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Effect of perturbations on the stability of triangular libration points of the robes restricted three-body problem when the primaries are oblate spheroid

AbdulRazaq AbdulRaheem[1], Faluyi Oludotun Omoniyi[2]

[1]Department of Mathematics and Statistics, Kwara State University, Malete, Nigeria
[2]Department of Physics and Material Science, Kwara State University, Malete, Nigeria

Email address

abdulrazaq.abdulraheem@gmail.com (A. R. A. Raheem)

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Abstract

The effect of small changes in the Coriolis and centrifugal forces on the stability of equilibrium points in the Robe restricted three body problem was studied. In this problem we considered both primaries as oblate spheroid. The critical mass obtained depends on the small changes in the Coriolis and centrifugal forces, oblateness of the rigid shell and the second the second primary as well as the density parameter k. The stability of the triangular points depends largely on the values of *k*. The destabilizing tendencies of the centrifugal force and oblateness factors were enhanced when $k>0$ and weakened for $k<0$.

1. Introduction

The restricted three body problem is to describe the motion of an infinitesimal mass when the primaries rotate around their center of mass under their mutual gravitational attraction and the infinitesimal mass moves in the orbital plane of the two primaries. The infinitesimal mass is attracted by the primaries but does not influence the motion of the primaries. The classical restricted three body problem has five equilibrium points; three collinear and two triangular. The collinear points are not stable for all values of the mass parameter μ while the triangular points are stable for a critical mass value $\mu < \mu_c = 0.03852..$ [1]. The properties of motion of a restricted three-body problem and their generalizations have been studied by several researchers (Szebehely, [1]; Bhatnagar and Hallan, [2]; Khanna and Bhatnagar, [3]; Singh and Ishwar, [4]; Sharma, Taqiv and Bhatnagar [5], AbdulRaheem and Singh, [6], Narayan and Kumar [16], Sharma and SubbaRao [17] and [18], Singh and Umar, [19], Raman and Sharma, [20]).

Robe [7] introduced a new kind of restricted three-body problem that incorporates the effect of buoyancy force. One of the primaries m_1 is a rigid shell of mass filled with homogeneous incompressible fluid of density ρ_1. The second primary m_2 is a point mass located outside the shell. The third body m is the particle of negligible mass of density ρ_3 which moves inside the shell under the influences of the gravitational attraction of the primaries and the buoyancy force of the fluid of density ρ_1. Robe studied the motion of the infinitesimal mass when m_2 describes both circular and elliptic orbits. He obtained the equilibrium points and showed that, for the circular case, the equilibrium point is linearly stable when $\rho_3 < \rho_1$ and unstable when $\rho_1 < \rho_3$.

The third body m which moves in the shell filled with the homogeneous incompressible fluid is a submerged body that experiences buoyancy force. The line of action of the buoyancy force passes through the centre of buoyancy. If the third body is neutrally buoyant it will remain at rest at any point where it is immersed in the fluid.
The stability of a submerged body depends on the centre of buoyancy and centre of gravity of the body. If the centre of buoyancy lies above the centre of gravity then a stable equilibrium is obtained. Unstable equilibrium arises when centre buoyancy lies below centre of gravity (Berger [14]).

Robes problem has been modified to define a new problem (Shrivastava and Garain [8], Plastino and Plastino [9], Giordano, Plastino and Plastino [10], Hallan and Rana [11] and Hallan and Mangang [12]).

The classical problem has been generalized by taking into account the shapes and luminous properties of the primaries as well as the small changes in the Coriolis and centrifugal forces acting on the satellites in orbits. The degree of flattening of the primaries and the changes in the Coriolis and centrifugal forces, due to rotational motion, cause perturbations in the motions of the primaries and affect the stability of the restricted three body problem.

In our model we consider both the first primary (rigid shell filled with an incompressible homogeneous fluid) and the second primary as oblate spheroids, together with small changes in the Coriolis and centrifugal forces to study the Stability of Triangular Equilibrium points of the Robes Restricted Three-Body Problem.

The astrophysical consideration of our model is that it may be used for studying the small oscillations of the Earth's inner core due to small changes in Coriolis and centrifugal forces as well as the oblateness of the primaries together with the attraction of the Moon. The model is also applicable to the study of the motion of an artificial satellite under the influence of Earth's attraction.

