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General law of universal gravitation

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Abstract

The “General Law of Universal Gravitation” is a study in Physics developed by the author in this paper in 2004. This study is the general case of the Universal Gravitation introduced by Newton since 1687. In fact, the universal gravitation was developed to describe the forces between two or more objects that exert gravitational forces from the center of these objects and result in their interaction with each other. The universal gravitation of Newton relates the masses of each object and the distance that separate them. Newton’s law of universal gravitation is considered as the basis of the classical mechanics and it resolves many problems such as the elliptic trajectory of planets around the sun, the free fall of objects on Earth and so on... However, this study remains limited, and it lacks accuracy for very big objects such as galaxies and very small objects such as particles and atoms, and so on. In this paper, the author proposed a new study that generalizes the study made by Newton, and that is much simpler than the General Theory of Relativity of Einstein, and it is developed to resolve any natural phenomenon in which is impossible to be resolved by applying classical physics (based on Newton’s laws) or the modern physics. For example, the proposed theory can resolve the unification of the gravitational field produced by a matter and an electromagnetic wave, it can predict the bending of light near a massive body, it can predict the variation of the frequency and energy of light near a massive body, it can predict the synchronization between photons of a source of light, it can explain many phenomena in the nature which are almost impossible to be explained by applying classical physics. Moreover, the proposed equation categorizes gravity into four main categories, which is not the case in classical physics and modern physics.

1. Introduction

In physics, “Newton's law of universal gravitation” states that every point mass in the universe attracts every other point mass with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them [1]. (It was demonstrated that any form of mass exerts a force, as this force is originated from its center). This is the law proposed by Newton derived from empirical observations. It is a part of classical mechanics and was formulated in Newton's work “*Philosophia Naturalis Principia Mathematica*” (“the Principia”), first published on 5 July 1687 [1]. In modern language, the law states the following [1]:

“Every point mass attracts every single other point mass by a force pointing along the line intersecting both points. The force is proportional to the product of the two masses, and inversely proportional to the square of the distance between them” [1].

After Newton introduced his law in physics, this law has been superseded by Einstein's theory of general relativity. However, Newton’s law is always used as an excellent approximation of the effects of gravity for medium scale objects and masses and because it is much simpler than the General Relativity of Einstein. Relativity is required only when there is need for extreme precision, or when dealing with gravitation for extremely

massive and dense objects [1-23].

Some observations conflicting with Newton's law and theory:

- This theory doesn't fully explain the precession of the perihelion of the orbits of the planets, especially of planet Mercury, which was detected long after Newton's time. There is a 43 arcsecond per century discrepancy between the Newtonian calculation, which arises only from gravitational attractions from the other planets, and from precession that was observed through advanced telescopes during the 19th Century [34].

- According to Newton's law, the deflection of the light ray due to gravity is calculated to be half of what the General theory of gravity predicts, and what was practically observed by astronomers.

In this paper, the author proposed a new theory (developed in 2004), called the "General Law of Universal Gravitation", that generalizes Newton's law of universal gravitation. It can resolve many problems which are impossible to be resolved using Newton's law of universal gravitation such as the unification of gravitational field yields by a matter and an electromagnetic wave, the gravitational force produced by an electromagnetic wave, the bending of the electromagnetic wave near a matter, the variation of the frequency of the electromagnetic wave near a matter. Basically, every energy has 3 types of fields (gravitational field, magnetic field, electric field).

This theory is an alternative to the General Theory of Relativity of Einstein in which the same phenomenon can be explained using the new theory introduced in this paper.

In Section 2, the universal gravitation of Newton is presented and it contains the main theories proposed by Newton and by the classical physics. In Section 3, the "General Law of Universal Gravitation" is proposed and presented. Then there is a comparative study between this new theory and the proposed theory by Newton and modern physics in Section 4. And finally, in Section 5, a conclusion is presented.

2. The Universal Gravitation of Newton

Before 1687, many scientists have collected a large number of data on the motion of planets such as the moon, and other planets in the solar system. But there is not any understanding of how these planets work and the concept of forces was not clear. In that same year, Newton has discovered the secret of physics and of the universe, and then he proposed the existence of forces between masses which act on these masses directed from the center of the mass toward other mass and vice versa, these proposed forces act on moon and planet and let them move in an orbit around the sun and around each other (case of moons) [1]. Newton said that this force is the gravitational force exerted by a mass such as earth on other masses such as moon. And moreover, he realized that the same force which let the moon turn

around the earth is the same force that let the apple fall from the tree to the ground [1].

2.1. Newton's Law of universe Gravitation

After analyzing data about the motion of moon around the earth, Newton made a bold assertion that the force that governs the motion of planet Earth is the same force that attracts the apple toward the earth [1]. In 1687, Newton published his work on the law of gravity in his treatise *Mathematical Principles of Natural Philosophy* [1]. In his book, he developed the law of universal gravitation, and he stated that "every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them." [1]. In case we have 2 particles of masses m_1 and m_2 and they are separated by a distance r , then the magnitude of this gravitational force is [1]:

$$F_g = G \frac{m_1 m_2}{r^2} \quad (1)$$

Where G is a constant, called the *universal gravitational constant*. Its value in SI units is:

$$G = 6.674 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$$

We can express this force in vector form by defining a unit vector $\vec{r}_{12}$ (Fig. 1) with [1]

$$\vec{r}_{12} = (x_{r_2} - x_{r_1})\vec{i} + (y_{r_2} - y_{r_1})\vec{j} + (z_{r_2} - z_{r_1})\vec{k}$$

Because this unit vector is directed from particle 1 toward particle 2, the force exerted by particle 1 on particle 2 is [1]

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \vec{r}_{12} \quad (2)$$

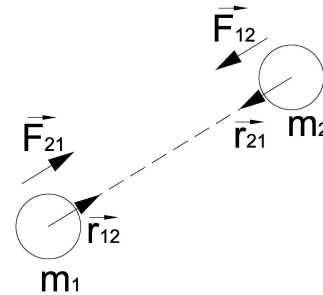


Fig. 1. The gravitational force between two particles is attractive. The unit vector $\vec{r}_{12}$ is directed from particle 1 toward particle 2. The gravitational force vector $\vec{F}_{12}$ is directed from particle 2 toward particle 1 [1].

The gravitational force $\vec{F}$ is a force that exist everywhere around a particle it is called a "field force", regardless of the medium that separates them [1]. This force decreases rapidly when the distance between the two particle increases [1].

Equation 2 shows us that any particle exerts a force from its center whatever is its form as the total mass of this particle is concentrated at its center, in the universe the most of objects have spherical forms.

For example, the magnitude of the force exerted by the

Earth on a particle of mass m near the Earth's surface is:

$$F_g = G \frac{M_E m}{R_E^2} \quad (3)$$

Where M_E is the Earth's mass and R_E its radius. This force is directed toward the center of the Earth.

