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A Study of Facilitated Oxygen Diffusion in Humans in ALBAHA (KSA)

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Abstract
The paper analyses the facilitated oxygen diffusion in skeletal muscle fibre in steady and extinction states at province AlBAHA of Kingdom of Saudi Arabia. The oxygen being continuously consumed by human body, diffuses in muscle fibre as well as binds with the myoglobin to produce oxy-myoglobin. With the increase of high altitude, oxygen concentration decreases exponentially and consequently, it results oxygen debt causing various illnesses. The altitude of ALBAHA varies from 1600 to 2500 meters above sea level. In case of insufficient oxygen supply at high altitude, oxy-myoglobin releases oxygen to meet the requirement in the human body to some extent. Subsequently, the point of zero oxygen concentration (anoxia) recedes towards the exterior surface of the human body (a worsen situation). The movement of point of anoxia depends upon the rate of absorption of oxygen by the human body and height from the sea level. An appropriate numerical method has been used to find the oxygen concentration at any time at any point in the human body as a function of rate of oxygen consumption and the facilitated diffusion parameter at different altitude at ALBAHA. The obtained results are compared with the results available in literature, which are in close agreement.

1. Introduction

AlBaha is the smallest province of the kingdom's of Saudi Arabia in the south west of area 11,000 square km with population of 533001 (in 2014). AlBaha is one of the Kingdom's prime tourist attractions and people enjoy a pleasant climate here. Geographically, AlBaha contains the high mountains and forests. The altitude of eastern hills of AlBaha varies from1600 meter to 2500 meters above sea level [1-2]. The oxygen concentration at different places of AlBaha is found to be low and hence human being suffers with the consequences of anoxia. Although, human body can have short and long term adaptation at high altitude that allow a partial compensation of lack of oxygen. Generally, athletes use these adaptations to help their performance at high altitude [3-6]. Roberto [29] also presented the same type of analysis in the direction of adaptation at high altitude. A comparitive studies 9-10 years old children in Tibet has been demonstrated by Bianda et.al. [30]. The human body can perform best at sea level, where the atmospheric concentration of oxygen (O_2) in air is 20.9%, so the partial pressure of O_2 (pO_2) is 21.136 kPa. It has been also seen that enhanced oxygen supply improves the viability in bio artificial pancreas Barkai [31]. But at very high altitude acclimatization is not possible due to insufficient oxygen in environment. Travel to high altitude regions can lead to medical problems such as cerebral edema, risk of permanent brain damage, cell anemia, strokes and congenital heart diseases in people [7-9]. Hunch et.al. also studied the mountain sickness in his research paper [32]. In a recent study, effect of height on humans in Saudi Arabia has been presented by al Tahan et. al. [2], which shows that the frequency of thrombotic stroke at high altitude was 93.4% as compared to 79.3% at low altitude ($P < 0.05$). The risk factors at high altitudes are hypertension, diabetes mellitus, ischemic heart disease and atrial fibrillation. The occurrence of stroke at high altitude is more, irrespective of gender, age and occupations. The present study gives idea about how to overcome the situation or to take precaution.

The oxygen being continuously consumed by human body, diffuses in muscle fibre as well as binds with the myoglobin to produce oxy-myoglobin. The problem was carried out by Crank and Gupta [10] by considering the rate of absorption as constant and solved the problem of oxygen diffusion. Martinez and Marquina [11] and Martinez et. al. [12] extended the work by considering that the rate of absorption as function of distance from outer surface, applicable to inhomogeneous media. The eminent Physiologists, such as Huxley [13], Hodgkin [14] etc. had taken a keen interest towards the Mathematical approach to solve physiological problems. Oxygen not only diffuses in human body but also binds with myoglobin. The subject is very useful to understand the facilitated diffusion process [15-17]. The above models can be obtained as particular cases, when rate of absorption is considered as constant and as a function of distance from outer surface respectively at the sea level only, where maximum oxygen concentration is available.

The facilitated oxygen diffusion is studied at different height along with different consumption rate in steady as well as extinction state. The transport of oxygen flow is greatly enhanced by myoglobin. But it is affected by the atmospheric oxygen concentration available at different height. Muscle fibre consumes oxygen even at rest (steady state) of the body, because of the biological process taking place in human body. This consumption of energy requires constant metabolisim of sugar, which consumes oxygen. The oxygen consumption in live tissues at rest is about 5×10^{-8} mol/cm^{3}s and the consumption of myoglobin is about 2.8×10^{-7} mol/cm^3 in human being. When myoglobin is fully saturated, it contains only 5% supply of oxygen [15]. The amount of oxygen at the exterior of the muscle cell must be sufficient to penetrate to the center of the cell to prevent the oxygen deficiency (i.e. oxygen debt), which is really a common problem at high altitude. Further, when a body is in dynamic state consumption rate of oxygen is further increased. Thus, the chance of illness due to oxygen debt in dynamic state is further increased at high altitude. In the present work, effect of height on people of AlBaha on facilitated oxygen diffusion has been studied. Free and bounded concentration of oxygen is obtained at any time as a function of rate of absorption of oxygen and the facilitated diffusion parameter by numerical methods in steady as well as in extinction state at different altitude of AlBaha. Results obtained are in consistent with available results in literature.

