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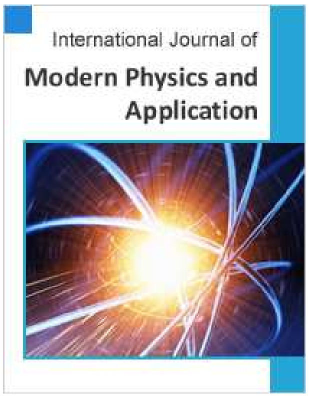

Optimization of Working Parameters of Magneto-Resonant Tomograph

Andrey N. Volobuev[1], Eugene S. Petrov[2], Sergey N. Chemidronov[3]

[1]Department of Physics, Samara State Medical University, Samara, Russia
[2]Deprartment of Operative Surgery, Samara State Medical University, Samara, Russia
[3]Deprartment of Anatomy, Samara State Medical University, Samara, Russia

Email address
volobuev47@yandex.ru (A. N. Volobuev)

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Abstract
The problem of increase in a signal providing process of internal organs visualization in the magneto-resonant tomograph is investigated on the basis of the some its parameters optimum choice. The principle of the tomograph work is analysed. On the basis of quantum-mechanical approach the interrelation between an angle of the magnetic moment precession of hydrogen nucleus in an organism, frequency of the variable magnetic field stimulating the precession and size of the used constant magnetic field is found. The given interrelation allows find the optimal parameters of the tomography work. The opportunity of calculation of the magnetic moment precession parameters on the basis of the classical approximation is shown.

1. Introduction

The magneto-resonant tomography (MRT- tomography) now is the most popular method of internal organs visualization at diagnostics of various diseases [1 - 7]. In this method, first of all, distribution of the hydrogen atoms concentration in various hydrogen-containing tissues of an organism is investigated.

The basic idea of this method consists in excitation of a magnetic moment resonant precession of a hydrogen atom nucleus (protons) which is taking place in a constant magnetic field. A resonant precession since the frequency of precession coincides with frequency of variable magnetic field this precession stimulating. Further registering of the EMF of the electromagnetic induction arising in the coil, fig. 1, taking place in the area of the magnetic field influence of the proton magnetic moment μ is carried out. In physics this phenomenon refers to a nuclear magnetic resonance (NMR).

The cyclic frequency of precession is defined by the Larmor's formula:

$$\omega = \gamma B,$$

where $\gamma = \dfrac{\mu}{\pi}$ there is constant for the given chemical element size - so-called hyromagnetic ratio; the proton magnetic moment divided in Planck's reduced constant is equal $\mu = \dfrac{\mu_p}{\hbar} = 1.34 \cdot 10^8 \dfrac{1}{T \cdot s}$, where $\mu_p = 1.406 \cdot 10^{-26} \dfrac{J}{T}$ - the proton absolute magnetic moment.

The purpose of this paper is finding-out of the optimum from the point of view of the visualization quality, the most essential parameters of the proton precession, such as the angle of precession, frequency of precession and external constant and variable magnetic fields.

2. Magneto-Resonant Tomograph

The scheme of a magneto-resonant tomograph is shown on fig. 1.

The patient is located in the tomography box 1 on the mobile couch. With the help of the coils which have been not shown on fig. 1, the constants (not varying in due course) magnetic fields in a direction of all three axes *OX*, *OY*, and *OZ* in the tomograph are created. On fig. 1 the basic constant magnetic field B_0 directed along axis *Z* is shown. Along this axis the patient lays. Constant magnetic fields change linearly along axes, i.e. has gradient character. Dependence of the basic constant magnetic field induction, for example, from coordinate *Z* looks like:

$$B_0 = kZ \ ,$$

where *k* there is factor of proportionality.

