
Frequency Prediction of a Von Karman Vortex Street Based on a Spectral Analysis Estimation

Rafael Bardera-Mora¹, Miguel Ángel Barcala-Montejano², Ángel Antonio Rodríguez-Sevillano^{2, *}, Miguel Ruiz de Sotro², Gonzalo González de Diego²

¹National Institute for Aerospace Technology (INTA), Torrejón de Ardoz, Spain

²Department of Aircraft and Space Vehicles, School of Aeronautics and Space Engineering, Universidad Politécnica de Madrid, Madrid, Spain

Email address

barderar@inta.es (R. Bardera-Mora), miguel.barcala@upm.es (M. A. Barcala-Montejano), angel.rodriguez.sevillano@upm.es (Á. A. Rodríguez-Sevillano), miguel.ruizdesotro@gmail.com (M. R. de Sotro), gonzalogddiego@gmail.com (G. G. de Diego)

*Corresponding author

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Abstract: Spectral analysis studies the power distribution over frequency of a signal. This allows the characterization of time signals by its harmonics. This article will establish a relationship between the autocorrelation function and the spectrum. The direct implementation of the theory when analyzing a finite time signal results in a raw periodogram or first estimation of the spectrum. However, owing to the biased nature of the autocorrelation function, the periodogram obtained will not be a good estimation. Thus, several estimation techniques are needed in order to acquire a reliable spectrum. Amongst the techniques handled are the averaging Welch method, the use of window functions or tapering and the implementation of Fast Fourier Transform algorithms. To validate the accuracy and improvements made with these techniques, an algorithm is implemented in Matlab. Several synthetic signals are assessed and the classical Kármán Vortex Street is performed in a wind tunnel experiment. The results obtained are proof of the need for a careful study of the different estimation techniques when analyzing a signal.

Keywords: Spectral Analysis, Welch Averaging, Tapering, Resampling, Kármán Vortex Street

1. Introduction

Spectral Analysis is a tool used in statistical signal processing that considers the problem of estimating the spectral distribution, i.e. the distribution of power in a frequency interval, of a time series obtained from measurements. In physics, the term used to describe the distribution of power over frequency is Power Spectral Density (PSD). The applications of spectral analysis cover diverse fields. In fields of Physics such as astronomy or meteorology, spectral analysis can be used to reveal hidden periodicities in the studied data which can be associated with recurring processes. In medicine, spectral analysis can be used for diagnosis and data can be obtained when applying

this technique to signals measured from the patient, such as an electrocardiogram or an electroencephalogram. In control systems, the need for characterizing the dynamical behavior of a given system makes spectral analysis an interesting option. Other interesting applications in the field of engineering could be the study of characteristic frequencies in order to avoid resonance problems, for instance, flutter problems in bridges and wings can cause severe damage in the structure and careful observation in their frequency behavior can avoid this.

This paper attempts to synthesis the complexity of spectral analysis into a simple and concise document where the basic mathematical theory is explained and the main techniques used in PSD estimation are developed. That way, a Kármán vortex street can be studied in the wind tunnel facility. In this

article, Fourier analysis will be used as a method to obtain the PSD. The use of Discrete Fourier Transform (DFT) establishes a relationship between the spectra and the signal. Alternative works which follow similar strategies can be found in [1] and [2].

Examples of specific applications of these techniques are the study of brain waves by periodogram estimation [3], vibration analysis of turbines using autoregressive methods [4] or testing if a time series is consistent with white noise using wavelets coefficients [5].

2. Spectrum Analysis Definitions

2.1. Autocorrelation and PSD

In order to determine the PSD of a signal, it is necessary to start by defining the autocorrelation function. The autocorrelation function of a real continuous signal $x(t)$ is defined in the following expression from [6],

$$R_{xx}(t_1, t_2) = E(x(t_1)x(t_2)) \quad (1)$$

where $E(x(t_1)x(t_2))$ represents the expectation of a generic signal $x(t)$, and t_1 and t_2 are two different instants in time. When $t_1 = t_2$, the autocorrelation function is equal to the variance of the signal. In the case of a stationary signal, the statistics of a temporally random process are independent of the time origin. This implies that mean values and statistical moments of the process are constant as they are not dependent upon the absolute value of time. Thus, the autocorrelation function only depends on the time lag $\tau = t_2 - t_1$.

$$R_{xx}(\tau) = E(x(t)x(t + \tau)) \quad (2)$$

In addition, if the random process is ergodic, every member of an ensemble in a stationary process has the same statistical behavior. Hence, a single sample function can determine the statistical behavior of the complete ensemble and the statistical moments can be determined by time averages over a period of time T as well as by ensemble averages.

$$R_{xx}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)x(t + \tau) dt \quad (3)$$

This function, as seen in expression [3], gives an idea of the similarity of a signal with itself displaced a lag of time. Its relation with spectrum analysis can now be understood. The harmonics with a characteristic period of time τ_0 will yield a high value in the integral, e.g. a sine wave displaced a value 2π would conserve its similarity with the original signal resulting in a peak in the autocorrelation function for that time lag.