2. Formulation of the Problem

Considering a muscle fibre as a long circular cylinder of radius ‘a’ where diffusion takes place in the radial direction and distribution of chemical species are radially symmetrical. When oxygen [O_2] passes through the muscles, it reacts with myoglobin [Mb] to produce oxy-myoglobin[MbO_2].

$$O_2 + Mb \underset{K^-}{\overset{K^+}{\rightleftharpoons}} MbO_2 \tag{1}$$

where K^+ and K^- are reaction constants in forward and backward directions respectively

The law of mass action for uptake of oxygen f into oxy-myoglobin is

$$f = - K^- C + K^+ Se \tag{2}$$

where S, e and C are the concentrations of oxygen [O_2], myoglobin [Mb] and oxy-myoglobin [MbO_2] respectively at radial distance r, from the centre of the medium at any time t.

Let the oxygen concentration at the surface (boundary) is

$$S_a = S_0\left(1-\frac{h}{h_0}\right), \tag{3}$$

where S_0 is the maximum oxygen concentration available in atmosphere, h the height of AlBaha that varies from 1600 to 2500 meter from sea level and h_0(8000 meter) is the maximum height [1].

Governing diffusion equations for oxygen, oxy-myoglobin and myoglobin are respectively given by

$$\frac{\partial S}{\partial t} = D_s \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial S}{\partial r}\right) - g - f\ , \tag{4}$$

$$\frac{\partial C}{\partial t} = D_c \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right) + f\ , \tag{5}$$

$$\frac{\partial e}{\partial t} = D_e \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial e}{\partial r}\right) - f\ , \tag{6}$$

where g be the constant consumption rate of oxygen per unit volume in the mediumand D_s, D_c and D_e are the diffusion constants of oxygen, oxy-myoglobin and myoglobin respectively.

The boundary conditions to solve the differential equations are

$$\text{at } r = a,\ S = S_a,\quad \frac{\partial C}{\partial r} = 0, \frac{\partial e}{\partial r} = 0\ , \tag{7}$$

$$\text{at } r = 0,\ \frac{\partial S}{\partial r} = 0,\quad \frac{\partial C}{\partial r} = 0, \frac{\partial e}{\partial r} = 0\ . \tag{8}$$

Total myoglobin e_0 in the medium is conserved by the reaction. Therefore,

$$e + C = e_0. \tag{9}$$

Let the non dimensionalised variables are

$$\sigma = \frac{K^+ S}{K^-},\quad U = \frac{C}{e_0}, V = \frac{e}{e_0}, y = \frac{r}{a}, T = te_0 K^+. \tag{10}$$

Introducing the non dimensional variables (10), equations (4), (5) and (6) reduce to

$$\frac{\partial \sigma}{\partial T} = \varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial y^2} + \frac{1}{y}\frac{\partial \sigma}{\partial y} \right) - \gamma + (U - \sigma V), \tag{11}$$

where $\varepsilon_1 = \frac{D_s}{a^2 K^+ e_0}, \gamma = \frac{g}{e_0 K^-}$.

$$\frac{e_0}{K}\frac{\partial U}{\partial T} = \varepsilon_2 \left(\frac{\partial^2 U}{\partial y^2} + \frac{1}{y}\frac{\partial U}{\partial y} \right) - U + \sigma V, \tag{12}$$

where $\varepsilon_2 = \frac{D_c}{a^2 K^-}, K = \frac{K^-}{K^+}$.

$$\frac{e_0}{K}\frac{\partial V}{\partial T} = \varepsilon_3 \left(\frac{\partial^2 V}{\partial y^2} + \frac{1}{y}\frac{\partial V}{\partial y} \right) + U - \sigma V, \tag{13}$$

where $\varepsilon_3 = \frac{D_e}{a^2 K^-}$.

And

$$U + V = 1. \tag{14}$$

The boundary conditions with respect to new variables becomes

$$\text{At } y = 1,\ \sigma = \sigma_1,\quad \frac{\partial U}{\partial y} = 0, \frac{\partial V}{\partial y} = 0. \tag{15}$$

And

$$\text{At } y = 0,\ \frac{\partial \sigma}{\partial y} = 0,\quad \frac{\partial U}{\partial y} = 0, \frac{\partial V}{\partial y} = 0. \tag{16}$$

Since molecular weight and structure of oxy-myoglobin and myoglobin are identical, so the constants ε_2 and ε_3 are approximately taken to be the same. Hence equation (13) is superfluous. Therefore, only differential equations to be solved are (11) and (12).

2.1. Steady State

Oxygen diffuses into the muscle fibre freely, where some of the oxygen is absorbed by the cells, thereby being removed from the diffusion process. The concentration of oxygen at the surface is maximum and constant at particular height, but as the height increases concentration of O_2 decreases. The first phase of the problem continues, until steady state reached in which the oxygen does not change any further in the medium with time at a particular height.