2.2. Free-Fall Acceleration and the Gravitational Force

The magnitude of the gravitational force on a mass or object near the Earth's surface is called the *weight* of the mass or object, where it is given by $\vec{F} = m\vec{g}$. Therefore

$$F = mg = G \frac{M_E m}{R_E^2} \Rightarrow g = G \frac{M_E}{R_E^2} \quad (4)$$

As one can see, the weight is independent of the mass of the object on the surface of the Earth; g is also called the acceleration of the object on the surface of the Earth. Its value is approximately 9.80 m/s^2 [1].

2.3. The Gravitational Field

Some scientists asked Newton "how is it possible for masses or objects to interact when they were not in contact with each other such as the sun and planets around the sun". In fact, Newton was not able to answer this question [1]. The answer of this question came after Newton's death [1]. In fact, the answer to this question is that any mass or object has a gravitational field around it exerted by the object such as its whole mass is concentrated in the center of the object and this gravitational field exists in every point in the space around the object even if it is void [1].

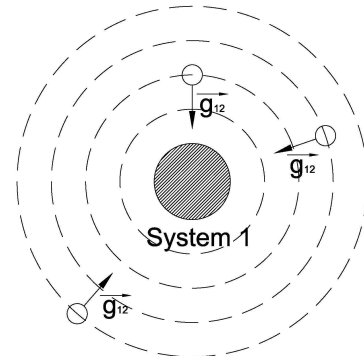
Any particle or object of mass m when it is placed at a point where the gravitational field is $\vec{g}$, the force created on this particle is $\vec{F}_g = m\vec{g}$. The gravitational field $\vec{g}$ is defined as $\vec{g} = \frac{\vec{F}_g}{m}$ [1].

The gravitational field at any point in the space is equal to the gravitational force experienced by a tested particle located at that point divided by the mass of the tested particle. We call the object creating the field the source particle. The gravitational field exists even if the tested particle is not presented; the source particle creates the gravitational field [1]. Moreover the gravitational field is created by both particles the source particle and the tested particle.

Let's consider for example an object of mass m near the Earth's surface. The gravitational force acts on the object and it has a magnitude $GM_E m/r^2$, the field $\vec{g}$ at a distance r from the center of the Earth is

$$\vec{g} = \frac{\vec{F}_g}{m} = -\frac{GM_E}{r^2} \vec{r} \quad (5)$$

The negative sign of the right side of the equation (5) indicates that $\vec{r}$ is a unit vector pointing radially outward from the earth and the field points toward the center of the Earth (fig. 2) [1].



Gravitational field of the System 1

Fig. 2. The gravitational field vector $\vec{g}$ of the system 1 in the space.

2.4. Gravitational Potential Energy

The gravitational potential energy function is defined as $U = mgh$ it is valid only when the particle is near the Earth's surface, in which we can consider that the gravitational force is constant [1].

The variation in the potential energy of a system is connected to a given displacement of the system, and it is defined as the negative of the internal work done by the force on this system during the displacement:

$$\Delta U = U_f - U_i = -\int_{r_i}^{r_f} F(r) dr \quad (6)$$

This equation can be applied to any particles. The gravitational potential energy between two particles of masses m_1 and m_2 separated by a distance r is

$$U = -\frac{Gm_1 m_2}{r} \quad (7)$$

This equation indicates that the gravitational potential energy between any two particles varies as $1/r$, whereas the force between them varies as $1/r^2$. The potential energy is negative because the force is attractive and one has chosen the potential energy as zero when the particle separation is infinite [1]. Because the force between the particles is attractive, an external agent must do positive work to increase the separation between the particles and to increase the potential energy [1]. That is, U becomes less negative as r increases [1].

2.5. Invalidity of the Universal Gravitation of Newton on Very Big and Very Small Scales

The Universal Gravitation of Newton is valid only for medium scale in the nature such as objects in our environment that we can see and also it can be applied on the solar system or any other system in the universe such as binary stars systems. But this can't be applicable on very big scales such as galaxies or very small scales such as particles or electromagnetic waves.

• For example: the Whirlpool galaxy is a very bright galaxy surrounded by the "arms" of the galaxy, which contains material in orbit around the central core. According

to Newton's law, and based on this distribution of matter in the galaxy, the speed of an object in the outer part of the galaxy would be smaller than that for objects closer to the center, just like for the planets of the solar system [1].

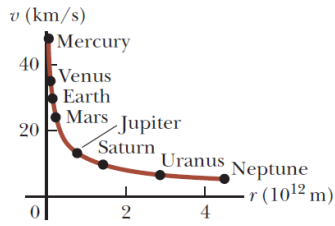


Fig. 3. The orbital speed v as a function of distance r from the Sun for the eight planets of the solar system. The theoretical curve is in red-brown, and the data points for the planets are in black (from [1] page 391).

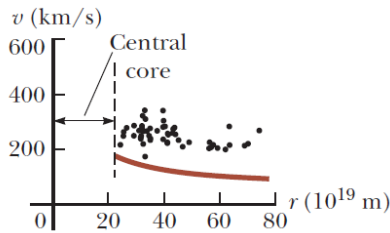


Fig. 4. The orbital speed v of a galaxy object as a function of distance r from the center of the central core of the Andromeda galaxy. The theoretical curve is in red-brown, and the data points for the galaxy objects are in black. No data are provided on the left because the behavior inside the central core of the galaxy is more complicated (from [1] page 391).

The observation is different from Newton's law. In figure 4, the red-brown curve shows the expected speeds for these objects if they were traveling in circular orbits around the mass concentrated in the central core [1]. The data for the individual objects in the galaxy shown by the black dots are all well above the theoretical curve [1]. These data, and many other data taken over the past years, show that for objects outside the central core of the galaxy, the curve of speed versus distance from the center of the galaxy is approximately flat rather than decreasing at larger distances [1]. To explain this phenomenon, the scientists proposed the existence of the Dark matter in galaxies and surround galaxies because objects are rotating faster than what is expected with Newton's law [1]. This means that there must be additional mass in a more extended distribution, causing these objects to orbit so fast [1]. This dark matter is proposed to exist in a large halo around each galaxy (with a radius up to 10 times as large as the visible galaxy's radius) [1]. The dark matter it is not luminous, it doesn't emit electromagnetic radiation and it must be either very cold or electrically neutral. Therefore, we cannot detect or see the dark matter, except through its gravitational effects [1].

Till now, no one knows what dark matter consists of. It is theoretically introduced just to explain why the behavior of very big scales is not the same as Newton's law.

In fact, this phenomenon can't be explained using classical physics, but it can be explained using the "General Law of Universal Gravitation" without introducing the term of dark matter or dark energy. This is developed in the section 3.

