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A mathematical model for different shapes of stenosis and slip velocity at the wall through mild stenosis artery

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Abstract

The Present investigation deals with the two-layered mathematical model of blood flow for a mild stenosis artery in the presence of axially variable, peripheral layer thickness and variable slip at the wall. The model consists of a core surrounded by a peripheral layer. It is assumed that the fluids of both the regions (core and peripheral) are Newtonian having different viscosity. The geometry of the interface between the peripheral layer and the core region has been determined and the result obtained in the analysis have been evaluated numerically and discussed briefly. In the present analysis, new analytic expression for the thickness of the peripheral layer has been obtained in terms of measurable quantities flow rate (Q), centerline velocity (U), pressure gradient $(-dp/dz)$, plasma velocity (μ_c). It is important to mention that in the present analysis, core viscosity has been obtained by two methods. Firstly, by calculating from the formula obtained in the present analysis; and second, by calculating the red cell concentration in the core and then using concentration versus relative viscosity curve. It is found that the agreement between the two is very good (error<1.4%). The significance of the present model over the existing models could be useful in the development of new diagnosis tools for many diseases.

1. Introduction

Atherosclerosis is the leading cause of death in many countries. There is considerable evidence that vascular fluid dynamics plays an important role in the development and progression of arterial stenosis, which is one of the most widespread diseases in human beings. The fluid mechanical study of blood flow in artery bears some important aspects due to the engineering interest as well as the feasible medical applications. Stenosis, a medical term which means narrowing of an artery, tube or orifice, is the abnormal and unnatural growth in arterial wall thickness that develops at various locations of the cardiovascular systems under diseased conditions. The actual causes of stenosis are not well

known but it has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissues may be responsible for the same (Young, 1968; Shukla et al. 1980b; Chaturani and Ponalagusamy, 1986). Cardiovascular system can cause circulatory disorders by reducing or occluding the blood supply which may result in serious consequences (myocardial infarction, cerebral stroke). The hemodynamic behavior of the blood flow is influenced by the presence of the arterial stenosis. If the stenosis is present in an artery, normal blood flow is disturbed. The actual causes of stenosis are not well known but its effects on the cardiovascular system can be understood by studying the blood flow in its vicinity Bugliarello and Hayden (1963), Bugliarello and Sevilla (1970) have experimentally observed that when blood flows through narrow tubes there exists a cell free plasma layer near the wall. In view of their experiments, it is preferable to represent the flow of blood through narrow tubes by a two-layered model instead of one-layered model. But Lee and Fung (1970) have obtained the numerical results for the streamlines and distribution of velocity, pressure, vorticity and the shear stress for different Reynolds number in blood flow through locally constricted tubes. For such type of models, the flow of blood is represented by one-layered model.

Many investigators (Chaturani and Kaloni, 1976; Chaturani and Upadhya, 1979; Shukla et al., 1980b) have theoretically studied the flow of blood through uniform and stenosed tubes and analyzed the influence of slip velocity or peripheral plasma layer thickness on the flow variables such as velocity, wall shear stress and flow resistance. In these models, the peripheral layer thickness and slip velocity are assumed a priori based upon the experimental observations. To understand the flow patterns in stenosed arteries. A fairly good number of theoretical and experimental studies on the blood flow through stenosed artery are available in the published literature, and some of them are Young (1968), has analyzed the flow of blood through an arterial stenosis. Shukla et al (1980a, b) have taken two-layered models and analyzed the influence of peripheral plasma viscosity on flow characteristics. Chaturani and Kaloni (1976), Chaturani and Ponalagusamy (1982) and Ponalagusamy (1986) considered the flow of blood represented by a two-layered model. Kumar and Kumar (2006) conducted a numerical study of the axi-symmetric blood flow in a constricted rigid tube. Sahu et.al (2010) obtained results of arterial blood flow in stenosed vessel using non-Newtonian couple stress fluid model. Basu et.al (2012) investigated a non- Newtonian two-phase fluid model for blood flow through arteries under stenotic condition. More recently Basu et.al (2013) discussed on a non-Newtonian fluid model for blood flow using power law through an atherosclerotic arterial segment having slip velocity. Ellahi et.al (2014) gave an idea on a mathematical study of non-Newtonian micropolar fluid in arterial blood flow through composite stenosis. Agarwal and Varshney

(2014) investigated that the slip velocity effect on MHD oscillatory blood flows through stenosed artery. Sharma et.al (2014) observed that the pulsatile MHD flow in an inclined catheterized stenosed artery with slip on the wall.