Concentration of oxygen at every point in the medium is constant. The differential equations governing the steady state process are

$$\varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial y^2} + \frac{1}{y}\frac{\partial \sigma}{\partial y} \right) - \gamma + (U - \sigma V) = 0. \tag{17}$$

$$\varepsilon_2 \left(\frac{\partial^2 U}{\partial y^2} + \frac{1}{y}\frac{\partial U}{\partial y} \right) - U + \sigma V = 0. \tag{18}$$

Steady state solution for concentration distribution in muscle fibre (Keener and Sneyd [18], pp. 43) is

$$\varepsilon_1 \sigma + \varepsilon_2 U = \frac{\gamma y^2}{4} + A \log y + B. \tag{19}$$

To have finite solution the constant of integration A must be zero. The value of the constant B can be obtained by using the condition (15) and (16).

The constants ε_1, ε_2 in equations (17) and 18) are small enough to warrant the approximation that Quasi steady state holds good in the interior of the muscle fibre. Thus

$$U = \frac{\sigma}{1+\sigma}. \tag{20}$$

Using equation (2.1.2) and (2.1.3), The solution for oxygen concentration becomes

$$\sigma(y,t) = \frac{1}{2}\left[\gamma_1 (y^2 - 1) + \sigma_1 + \rho U_1 - 1 - \rho + \sqrt{\left(\gamma_1 (y^2 - 1) + \sigma_1 + \rho U_1 - 1 - \rho\right)^2 + 4\left(\gamma_1 (y^2 - 1) + (\sigma_1 + \rho U_1)\right)} \right] \tag{21}$$

where σ_1 and U_1 are free and bounded oxygen concentrations at the surface respectively and $\rho = \varepsilon_2/\varepsilon_1$, $\gamma_1 = \gamma/4\varepsilon_1$ are facilitated oxygen diffusion parameter and consumption parameter respectively.

Oxygen decreases with the increase of height. Oxygen debt occurs, when σ becomes zero, while marginal oxygen debt occurs when total oxygen concentration falls to zero. The free oxygen concentration at the boundary and oxygen content in the oxy-myoglobin just enough to prevent oxygen debt at the center is called critical oxygen concentration σ_0,

$$\sigma_0 + \rho \frac{\sigma_0}{1+\sigma_0} = \frac{\gamma}{4\varepsilon_1}. \tag{22}$$

2.2. Extinction State

At high altitude the perfusion level falls to critically low values (case of anoxia). Thus no oxygen passes in the skeletal muscle fibre. Myoglobin releases the bound oxygen to prevent the onset of anoxia for a small period of time. The medium continues to absorb the available oxygen already in it due to biological process and as a consequence, oxygen debt occurs at the farthest distance (at the center) of the muscle fibre. Thus the point of anoxia (zero oxygen concentration) recedes towards the exterior surface. The oxygen concentration obtained in steady state can be taken as the initial distribution of oxygen for the solution of moving boundary value problem (extinction state.) The governing differential equations of extinction state are

$$\frac{\partial \sigma}{\partial T} = \varepsilon_1 \left(\frac{\partial^2 \sigma}{\partial y^2} + \frac{1}{y}\frac{\partial \sigma}{\partial y} \right) - \gamma + (U - \sigma V), \tag{23}$$

$$\frac{e_0}{K}\frac{\partial U}{\partial T} = \varepsilon_2 \left(\frac{\partial^2 U}{\partial y^2} + \frac{1}{y}\frac{\partial U}{\partial y} \right) - U + \sigma V, \tag{24}$$

and the boundary conditions are

$$\text{at } y = 1,\ \sigma = 0,\quad \frac{\partial U}{\partial y} = 0, \frac{\partial V}{\partial y} = 0, \tag{25}$$

$$\text{at } y = 0,\ \frac{\partial \sigma}{\partial y} = 0,\quad \frac{\partial U}{\partial y} = 0, \frac{\partial V}{\partial y} = 0, \tag{26}$$

$$\text{and } \sigma(y,0)=\sigma,\ U(y,0)=U. \tag{27}$$

We investigate the effect of myoglobin in facilitated oxygen diffusion at different points in the body as a function of time, facilitated oxygen diffusion parameter and consumption parameter at different heights of AlBaha in extinction state. The rate of change of concentration of oxygen and movement of boundary point (separating oxygen and non oxygen medium in skeletal muscle fibre) is found to be different at different time and different height. Different numerical methods are proposed for different time region by earlier researchers [22-27]. Crank and Gupta [10] suggested that before the disturbance at the surface concentration of oxygen has an effect on the concentration of oxygen in the neighbourhood of center of the muscle (boundary does not move within specified degree of accuracy), suitable approximation can be obtained using Lagrange method. The present solution is obtained by finite difference method incorporating the suitable approximations at the point of inconsistency in diffusion process with time and height.

3. Numerical Methods

The percentage of oxygen concentration of oxy-myoglobin determines the content of oxygen in our blood. At high altitude saturation of oxy-myoglobin decreases [19-21]. The result was found to be more pronounced with the increase of height as well as that of consumption rate. The continuous absorption of oxygen in skeletal muscle fibre due to biological process in anoxia results in oxygen debt at the centre(farthest distance from surface) of muscle fibre. Subsequently, the boundary of zero concentration of oxygen moves towards exterior surface. Major problem to employ Numerical Methods arises because of abrupt imposition of zero oxygen concentration at the surface in case of anoxia for time $T \geq 0$ in extinction state, which gives discontinuity in the derivative boundary condition.