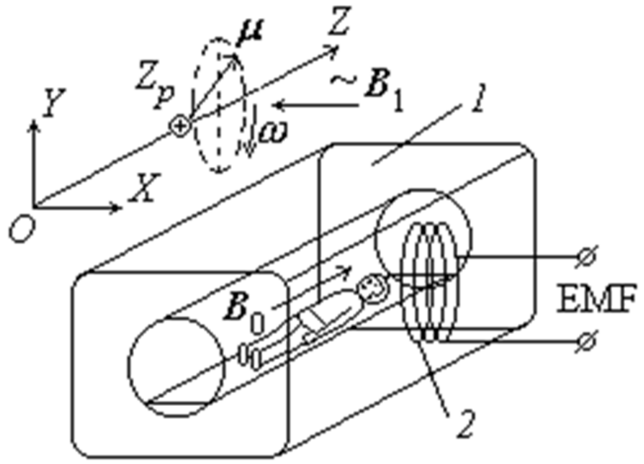

Figure 1. *Patient in magneto-resonant tomograph.*

Magneto-resonant tomograph usually works in an impulse conditions. It means that the solenoid 2 on fig. 1 (the radio-frequency coil) submits impulses of the variable magnetic field B_1 which turn the nucleus magnetic moments μ of the researched element in an organism and they start to rotate (precession) under some angle θ to axis *Z* – an angle of precession. During a pause in the same solenoid, due to the phenomenon of electromagnetic induction, arises EMF (MRT-signal) which goes on the computer for formation of the MRT-image.

The frequency of the submitted impulse signal will cause resonant (the same frequency) precession of the nucleus magnetic moment μ with the big angle of precession θ only on the certain coordinate Z_p. Really, if in Larmor's formula for the frequency of precession to substitute the dependence of the constant magnetic field induction on coordinate we shall receive:

$$\omega = \gamma B_0 = \gamma k Z \ .$$

Hence, submitting on an organism the certain frequency ω of the variable magnetic field B_1, we actually choose, at set γ (i.e. the chosen element which distribution is investigated) corresponding to this frequency the section Z_p of an organism from which the MRT-signal is registered. The definition of size γ is determined by an allowable interval of frequencies ω of the variable magnetic field B_1 generated by the radio-frequency coil 2, fig. 1. This interval of frequencies $\Delta\omega = \gamma\Delta B_0$ determines the created gradient of the basic magnetic field induction B_0. If frequency ω leaves the allowable interval it starts to be raised resonant precession of nucleus the magnetic moments of elements with other value of gyromagnetic ratio γ that is inadmissible. Thus, for change of researched section it is necessary to change a little the frequency ω of the variable magnetic field B_1.

For the defined of two other coordinates of an organism section researched area are created similar auxiliary gradient constant magnetic fields along coordinates *OX* and *OY*.

Size EMF induced in the solenoid 2, fig. 1, (i.e. the MRT-signal), first of all, it is proportional to nucleus quantity of element which magnetic moment resonant precession with the maximal angle of precession θ on frequency of the variable magnetic field used in a tomograph. It is connected by that gyromagnetic ratio γ depends on a kind of the element.

Now it is used basically hydrogen MRT $\left(\gamma = 42.67\dfrac{MHz}{T}\right)$ which the image various hydrogen-containing tissues, first of all, waters is shows. Percentage of water in various tissues is various. There are in grey substance of a brain 83 %, in muscles 78 %, in a skin 68 %, etc. Brightness of these tissues on the MRT-image will be various. Anatomic areas with small density of the hydrogen atoms, for example, air, bones induce very small MRT-signal and see on the MRT-image by more dark. The areas with the big density of hydrogen atoms see more light. Thus, brightness of the image of the organism researched area depends on a kind of the tissue.

Program-computer methods usually invert, change shades of the image, making its more similar to the roentgenogram (a bone tissues it is light, muscular dark).

A magneto-resonant tomography it is basically research of the tissues morphology, but not only.

During a pause in submission of the high-frequency magnetic field B_1 the coil 2 registers the MRT-signal. The nucleus magnetic moment μ starts to be oriented gradually along power lines of constant magnetic field – relaxed (by fig. 1 along axis *Z*) since there is no receipt of energy from the outside. It results attenuates EMF received in the solenoid 2. Speed of attenuation (relaxation) depends on lines of factors.

First, depends from a phase condition of substance and its density. In solid bodies (bones) where tight coupling between atoms the precession nucleus magnetic moment during a pause easily gives the energy to surrounding nucleus. Therefore, attenuation registered EMF will be fast. In liquids because of weaker coupling of atoms there is attenuation more slowly.