The paper consists of four sections. Section one establishes the relevant equations of motion that incorporates the effect of buoyancy force using some basic assumptions. In the second section we obtained the equilibrium points. Section three deals with the variational equations of motion of the problem and solutions of the resulting characteristic equation obtained. In section four, we obtained the critical mass of the mass parameter. This is followed by the conclusion on the findings.

2. Equations of Motion

Let the mass of the rigid shell and that of the point mass be m_1 and m_2 respectively. Let the density of the incompressible fluid inside the shell be ρ_1 and that of the infinitesimal mass be ρ_3 with mass m. Let A_1 and A_2 denote the oblateness coefficients of the rigid shell and the second primary respectively such that $0 < A_1, A_2 << 1$ (McCuskey, [13]).

Let M_1, M_2 and M_3 be the centers of m_1, m_2 and m_3 respectively such that $M_1M_3 = r_1$ and $M_2M_3 = r_2$. Let G be the gravitational constant and (x, y) the coordinates of the infinitesimal mass m in the orbital plane. Let the line joining m_1 and m_2 be the x-axis. Then the total potential acting on m is

$$-\frac{Gm_1A_1}{2r_1^3}-\frac{Gm_1}{r_1}+\frac{4}{3}\pi G\rho_1\left(1-\frac{\rho_1}{\rho_3}\right)r_1^2-\frac{Gm_2}{r_2}-\frac{Gm_2A_2}{2r_2^3} \qquad (1)$$

where

$$r_1^2=(x+x_1)^2+y^2$$

$$r_2^2=(x+x_2)^2+y^2$$

Let the coordinates of m_1 and m_2 be $(x_1, 0)$ and $(x_2, 0)$ respectively. In the dimensionless rotational coordinate system we choose the unit of mass to be the sum of the masses of the primaries. We take the unit of length equal to the distance between the primaries and the unit of time is chosen such that $G = 1$.

The equations of motion of the infinitesimal body are (AbdulRaheem and Singh, [1])

$$\ddot{x}-2n\varphi\dot{y}=\Omega_x$$

$$\ddot{y}-2n\varphi\dot{x}=\Omega_y \qquad (2)$$

where

$$\Omega=\frac{1}{2}n^2\psi\left(x^2+y^2\right)-kr_1^2+\frac{1-\mu}{r_1}+\frac{\mu}{r_2}+\frac{1-\mu}{2r_1^3}A_1+\frac{\mu}{2r_1^3}A_2 \qquad (3)$$

with

$$r_1^2=(x+\mu)^2+y^2$$

$$r_2^2=(x-1+\mu)^2+y^2 \qquad (4)$$

We suppose that the origin is at the barycentre of m_1 and m_2 be $(x_1, 0)$ and $(x_2, 0)$ so that $x_1 = -\mu$ and $x_2 = 1-\mu$. The mean motion is given as

$$n^2=1+\frac{3}{2}(A_1+A_2),\ k=\frac{4}{3}\pi\rho_1\left(1-\frac{\rho_1}{\rho_3}\right), \rho_1\neq\rho_3.$$

Parameters φ and ψ denote the Coriolis and centrifugal forces. We introduce small perturbations ε and

ε' in φ and ψ respectively such that $\varphi = 1+\varepsilon$, $|\varepsilon| << 1$ and $\psi = 1+\varepsilon'$, $|\varepsilon'| << 1$.

3. Equilibrium Points

The equilibrium points occur when the velocity and acceleration of the system are zeros. That is,

$$\Omega_x = 0, \Omega_y = 0 \tag{5}$$

3.1. Triangular Points

The triangular points are given by the equations

$$\Omega_x = 0, \Omega_y = 0, \; y \neq 0$$

That is, for $k \neq 0$, we have

$$x\left[n^2\psi - 2k - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3}{2}\frac{(1-\mu)}{r_1^5}A_1 - \frac{3}{2}\frac{\mu}{r_2^5}A_2\right]$$

$$-\mu\left[2k + \frac{1-\mu}{r_1^3} - \frac{1-\mu}{r_2^3} - \frac{3}{2}\frac{(1-\mu)}{r_1^5}A_1 - \frac{3}{2}\frac{\mu}{r_2^5}A_2\right] = 0 \tag{6}$$

and

$$y\left[n^2\psi - 2k - \frac{1-\mu}{r_1^3} - \frac{\mu}{r_2^3} - \frac{3}{2}\frac{(1-\mu)}{r_1^5}A_1 - \frac{3}{2}\frac{\mu}{r_2^5}A_2\right] = 0$$