Douglas & Gallie [22] introduced a Numerical method of variable time step, keeping the size of the space mesh fixed. Murray &Landis [23] used a variable space mesh and kept the time step fixed. Ehlrich [24] employed implicit formula at the intermediate points and Taylor's expansions near the moving boundary in both time and space directions. Lotkin [25] used subdivided differences, while Crank [26] suggested a three-point Lagrange interpolation formula near the moving boundary.

In the present analysis, the concentrations at the intermediate points between the two boundaries have been calculated by using simple explicit finite-difference formula. Near the moving boundary a Lagrange formula has been used, as suggested by Crank [26] to get consistency with the diffusion process. The whole region, $0 \leq y \leq 1$, is subdivided into M intervals each of width δy and taking $y_r = r\delta y$ where $0 \leq r \leq M$ ($M\delta y = 1$).

Let the concentrations at each of the grid points at the j^{th} time level of human being are known and the position of the moving boundary at that time is somewhere in the r^{th} interval between y_{r-1} and y_r, given by $y_b = (r-1)\,\delta y + p^j\delta y$, $0 \leq p^j < 1$ along radial direction. Fig. 1(a) presents the boundary point y_b(separating anoxia and without anoxia), where as Fig 1(b) presents the oxygen layer in Albaha in atmosphere. The parameter values h= 0 and h= h_0 are the height at sea level and death zone respectively for oxygen layer.

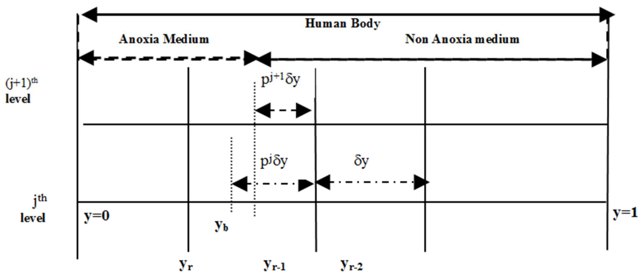

Fig. 1 (a). *Discretisation of oxygen medium in human body.*

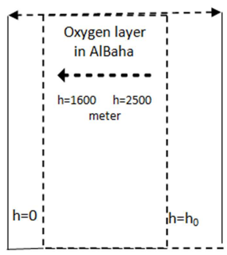

Fig. 1 (b). *Oxygen layer of AlBaha in the atmosphere.*

Then the concentrations at the $(j+1)^{th}$ time level can be calculated using the well known explicit formula.

The oxygen concentration at the sealed surface is given by

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1(\sigma_{i+1j} - \sigma_{ij})}{dy}\left(\frac{2}{dy}+1\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \quad (28)$$

while at the other intermediate points the explicit formula becomes,

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dy}\left(\frac{(\sigma_{i+1j} - 2\sigma_{ij} + \sigma_{i-1j})}{dy} + \frac{(\sigma_{i+1j} - \sigma_{i-1j})}{2y_i}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right]. \quad (29)$$

Concentrations in the Neighbourhood of the boundary as suggested by Crank and Gupta [10] using the appropriate finite-difference replacement leads to have the following equations.

When boundary does not move within the specified degree of accuracy

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dy}\frac{2}{dy}\left(\frac{\sigma_{i-1j}}{p_s+1} - \frac{\sigma_{ij}}{p_s}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \quad (30)$$

where $p_s = \dfrac{\sqrt{2\sigma_{ij}}}{dy}$,

when boundary starts moving

$$\sigma_{ij+1} = \sigma_{ij} + \delta t\left[\frac{\varepsilon_1}{dy}\left(\frac{2}{dy}\left(\frac{\sigma_{i-2j}}{p_s+1} - \frac{\sigma_{i-1j}}{p_s}\right) + \frac{\sigma_{i-1j} - \sigma_{i-2j}}{y_i}\right) - \gamma + U_{ij} - \sigma_{ij}V_{ij}\right], \quad (31)$$

where $p_s = \dfrac{\sqrt{2\sigma_{i-1j}}}{dy}$.

Similarly, the bounded oxygen concentration at the surface, intermediate points and in the neighbourhood of moving boundary can be obtained respectively.

4. Results and Discussion

Finite difference technique numerical technique has been used to find the concentration of oxygen at different points in the medium for various values of parameters, such as facilitated parameter ρ, oxygen consumption rate parameter γ_1, time T and space y. An approximate numerical method has been utilized to obtain the results. Result illustrates contribution of myoglobin to a muscle cell to prevent the anoxia. The combined effect of diffusion and myoglobin in oxygen consumption is presented in graphs 1- 9. Myoglobin facilitates the diffusion of oxygen into the tissues in skeletal muscle. It serve as a source of oxygen to delay or prevent the onset of anoxia, when perfusion level falls critically to low value. Oxy-myoglobin alone supplies to meet the metabolic need of oxygen of skeletal muscle for a small period of time.