- The equation (1) developed by Newton is not correct

when the distance (r) is nearly equal to zero. In fact, if the distance becomes near to zero, therefore the force becomes near to infinite which is not correct because the force is limited and it can't be equal to infinite when the distance is equal to zero. This problem is resolved in this paper when it introduces other terms in the equation (9) which will be explained in the next section, the introduced parameter " a_i " has a role to eliminate the value of zero when the distance is equal to zero without affecting the equation and the whole curve. The same for the velocity of the tested system (or object) which can't be equal to infinite when the distance is nearly equal to zero.

- According to Newton, if a particle with very small dimension yields an electromagnetic wave then the wave can't go out of the particle because the distance is near to zero which means the force is near to infinite. This is incorrect even if the distance between the produced wave and the center of the particle is near to zero, the wave goes out of the particle independently of the massive force exerted by the particle on the wave. This is resolved by using the equation (9).

- Attempting to explain the Moon's apsidal motion, Newton proposed in his "Principia" an extended expression of his law of gravity including an inverse-cube term of the form

$$F_g = G \frac{m_1 m_2}{r^2} + B \frac{m_1 m_2}{r^3} \quad (8)$$

In fact, even with this attempt, it still a particular case of the equation (9).

3. General Law of Universal Gravitation

In this section, the author proposed a General Law of Universal Gravitation which is the general case of the Newton's Universal Gravitation. It is proposed for many reasons, and the most important ones are as following:

- The matter and light have the same behavior in a gravitational field.

- The light has a weak Gravitational field, and the matter has a strong Gravitational field.

- The Light and matter are measured as units of energy.

- It is possible to unify the gravitational field of the light and the matter under one form.

- The proposed theory predicts the variation of the Energy of the electromagnetic wave near a massive body.

- Every energy has 3 types of fields (gravitational field, magnetic field, and electric field).

- As the 3 types of fields (gravitational field, magnetic field, and electric field) are attached to the source of energy (i.e. electromagnetic wave, or matter), therefore their speed of propagation is also equal or higher than the speed of the source of energy (i.e. electromagnetic wave or matter).

- It explains the "Extra energetic photons": Photons travelling through galaxy clusters should gain energy and then lose it again on the way out. The accelerating expansion of the universe should stop the photons returning all the

energy, but even taking this into account photons from the cosmic microwave background radiation gain twice as much energy as expected. This may indicate that gravity falls off faster than inverse-squared at certain distance scales.

-It explains the “Accelerating expansion”: The metric expansion of space seems to be speeding up. Dark energy has been proposed to explain this. A recent alternative explanation is that the geometry of space is not homogeneous (due to clusters of galaxies) and that when the data are reinterpreted to take this into account, the expansion is not speeding up after all, however this conclusion is disputed.

- It explains the “Dark flow”: Surveys of galaxy motions have detected a mystery dark flow towards an unseen mass. Such a large mass is too large to have accumulated since the Big Bang using current models and may indicate that gravity falls off slower than inverse-squared at certain distance scales.

- It explains the “Extra massive hydrogen clouds”: The spectral lines of the Lyman-alpha forest suggest that hydrogen clouds are more clumped together at certain scales than expected and, like dark flow, may indicate that gravity falls off slower than inverse-squared at certain distance scales.

-It explains the expansion of the universe without introducing the dark energy.

-It categorizes the gravity into four main forms which are developed in the section 4.6.

The proposed General Law of Universal Gravitation states the following:

“Every Energy in the Universe interacts with every other Energy with a force that is directly proportional to the product of their energies and proportional to the function of distance and to the ratio of the variation of the force.”

This proposition is the general case of the law proposed by Newton in which it stated:

“Every particle in the Universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.”

So if two systems have energies E_1 and E_2 and are separated by a distance r , the magnitude of this gravitational force is

$$F_g = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t) \quad (9)$$

Where,

- b_j is the parameter that determines the universal gravitational constants. Its value in SI units is $[N/m^j kg^2]$. This parameter can be positive or negative; if it is positive, these two energies attract each other; if it is negative, these two energies repulse each other. In the negative case, it will explain the expansion of the universe due to a negative force.

- B_1 is a parameter of the first system with energy E_1 . It is expressed in $[J/Kg]$.

- E_1 is the energy of the first system. It is expressed in $[J]$.

- B_2 is a parameter of the second system with energy E_2 . It is expressed in $[J/Kg]$.

- E_2 is the energy of the second system. It is expressed in $[J]$.

- r^i and r^j are the distance between the two systems of

energies. They are expressed in $[m^i]$ and $[m^j]$ respectively.

- a_i is the correction value of the distance between two systems of energies. Its unit is $1/m^i$. (This parameter is very important because it eliminates the infinite value of the force F when r^i is very close to zero, it resolves the speed of particles such as electrons around a nuclei without using the Modern Physics.

- I and J are integer numbers $\in \mathbb{N}$.

- $f(t)$ is the ratio of the variation of the force due to a sudden perturbation in its form or movement, it is a function of time and its value is between 0 and 1. In stable and permanent conditions it is equal to 1.

The expression $\frac{E}{B}$ is called system energy; it can take the form of matter or light.

• Particular case, when one takes $b_0 = G$ and $b_j = 0$ for $j \neq 0$, $a_2 = 1$ and $a_i = 0$ for $i \neq 2$, and $f(t) = 1$ then

$$F_g = G \frac{\frac{E_1}{B_1} \frac{E_2}{B_2}}{r^2} \quad (10)$$

Where,

- G is the universal gravitational constant. Its value in SI unit is $G = 6.674 \cdot 10^{-11} \text{ Nm}^2/\text{kg}^2$

Remark: $b_0 = G = [N/\text{kg}^2]$ and $a_2 = [1/\text{m}^2]$ therefore,

the unit of $G = \frac{[N]}{[1/\text{m}^2]} = \text{Nm}^2/\text{kg}^2$

• For a particular case, if one takes the equation (10), when the attraction between two systems is the attraction between two matters, therefore, one has

$\frac{E_1}{B_1} = m_1$ with m_1 is the mass of the first system. They share the same SI unit of the mass $[kg]$, $\frac{E_2}{B_2} = m_2$ with m_2 is the mass of the second system. They share the same SI unit of the mass $[kg]$.

Therefore the particular case has the following form $F_g = G \frac{m_1 m_2}{r^2}$ which is the same as the Newton's law of universal gravitation.

• For a particular case, if one takes the equation (10), when the attraction between two systems is the attraction between matter and light, therefore, he has:

$\frac{E_1}{B_1} = m_1$ with m_1 is the mass of the first system. They share the same SI unit of the mass $[kg]$.

$\frac{E_2}{B_2}$ is the system energy of the light and it reserves the SI unit of the mass $[kg]$.

Therefore the particular case has the following form

$$F_g = G \frac{m_1 \cdot \frac{E_2}{B_2}}{r^2}$$

In fact, light is attracted by the gravitational field of the matter, this is indicated in the General Relativity, and the experience made in the year 1919 demonstrated that the light bends when it passes near a heavy gravitational field as the sun [1]. This can be applied for any system of energy. But for small masses the light is not affected because of the weak gravitational field between them and because of the high speed of the light.