Using equations (6) and (7) we obtain

$$n^2\psi - 2k - \frac{1}{r_1^3} - \frac{3}{2}\frac{A_1}{r_1^5} = 0$$

$$n^2\psi - 2k - \frac{\mu}{r_2^3} - \frac{3}{2}\frac{A_2}{r_2^5} = 0 \tag{7}$$

When the primaries are neither oblate nor radiating and $\rho_1 = \rho_3$

$$r_i = \frac{1}{\psi^{\frac{1}{3}}} . \; (i = 1,2) \tag{8}$$

We assume the solutions of equations (7) are

$$r_1 = \frac{1}{\psi^{\frac{1}{3}}} + \alpha_1$$

$$r_2 = \frac{1}{\psi^{\frac{1}{3}}} + \alpha_2 \tag{9}$$

where α_i , $(i = 1,2)$ are very small perturbations. Knowing r_1 and r_2 from equations (7) the exact coordinates of the triangular points are obtained by solving equations (4) for *x* and *y*. Thus

$$x = \frac{1}{2} - \mu + \frac{1}{2}\left(r_1^2 - r_2^2\right)$$

$$y = \pm\left[\frac{1}{2}\left(r_1^2 + r_2^2\right) - \frac{1}{4}\left(r_1^2 - r_2^2\right)^2\right]^{\frac{1}{2}} \tag{10}$$

Using equations (9) in (10) and restricting ourselves to linear terms in A_1 , A_2 and *k*, we obtain

$$\alpha_1 = \frac{1}{\psi^{\frac{1}{3}}}\left[\frac{2k}{3\psi} - \frac{1}{2}(A_1 + A_2) + \frac{1}{2}A_1\psi^{\frac{2}{3}}\right]$$

Putting the values of r_1 and r_2 in equation (10) we have

$$x = \frac{1}{2} - \mu + \frac{1}{2\psi^{\frac{2}{3}}}\left(\frac{3\psi + 4k}{3\psi + 2k}\right)(A_1 - A_2)^{\frac{2}{3}}$$

$$y = \pm\sqrt{\frac{4(3\psi + 4k) - 3\psi^{\frac{5}{3}}}{12\psi^{\frac{5}{3}}}}\left[1 + \frac{3\psi}{2(3\psi + 2k)}\left\{-(A_1 + A_2) + \frac{1}{2}(A_1 + A_2)\psi^{\frac{2}{3}}\right\}\right] \tag{11}$$

The values of x and y obtained in equations (11) are the triangular points and are denoted by L_4 and L_5 respectively. They are located symmetrically with respect to the horizontal axis. It is seen from equations (11) that their location is affected by the perturbation in the centrifugal forces, oblateness of the two primaries and the density parameter.

4. Stability of Triangular Points

Let (ξ, η) denote a small displacement in the triangular points (x_0, y_0) . Then we write $x = x_0 + \xi$, $y = y_0 + \eta$. Substituting these values into equations (2), gives the variational equations of the problem as

$$\ddot{\xi} - 2n\varphi\dot{\eta} = \Omega_{xx}(x_0, y_0)\xi + \Omega_{xy}(x_0, y_0)\eta + O(2)$$

$$\ddot{\eta} + 2n\varphi\dot{\xi} = \Omega_{xy}(x_0, y_0)\xi + \Omega_{yy}(x_0, y_0)\eta + O(2)$$

