizing	913	584.3	1777	946.8	1.121
40Cr(3)	940	805	1592	1305	0.474
60Si2Mn, quench, medium-temperature tempering (3)	1504.8	1369	1721	2172.4	0.20

Note: σ_b is a strength limit; σ_s is an yield limit; (A)-Bar $\varphi = 30$; (B)-Y-type test specimen;

(1)---The Masing's materials; (2)---The cycle-harden materials; (3)-Cyclic softening materials.

3.4. Calculations on Cracking Strength in Whole Process

Due to the behaviors shown by the materials are different at each stage, their dimensions and units in computing models about strength problem are also differences. But the author recently researches finding, on account of the critical sizes in each stage are all inherent constants, they are the only depended on the material properties, therefore which exist necessary the critical sizes of crack in each stage. So that the total critical sizes a_{wc} as the material constants in whole process should be applied as the important parameter for the assessment, here it proposes as follow:

$$a_{wc} = a_{th} + a_{1c} \tag{19}$$

Or

$$a_{wc} = a_{tr} + a_{1c} \tag{20}$$

It shoud point that the data in table 4 is the material constants for six kinds of materials, which they are under the monotonous loading. And the data in table 5 is called as the critical size of crack, which is calculated with calculable formulas.

***Table 4.** Data of material's performances.*

Materials [15-16]	**Heat- treatment**	σ_b MPa	σ_s MPa	K MPa	n_1	E MPa	σ_f MPa	b_1	m_1'
(1)30CrMnSiA	Hardeningand Tempering	1177	1104.5	1475.76	0.063	203005	1795.1	-0.0859	11.64
(1)LC4CS	CS	613.9	570.8	775.05	0.063	72571.8	710.62	-0.0727	13.76
2024-T3 Aluminum		469	379	455	0.032	70329	558	-0.124	8.06
(2)QT800-2	Normalization	913	584.3	1777	0.2034	160500	946.8	-0.083	12.05
4340	Quenching and tempering	1241	1172	1579	0.066		1655	-0.076	13.16
(3)40Cr	in oil-quench 850°C, temper 560°C, air cooling	940	805	1592	0.173		1305	0.12	8.33

(1)---The Masing's materials; (2)---The strain harding material; (3)---it is a material happened the strain softening under cyclic loading.

***Table 5.** Calculated data by means of the calculable formulas.*

Materials	**Heat- treatment**	a_{th} / a_{tr} (5)	K_{1c} (6)	a_{1c} (5)	K_{2c} (6)	a_{2c} (5)	K_{Wc} ⑥	D_{Wc} (5)	K_c (4)
(1)30CrMnSiA	Hardeningand Tempering	0.251/ 0.291	46.64	0.568	46.64	0.212	92.05/ 94.2	0.837/ 0.877	98.9
(1) LC4CS,	CS	0.262	24.51	0.587	24.51	0.3786	36.7	0.849	38.5
2024-T3	Aluminum	0.218	14.4	0.459	14.4	0.212	25.8	0.68	31
(2)QT800-2	Normalization	0.253	56.2	2.944	56.2	1.121	58.56	4.318	47.6
4340	Quenching and tempering	0.259	49.94	0.578	49.94	0.29	60	1.127	50
40Cr (3)	in oil-quench 850°C, temper 560°C, air cooling	0.222	50.36	1.245	50.36	0.475	58.64	1.941	154

(1)---The Masing's materials; (2)---The cycle-harden material; (3)---it is a material happened the strain softening under cyclic loading;
(4) K_c is the experiment data of critical stress intensity factor ($MPa\sqrt{m}$), $MPa\sqrt{m}$; (5)—Units are the " mm "; (6)--- Units are the " $MPa\sqrt{m}$ ".

It can seen from the above table 5, for the critical factors K_{1c} and the K'_{2c} of the crack, $K_{1c} = K_{2c}$, because corresponding to end point of the K_{1c} -value just is the starting point of the K_{2c}-value, where they are the same at point A2 on abscissa axis O4 IV; but for their critical sizes of sizes, $a_{2c} \neq a_{1c}$. So, when to take the value for the $[K]$ it should only be caculated by the K_{1c}/n or K_{2c}/n with the safe factor n .

Here can be compared in calculated data with the experiment ones K_c , and can be seen out: 1) the calculated value for the nodular cast iron QT800-2 which is materials happened strain harding, the calculated K_{2c} of it closes to the experimental data Kc; the K_{2c} for the steel 4340 by quenching and tempering is also close to the experimental Kc; the calculated values K_{Wc} for Masing's materials 30CrMnSiA and LC4CS, between the calculated K_{Wc} and the Kc, both is approximating; But for the steel 40Cr of shown strain softening, both error between the calculated and the experimental data is bigger.