The Wiener–Khinchine theorem presents a relationship between the autocorrelation function and the spectrum. Being the latter the Fourier Transform of the former [7].

$$S(\omega) = \int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega\tau} d\tau \quad (4)$$

Reciprocally, the autocorrelation function can be retrieved with the definition of the inverse Fourier Transform.

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S(\omega)e^{i\omega\tau} d\omega \quad (5)$$

It is interesting to observe that for the case of $\tau = 0$ the spectrum integral over the frequency range is equal to the autocorrelation function, that is, the variance of the signal.

$$\sigma^2 = \int_{-\infty}^{\infty} S(\omega)d\omega \quad (6)$$

where $\sigma^2 = E(x(t)^2)$ is the variance of the signal. This shows that the integrand $S(\omega)d\omega$ is the contribution to the variance (that is, the fluctuation power) of $x(t)$ from the frequency band $d\omega$ centered at ω . This is the reason why the spectrum of a signal is also known as the Power Spectral Density (PSD).

Using the convolution property of the Fourier Transform, that states that the transform of the convolution of two signals is the product of the transform of those two signals [8], the PSD can also be obtained with the following equation [7],

$$S(\omega) = \lim_{T \rightarrow \infty} \frac{|X(\omega)|^2}{T} \quad (7)$$

where $X(\omega)$ is the Fourier Transform of the time signal $x(t)$.

2.2. Discrete Formulation

In real time history records, the signals obtained are not continuous but are sampled at discrete time instants. Thus, it is necessary to redefine the previous expressions for the discrete case. The Discrete Fourier Transform (DFT) of a signal can be defined as

$$X_k = \sum_{n=0}^{N-1} x_n e^{-i\frac{2\pi}{N}kn} \quad (8)$$

where the subscript x_n refers to the sample n of the discrete time signal and X_k refers to the Fourier Transform at the frequency $\frac{2\pi}{N}k$, being N the number of samples for the discrete signal [9]. Hence, to relate this expression to the continuous case of the Fourier Transform, it is necessary to multiply the DFT by the time between samples Δt .

$$X_k|_{real\ case} = \Delta t \cdot X_k \quad (9)$$

where X_k is defined in equation (8) and $X_k|_{real\ case}$ corresponds to the discrete approximation of the continuous Fourier Transform of the signal $x(t)$ [10] Finally, the PSD expression for discrete time signals results from the following expression

$$S_k = \frac{\Delta t^2 X_k^2}{T} = \frac{\Delta t X_k^2}{N} = \frac{X_k^2}{N f_s} \quad (10)$$

where f_s is the sampling frequency at which the discrete signal has been recorded, i.e., the inverse of the time difference between samples Δt . The estimation of the spectrum in equation (10) is called the periodogram of a discrete signal x_n . However, this periodogram is not the best estimator for the spectra owing to the biasing of the

autocorrelation function in the discrete case. The biasing of the autocorrelation function is the result of the availability of few elements in the estimation for large lags of time in the discrete case (there is little overlapping of the two signal vectors, the original, and the displaced one).

The complexity of a DFT algorithm is $O(N^2)$, meaning that the computational time required for the calculation of the transform increases in a quadratic manner with the number of elements N involved. To improve this, Fast Fourier Transform (FFT) algorithms are used in order to reduce computational time due to their lower complexity. The complexity of these algorithms is $O(N \log_2 N)$ [11] allowing a decrease in the necessary computational resources.

2.3. Resampling

Many types of measurement techniques acquire data in random time intervals. Therefore, it is not possible to control

the time history of the sampling. In the vortex street application, velocity measurements are taken by detecting particles within the flow, and the arrival of these cannot be controlled nor predicted. This is an issue for the estimation of Fourier Transforms due to the requirement of evenly distributed time signals in FFT algorithms. To solve this problem, a resampling technique is needed.

Many resampling methods have been suggested but the Sample & Hold algorithm is the most wide spread and one of the easiest to implement [12]. This method consists of a zero order interpolation that follows the expression.

$$x_{resampled}(t) = x(t_i) | (t_i < t < t_{i+1}) \tag{11}$$

where $x(t_i)$ corresponds to the discrete sample recorded in the instant t_i . Figure 1 shows the resampling process of the Sample & Hold method of an arbitrary non-dimensional signal.