Steady state: Fig. 2 represents the oxygen distribution throughout the skeletal muscle fiber from centre (y = 0) to the outer surface (y = 1) for consumption parameter γ_1, = 3.587 and facilitated parameter ρ= 3.00,5.00 and 8.00at different height h = 0.200(0.025)0.300. Oxygen concentration is found to be highest at height h =0.0 i.e. at the sea level and hence the best performance of human is found to be at sea level only. As the human body climbs at higher altitude, oxygen concentration decreases and becomes zero at h=1.0, called the *Death zone*, where survival is not possible. The oxygen concentration is very low at height h=0.300(of AlBaha). At high altitude, deficiency of oxygen causes several illness [2, 7-8], as evidenced by medical science. Fig.2 shows how the oxygen concentration decreases with the increase of height and hence one can take precautionary measure accordingly. At high altitude, it is advised to increase the breathing rate so that more oxygen enters into the body. At a fixed oxygen concentration maintained at the surface, the distance to which oxygen can penetrate and free oxygen delivered to the tissues are increased in presence of myoglobin. Thus with increase of facilitated oxygen diffusion parameter ρ =5.00, 8.00 oxygen distribution in human is found to be increased, keeping other parameter fixed.

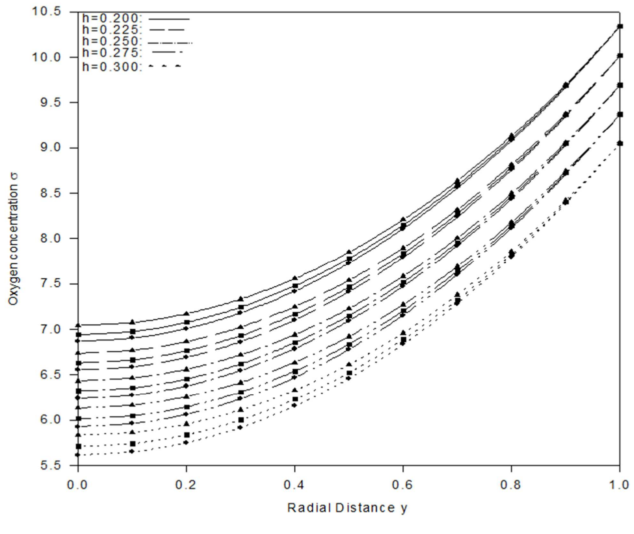

Fig. 2. *Oxygen concentration σ vs. radial distance y for consumption parameter γ_1= 3.587 and facilitated parameter ρ=3.0:●●●,ρ=5.0: ■■■ and ρ=8.0:▲▲▲.*

The effect of physical exercise and daily physical work at high altitude has been presented in Fig. 3. In fact it increases the consumption rate of oxygen in the body. Fig. 3 shows the oxygen distribution as the consumption rate γ_1 of oxygen increases and hence oxygen distribution is further decreased with the increase of height. All these observations gives very important information of crisis of oxygen to take care, before one collapse due to chest pain and effect on heart. And, if he continues to do physical work, where consumption parameter γ_1 increases situation becomes worsed. It is further reduced as the height increases (see Fig. 4). Fig. shows that a part of the body does not get any oxygen, as marginal oxygen concentration becomes zero. That may cause several severe medical problems.

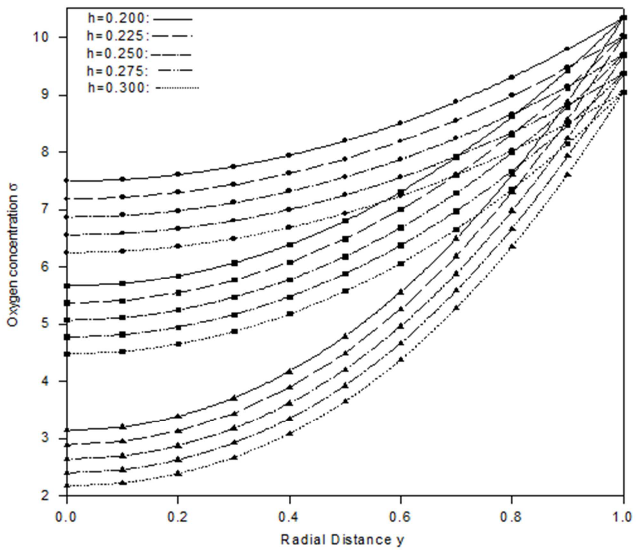

Fig. 3. Oxygen concentration σ vs. radial distance y for facilitated parameter ρ= 5.217 and consumption parameter γ_1= 3.0:●●●, γ_1=5.0: ■■■ and γ_1=8.0:▲▲▲.

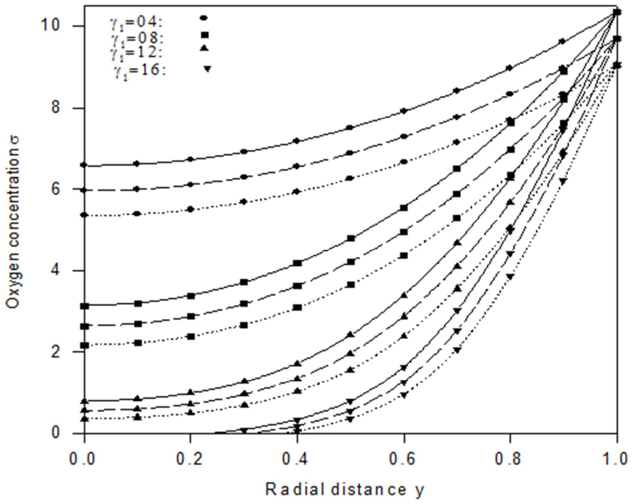

Fig. 4. Oxygen concentration σ vs. radial distance y for facilitated parameter ρ= 5.217 and height h= 0.20:———, h= 0.25: – – –and h =0.30: …….