Their characteristic equation is

$$\lambda^4 - \left(\Omega_{xx}^0 + \Omega_{yy}^0 - 4n^2\varphi^2\right)\lambda^2 + \Omega_{xx}^0\Omega_{yy}^0 - \left(\Omega_{xy}^0\right)^2 = 0 \tag{12}$$

where the superscript ∘ indicates that the partial derivatives are evaluated at the triangular points (x_0, y_0) . The partial derivatives are given by

$$\Omega_{xx}^0 = \psi^{\frac{2}{3}}(\psi - 2k)\left(\frac{3\psi}{3\psi + 4k}\right)\left(\frac{3}{4} + a_1 + \mu b_1\right)$$

$$\Omega_{yy}^{0} = (\psi - 2k)\left(\frac{3\psi}{3\psi+4k}\right)\left[\frac{3}{4}\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)+a_2+\mu b_2\right] \quad (13)$$

where

$$a_1 = \frac{1}{4}\left[\frac{9}{2}\left(\frac{5\psi-2k}{3\psi-4k}\right)(A_1+A_2)+\left(\frac{9\psi\left(2-\psi^{\frac{2}{3}}\right)+24k}{3\psi+2k}\right)A_1+\frac{9\psi^{\frac{5}{3}}A_1}{\psi(3\psi+4k)}-6\left(\frac{3\psi+4k}{3\psi+2k}\right)A_2\right]$$

$$b_1 = \frac{1}{4}\left[-\frac{9\psi^{\frac{5}{3}}A_1}{\psi(3\psi+4k)}-3\left(\frac{6\psi+16k+3\psi\left(2-\psi^{\frac{2}{3}}\right)}{3\psi+2k}\right)A_1-\frac{9\psi^{\frac{5}{3}}A_2}{(3\psi+4k)}+12\left(\frac{3\psi+4k}{3\psi+2k}\right)A_2+\frac{3\psi^{\frac{8}{3}}(3\psi+2k)}{\psi(3\psi+8k)}A_2\right]$$

$$a_2 = \frac{1}{4\left(4-\psi^{\frac{2}{3}}\right)}\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\left[3\left\{\left(\frac{18\psi+8k}{3\psi-2k}\right)-\frac{1}{2}\left(\frac{15\psi+4k}{3\psi-2k}\right)\psi^{\frac{2}{3}}\right\}(A_1+A_2)+6\left(\frac{3\psi+4k}{3\psi+2k}\right)\psi^{\frac{2}{3}}(A_1+A_2)\right.$$

$$\left.+3\frac{\left(4-\psi^{\frac{2}{3}}\right)(3\psi+4k)}{\psi(3\psi+8k)}A_1-9\psi^{\frac{5}{3}}\frac{\left(4-\psi^{\frac{2}{3}}\right)}{3\psi+2k}A_1(\ )\right]$$

$$b_2 = \frac{1}{4\left(4-\psi^{\frac{2}{3}}\right)}\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\varsigma\psi}\right)$$

$$a_3 = \frac{1}{4\left(4-\psi^{\frac{2}{3}}\right)}\left[\left[\left\{\frac{3}{2}\left(\frac{15\psi-4k}{3\psi-2k}\right)\psi^{\frac{2}{3}}-24\left(\frac{3\psi-k}{3\psi-2k}\right)\right\}(A_1+A_2)-\frac{3\left(4-\psi^{\frac{2}{3}}\right)}{\psi}\left(\frac{3\psi+4k}{3\psi+8k}\right)A_1+9\frac{\left(4-\psi^{\frac{2}{3}}\right)\psi^{\frac{5}{3}}}{3\psi+2k}A_1\right.\right.$$

$$-3\frac{\left(5-\psi^{\frac{2}{3}}\right)(3\psi+4k)}{3\psi+2k}A_1-6\frac{\left(\psi^{\frac{2}{3}}-2\right)(3\psi+4k)}{3\psi+2k}A_2(\ )$$

$$-9\psi^{\frac{5}{3}}\frac{\left(4-\psi^{\frac{2}{3}}\right)}{3\psi+2k}(A_1+A_2)+3\left(1+\psi^{\frac{2}{3}}\right)\frac{3\psi+4k}{3\psi+2k}(A_1+A_2)(\ ].$$

Each a_i, b_i $i=1,2,3$ is very small.