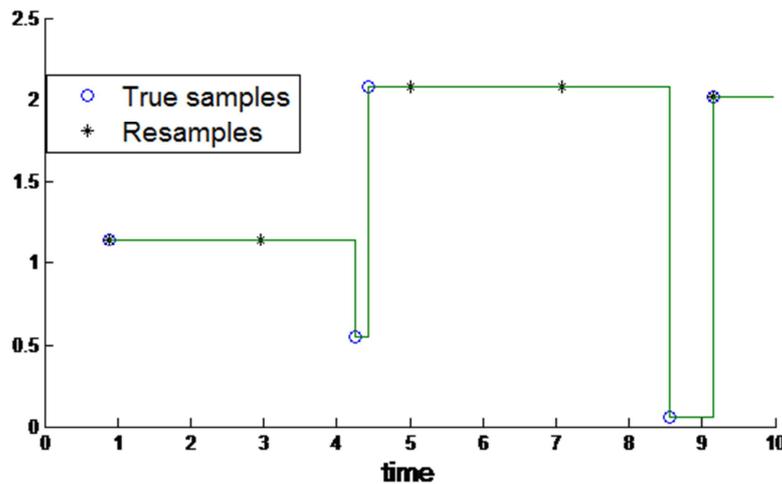


Figure 1. Sample & Hold resampling method.

From this figure, two immediate problems can be observed. Firstly, if two neighboring true samples are further apart than the time between resamples, the first one will be resampled several times although no new information about the signal is available. Secondly, if various samples are between two resamples, the information contained in all but the last one will be lost.

The study carried out by [13] shows that the estimation of the spectrum is influenced by two phenomena.

- a) The Sample & Hold method acts like a low pass filter due to the fact that not all samples are taken into account during the resampling. The cutoff frequency at which the spectrum is attenuated by half is $\hat{n}/2\pi$.
- b) The Sample & Hold method introduces white noise owing to the discontinuity of the steps formed during the resampling. The level of noise will be reduced by increasing the mean sampling rate of the original signal.

3. Estimation Techniques

3.1. Welch Averaging Method

As mentioned in the previous section, the periodogram is

not a good estimator for the PSD of a signal. This means that even if the number of elements in a signal is increased, the autocorrelation function for very large time lags will have fewer elements than for smaller lags. Hence, the estimation will be poor.

To solve this, the estimation of the PSD can be improved by averaging different periodograms of the signal. This can be achieved by splitting the time signal into different portions upon which the periodograms are estimated [14]. This technique considerably reduces the noise in a signal in exchange of reducing the frequency resolution due to the fact that the minimum frequency detected in a spectrum is the inverse of the duration of the signal [9]. Thus, for a higher amount of segments used, the shorter their lengths will be,

$$\hat{S} = \frac{1}{L} \sum_{l=1}^L S^l \tag{12}$$

where \hat{S} is the estimation of the PSD from L different periodograms calculated for L segments of the original signal.

Bartlett's method consists in averaging periodograms corresponding to segments of a signal that are not overlapped. On the other hand, Welch's method improves the estimation

of spectra by overlapping the segments in the signal so as to increase the number of periodograms for averaging.