Fig. 5 presents the oxygen concentration at the highest altitude of AlBaha at various points of human body with the increase of consumption rate γ_1. Point at the farthest distance from the source has deficiency of oxygen at first and continues to the next with the increase of consumption parameter γ_1. Therefore, when one is at high altitude and such situation arises, he should reduce or stop the physical work at once and breath fast to meet the immediate requirement of oxygen. Technically, decrease of rate of consumption of oxygen γ_1 and increase of facilitated oxygen diffusion parameter ρ increases the oxygen distribution in human body. Although, when free oxygen reduces to very low level either due to increase to height or demand of uptake of oxygen due to physical work, in such cases oxy-myoglobin releases the bounded oxygen to meet the requirement of oxygen in static state as well as in dynamic states both (see Fig 6).

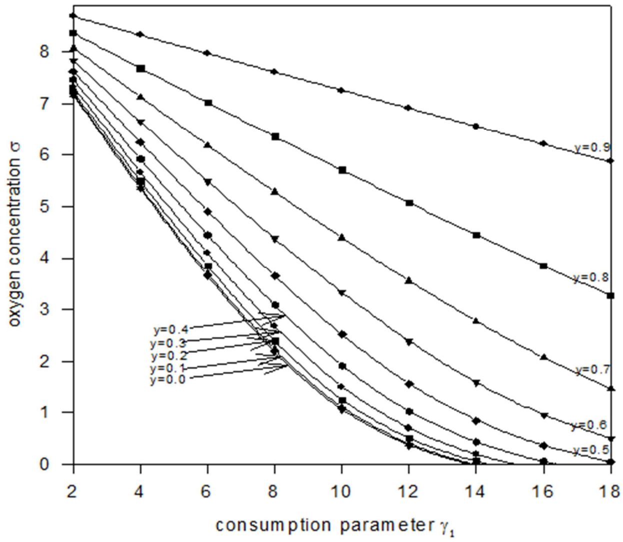

Fig. 5. Oxygen concentration σ vs. consumption parameter γ_1 at height h= 0.30 for facilitated parameter ρ= 5.217.

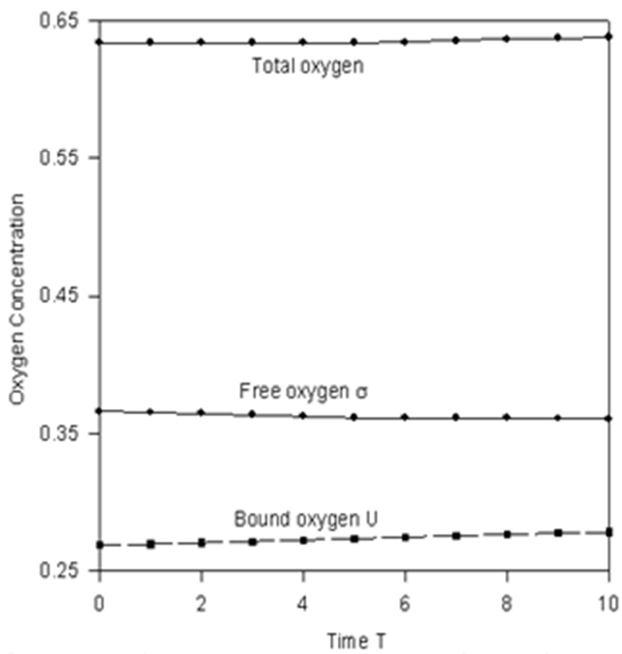

Fig. 6. Release of bounded oxygen by oxy-myoglobinat sea level, when free oxygen is not sufficient for facilitated parameter ρ= 5.217 and. consumption parameter γ_1 =16.0.

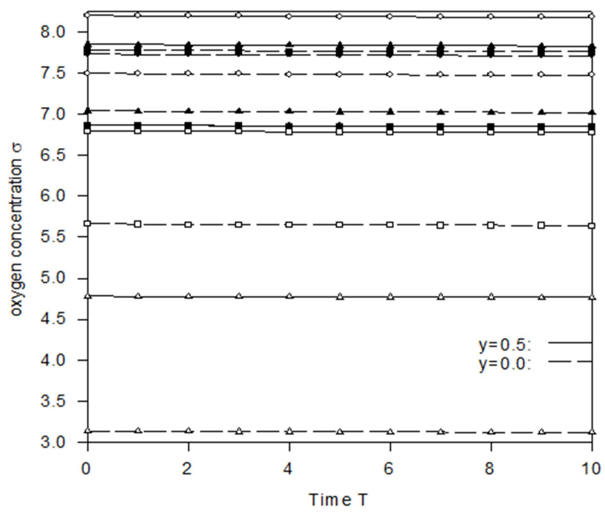

Fig. 7. *Oxygen concentration σ vs. time T at h= 0.2 facilitated parameter ρ=3.0: ●●●,ρ=5.0: ■■■ and ρ=8.0: ▲▲▲ and consumption parameter γ_1= 3.0: ○○○ γ_1= 5.0: □□□ and γ_1= 8.0: △△△.*