In all these models, the peripheral layer thickness is assumed a priori. It would be of interest to obtain the analytic expression for peripheral layer thickness in terms of the measurable flow variables (flow rates, pressure gradient, etc.).

2. Formulation of the Governing Equations

Consider an axially axisymmetric model of blood flow in a uniform circular tubes or vessels of radius R. The Symmetric flow of blood assumed to be incompressible in the two directions through a rigid walled artery with an axially symmetric paired stenosis. Also we consider the flow of a plasma layer and a core layer with viscosity coefficient (μ_p) and (μ_c) respectively. Thus the geometry of stenosis, (in non-dimensional form) is given by (Ponalagusamy, 1986), which is assumed to be symmetrical:

$$R(z) = 1 - \frac{\delta_s}{R_0 L_0{}^n} \frac{n^{n/n-1}}{(n-1)} \left[L_0{}^{n-1}(z-d) - (z-d)^n \right] \quad for \ \ d \le z \le d + L_0$$
$$= 1, \qquad\qquad\qquad\qquad\qquad\qquad otherwise \qquad (1)$$

where $n (\ge 2)$ is a parameter determining the shape of the stenosis, $R(z)$ is radius of the artery in the stenotic region, L_0 is length of the stenosis, d is the location, δ_s is the maximum height of the stenosis at $z = d + \dfrac{L_0}{n^{1/n-1}}$.
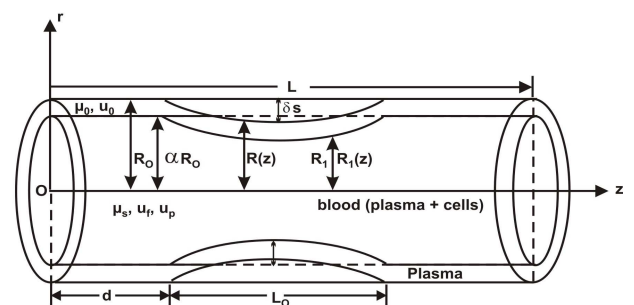

Figure (1). *Geometry of two fluid layers.*

We shall take cylindrical coordinate system (z, r, θ) whose origin is located on the vessels axis. The problem is investigated under the following assumptions (Philip and Chandra, 1996):
i.   Nobody forces act on fluid.
ii. The motion is slow, so the inertia effects can be neglected.
iii. Flow, which is due to the pressure gradient, is one-

dimensional and fluid is incompressible.

iv. The variation of cross-section of artery is considered to be very small.

The boundary conditions are:

$$
\begin{aligned}
&(i)\ u_p = u_S(z) && at && r = R(z)\\
&(ii)\ u_p = u_C && at && r = R_1(z)\\
&(iii)\ \tau_p = \tau_C && at && r = R_1(z)\\
&(iv)\ \frac{\partial u_P}{\partial r} = 0 && at && r = R_P
\end{aligned}
\qquad (2)
$$

where $u_S = \dfrac{\overline{u}_S}{\overline{U}_0}$ is the non-dimensional axially variable slip velocity, τ is the shear stress and R_P is the plug core radius.

The consistency function $\overline{\mu}\left(\overline{r}\right)$ may be written as:

$$
\overline{\mu}\left(\overline{r}\right) = \begin{array}{ll}
\overline{\mu}_C & at \quad 0 \le \overline{r} \le \overline{R}_1\left(\overline{z}\right)\\
\overline{\mu}_P & at \quad \overline{R}_1\left(\overline{z}\right) \le \overline{r} \le \overline{R}\left(\overline{z}\right)
\end{array}
\qquad (3)
$$

where $\overline{\mu}_C$ and $\overline{\mu}_P$ are the viscosities of central core fluid and plasma respectively and $\overline{R}_1\left(\overline{z}\right) \le \overline{r} \le \overline{R}\left(\overline{z}\right)$ are the radii of central core region and the artery in the stenotic region.