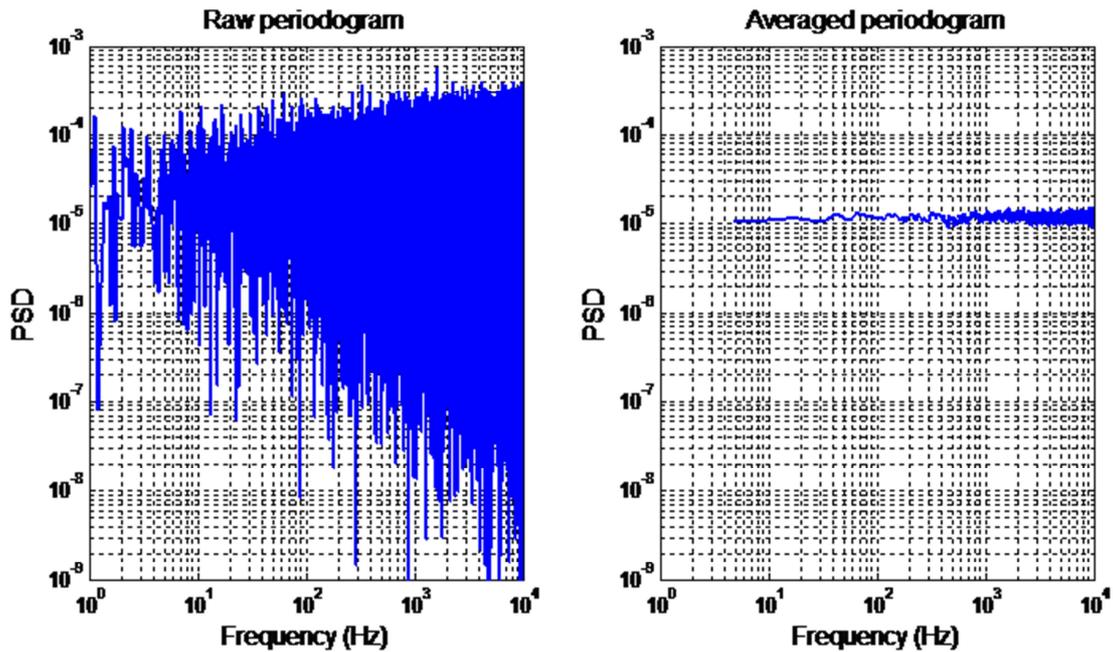


Figure 2. Averaging Spectra method.

Figure 2 shows the improvements made with this technique. The spectra correspond to a white noise non-dimensional function. This function is known to have a constant PSD, i.e., it has the same energy content for all frequencies. The averaged periodogram has been obtained by splitting the signal into 150 different segments and a periodogram has been calculated for each one of them.

3.2. Tapering

Another problem encountered during the estimation of the PSD of a signal is the fact that DFT algorithms treat the signal

as a period of a periodic function. This is due to the nature of the Discrete Fourier Transform, which is the summation of a finite number of complex exponentials or harmonics [15].

Owing to this periodicity, there will generally be a discontinuity between the first and the last elements of the signal, resulting in undesirable noise. To solve this problem, the original signal is multiplied by a window function so that both ends of the signal have zero values, avoiding the mentioned discontinuity [16]. In Figure 3 some non-dimensional window functions are shown. A comparison in the effect of the different windows can be seen in [17].

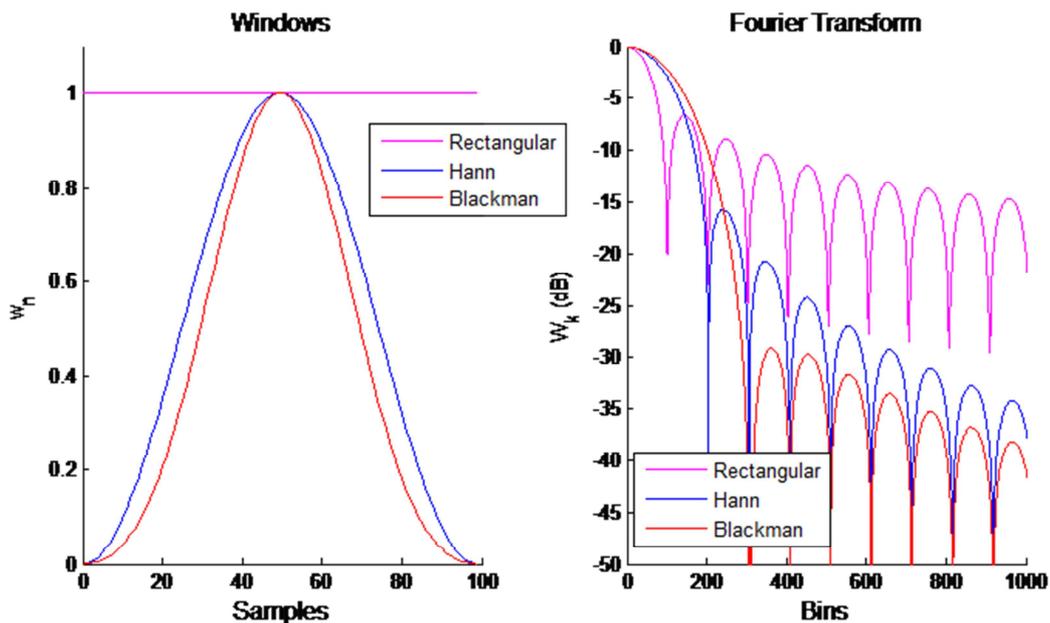


Figure 3. Window function examples.