The second prove of the proposed formulae (9) and (10) is that the light bends more when its frequency is high, and bends less when its frequency is low. This means that when the energy of the light is high, then the attraction force is high. And when the energy of the light is low, then the attraction force is low. In this way, the phenomena of attraction between light and matter is resolved.

One can express the “General Law of Universal Gravitation” in a vector form by defining a unit vector $\vec{r}_{12}$ as shown in figure 5. Because this unit vector is directed from system 1 toward system 2, the force exerted by system 1 on the system 2 is

$$\vec{F}_{12} = -\frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t) \cdot \vec{r}_{12} \quad (11)$$

• Particular case, when one takes $b_0 = G$ and $b_j = 0$ for $j \neq 0$, $a_2 = 1$ and $a_i = 0$ for $i \neq 2$, $f(t) = 1$ then

$$\vec{F}_{12} = -G \frac{E_1 E_2}{B_1 B_2 r^2} \vec{r}_{12} = G \frac{E_1 E_2}{B_1 B_2 r^2} \vec{r}_{21} \quad (12)$$

When a positive sign ($\vec{r}_{12}$) indicates that system 2 is attracted by system 1 and vice versa.

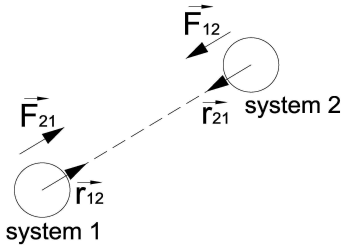


Fig. 5. The gravitational force between two systems is directed from the second system toward the first system. $\vec{r}_{12} = r_2 - r_1$ is a unit vector from system 1 toward system 2.

3.1. The Gravitational Field

The gravitational field exists in each point in the space. When a system of energy exists at a point where the gravitational field is $\vec{g}$, the system experiences a force $\vec{F}_g = \frac{E}{B} \vec{g}$ caused by the source of gravitation.

Where,

-B is a parameter of the system with energy E. It is expressed in J/Kg.

-E is the energy of the system. It is expressed in J.

So the gravitational field is equal to $\vec{g} = \frac{B \vec{F}_g}{E}$.

That is, the gravitational field at a point in space equals the gravitational force experienced by a tested system placed at that point divided by the System Energy ($\frac{E}{B}$) of the tested system. The system creating the field is called the “Source system”.

Considering now a tested system placed at a distance r from the “Source system” and it has system energy equal to ($\frac{E_2}{B_2}$). So one has $\vec{g} = \frac{B_2 \vec{F}_{12}}{E_2}$ and

$$\vec{F}_{12} = -\frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t) \cdot \vec{r}_{12}$$

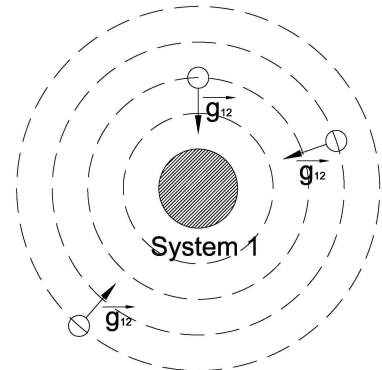
Therefore,

$$\vec{g} = \frac{B_2 \vec{F}_{12}}{E_2} = \frac{\frac{E_1 E_2}{B_1 B_2} \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t) \cdot \vec{r}_{12}}{\frac{E_2}{B_2}} = -\frac{E_1}{B_1} \cdot \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t) \cdot \vec{r}_{12} \quad (13)$$

• Particular case, when we take $b_0 = G$ and $b_j = 0$ for $j \neq 0$, $a_2 = 1$ and $a_i = 0$ for $i \neq 2$, $f(t) = 1$ then

$$\vec{g} = -G \frac{E_1}{B_1 r^2} \vec{r}_{12} \quad (14)$$

With the direction of the vector $\vec{g}$ is towards the center of the “Source system” as shown in figure 6.



Gravitational field of the System 1

Fig. 6. The gravitational field of the system 1 in the space. It depends on the Total energy of the system 1 and on the distance that separate it from the other systems.

For a particular case, when the tested system is a matter, therefore, $\frac{E}{B} = m$, the system energy is equal to the mass of the matter. Therefore, $\vec{g} = \frac{\vec{F}_g}{m} = G \frac{E_1}{B_1 r^2} \vec{r}_{12}$

If the Source System is a matter, then $\frac{E_1}{B_1} = m_1$ which is the mass of the Source System, therefore one has $\vec{g} = \frac{\vec{F}_g}{m} = G \frac{m_1}{r^2} \vec{r}_{12}$ which is the same as the Newton’s gravitational field.

The Electromagnetic radiation has a weak gravitational field.

So, in conclusion, the Gravitational field of Newton is a particular case of the General Gravitational field expressed by the equation (13).

3.2. The Gravitational Field Applied on an Electromagnetic Wave

3.2.1. Frequency of an Electromagnetic Wave Increases when Approaching to a Matter

Now let’s suppose the “Source System” is a matter and the tested system is an electromagnetic wave (system 2) that is

going toward the center of the “Source System” (figure 7, right wave).

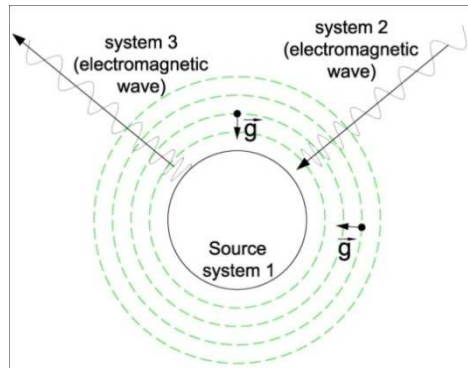


Fig. 7. an electromagnetic wave (system 2) is going towards a matter (Source System 1); the energy of the wave varies while it is approaching to the Source system 1, it increases. Another electromagnetic wave (system 3) is going out of the Source System 1, the energy of the wave varies while it is leaving the Source system 1, and it decreases.

The “General Law of Universal Gravitation” predicts the variation of the frequency of the electromagnetic wave to a higher level. In fact, if one uses the equation (9), the system energy of the system 2 is $\frac{E_2}{B_2}$ which depends mainly on the energy E_2 . If E_2 increases, B_2 will increase also to maintain the system energy constant. So in reality, the energy E_2 increases and the parameter B_2 increases, and the gravitational force $\vec{F}_{12}$ increases when the electromagnetic wave is going toward the Source System 1.

Demonstration:

Suppose the force of the equation (7) for an instant t is as following:

$$F = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot f(r) \cdot f(t) \quad (15)$$

Where,

$$f(r) = \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}$$

After a short period dt , the electromagnetic wave travelled a distance dr towards the “source system”. Then the equation (15) becomes

$$F' = \frac{E'_1}{B'_1} \frac{E'_2}{B'_2} f(r - dr) f(t + dt) \quad (16)$$

Where F' is the force applied on the electromagnetic wave after travelling a distance dr , (Figure 8).