The non-dimensional variables are:

$$
r = \frac{\overline{r}}{R_0}, \quad z = \frac{\overline{z}}{z_0}, \quad R = \frac{\overline{R}}{R_0}, \quad p = \frac{\overline{p}}{\rho_0 U_0}, \quad u_c = \frac{\overline{u}_c}{U_0}, \quad u_p = \frac{\overline{u}_p}{U_0}, \quad \delta_s = \frac{\overline{\delta}_s}{R_0}
$$

where $\overline{u}$ and $\overline{v}$ are velocity components in the axial $\overline{z}$ and radial $\overline{r}$ directions, $\overline{p}$ the pressure, $\overline{\rho}$ is the density, $\overline{R}_0$ is the radius of the normal artery, $\overline{z}_0$ the one-fourth length of the stenosis $\overline{L}_0$, $\overline{U}_0$ the average velocity in the normal artery region and $\overline{\delta}_s$ is the maximum height of the stenosis (Figure 1).

As per discussion made by Young (1968), the appropriate equations describing the flow in the case of a mild stenosis $(\delta_s / R \ll 1)$, subject to the additional condition (a) $\mathrm{Re}_p\,(\delta_s / L_0) \ll 1$ (b) $2R_0 / L_0 \sim o(1)$ are:

For region, $0 \le r \le R_1(Z)$,

$$
\frac{\partial p}{\partial z} = \frac{\beta}{\mathrm{Re}_p\,\mu}\left[\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u_c}{\partial r}\right]
\qquad (4)
$$

$$
-\frac{\partial p}{\partial r} = 0
\qquad (5)
$$

For region, $R_1(z) \le r \le R\,(Z)$

$$
\frac{\partial p}{\partial z} = \frac{\beta}{\mathrm{Re}_p}\left[\frac{\partial^2 u_c}{\partial r^2} + \frac{1}{r}\frac{\partial^2 u_p}{\partial r}\right]
\qquad (6)
$$

where $\mu = \dfrac{\overline{\mu}_p}{\overline{\mu}_c}, \beta = \dfrac{\overline{z}_0}{R_0}$ and $\mathrm{Re}_p = \dfrac{\overline{U}_0\,\overline{R}_0\,\overline{\rho}_p}{\overline{\mu}_p}$

3. Numerical Approach

Using boundary condition (2), the solution of (4) and (6) can be written as

$$
u_c = c(z)\left[R_0{}^2 - R_1{}^2 - (1-\mu)ur^2 + u_s(z)\right]
\qquad (7)
$$

$$
u_p = c(z)\left[R^2 - r^2\right] + u_s(z)
\qquad (8)
$$

where $c(z) = \dfrac{q(z)\mathrm{Re}_p}{4\beta}$ with $q(z) = -\dfrac{\partial p}{\partial z}$

$$
Q = \int_{R_1(z)}^{R(z)} 2r\,dr
$$

The Flow rate Q may be obtained as:

Flow rate for peripheral layer Q_p by using equation (8) is

$$
Q_p = \int_{R_1(z)}^{R(z)} 2r\,dr = \frac{1}{2}c(z)\left[\left(R^2 - R_1{}^2\right)^2 + u_s R_1{}^2\right]
\qquad (9)
$$

Similarly, flow rate for core region Q_C as:

$$
Q_C = \frac{1}{2}c(z)\left[2R^2 R_1{}^2 - 2R_1{}^4(1-\mu) - \mu R_1{}^4\right] + u_s R_1{}^2
\qquad (10)
$$

The total flow rate Q is: $Q = Q_C + Q_p$

$$
Q = \frac{q(z)\mathrm{Re}_p}{8\beta}\left[R^4 - R_1{}^4 - (1-\mu)\right] + R^2 u_s(z)
\qquad (11)
$$

$$
-\frac{\partial p}{\partial z} = \frac{8\beta}{\mathrm{Re}_p}\left[\frac{Q - R^2 u_s(z)}{R^4 - R_1{}^4 - (1-\mu)}\right]
\qquad (12)
$$

Integrating above equation and using the condition $p = p_0$ at $z = 0$ and $p = p_1$ at $z = L$ (From Fig.1) and simplifying, we obtained as:

$$
\lambda = \frac{p_0 - p_1}{Q} = \frac{8\beta}{\mathrm{Re}_p}\int_0^L \frac{dz}{\left[R^4 - R_1{}^4(1-\mu)\right]} - \frac{8\beta}{\mathrm{Re}_p}\int_0^L \frac{R^2 u_S}{\left[R^4 - R_1{}^4(1-\mu)\right]}dz
\qquad (13)
$$

where $L = \dfrac{\overline{L}}{z_0}$ and resistance to flow.