However, the tapering technique generates a spectral leakage where energy content in a frequency is spread over its surroundings. To explain this, it is necessary to take into

account the Fourier Transform convolution property, i.e., the product of two signals is transformed into the convolution of the transformations of both signals (equation (13)).

$$F(x(t) \cdot w(t)) = X(\omega) * W(\omega) = \int_{-\infty}^{\infty} X(\omega - \Omega) \cdot W(\Omega) d\Omega \tag{13}$$

This means that for the estimation of the PSD at the frequency ω , the energy content of the signal at frequency $\omega - \Omega$ is added due to the lobes of the window function transform present in the frequency Ω . The ideal case for a window is the one with a Dirac delta function as a Fourier Transform. However, in this case the convolution would not modify the original signal. The window with this transform corresponds to a constant function of unitary value which does not attenuate both ends of the time signal.

The window function choice must depend on the spectral form of the original signal. For instance, the number of harmonics and its amplitude must be taken into account when choosing a window. Preferably, windows should have their main lobe as narrow as possible because this leads to a higher frequency resolution. They should also have their secondary lobes as low as possible so as to mitigate the leakage, although this contributes to a wider main lobe, meaning that this effect compromises the first point.

3.3. Zero Padding

Zero-padding is a technique where zeros are added to the end of the original signal in order to increase its length. This could seem to result in an increase of frequency resolution but it is false. Adding zeros to the signal will not contribute with new information and the effect is the same as applying a rectangular window, which can lead to an excessive leakage

[15].

Nevertheless, due to the fact that the efficiency of FFT algorithms is highly influenced by the number of elements in the signal, zero padding is used to obtain a signal with a number of elements that is a power of two so that FFT efficiency is maximized [11].

4. Applications

4.1. The Algorithm

The previous techniques have been implemented in an in-house Matlab program to test its efficiency and accuracy.

Figure 4 shows the flow chart which includes the main aspects of the software. In this program the signal $x(t)$ is resampled for the case where it is not evenly distributed in time. The Welch averaging method is used by splitting the signal into a number of portions predefined by the user. Then, for each portion, the Hann window function is applied. This window is used mainly because of its widespread use but it can be changed by the user. In order to maximize computational efficiency, zeros are added to the different portions so as to acquire a number of samples corresponding to a power of two. FFT is applied to each of the portions to calculate several periodograms which are then averaged to obtain the final estimation of the spectrum $S(f)$.

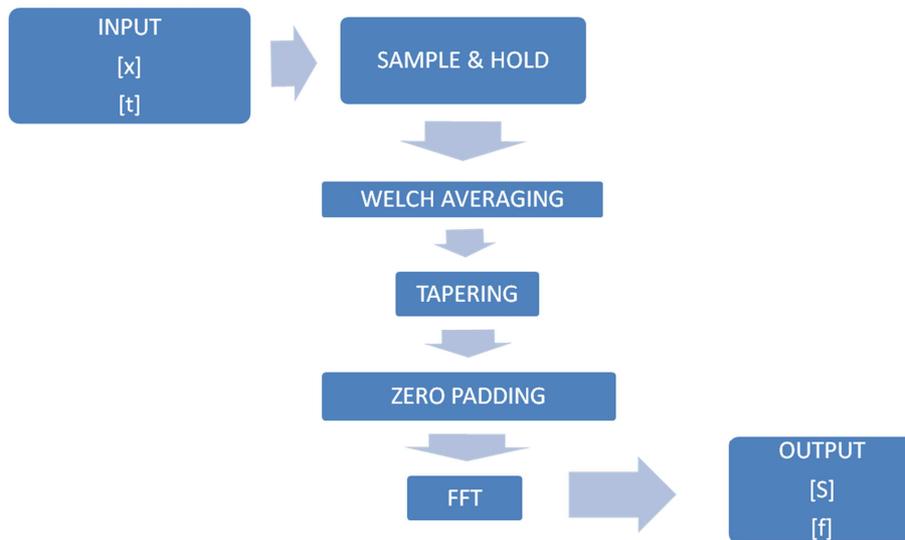


Figure 4. Spectral analysis program flow chart.

4.2. Synthetic Signals

A way to prove the software’s reliability is to create signals based on mathematical functions and white noise. The goal is to minimize the effect that the noise produces in the spectra obtained.

The signal that will be analyzed is a non-dimensional periodic function with fixed harmonics and white noise. Its

mathematical expression is the following

$$x(t) = 4 \sin(20\pi t) + 2 \sin(50\pi t) + 8 \sin(60\pi t) + x_{noise} \tag{14}$$

Figure 5 shows the results of using this software for obtaining the spectra. On the left, the raw spectrum has been represented. It corresponds to the case where none of the techniques have been applied and the signal has been directly transformed to the frequency domain. On the right, the estimation techniques have been applied showing a clearer

and more accurate spectrum. The frequencies of the peaks shown in the spectrum perfectly match the frequencies corresponding to each of the harmonics present in the original signal. In addition, the intensity of these peaks is directly related to the amplitudes of the respective harmonics.