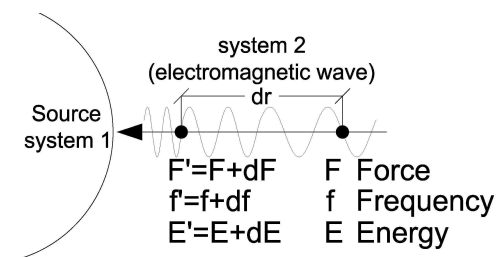


Fig 8. Electromagnetic Wave approaching to the Source System 1.

And it is considered that there is no perturbation in the source system between the interval dt , therefore,

$$f(t + dt) = f(t)$$

$$F' = F + dF$$

The mass of the source system doesn't change, therefore,

$$\frac{E'_1}{B'_1} = \frac{E_1}{B_1}$$

$$\Rightarrow \frac{F}{\frac{E_2}{B_2} f(r) f(t)} = \frac{(F + dF)}{\frac{E'_2}{B'_2} f(r - dr) f(t + dt)}$$

$$\Rightarrow \frac{E_2}{B_2} = \frac{E'_2}{B'_2} \frac{f(r - dr) f(t + dt) F}{(F + dF) f(r) f(t)}$$

$$\Rightarrow \frac{E_2}{B_2} = \frac{E'_2}{B'_2} \frac{f(r - dr) F}{(F + dF) f(r)}$$

The mass of the tested system doesn't change also

$$\Rightarrow \frac{f(r - dr) F}{(F + dF) f(r)} = 1$$

And

$$\frac{E'_2}{B'_2} = \frac{E_2}{B_2}$$

Where,

$$E'_2 = E_2 + dE_2$$

$$B'_2 = B_2 + dB_2$$

$$\Rightarrow \frac{E_2 + dE_2}{B_2 + dB_2} = \frac{E_2}{B_2}$$

$$\Rightarrow B_2 E_2 + B_2 dE_2 = B_2 E_2 + E_2 dB_2$$

$$\Rightarrow \frac{dE_2}{E_2} = \frac{dB_2}{B_2}$$

$$\Rightarrow \ln(E_2) = \ln(B_2) + \text{constant}$$

$$\Rightarrow \ln\left(\frac{E_2}{B_2}\right) = \text{constant}$$

$$\Rightarrow \frac{E_2}{B_2} = \text{constant Which is true.}$$

It means the variation of the energy of the electromagnetic wave E_2 is related proportional to the variation of the parameter B_2 . And more precisely one has

$$\frac{E_2}{B_2} = \text{constant}$$

This means when the energy E_2 increases, the parameter B_2 increases.

3.2.2. Frequency of an Electromagnetic Wave Decreases when Going Away from the Matter

Now let's suppose the "Source System" is a matter and the tested system is an electromagnetic wave (system 3) that is going out or running away from the center of the "Source System" (figure 7, left wave) and figure 9.

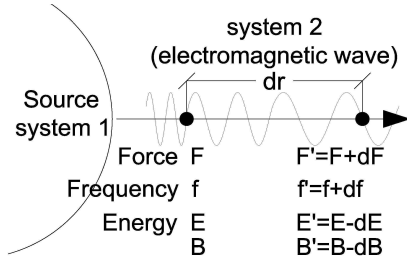


Fig 9. Electromagnetic Wave running away from the Source System 1.

The "General Law of Universal Gravitation" predicts the variation of the frequency of the electromagnetic wave to a lower level. In fact, if one uses the equation (9), the system energy of the system 3 which is E_2/B_2 which depends mainly on the energy E_2 . If E_2 decreases, B_2 decreases. So in reality, the energy E_2 decreases and the parameter B_2 decreases, the gravitational force $\vec{F}_{12}$ decreases, when the electromagnetic wave is going out of the Source System 1.

The same method as the previous sub-section can be used to demonstrate that

$$\frac{E'_2}{B'_2} = \frac{E_2}{B_2} = \text{constant}$$

Where,

$$E'_2 = E_2 - dE_2$$

$$B'_2 = B_2 - dB_2$$

$$\Rightarrow \frac{E_2 - dE_2}{B_2 - dB_2} = \frac{E_2}{B_2}$$

$$\Rightarrow B_2 E_2 - B_2 dE_2 = B_2 E_2 - E_2 dB_2$$

$$\Rightarrow \frac{dE_2}{E_2} = \frac{dB_2}{B_2}$$

$$\Rightarrow \ln(E_2) = \ln(B_2) + \text{constant}$$

$$\Rightarrow \ln\left(\frac{E_2}{B_2}\right) = \text{constant}$$

$$\Rightarrow \frac{E_2}{B_2} = \text{constant Which is true.}$$

Briefly, the new introduced formulae (9), (11), and (13) are the general case of the Newton's law of universe Gravitation. And moreover, they can explain any phenomena in the nature such as the curvature of the light near a massive body, the variation of the frequency of the light near a massive body, the gravitational lenses, and many others phenomena. These phenomena are impossible to be explained using the traditional Newton's law.

The next section proposes 4 types of gravity.

4. Comparison between the Proposed Theory, Classical Theory, and Modern Theory

4.1. Gravitational Field of Light

Classical Physics:

The light is a form of electromagnetic wave and it has not any gravitational field because it is not formed by matter [1].

Modern Physics:

The light has two forms, the first one is an electromagnetic field (wave form) and the second one is a particle called photon or Quanta. So as it is a particle, it must have a gravitational field [1].

Proposed Theory in this paper:

It defines that every energy has a gravitational field. And the force created by the gravitational field is as described by the equation (9) and (13).

4.2. Speed of the Gravity

Classical Physics:

In the simple Newtonian model, gravity propagates instantaneously: the force exerted by a massive object points directly towards that object's present position. For example, even though the Sun is 500 light seconds from the Earth, Newtonian gravity describes a force on Earth directed towards the Sun's position "now," not its position 500 seconds ago. Putting a "light travel delay" (technically called "retardation") into Newtonian gravity would make orbits unstable, leading to predictions that clearly contradict Solar System observations [28-30].

Modern Physics:

In general relativity, on the other hand, gravity propagates at the speed of light; that is, the motion of a massive object creates a distortion in the curvature of spacetime that moves outward at light speed. This might seem to contradict the Solar System observations described above, but remember that general relativity is conceptually very different from Newtonian gravity, so a direct comparison is not so simple. Strictly speaking, gravity is not a "force" in general relativity, and a description in terms of speed and direction can be tricky. For weak fields, though, one can describe the theory in a sort of Newtonian language. In that case, one finds that the "force" in General Relativity is not quite central-it does not point directly towards the source of the gravitational field-and that it depends on velocity as well as position. The net result is that the effect of propagation delay is almost exactly cancelled, and general relativity very nearly reproduces the Newtonian result [1-6].