The calculating criterions about the crack strength in whole process there are two kinds of ways: 1) The assessment method for the stress intensity factor; 2) The assessment method for the crack size. It should explain, if to apply the assessment method for the crack size, it must use the variable a in whole process and those relevant material conatants, it has to adopt their critical sizes shown different performances at each stage.

For instance, for a material there are following cases:

1. The assessment method for the stress intensity factor

(1) The condition 1

If the stress factor of a short crack may be,

$$H_1 = \sigma \times \sqrt[m_1]{a_1} \leq H_{th}, (MPa\sqrt[m_1]{m}) ,$$

Or $H_1 = \sigma \times \sqrt[m_1]{a_1} \geq H_{th}, (MPa\sqrt[m_1]{m}).$

(2) The condition 2

If to use the assessment method of the stress intensity factor in the first stage, it is as below,

$$H_1 = \sigma \times \sqrt[m_1]{a} \le [H\ ] = H_{1c} / n, (MPa\sqrt[m_1]{m})$$

$$H_{1c} = \sigma_s \times \sqrt[m_1]{a_{tr}}$$

Here, for masing's material, $n = 1.6$; for the materials of happened strain hardening, $n = 3$.

(3) Condition 3

1) If to use the assessment method of the stress intensity factor K'_1 at earlier time in the second stage, that is as following formula,

$$K_1 = y(a/b) \times \sigma \times \sqrt{\pi a} \le [K_1] = K_{1c} / n$$

Here, for the materials of happened strain harding, that the $[K_1]$ is

$$[K_1] = K_{1c}/n = \sigma_s \sqrt{\pi a_{1c}} / n,\ n = 3.$$

For the Masing's materials, it is

$$[K_1] = K_{1c}/n = \sigma_s \sqrt{\pi a_{1c}} / n, n = 1.6$$

2) If to calculate the stress intensity factor K_2 at the latter time in the second stage, that is as below,

$$K_2 = y(a/b) \times \sigma \times \sqrt{\pi a} \le [K_2] = K_{2c} / n$$

Here, for the materials of happened strain harding, that the $[K_2]$ is

$$[K_2] = K_{2c}/n = \sigma_f \sqrt{\pi a_{2c}} / n,\ n = 3.$$

where it can see, that the a_{2c} of the steels QT800-2 and 4340 in the table 5.

For the Masing's materials, it is

$$[K_2] = K_{2c}/n = \sigma_f \sqrt{\pi a_{2c}} / n,\ n = 1.6$$

Where, it can see, that the a_{2c} of the steels 30CrMnSiA and LC4CS in the table 5.

3) If to use the assessment method of the stress intensity factor K_{wa} in whole process, that is as follow,

$$K_w = y(a/b) \times \sigma \times \sqrt{\pi a_w} \le [K_w] = K_{wc} / n \tag{21}$$

$$[K_w] = K_{wc}/n = \sigma_f \sqrt{\pi a_{wc}} / n, n = 3 \tag{22}$$

Where, total size of a crack in whole process, $a_w = a_1 + a_2$; total critical size a_{wc} of a crack, $a_{wc} = a_{th} + a_{1c}$ or $a_{wc} = a_{tr} + a_{1c}$, here to take smaller one of both data; $n = 3$.

2. The assessment method for the crack size

(1) The cases 1

If the crack size is

It may be: $a_1 \le a_{th}, (mm)$, then the short crack will grow; or $a_1 \ge a_{th}, (mm)$, then it will not grow.

(2) The condition 2

1) To apply the assessment method of the crack size in the first stage, it is as below,

$$a_1 \le [a] = a_{tr} / n, (mm), \tag{23}$$

Here, for Masing's material, $n = 1.6$; for the materials of happened strain hardening, $n = 3.0$.

2) To apply the assessment method of the crack sizes at earlier time in the second stage, that is

$$a_2 \le [a], (mm),\ [a] = a_{1c} / n \tag{24}$$

Here for Masing's material, $n = 1.6$;

Here, for the materials of happened strain hardening, $n = 3$.

(3) The condition 3

To apply the assessment method of the total crack size in the whole process, that is

$$a \le [a] = a_{wc} / n, \tag{25}$$

Here, for Masing's materials, $a_{wc} = a_{th} + a_{1c}$ or $a_{wc} = a_{tr} + a_{1c}$, to take smaller one of both data; $n = 3$.