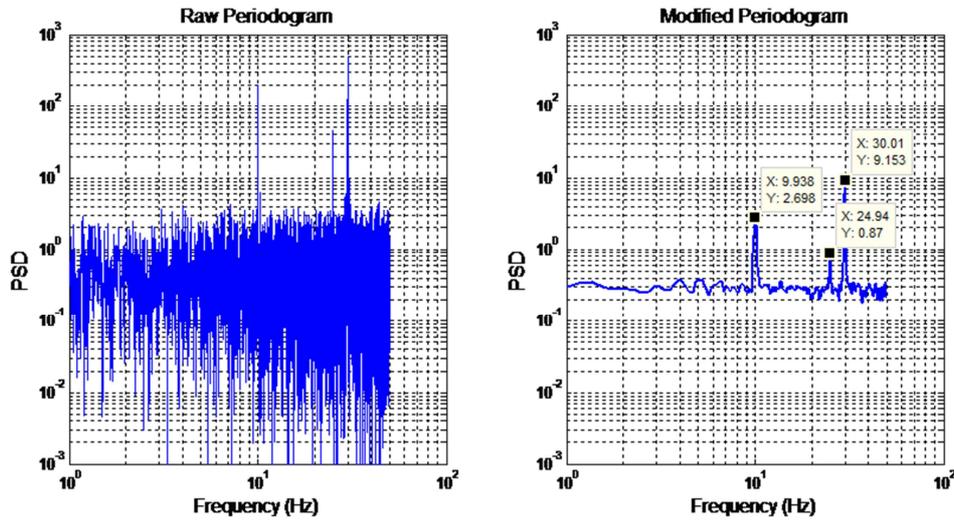


Figure 4. PSD comparison of evenly distributed in time signal.

The next synthetic signal, with the same harmonics as before, simulates a stochastic signal where the samples are not evenly distributed in time. To achieve this, the samples have been generated according to a time vector that follows a Poisson statistical distribution [18].

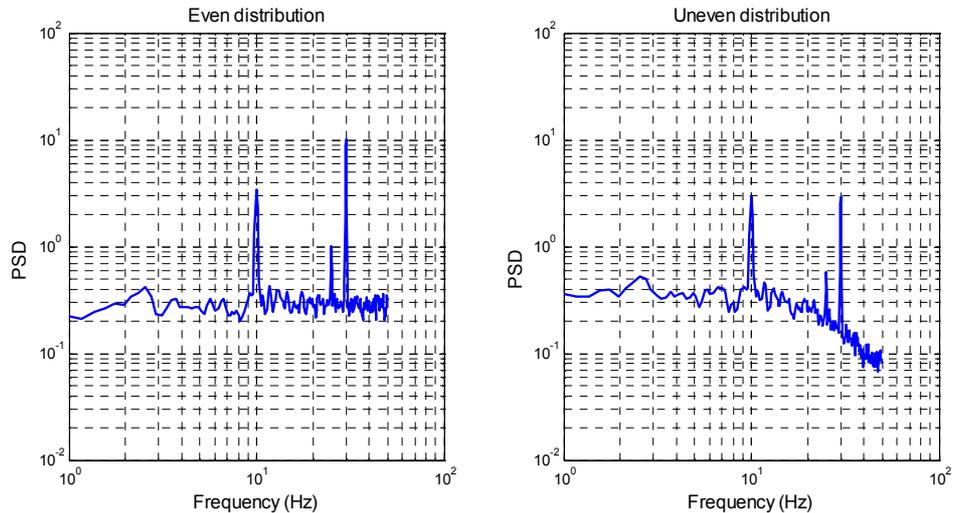


Figure 5. PSD comparison between even and uneven samples distributions in time.

Figure 6 shows the result of analyzing a signal which is not evenly distributed in time, meaning that resampling is necessary. The harmonics are still detected with relatively high precision but, due to the Sample & Hold method, the resampling behaves as a low pass filter, attenuating high frequencies as shown in the spectrum of the right.

4.3. Kármán Vortex Street

A real application of the developed software is done by reproducing the Kármán Vortex Street flow using a wind tunnel and analyzing the characteristic frequencies involved in it. This phenomenon consists in the shedding of eddies

behind a cylinder which is exposed to a uniform flow of velocity V [19] as shown in Figure 7. The characteristic frequencies will be those corresponding to the eddies generated in the flow.