Proposed Theory in this paper:

In the equation (9), $f(t)$ is the ratio of the variation of the force due to a sudden perturbation in its form or movement, it is a function of time and its value is between 0 and 1. In stable and permanent conditions it is equal to 1. In fact, this is the ratio of the velocity of the gravity.

gravity as following:

$$F_g = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^I (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} f(t)$$

With $a_0 \neq 0$

a_0 is described as the limit of the gravity for small scales particles.

• For example: let's consider Newton's law of gravity applied on small scale particles

$$F_{12} = G \frac{m_1 m_2}{r^2}$$

$\lim_{r \rightarrow 0} F_{12} = \infty$ which is not true at all.

Let's taking the equation (9) for two particles have masses, therefore

$$F_g = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^I (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}$$

With $a_0 \neq 0$, $a_2 = 1$ and $a_i = 0$ for $i \neq [0; 1]$

And $b_0 = G$, $b_j = 0$ for $j \neq 0$,

Therefore,

$$F_g = m_1 m_2 \frac{G}{a_0 + r^2}$$

And

$\lim_{r \rightarrow 0} F_{12} = m_1 m_2 \frac{G}{a_0}$ which is a limited quantity.

This is true because the force is limited, the same for the speed of a particle can't overpass the speed of light.

To determine the quantity a_0 , one can consider an electron rotating around a nuclei with a distance $r = 10^{-12}$. The velocity of this electron is 0.99c

Therefore,

$$F_g = m_1 m_2 \frac{G}{a_0 + r^2} = m_2 \frac{V^2}{r}$$

$$\Rightarrow a_0 = \frac{m_1 G r}{V^2} - r^2$$

4.4. Force between Stars and the Center of a Galaxy

Classical Physics:

The classical physics failed to describe forces between stars and the center of galaxies. The classical method determines the velocity of stars as following

$$F_g = m_1 m_2 \frac{G}{r^2} = m_2 \frac{V^2}{r}$$

$$\Rightarrow V = \sqrt{\frac{m_1 G}{r}}$$

This will result as the following figure 13:

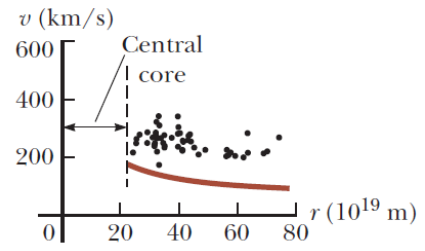


Fig 13. The orbital speed v of a galaxy object as a function of distance r from the center of the central core of the Andromeda galaxy. The theoretical curve is in red-brown, and the data points for the galaxy objects are in black. No data are provided on the left because the behavior inside the central core of the galaxy is more complicated (from [1] page 391).

In figure 13 the predicted velocity by Newton is much lower than the detected velocity. So the equation $V = \sqrt{\frac{m_1 G}{r}}$ doesn't work.

Modern Physics:

The introduction of dark matter has resolved the problem, but it still incomprehensible and enigmatic, because such matter is not possible to be detected.

Proposed Theory in this paper:

The proposed theory has resolved the problem because the behavior in a galaxy is not the same as the behavior of two objects or two particles. It is much more complicated because there are billions of stars and massive bodies which are close to each other and affect each other. Therefore the velocity of

a star in the galaxy is not $V = \sqrt{\frac{m_1 G}{r}}$.

The equation (9) predicts the behavior of stars from the center of a galaxy which rotate at a constant speed as described in the following equations and figure:

$$F_g = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^I (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)} = \frac{E_2}{B_2} \cdot \frac{V^2}{r}$$

$$\Rightarrow V = \sqrt{\frac{m_1 r \sum_{j=0}^I (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}}$$

For example it can have the following form:

$$V = \sqrt{\frac{b_0 + b_1 r + b_2 r^2 + b_3 r^3}{a_1 r + a_2 r^2 + a_3 r^3}}$$

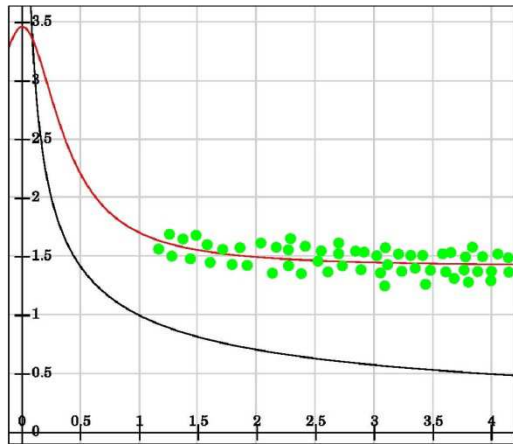


Fig 14. the curve in black color is predicted by Newton's equation, the curve in red color is predicted by the equation (9) proposed by the author, the dots in green color are the velocity of stars in the Andromeda galaxy. This figure is not to scale but it gives us an idea about how we can obtain a result close to the true experiments.

4.5. Acceleration of the Universe

Classical Physics:

The acceleration of the universe has no answer in classical physics because it violates the principles developed by Newton. It is not possible to have an accelerating universe because the velocity should decrease with distance and not increase. But this is not what scientists observed in reality, the universe is accelerated, and it violates the principles of Newton.

Modern Physics:

In Modern physics, the answer to the accelerating universe is to introduce a negative energy which role is to accelerate the universe; this energy is called the dark energy. But till now there is no experimental evidence about this energy. It is only a theory developed to answer the acceleration of the universe which contradicts the laws of Newton.

Proposed Theory in this paper:

In the following sub-section, more details about the types of gravity are presented, but for instance. If one takes the equation (9), it will answer the acceleration of the universe because such acceleration is due to a negative force that increases with distance, such force is presented in the nature and it is described in equation (9) where $f(r) = \frac{\sum_{j=0}^l (b_j \cdot r^j)}{\sum_{i=0}^l (a_i \cdot r^i)}$ must obey the following conditions:

- $f(r) \leq 0$
- $\lim_{r \rightarrow \infty} f(r) \leq A$ with $A < 0$
- $\frac{df(r)}{dr} \leq 0$

4.6. Types of Gravity

Classical Physics:

In classical physics Newton discovered the gravity which has a positive sign. This category of gravity is the first type of gravities in this paper.

Modern Physics:

In modern physics, to resolve the problem of Newton's law for galaxies, they have introduced the dark matter [31-33]. Astrophysicists hypothesized dark matter due to discrepancies between the mass of large astronomical objects

determined from their gravitational effects and the mass calculated from the "luminous matter" they contain: stars, gas, and dust [31-33]. It was first postulated by Jan Oort in 1932 to account for the orbital velocities of stars in the Milky Way and by Fritz Zwicky in 1933 to account for evidence of "missing mass" in the orbital velocities of galaxies in clusters. Subsequently, many other observations have indicated the presence of dark matter in the universe, including the rotational speeds of galaxies by Vera Rubin, in the 1960s–1970s, gravitational lensing of background objects by galaxy clusters such as the Bullet Cluster, the temperature distribution of hot gas in galaxies and clusters of galaxies, and more recently the pattern of anisotropies in the cosmic microwave background. According to consensus among cosmologists, dark matter is composed primarily of a not yet characterized type of subatomic particle. The search for this particle, by a variety of means, is one of the major efforts in particle physics today [31-33].