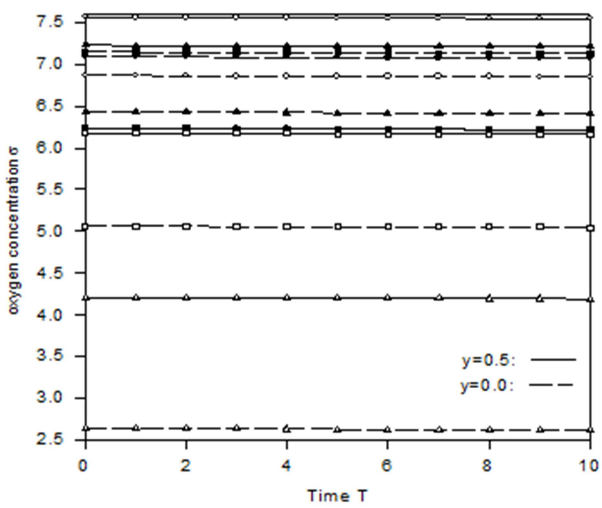

Fig. 8. *Oxygen concentration σ vs. time T at h= 0.25 facilitated parameter ρ=3.0: ●●●,ρ=5.0: ■■■ and ρ=8.0: ▲▲▲ and consumption parameter γ_1= 3.0: ○○○ γ_1= 5.0: □□□ and γ_1= 8.0: △△△.*

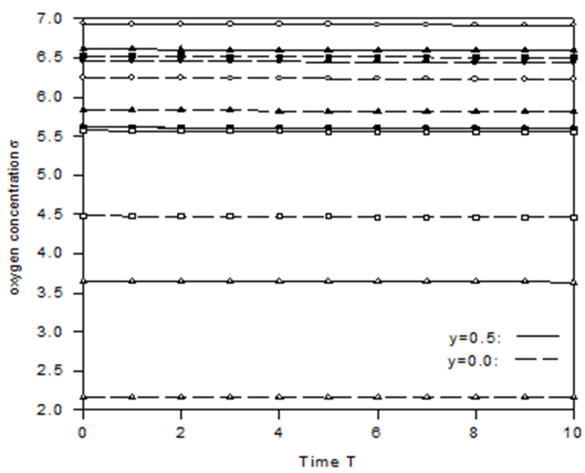

Fig. 9. *Oxygen concentration σ vs. time T at h= 0.30 facilitated parameter ρ=3.0: ●●●,ρ=5.0: ■■■ and ρ=8.0: ▲▲▲ and consumption parameter γ_1= 3.0: ○○○ γ_1= 5.0: □□□ and γ_1= 8.0: △△△.*

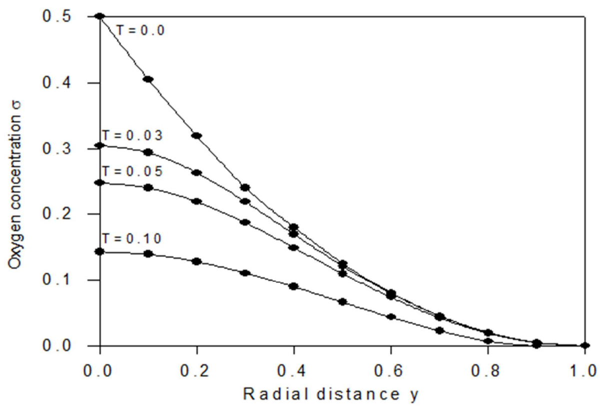

Fig. 10. *Comparision of oxygen concentraion σ (●) with Crank and Gupta [10].*

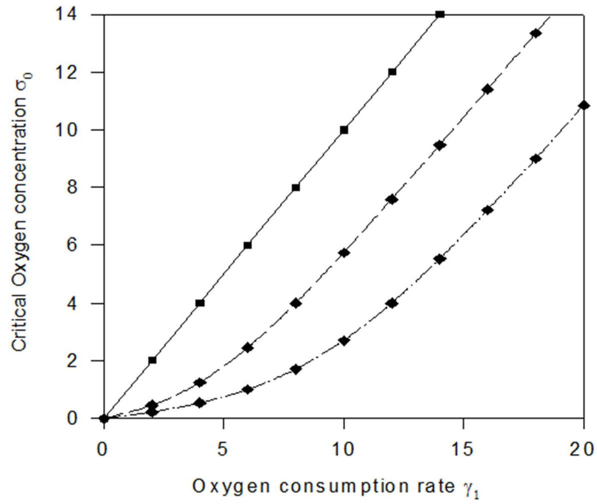

Fig. 11. *■ values shows the comparision of critical oxygen concentration with Keener and Sneyd [18] for ρ=0: ——, ρ=5: ——, ρ=10: ------.*