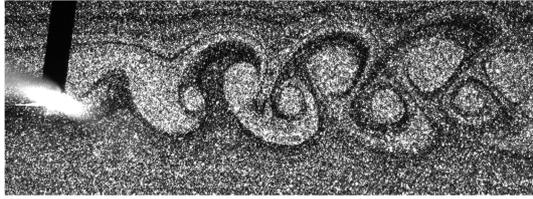


Figure 7. Kármán vortex street visualization at $Re=220$ (INTA).

The most important non dimensional parameters involved in this fluid-dynamic process are the Reynolds number and the Strouhal number, defined as the following expressions,

$$Re = \frac{\rho VL}{\mu} \quad (15)$$

$$St = \frac{fL}{V} \quad (16)$$

where f and L are characteristic values of frequency and length, and ρ and μ are the density and dynamic viscosity of the fluid respectively. The first number indicates the relationship between the convective and viscous forces in a fluid and quantifies the turbulence a flow has. On the other hand, the Strouhal number describes the oscillating flow mechanisms in the process.

The analysis of characteristic frequencies is done by calculating the PSD of the flow velocity behind the cylinder. In order to obtain the velocity measurements, a Laser Doppler Anemometry system (LDA) is used during the experiment. LDA is based on the Doppler effect to establish a relationship between flow velocity and the frequency changes in a laser beam [20]. Figure 8 shows the experiment set up in which two LDA systems are used. The green one measures the upstream velocity in order to regulate the wind tunnel speed. Its measurements are done more than 10

diameters ahead of the cylinder in order to minimize the perturbations induced by this instrument. The red one measures five diameters behind the cylinder and its data is analyzed so as to find characteristic frequencies in the fluid dynamic process.

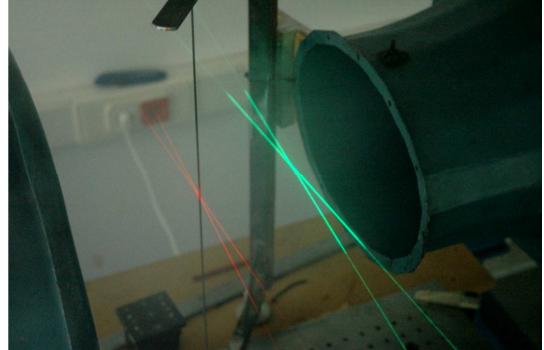


Figure 8. Experimental setup.

The rod diameter is 2 mm and the wind tunnel velocities under the measurements taken are 1 m/s and 2 m/s. Taking into account a temperature of 293 K, the Reynolds number can be estimated to be

$$\begin{aligned} Re_1 &= 132 \\ Re_2 &= 265 \end{aligned} \quad (17)$$

According to previous experiments, the Reynolds number obtained can be related to the Strouhal number [21].

$$\begin{aligned} Re_1 = 132 &\rightarrow St_1 = 0.18 \\ Re_2 = 265 &\rightarrow St_2 = 0.2 \end{aligned} \quad (18)$$

Once the Strouhal number is known, the shedding frequency behind the cylinder can be calculated. This first estimation allows us to determine whether or not the spectral analysis is successful.

$$\begin{aligned} U_\infty = 1 \text{ m/s} &\rightarrow f_1 = 90 \text{ Hz} \\ U_\infty = 2 \text{ m/s} &\rightarrow f_2 = 200 \text{ Hz} \end{aligned} \quad (19)$$

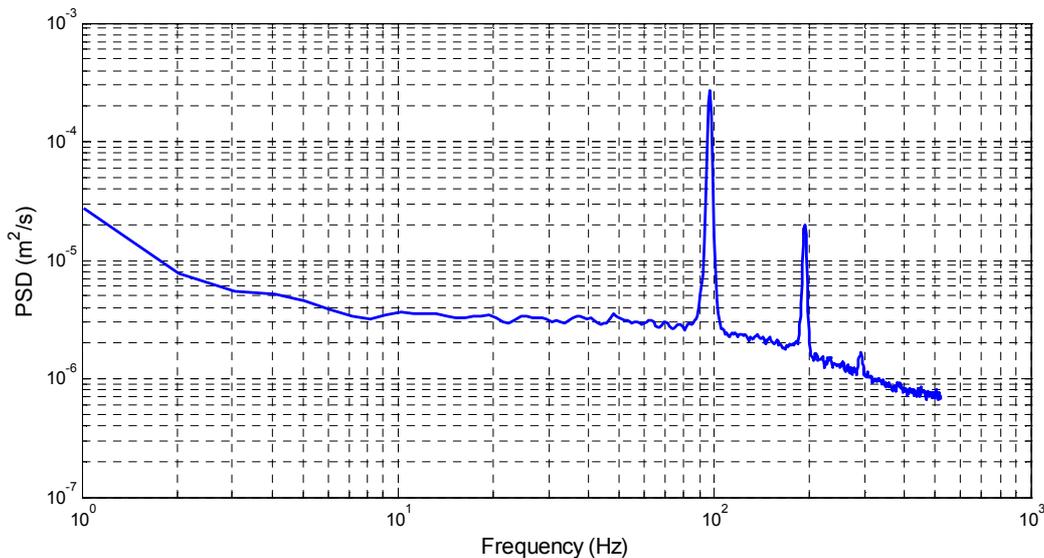


Figure 9. Spectral analysis of Kármán Vortex Street ($U_\infty = 1 \text{ m/s}$; $Re = 132$).