Proposed Theory in this paper:

The gravity in this paper has four main forms in the universe. This is deduced from the equation (9) and (13). In fact,

- The first form is called positive near attractive gravity.

It is defined as the gravity which results in a positive attraction between two energies. The attraction increases when the distance between them decreases, it is derived from equation (9)

$$F_g = \frac{E_1}{B_1} \cdot \frac{E_2}{B_2} \cdot \frac{\sum_{j=0}^l (b_j \cdot r^j)}{\sum_{i=0}^l (a_i \cdot r^i)} f(r) \quad (9)$$

Where,

$$f(r) = \frac{\sum_{j=0}^l (b_j \cdot r^j)}{\sum_{i=0}^l (a_i \cdot r^i)}$$

And the conditions of this gravity are defined as below:

- $f(r) \geq 0$
- $\lim_{r \rightarrow \infty} f(r) = 0$
- $\frac{df(r)}{dr} \leq 0$

This gravity is similar to the gravity discovered by Newton, but it is more general. The concept is the same but the result is not necessary the same.

Its graph can be simplified as following:

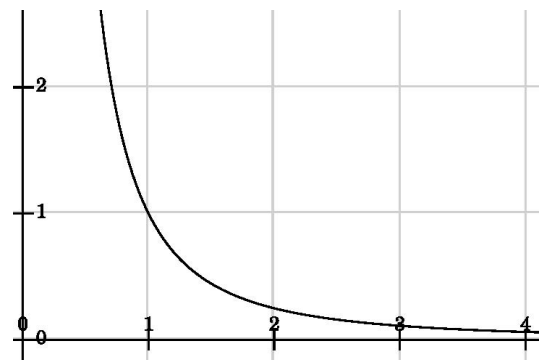


Fig 15. represents a symbolic graph of the first form of gravity.

• The second form is called positive far attractive gravity
It is defined as the gravity which results in a positive far attraction between two energies. The attraction increases when the distance between them increases, it is derived from equation (9)

Where,

$$f(r) = \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}$$

And the conditions of this gravity are defined as below:

- $f(r) \geq 0$
- $\lim_{r \rightarrow \infty} f(r) \geq A$ with $A > 0$
- $\frac{df(r)}{dr} \geq 0$

This gravity is not discovered yet.

Its graph can be simplified as following:

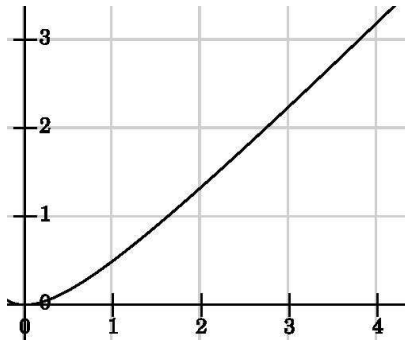


Fig 16. represents a symbolic graph of the second form of gravity.

• The third form is called negative near repulsive gravity
It is defined as the gravity which results in a negative repulsion between two energies. The repulsion increases when the distance between them decreases, it is derived from equation (9)

Where,

$$f(r) = \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}$$

And the conditions of this gravity are defined as below:

- $f(r) \leq 0$
- $\lim_{r \rightarrow \infty} f(r) = 0$
- $\frac{df(r)}{dr} \geq 0$

This gravity is not discovered yet.

Its graph can be simplified as following:

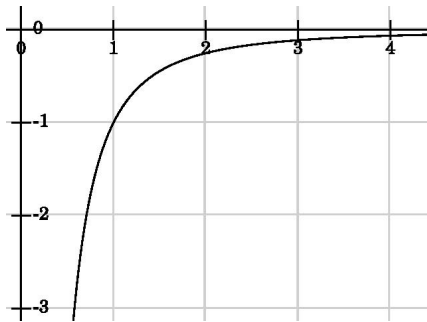


Fig 17. represents a symbolic graph of the third form of gravity.

• The fourth form is called negative far repulsive gravity
It is defined as the gravity which results in a negative far repulsion between two energies. The repulsion increases when the distance between them increases, it is derived from equation (9)

Where,

$$f(r) = \frac{\sum_{j=0}^J (b_j \cdot r^j)}{\sum_{i=0}^I (a_i \cdot r^i)}$$

And the conditions of this gravity are defined as below:

- $f(r) \leq 0$
- $\lim_{r \rightarrow \infty} f(r) \leq A$ with $A < 0$
- $\frac{df(r)}{dr} \leq 0$

This gravity is responsible to the acceleration of the universe.

Its graph can be simplified as following:

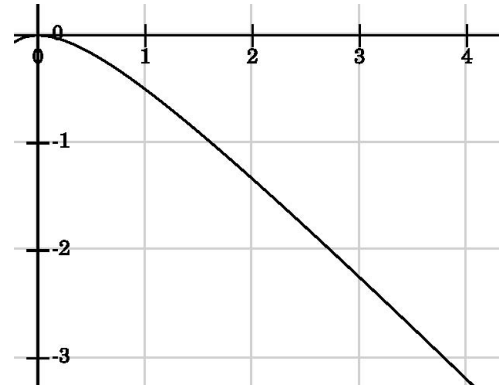


Fig 18. represents a symbolic graph of the fourth form of gravity.

Briefly, this section proposed different forms of gravities, these forms of gravities generalize all previous studies and moreover, it categorizes the gravities in 4 main forms. Two of them are discovered in the nature, the first one is discovered by Newton in a particular case, and the fourth one is proposed to be a form of dark energy, dark matter, and anti-gravity which are responsible for the acceleration of the universe, the constant speed of rotation of far stars from the center of galaxies, and for anti-gravity phenomena respectively. The other forms of gravity are not discovered yet, but they exist in the nature.

Remark: the nature can have a mixture of many types of gravities at the same moment, this can be possible and this is why the velocity of peripheral stars in a galaxy has approximately a linear form in contradiction to Newton's law.