If it can be corresponded under all conditions mentioned above, then is placed to a safe state in a definite time; Otherwise, it isn't a safe case.

4. Calculating Example

The steel 30CrMnSiA is a Masing's material, its strength limit $\sigma_b = 1177MPa$, yield limit $\sigma_s = 1104.5MPa$, $E = 203005MPa$, $K = 1475.76MPa, \sigma_f = 1795.1MPa,\ n' = 0.063, b'_1 = -0.0859$; Working stress $\sigma_{max} = 960MPa$ under monotonous loading, and supposing $\sigma_{pr} \approx 0.97\sigma_y = 0.97 \times 1104.5 = 1071(MPa)$; $y(a/b) = 1$. If it is being calculated by a designer in design, to try to calculate respectively following data:

(1) Calculate the threseld size a_{th} of a crack, the critical size a_{1c}, the a_{2c} and the total a_{wc} for the material;

(2) Calculate the factors H_{th}, K_{1c}, K_{2c} and K_{wc} of stress intensity corresponded to each critical crack size mentioned above, respectivaly;

(3) Use two kind of assessment methods to make evaluating for the security of the material.

The processes and steps of calculations are as below.

1. To adopt the assessment method of crack size

1) Calculate each critical size a_{th}, a_{1c}, a_{2c} and a_{wc} for the crack, and make an assessment for safety of the material

According to the formulas (8), (1) and (10), the crack size a_1, the threshold a_{th} and the transitional a_{tr} are respectively calculated as below,

To take $\sigma_{pr} \approx 0.97\sigma_y = 0.97 \times 1104.5 = 1071(MPa)$.

$$a_1 = \frac{\sigma^2}{\sigma_{pr}^2 \times \pi} v = \frac{960^2}{1071^2 \times \pi} 1mm = 0.256, (mm)$$

$$a_{th} = \left(\frac{1}{(\pi mm)^{0.5}} \right)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+b_1}} = (0.564)^{\frac{1}{0.5+(-0.0859)}} = 0.251(mm),$$

The result $a_1 = 0.256(mm) > a_{th} = 0.251(mm)$, then it can predict that the micro crack must initiate on the material.

And $a_{tr} = \left(\sigma_s^{(1-n')/n'} \times \frac{E \times \pi^{1/2\times n'}}{K'^{1/n'}} \right)^{\frac{2m_1n_1}{2n_1-m_1}} = \left(\sigma_s^{(1-0.063)/0.063} \times \frac{203005 \times \pi^{1/2\times 0.063}}{1475.76^{1/0.063}} \right)^{\frac{2\times 11.64\times 0.063}{2\times 0.063-11.64}} = 0.291(mm)$;

$$[a] = a_{tr} / n = 0.291/1.6 = 0.182(mm).$$

The results $a_1 = 0.256(mm) > a_{th} = 0.251(mm)$ and $a_1 = 0.256 > [a_1] = 0.181(mm)$, so the crack must also grow in the material.

2) According to the formula (18), its growed size of the crack under work stress σ in the second stage is

$$a_2 = \frac{\sigma^2 \times \pi}{\sigma_y^2}, (mm) = \frac{960^2 \times \pi}{1104.5_y^2} = 2.373mm$$

3) According to the formula (14), its critical size during long crack growth should be,

$$a_{1c} = \frac{K^2}{\sigma_s^2 \times \pi} = \frac{1475.8^2}{1104.5^2 \times \pi} = 0.568(mm)$$

$$[a\ ] = a_{1c} / n = 0.568/1.6 = 0.355.$$

The crack both $a_2 = 2.373 > [a] = 0.355(mm)$ and $a_2 = 2.373 > a_{1c} = 0.568(mm)$,

4) The total crack size in the whole process is:
$a_w = a_1 + a_2 = 0.256 + 2.373 = 2.629(mm)$

According to the formula (19), the total critical size of crack should be as below,

$$a_{wc} = a_{th} + a_{1c} = 0.251 + 0.568 = 0.819(mm).$$

$$[a] = a_{wc}/n = 0.819/3 = 0.273(mm)$$

Now, the crack not only $a_w = 2.629 > [a_{wc}] = 0.273(mm)$, but also $a_w = 2.629 > a_{wc} = 0.819(mm)$, so that the material of the structure must be fracture.