Extinction state: Extinction state is an analysis of oxygen diffusion, when perfusion level of oxygen falls critically to very low values. Oxy-myoglobin releases the oxygen to meet the metabolic need of oxygen of skeletal muscle for a small period of time. The human body continuously consumes the total oxygen available inside the body, due to biological process even in static state. After some time, oxygen debt arises at the centre (Farthest distance from the source) of the human body. Subsequently, the zero oxygen concentration point recedes towards the outer surface of the body. This is the actual situation of death with time, due to lack of oxygen. A time analysis of oxygen distribution in human body is presented in Fig. 7 and consumption parameter γ_1= 3.00, 5.00 and 8.00 and consumption parameter γ_1= 3.00, 5.00 and 8.00 at height h =0.200 of AlBaha. Figure shows that oxygen concentration decreases also with time T. The rate of decrease of oxygen concentration at the midpoint (y= 0.5) of the medium is less than that at the farthest distance y =0.0.The effect was found to be more pronounced, if one is involved in some physical work (i.e. increase of consumption parameter).

Fig. 8 and 9 present the oxygen distribution against Time T at height h =0.300 and h =0.250 of AlBaha respectively at consumption parameter γ_1= 3.00, 5.00 and 8.00 and facilitated diffusion parameter γ_1= 3.00, 5.00 and 8.00 at mid point y=0.5 and centre y= 0.0(centre) of the medium. Oxygen concentration reduces with time and effect is found to be more at the exterior surface at sea level. The effect is less with time at high altitude. When oxygen concentration σ reduces to very low level at high altitude, oxy-myoglobin releases the bounded oxygen to meet the oxygen requirement in the body rapidly with time.

Figure 10 compares oxygen concentration with the result obtained by Crank and Gupta [10], when rate of consumption per unit volume is constant in one-dimension cartesian coordinate system, in the absence of oxygen uptake function *f*. The obtained results are in good agreement with those of Crank and Gupta [10]. The model presented by Martinez *et al*.[12] can be obtained as a special case of the present model, by taking rate of consumption per unit volume as a function of distance (i.e. √distance) in the one-dimension. A comparative study with different models presented by Martinez *et al*. [12], Crank and Gupta [10] and Hansen and Hougaard [27] is shown in Table 1, which shows the close agreement with the results. Figure 11 compares the critical oxygen concentration at the surface against oxygen consumption rate parameter γ_1 for facilitated parameter ρ= 0, 5 and 10 with the result of Keener and Sneyd [18] at sea level. Figure shows that as the consumption rate increases, the critical oxygen concentration at the surface has to be increased to prevent oxygen debt.

Table 1. *Comparative study of free oxygen concentration in extinction state as a function of time T at the surface.*

Time T	Matinez et. al. Model		Crank and Gupta Model		Hansen and Hougaard Model	
	Ref. [12]	Present	Ref. [10]	Present	Ref. [27]	Present
0.04	0.2759	0.2758	0.2759	0.2743	0.2743	0.2741
0.10	0.1449	0.1445	0.1449	0.1432	0.1432	0.1430
0.18	0.0235	0.0238	0.0235	0.0213	0.0218	0.0217
0.19	0.0109	0.0111	0.0109	0.0082	0.0090	0.0091

5. Conclusion

The study of oxygen diffusion in skeletal muscle fibre at mountain has wider applications from medical point of view. Oxygen concentration decreases with the increase of height. The problem is of immediate interest in medical research concerning uptake of oxygen in humans living at high altitude like AlBaha. The present problem analyses the combined effect of consumption of oxygen and along with the increase of altitude of AlBaha. The role of myoglobin is to prevent the deficiency of oxygen either due to high altitude or increase of demand of oxygen consumption in the muscle fibre. Results show that boundary moves faster at the end of the process but slower in the beginning of the process. Further finite difference technique does not give better accuracy for fast movement of the boundary. The results obtained are in good agreement with the modeling results available in literature and will be useful for practical purposes.

Nomenclature

S: Oxygen concentration [O_2]
S_a: Oxygen concentration at the surface
C: Oxy-myoglobin concentration [Mb]
e: Myoglobin concentration [MbO_2]
e_0: Total myoglobin
f : Rate of uptake of oxygen
D_s: Oxygen diffusion constant
D_c: Oxy-myoglobin diffusion constant
D_e: Myoglobin diffusion constant
T: time
r: radius
S_0: Oxygen concentration at x = 0
S_l: Oxygen concentration at x = *l*
C_0: Oxygen concentration in oxy-myoglobin at x = 0
C_l: Oxygen concentration in oxy-myoglobin at x = *l*
σ: SK^+/K^-
σ_1: Dimensionalised oxygen concentration at the surface
U: C/e_0
V: e/e_0
J : flux of oxygen
J’: Jl/ε_1
ε_1: $D_s/K^+e_0l^2$
ε_2: D_c/K^-l^2
ε_3: D_e/K^-l^2
K: K^+/K^-
T: K^+e_0t
y: x/l
ρ: $\varepsilon_2/\varepsilon_1$
γ: g/e_0K^-
γ_1: γ/4.0:
h: height of experiment performed
h_0: Maximum height of oxygen available in atmosphere.
h’: h/h_0

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