The wall shear stress τ_w can be defined as:

$$
\tau_w = \frac{1}{\mathrm{Re}_p}\left[\frac{\partial u_P}{\partial r}\right]_{r=R}
\qquad (14)
$$

Using equation (8) and (12) then:

$$\tau_w = \frac{4R}{\text{Re}_P}\left[\frac{Q - R^2 u_S(z)}{R^4 - R_1^4(1-\mu)}\right] \qquad (15)$$

4. Analytical Solution for Velocity, Core Viscosity and Thickness

Case-I For one layered model with the slip at the wall (when $R = R_1$), then calculate the flow rate Q_{1L} and wall shear stress τ_{1L} from equation (11) and equation (15) as:

$$Q_{1L} = \frac{\rho^* q(z)\text{Re}_p}{8\beta}.R^4 + R^2 u_C \qquad (16)$$

$$\tau_{1L} = \frac{4(Q_{1L} - R^2 u_S)}{\rho^* \text{Re}_p R^3} \qquad (17)$$

Case-II For two layered model with the slip at the wall (when $u_s = 0$), then calculate the flow rate Q_{2L} and wall shear stress τ_{2L} from equation (11) and equation (15) as:

$$Q_{2L} = \frac{q(z)\text{Re}_p R^4}{8\beta}\left[1 - A^4(1-\mu)\right] \qquad (18)$$

$$\tau_{2L} = \frac{4Q_{2L}}{\text{Re}_p R^3\left[1 - A^4(1-\mu)\right]} \qquad (19)$$

where $A = 1 - \dfrac{\delta}{R}$ and $\dfrac{\delta}{R}$ is the thickness of peripheral layer which is a function of axial distance z. Since the one-layered with slip and two-layered without slip represent the same phenomena.

The flow rates and wall shear stresses can be equated as:

$$Q_{1L} = Q_{2L} \qquad and \qquad \tau_{1L} = \tau_{2L}$$

$$\frac{\rho^* q(z)\text{Re}_p .R^4}{8\beta} + R^2 u_S = \frac{q(z)\text{Re}_p R^4}{8\beta}\left[1 - A^4(1-\mu)\right] \quad (20)$$

and

$$\frac{4(Q_{1L} - R^2 u_S)}{\rho^* \text{Re}_p R^3} = \frac{4Q_{2L}}{\text{Re}_p R^3\left[1 - A^4(1-\mu)\right]} \quad (21)$$

Solving above equation (20) and (21) we obtain u_S as:

$$u_S = \frac{q(z)R^2}{8\beta}\left[\text{Re}_p\{1 - A^4(1-\mu)\} - \rho^* \text{Re}_p\right] \qquad (22)$$

Using equation (19), the expression of core viscosity as:

$$\bar{u}_C = \frac{\bar{\mu}_P A^4}{\left[A^4 + \dfrac{8\beta Q^*}{q(z)\text{Re}_p R^4} - 1\right]} \qquad (23)$$

For a two-layered model (plasma layer and a core layer) without slip $u_S = 0$, the expression for velocity in the core region is obtained from equation (7) as:

$$u_C = \frac{q(z)\text{Re}_p R^2}{4\beta}\left[1 - A^2(1-\mu) - \mu\left(\frac{r}{R}\right)^3\right] \qquad (24)$$

The centerline velocity (U) from equation (24) can be obtained as:

$$U = \frac{q(z)\text{Re}_p R^2}{4\beta}\left[1 - A^2(1-\mu)\right] \qquad (25)$$

Elimination μ from (18) and (24) gives:

$$\frac{\delta}{R} = 1 \pm \left[\frac{q(z)\text{Re}_p R^4 - 8\beta Q^*}{q(z)\text{Re}_p R^4 - 4\beta U R^2}\right]^{1/2} \qquad (26)$$

From equation (25), we notice that the centerline velocity (U) is always less than $\dfrac{q(z)\text{Re}_p R^2}{4\beta}$. Since only those values of $\dfrac{\delta}{R}$ are of interest which are real and less than or equal to unity. The following condition can be obtained from (26).

$$q(z)\text{Re}_p R^2 \geq 8\beta Q^* \qquad (27)$$

The equation (26) becomes:

$$\frac{\delta}{R} = 1 - \left[\frac{q(z)\text{Re}_p R^4 - 8\beta Q^*}{q(z)\text{Re}_p R^4 - 4\beta U R^2}\right]^{1/2} \qquad (28)$$