2. To adopt the assessment method of stress intensity factor.

1) According to the formulas (6) and (7), to calculate the stress factor H_1 and its critical value in the first stage is as below,

$$H_1 = \sigma \times \sqrt[m_1]{a_1} = 960 \times \sqrt[11.64]{2.56\times 10^{-4}} = 471.74(MPa \cdot m^{1/m_1}),$$

$$H_{1c} = \sigma_s \times \sqrt[m_1]{a_{1c}} = 1104.5 \times \sqrt[11.64]{5.68\times 10^{-4}} = 581.2, (MPa \cdot m^{1/m_1}),$$

$$[a_1] = H_{1c}\ / n_1 = 581.2/1.6 = 363.25, (MPa \cdot m^{1/m_1}),$$

$$H_1 = 471.74 > [H_1] = 363.25(MPa \cdot m^{1/m_1})$$

Therefore, it is already insecurity in the first stage.

2) According to the formulas (13), (12) and (14), the factor values K, K_y and total critical K_{wc} of crack are calculated as below.

Calculate the values K, K_y and K_{wc} of stress intensity factors of the crack in the second stage, respectivaly;

a) The stress intensity under work stress is

$$K_2 = y(a/b)\sigma \times \sqrt{\pi a_2} = 1\times 960 \times \sqrt{\pi 2.373\times 10^{-3}} = 82.89(MPa\sqrt{m})$$

b) The threshold values K_{tr} of the stress intensity at transitional point under yield stress is as below

$$K_y = \sigma_s \times \sqrt{\pi a_{tr}} = 1104.5 \times \sqrt{\pi 2.51\times 10^{-4}} = 31(MPa \cdot \sqrt{m}),$$

$$[K_y] = K_y / n = 31/1.6 = 19.4(MPa \cdot \sqrt{m});$$

c) The critical values K_{1c} in second stage is,

$$K_{1c} = \sigma_s \cdot \sqrt{\pi a_{1c}} = 1104.5\sqrt{\pi 5.68\times 10^{-4}} = 46.66(MPa\sqrt{m}),$$

$$[K_1] = K_{1c} / n = 46.6/1.6 = 29.1(MPa \cdot \sqrt{m})$$

So the stress factor values both $K_2 = 82.89(MPa\sqrt{m} > [K_y] = 19.4$ and $[K_1] = 29.1$, and the $K_2 = 82.89 > K_y = 31$ and $K_{1c} = 46.66(MPa\sqrt{m})$.

3) Calculations for the factor method in whole process.

To calculate total size of crack for formula, that is,

$$a_w = a_1 + a_2 = 0.256 + 2.373 = 2.629(mm)$$

According to the formulas (21) and (22), the total factor in whole prosess is

$$K_w = y(a/b)\sigma \times \sqrt{\pi a_w} = 1\times 960\sqrt{\pi 2.629\times 10^{-3}} = 87.25(MPa\sqrt{m})$$

the critical factor of total crack size in the whole process is

$$K_{wc} = \sigma_f \times \sqrt{\pi a_{wc}} = 1795.1 \times \sqrt{\pi 8.19\times 10^{-4}} = 91.1(MPa\sqrt{m}).$$

Here can see, this critical size of crack with the experimental data ($K_c = 98.9MPa\sqrt{m}$) of that stress intensity factor in table 5 is relatively approximating.

4) Its allowable value of stress intensity factor is,

$$[K] = K_{wc} / n = 91.1/3 = 30.4(MPa\sqrt{m}),$$

Now, $K_w = 87.25 > [K] = 30.4(MPa\sqrt{m})$.

So that the material must be fracture.

Here it can find out, the results for two kinds of calculating and assessment that is coincident.

5. Conclusions

(1) Due to the b_1 is the only constants for the materials behaviours shown inherent properties, so that the new threshold values a_{th} depended on them, that are also the sole, and calculable one.

(2) The equation that crack size a_{tr} at transitional point between the elastic and plastic strain, that it is suited to calculate for the materials shown strain hardening and Masing's ones.

(3) As the yield stress $\sigma_s(\sigma_y)$ is the only the constant shown own inherent property, so that the new critical value a_{1c} depended on the σ_s, that is also the sole, and the a_{1c} is a calculable parameter. Similarly, because the fracture stress σ_f is the only the constant shown own inherent one, so that the new critical value a_{2c} depended on the σ_f that is also the sole.

(4) The critical a_{1c} and a_{2c} of a crack are inherent constants shown the materials' characters; therefore the critical stress factors K_{1c} and K_{2c} based on a_{1c} and a_{2c} are also sole values, and are all calculable ones.