Then the characteristic equation becomes

$$\lambda^4-\left[\left\{3\psi\left(\frac{\psi-2k}{3\psi+4k}\right)\left(\psi^{\frac{2}{3}}b_1+b_2\right)\right\}\mu+9\psi\left(\frac{\psi-2k}{3\psi+4k}\right)-4\varphi^2+16\left(\frac{\psi-2k}{3\psi+4k}\right)k\right.$$

$$+3\psi^{\frac{5}{3}}\left(\frac{\psi-2k}{3\psi+4k}\right)a_1+3\psi\left(\frac{\psi-2k}{3\psi+4k}\right)a_2-6(A_1+A_2)\varphi^2\ ]\lambda^2-\frac{9}{4}\psi^{\frac{8}{3}}\left(\frac{\psi-2k}{3\psi+4k}\right)^2\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\times$$

Its roots are

$$\lambda^2=\frac{1}{2}\left[\left\{3\psi\left(\frac{\psi-2k}{3\psi+4k}\right)\left(\psi^{\frac{2}{3}}b_1+b_2\right)\right\}\mu+9\psi\left(\frac{\psi-2k}{3\psi+4k}\right)-4\varphi^2+16\left(\frac{\psi-2k}{3\psi+4k}\right)k\right.$$

$$\left.+3\psi^{\frac{5}{3}}\left(\frac{\psi-2k}{3\psi+4k}\right)a_1+3\psi\left(\frac{\psi-2k}{3\psi+4k}\right)a_2-6(A_1+A_2)\varphi^2\pm\sqrt{\Delta}\right] \quad (14)$$

where

$$\Delta = 3\psi^{\frac{5}{3}}\left(\psi-\frac{20k}{3}\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)(3+4b_3)\mu^2$$

$$+\left[\left\{\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\left(\psi-\frac{10k}{3}\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\right)\right\}b_1\right.$$

$$+\left\{\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\right)\right\}b_2$$

$$-6\psi^{\frac{2}{3}}\left(\psi^2-\frac{20}{3}\psi k\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)b_3+12\psi^{\frac{2}{3}}\left(\psi^2-\frac{20}{3}\psi k\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)a_3$$

$$-9\psi^{\frac{2}{3}}\left(\psi^2-\frac{20}{3}\psi k\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\mu+\left(3\psi-4\varphi^2-\frac{14}{3}k\right)^2$$

$$+\left\{\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\left(\psi-\frac{10k}{3}\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\right)\right\}a_1$$

$$+\left\{\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\right)\right\}a_2$$

$$-6\psi^{\frac{2}{3}}\left(\psi^2-\frac{20}{3}\psi k\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)a_3+\left(48\varphi^4+56\varphi^2k-36\psi\varphi^2\right)\left(A_1+A_2\right) \quad (15)$$

The roots of the characteristic equation (12) are functions of μ, A_1, A_2, k and are controlled by Δ. Three cases can be discussed for Δ:

I. When $\Delta > 0$, we have that the roots are negative showing that the triangular points are linearly stable.
II. When $\Delta < 0$, we have that the real parts of two of the four roots are positive and equal showing that the triangular points are unstable.
III. When $\Delta = 0$, we have that the double roots give secular terms, showing that the triangular points are unstable.

5. Critical Mass

The solution of the equation $\Delta = 0$ gives the critical mass value μ_c of the mass parameter. That is

$$\mu_c=\frac{1}{2}\left(1-\sqrt{\frac{A-4B}{A}}\right)$$

$$+\frac{1}{2}\left[\left\{\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\left(\psi-\frac{10k}{3}\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)\right)\right\}\right]\left[\frac{b_1+2a_1}{\sqrt{A(A-4B)}}-\frac{b_1}{A}\right]$$

$$+\frac{1}{2}\left[\left\{\left(\psi-\frac{10k}{3}\right)\left(6\psi-8\varphi^2-\frac{28k}{3}-3\psi^{\frac{2}{3}}\left(\psi-\frac{10k}{3}\right)\right)\right\}\left[\frac{b_2+a_2}{\sqrt{A(A-4B)}}-\frac{b_2}{A}\right]\right]$$

$$\frac{1}{3}\left[\frac{4B}{\sqrt{A(A-4B)}}+\frac{2\sqrt{A-4B}}{\sqrt{A}}-\frac{\sqrt{A}}{\sqrt{A(A-4B)}}-1\right]b_3-\frac{2}{3}a_3+\frac{\left(48\varphi^4+56k\varphi^2-36\psi\varphi^2\right)\left(A_1+A_2\right)}{\sqrt{A(A-4B)}} \quad (16)$$

where $A=9\psi^{\frac{2}{3}}\left(\psi^2-\frac{20}{3}\psi k\right)\left(4-\psi^{\frac{2}{3}}+\frac{16k}{3\psi}\right)$ and $B=\left(3\psi-4\varphi^2-\frac{14}{3}k\right)^2$.