The speed of MRT-signal attenuation determines on-off time $Q = \frac{T}{t_i}$ of the used impulse variable magnetic field B_1 (T there is period of the impulse field, t_i - duration of impulse) since attenuation of the MRT-signal should have time to be shown. For various tissues the duration of relaxation (and hence and used pauses) are various and make from 50 *ms* up to 2.5 *s*. For example, time of the relaxation for substance of malignant tumours on $30 \div 35\%$ less than normal tissues due to the greater density of the tumoral tissues. Therefore, for reception of the qualitative MRT-image of the tissue certain kind it is necessary to regulate on-off time of the variable magnetic field B_1 submitted on the tissue. All this is taken into account at formation of the MRT-image.

Second, the motionless liquid (intercellular, cytoplasm, etc.) gives rather powerful MRT-signal. The moving liquid does not give opportunity of the necessary geometrical condition occurrence between directions of the external constant magnetic field B_0, the precession magnetic moment μ of moving nucleus and the vector of the variable magnetic field induction B_1. A nucleus of atoms, and hence their magnetic moments during movement of a liquid chaotically rotate. In this connection moving blood does not generate the MRT-signal. Hence, of vessels space and of heart chambers it is well visible concerning motionless tissues on the MRT-image. From this point of view MRT characterizes also a functional condition of a tissue in particular movement of a blood.

Thus, on the MRT-image we see low-mobile hydrogen-containing tissues.

In MRT the following parameters of physical sizes are used. An induction of the basic constant magnetic field there is in limits $B_0 = 0.02 \div 2\,T$. The interval of the variable magnetic field B_1 frequencies is determined by the gyromagnetic ratio γ i.e. a kind of the researched element (atom of hydrogen). This interval of frequencies is in the radiorange. Sensitivity of MRT allows distinguish area in the size $2 \div 4\ mm$ on the MRT-image.

On fig. 2 for example the MRT-image of the brain blood vessels in the cervical department of man is shown.

The big prospects are available for phosphoric MRT since phosphorus P_{15}^{31} $\left(\gamma = 17.2 \frac{MHz}{T}\right)$ has the important role in organism exchange processes. But sensitivity of the method to phosphorus makes $\sim 0.14\%$ from hydrogen MRT, that is connected to low concentration of phosphorus in the organism and the small magnetic moment of its nucleus (the magnetic moments of protons and neutrons in a phosphorus nucleus substantially compensate each other). It complicates use phosphoric MRT because own thermal fluctuations of an atoms of the radio-frequency coil 2, fig. 1, result in necessity application of superconducting registration magnetic systems that is reached at temperatures close to absolute zero on Calvin.

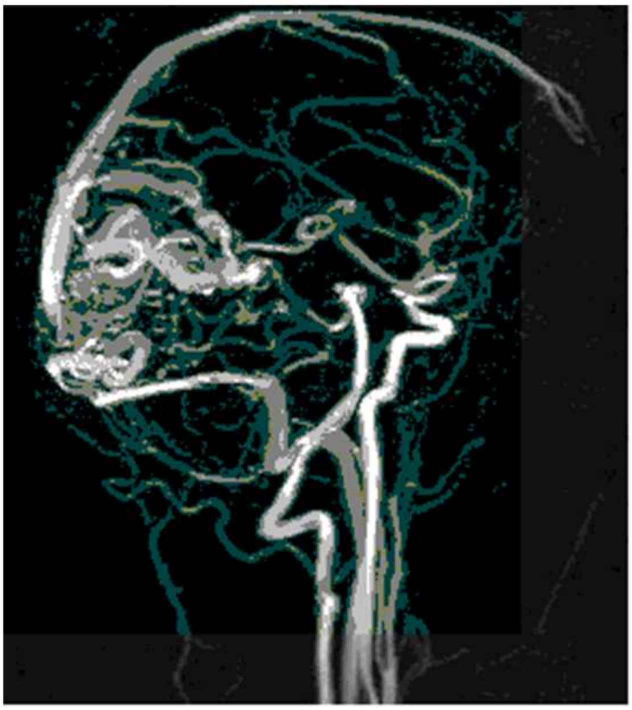

Figure 2. *MRT-image of the brain blood vessels in the cervical department of man.*

3. Quantum-Mechanical Analysis of MRT-Signal Occurrence

Parameters of the protons magnetic moment precession there are essentially influence the size registered EMF, and hence define quality visualization pictures of internal organs.