(5) Because corresponding to the factor at end point of the K_{1c} is the very one at starting point of the K_{2c}, where they are the same value at point A2 on abscissa axis O4 IV; but for their critical values of cracks, $a_{2c} \neq a_{1c}$. So, for some materials of the elastic-plastic and happened strain harding when to take the value for the $[K]$, it must only be calculated by the K_{1c}/n or K_{2c}/n in the safe factor n

(6) In assessment for crack strength, there are two kinds of methods that are calculated by the stress factor and by the crack size, both is consistent. For Masing's materials, it must be calculated by $K_{wc}/n = [K]$ or by $a_{wc}/n = [a]$; for the materials happened strain hardening, it must be calculated by $K_{1c}/n = [K]$ or $a_{1c}/n = [a]$; by the way, the assessment way of the crack size is more simpleness.

(7) For Masing's materials, the total critical a_{wc} of a crack is also the only, and it is another new constant shown a material's propertiy; therefore its critical stress factor K_{wc} based on a_{wc} is also the sole value, and also calculable; Their computing models can be used to calculate both for the safe assessment to some materials preexistted a flaw and for predicting crack in the structure designs; But for as the steel 40Cr shown strain softening, both error between the calculated and the experimental data is bigger.

(8) If to take same the yield stress and the calculating unit (mm) to calculate the strength coefficient K in fatigue subject, then the K is virtually the very the critical stress intensity factor K_{1c} on fracture mechanics under monotonous loading.

(9) In those computing models are proposed by the author, if readers want to apply in engineering calculations, it must yet be verified to combine experiments, and it has to consider the influences for the shape and size to the crack and the structure.

References

[1] Yan Gui Yu. Damage Growth Rate Calculations Realized in Whole Process with Two Kinks of Methods. American Journal of Science and Technology. Vol. 2, No. 4, 2015, pp. 146-164.

[2] Yangui Yu. The Life Predicting Calculations Based on Conventional Material Constants from Short Crack to Long Crack Growth Process. International Journal of Materials Science and Applications. Vol. 4, No. 3, 2015, pp. 173-188. doi: 10.11648/j.ijmsa.20150403.15.

[3] Yangui Yu. The Predicting Calculations for Lifetime in Whole Process Realized with Two Kinks of Methods for Elastic-Plastic Materials Contained Crack. Journal of Materials Sciences and Applications. Vol. 1, No. 2, 2015, pp. 15-32.

[4] Yangui Yu. The Calculations of Crack Propagation Rate in Whole Process Realized with Conventional Material Constants. Engineering and Technology. Vol. 2, No. 3, 2015, pp. 146-158.

[5] Yangui Yu. Multi-Targets Calculations Realized for Components Produced Cracks with Conventional Material Constants under Complex Stress States. «AASCIT» Engineering and Technology. Vol. 3, No. 1, 2016, pp. 30-46.

[6] China Machine Press, Mechanical design handbook, New Edition, Volume 5.31~124~135; 31-57~60; 31-134~136. (in Chinese).

[7] Shaobian Zhao and Zhongbao Wang, Anti-Fatigue Design---Methods & Data, China Machine Press, P. 90-109, 469-489 (in Chinese).

[8] Masing, G. Eigerspannungen and Verfestigung bein Messing, Proceeding of the 2nd International Congress of Applied Mechanics, Zurich, 1976, pp. 332-335.

[9] Yung-Li Lee, Jwo Pan, Richard B. Hathaway, Mark E. Barkey. Fatigue Testing and Analysis Theory and Practice. The first edition, Elsever Inc., USA. 142-152.

[10] David Broek Elementary Engineering Fracture Mechanics, Third revised edition Martinus Nijhoff Publishers, 1982, 6-14.

[11] Michael F. Ashby and David R. H. Jones, Engineering Materials, An introduction to their propertied and applications, Pergamon Press, 1980, P. 145-147.

[12] B. P. Kogaev, H. A. Maxymov, A. P. Gycenkov. Calculations on Strength and Durability for the Structure and Mechanical parts, Handbook, Machinery Industry Press, Moscow, 1985 70~75. (in Russian) (В. П. Когаев, Н. А. Махутов, А. П. Гусенков. Расчеты Деталей Машин и Конструкций на Прочность и долговечность, Справочник, Машиностроение 1985, Москва, 70-75).

[13] S. V. Doronin, et al., Ed. RAN U. E. Soken, Models on the fracture and the strength on technology systems for carry structures, (Novosirsk Science, 2005), PP. 160-165.(in Russian).

[14] U. Zerbst, S. Beretta, G. Kohler, A. Lawton, M. Vormwald, H. Th. Beier, C. Klinger, I. C erny´, J. Rudlin, T. Heckel a, D. Klingbeil, Safe life and damage tolerance aspects of railway axles – A review. Engineering Fracture Mechanics. 98, 214–271 (2013).