For simplicity we substitute $\varphi = 1+\varepsilon$, $\psi = 1+\varepsilon'$ and restrict ourselves to linear terms in $\varepsilon, \varepsilon'$ and k. Neglecting the product $A_i\varepsilon$ and $A_i\varepsilon'$ in equation (16) we find

$$\mu_c = \mu_0 + \mu_p + \mu_{br} + \mu_{kp} + \mu_k \quad (17)$$

where

$$\mu_0=\frac{1}{2}\left(1-\sqrt{\frac{23}{27}}\right)$$

$$\mu_p=\frac{4(36\varepsilon-19\varepsilon')}{27\sqrt{69}}$$

$$\mu_{br}=-\frac{1}{9}\left(1+\frac{13}{\sqrt{69}}\right)A_1+\frac{1}{9}\left(1-\frac{13}{\sqrt{69}}A\right)A_2$$

$$\mu_{kp}=\frac{3040(36\varepsilon-19\varepsilon')}{1863\sqrt{69}}$$

$$\mu_k=-\frac{128}{27\sqrt{69}}k-\frac{98}{81}\left(1+\frac{38990441}{18032\sqrt{69}}\right)kA_1+\frac{62}{81}\left(1-\frac{38990441}{11408\sqrt{69}}\right)kA_2.$$

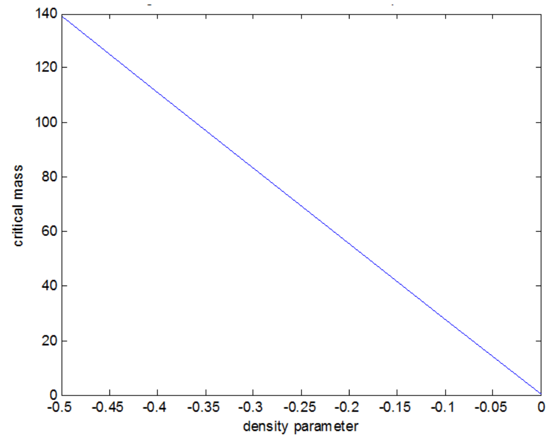

Fig 4. *Variation of critical mass with the parameters.*

Equation (17) gives the critical mass value of the mass parameter. It reflects the effect of perturbations in the Coriolis and centrifugal forces and the oblateness of the first (rigid mass) and second primaries on the critical mass of the Robes restricted three-body problem, indicating a destabilizing effect on the triangular equilibrium points.

The destabilizing tendencies of the centrifugal force and oblateness factors are further enhanced while the stabilizing property of the Coriolis is overpowered when $k > 0$. The stabilizing tendency of the Coriolis force and the destabilizing powers of the centrifugal force and oblateness factors are weakened when $k < 0$. Fig.4 shows the variation of the critical mass μ_c with the density parameter k for constant values ε, ε', A_1 and A_2.

When $k = 0$ we confirm the result of AbdulRaheem and Singh (2006) for $q_1 = 0, q_2 = 0$.

6. Conclusion

The effect of perturbations in Coriolis and centrifugal force, oblateness of the first (rigid shell) and second primaries on the stability of the triangular equilibrium points of the Robes restricted three-body problem was studied. The value of the critical mass obtained depends on the small change in the Coriolis and centrifugal forces, oblateness coefficients of the rigid shell and second primary as well as the density of the fluid and that of the infinitesimal mass in the shell.

It was observed that the perturbation in the centrifugal force and oblateness factors have destabilizing tendencies on the triangular equilibrium points. Though the Coriolis force has stabilizing power, the overall effect overwhelms this power. These destabilizing tendencies are further enhanced or weakened, depending on whether the density of the fluid in the shell is less than that of the infinitesimal mass or the density of the infinitesimal mass is less than that of the fluid.

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