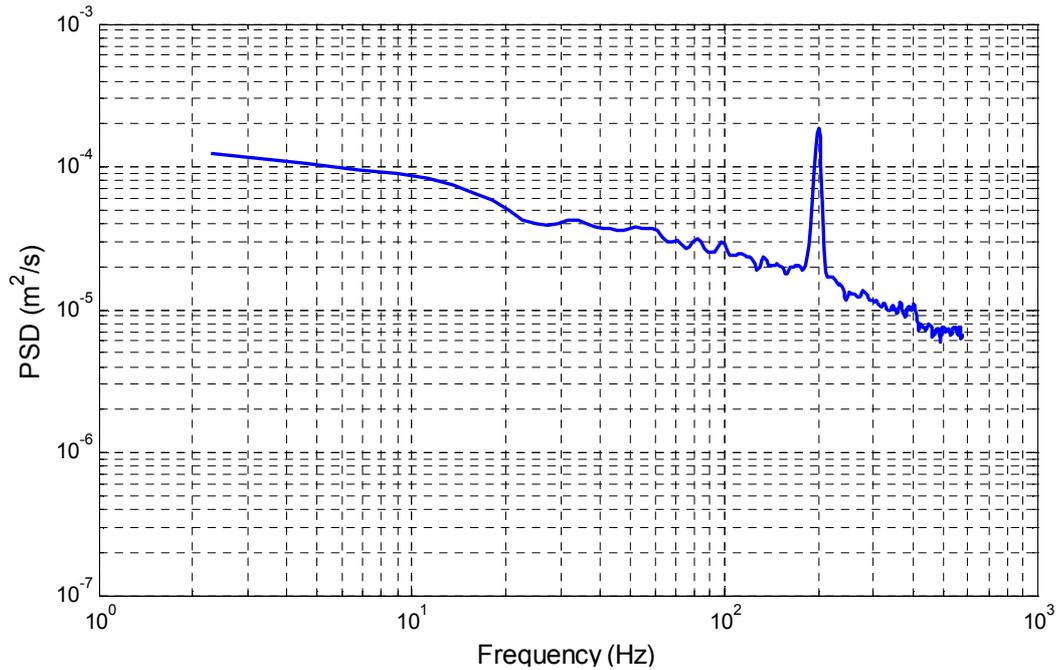


Figure 10. Spectral analysis of Kármán Vortex Street ($U_\infty = 2 \text{ m/s}$; $Re = 265$).

Results show a great success in estimating the vortex shedding frequency. In the first case, this frequency results to be 95 Hz, very close to the 90 Hz estimated. The slight difference can be due to temperature and wind speed variations. As for the second case, the frequency is exactly 200 Hz as predicted before. Figure 9 also shows a second and a third harmonics (they are multiples of the fundamental harmonic) that can be explained as reshaping of the fluid process from a perfect oscillation.

The spectra also shows more energy content in Figure 10 as the speed is greater and, thus, there is more kinetic energy. In both figures, a decrease in energy as frequency rises can be observed. This can be related to the energy cascade of turbulent flows characterized by Kolmogorov [22]. For very high frequencies this attenuation gets more noticeable owing to the low pass filter effect from the resampling.

5. Conclusions

Fourier analysis is one of the simplest methods for the spectra investigation of discrete time signals. Despite its simplicity, results obtained are reliable, accurate and easy to interpret. As proved in this article, a raw periodogram obtained from a direct transformation with FFT is not helpful due to the lack of smoothness that prevents the visualization of the power content of the PSD. This has led to the development of estimation techniques that enable an adequate correction of the curves regarding smoothness and resolution.

Results obtained from different experiments prove the reliability of the in-house developed software. In the first place, synthetic signals have allowed us to find known harmonics and to distinguish them from random noise by improving the quality of spectra. On the other hand, the

Vortex Street experiment was carried out successfully, achieving great levels of similarity with previous experimental results.

Once the software has been tested, it can be used in many applications of physics and engineering which need a careful study of signals not only in the time domain but also in the frequency domain.

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