We have used experimental data of flow through uniform tube. We write equation (22), (23) and (28) in the dimensional form as:

$$u_C = \frac{q(z)\text{Re}_p R^2}{4\beta}\left[1 - A^2(1-\mu) - \mu\left(\frac{r}{R}\right)^3\right] \qquad (29)$$

$$\bar{u}_c = \frac{\bar{\mu}_p(\bar{A})^4}{\left[+\dfrac{8\bar{Q}^* \bar{\mu}_p}{\pi \bar{q}_0(\bar{R}_0)^4} - 1\right]} \qquad (30)$$

where $(\bar{A}) = 1 - \dfrac{\bar{\delta}_0}{\bar{R}_0}$, $\bar{\delta}_0$ is the peripheral layer thickness in the normal artery region, $\bar{q}_0$ is the pressure gradient.

5. Numerical Result and Discussion

A two-layer model consisting of a core region of suspension of all the erythrocytes in plasma and a peripheral layer of plasma (Newtonian fluid) has been proposed to describe blood flow in small diameters vessels. We have the following data from Bugliarello and Sevilla (1963) and Bugliarello and Hayden (1970).

For $40\,\mu m$ *tube diameter* $C = 40\%$, $\overline{Q}* = 19.23 x10^{-6} cm^3 / \sec, \overline{\delta}_0 = 3.2\mu m, \overline{\mu}_0 = 0.144 \ and \ \overline{U} = 2.3 cm/\sec$

For $66.6\,\mu m$ *tube diameter* $C = 6\%$, $\overline{Q}* = 45.6546 x10^{-6} cm^3 / \sec, \overline{\delta}_0 = 12.87\mu m, \overline{\mu}_0 = 0.0143 \ and \ \overline{U} = 2.38 cm/\sec$

Using these values, the peripheral layer thickness is calculated for blood flow in 40 and 66.6 μm and obtained difference 0.094% from equation (12). With the help of the obtained values of peripheral layer thickness, the core viscosity and red blood cell in the core have been computed from equation (22) –(24). The volumetric flow rate Q vs. pressure gradient $\partial p / \partial z$ have proposed in the model through (12) have been plotted in Figure (2)
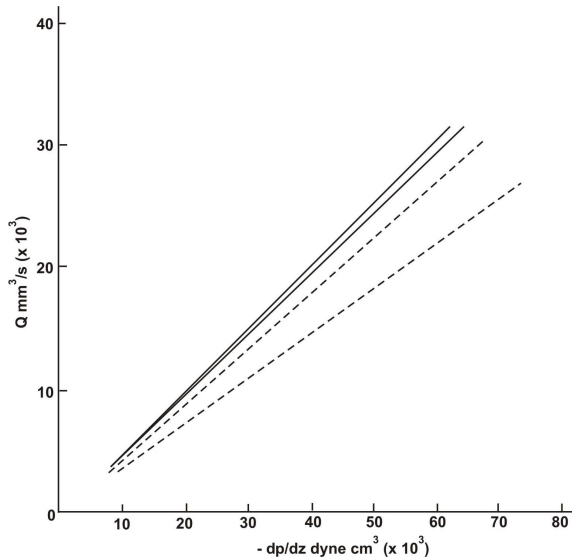

Figure (2). pressure-flow rate relationship in a vessel

6. Conclusion

The present study investigates a two-layered mathematical model of blood flow for a mild stenosis artery which has been modeled by core region with a Newtonian fluid peripheral layer. It is observed that resistance to the flow and wall shear stress increases when height of stenosis increases. The values of the apparent viscosity of blood, agreeability, rigidity and deformability of red cells can be determined by the present analysis more accurately than the other existing models because in the present study, core viscosity is obtained by calculating the actual red cell concentration in the core which is different from the concentration of whole blood. The present analysis could also serve as the check for experimentally measured rheologic values of blood. It may be mentioned at this stage that the variation of peripheral layer thickness, core viscosity and slip velocity with the axial distance in stenotic region has not been analyzed due to the non-availability of the experimental values of pressure gradient. This rheological information of blood in turn could be exploited for the development of new diagnostic tools for many diseases such as hypertension, renal, myocardial infarction etc.

AMS Subject Classification (1991): 760z05, 92c35

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