5. Conclusion

In conclusion, The "General Law of Universal Gravitation" is a study developed by the author in Physics in 2004. It is the general case of the Universal Gravitation introduced by Newton since 1687. As it is presented in the previous sections, the new theory proposed can resolve big problems in physics which are difficult or impossible to be resolved using the classical physics or the modern physics. The advantage of the proposed theory compared to the General

Theory of Relativity of Einstein is that, the proposed theory is much simpler, and it can resolve any phenomenon in the nature, even if it is difficult to be resolved using the General Theory of Relativity. The other advantage is that, the proposed theory has many parameters which can be adapted for each phenomenon in the nature, because each phenomenon works in a different way from other phenomena, so this will be more accurate compared to the classical physics. For example, the velocity of stars in a peripheral location in a galaxy is approximately equal to the velocity of stars near the center of the galaxy. So the classical physics can't answer to this phenomenon even in the General Relativity of Einstein, but this phenomenon is resolved using the proposed theory in this paper. So, the introduced equations represent the general case of the classical physics, in which any phenomenon in the nature can be resolved using these equations. Other example, the proposed theory can resolve the unification of the gravitational field produced by a matter and an electromagnetic wave, it can predict the bending of the light near a massive body, it can predict the variation of the frequency and energy of the light near a massive body, it can predict the synchronization between photons of a source of light, it can explain many phenomena in the nature which are almost impossible to be explained using the classical physics.

References

- [1] Raymond A. Serway, John W. Jewett Jr., Physics for Scientists and Engineers, Published by Brooks/Cole, ISBN-13: 978-0-495-82781-8, (2010).
- [2] Jacob Bunningham, Vlatko Vedral, Introductory Quantum Physics and Relativity, Published by Imperial College Press, ISBN-13: 978-1-84816-514-4, (2011).
- [3] Bertrand Berche, "Relativite", Laboratoire de physique des matériaux, université Henri Poincaré, Nancy1.
- [4] J. D. Jackson, "Classical electrodynamics", Wiley, New York, (third edition), (1999).
- [5] I. R. Kenyon, "General relativity", Oxford University Press, New York, (1990)
- [6] M. V. Berry, "Principle of cosmology and gravitation", institute of Physics publishing, Bristol, (1989).
- [7] P. M. Schwarz, J. H. Schwarz, "Special relativity, from Einstein to strings", Cambridge University Press, Cambridge, (2004).
- [8] M. Longair, "Theoretical concept in physics", Cambridge University Press, Cambridge, (2003).
- [9] W. Pauli, "Theory of relativity", Dover, New York, (1958).
- [10] F. Rohrlich, "Classical charged particles", Addison-Wesley, Redwood, (1965).
- [11] P. G. Bergmann, "Introduction to the Theory of relativity", Dover, New York, (1976).
- [12] T. Adam, N. Agafonova,... A. Zghiche, "Measurement of the neutrino velocity with the OPERA detector in the CNGS beam".
- [13] Mehran Rezaei, "Imaginary Relativity", WSEAS Transactions on Communications, ISSN: 1109-2742, Issue 2, Volume 9, (February 2010), pp. 127-142.
- [14] Mircea Neagu, "A relativistic approach on 1-jet spaces of the rheonomic Berwald-Moor metric", Balkan Society of Geometers, Geometry Balkan Press, (2011).
- [15] Chunxiao Zhang, Junjie Yue, "Application of an improved adaptive chaos prediction model in aero-engine performance parameters", WSEAS Transactions on Mathematics, E-ISSN: 2224-2880, Issue 2, Volume 11, (February 2012), pp. 114-124.
- [16] Franc Rozman, The EM properties of light, url: http://www.anti-energija.com/The_properties_of_light.pdf
- [17] Siddiqi Asif, "Sputnik and the Soviet Space Challenge", Gainesville, Florida. The University of Florida Press, ISBN 0-8130-2627-X, (2003), p. 155.
- [18] Linda J. Vandergriff, "Nature and Properties of Light", *Fundamentals of Photonics*, Publisher SPIE , pp. 1-38 .
- [19] C. V. Raman and S. Bhagavantam, "Experimental proof of the spin of the photon", *Indian J. Phys.* 6 353-366 ,(1931).
- [20] Ford Kenneth, "Basic Physics". Walton, Massachusetts: Blaisdell Publishing Co., (1968).
- [21] Jenkins and White. "Fundamentals of Optics", New York: McGraw-Hill, (1976).
- [22] National Photonics Skills Standard for Technicians, Pittsfield, Massachusetts: Laurin Publishing Company, Inc., (1995).
- [23] Seyrafi Khalil, "Electro-Optical Systems Analysis", Los Angeles: Electro-Optical Research Company, (1985).
- [24] Claude Ziad Bayeh, Nikos E. Mastorakis, "Absolute Zero Velocity Machine", in the book "Mathematical models and Methods in Modern Science", Proceedings of the 14th WSEAS International Conference on Mathematical Methods, Computational Techniques and Intelligent Systems (MAMECTIS '12), Porto, Portugal, Published by WSEAS Press, July 1-3, 2012, ISBN: 978-1-61804-106-7, pp. 302-306.
- [25] Claude Ziad Bayeh, Nikos E. Mastorakis, "Transformation between a fixed frame and a running frame at constant velocity which contains the "Absolute zero velocity machine"", in the book "Advances in Mathematical and Computational Methods", Proceedings of the 14th WSEAS International Conference on Mathematical and Computational Methods in Science and Engineering (MACMESE '12), Sliema, Malta, Published by WSEAS Press, September 7-9, 2012, ISBN: 978-1-61804-117-3, pp. 44-52.
- [26] Claude Ziad Bayeh, Nikos E. Mastorakis, "Measuring the speed of a spaceship using the "Absolute zero velocity machine"", in the book "Advances in Mathematical and Computational Methods", Proceedings of the 14th WSEAS International Conference on Mathematical and Computational Methods in Science and Engineering (MACMESE '12), Sliema, Malta, Published by WSEAS Press, September 7-9, 2012, ISBN: 978-1-61804-117-3, pp. 72-78.
- [27] Claude Ziad Bayeh, "Introduction to the Absolute Zero Velocity Machine", NAUN International Journal of Mathematics and Computers in Simulation, Issue 3, Volume 6, (2012), pp.307-314.

- [28] T. Damour, "Three Hundred Years of Gravitation", Cambridge Univ. Press, 1987.
- [29] S. Carlip, "Aberration and the Speed of Gravity," Phys. Lett. A267, pp.81–87, 2000.
- [30] R.P. Feynman, R.B. Leighton, and M. Sands, "*The Feynman Lectures on Physics*", Chapter II-21, Addison-Wesley, 1989.
- [31] V. Trimble, "Existence and nature of dark matter in the universe". *Annual Review of Astronomy and Astrophysics*, 25: pp. 425–472. (1987).
- [32] C. J. Copi, D. N. Schramm, M. S. Turner, "Big-Bang Nucleosynthesis and the Baryon Density of the Universe". *Science*, 267 (5195): pp. 192–199, (1995).
- [33] L. Bergstrom, "Non-baryonic dark matter: Observational evidence and detection methods". *Reports on Progress in Physics*, 63 (5): pp. 793–841, (2000).
- [34] Max Born, "Einstein's Theory of Relativity" (The 1962 Dover edition, page 348 lists a table documenting the observed and calculated values for the precession of the perihelion of Mercury, Venus, and the Earth.), *Dover Edition*, p. 348, (1924).