Let's consider the proton with the magnetic moment μ (precession with the angle θ to axis Z) taking place in constant vertical (along axis Z) B_0 and perpendicular to it rotating horizontal B_1 magnetic fields, fig. 3. At first look the maximal EMF (MRT-signal) will be at the angle of precession $\theta = 90^0$.

As against fig. 1 for convenience of the analysis the constant magnetic field B_0 on fig. 3 is shown vertical, and variable B_1 horizontal.

We research the problem on an opportunity of such angle achievement $\theta = 90^0$ in more detail on the basis of the quantum-mechanical approach.

Hamiltonian of systems (an external constant magnetic field - the precession magnetic moment of the proton) we shall write down as:

$$\widehat{H} = \widehat{H}_0 + \widehat{H}_{\text{int}}, \tag{1}$$

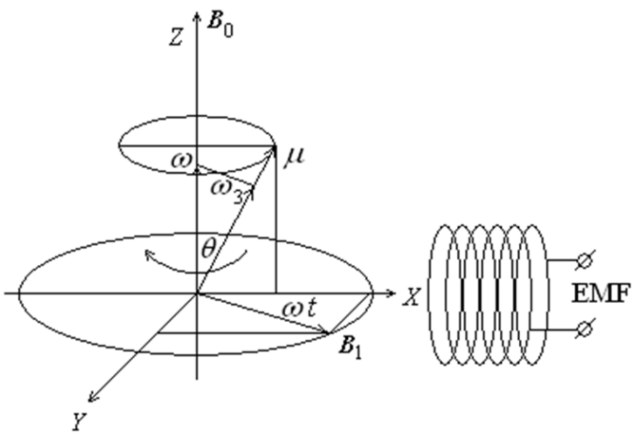

Figure 3. *Precession of magnetic moment in the constant and variable magnetic fields.*

Believing the horizontal variable magnetic field B_1 as some "perturbation", we assume $\widehat{H}_0$ there is not perturbation part of Hamiltonian, and $\widehat{H}_{\text{int}}$ - the "perturbation" connected to influence of the field B_1 on not perturbed part of system.

Not perturbed part of Hamiltonian, using Pauli's matrixes, we shall write down as [8, 9]:

$$\widehat{H}_0 = \mu B_0 \sigma_Z \cos\theta = \begin{pmatrix} \mu B_0 \cos\theta & 0 \\ 0 & -\mu B_0 \cos\theta \end{pmatrix}, \quad (2)$$

where

$$\sigma_X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (3)$$

Hamiltonian of interactions, taking into account $B_1 \perp B_0$, see fig. 3, looks like:

$$\widehat{H}_{\text{int}} = \mu B_1 \sin\theta\left(\sigma_X \cos\omega t + \sigma_Y \sin\omega t\right) = \begin{pmatrix} 0 & \mu B_1 e^{-i\omega t}\sin\theta \\ \mu B_1 e^{i\omega t}\sin\theta & 0 \end{pmatrix}, \quad (4)$$

where ω there is frequency of the variable magnetic field equal to frequency of the proton magnetic moment precession, t - time.

Also it is taken into account:

$$\sin\theta\left(\sigma_X \cos\omega t + \sigma_Y \sin\omega t\right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\sin\theta\cos\omega t + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\theta\sin\omega t = \begin{pmatrix} 0 & \sin\theta\left(\cos\omega t - i\sin\omega t\right) \\ \sin\theta\left(\cos\omega t + i\sin\omega t\right) & 0 \end{pmatrix} = \begin{pmatrix} 0 & e^{-i\omega t}\sin\theta \\ e^{i\omega t}\sin\theta & 0 \end{pmatrix}$$

The equation of Schrodinger for the operator of interaction (it is accepted where it is not point out especially, $\hbar = 1$) looks like:

$$i\frac{\partial\psi}{\partial t} = \widehat{H}_{\text{int}}(t)\psi, \quad (5)$$

where $\psi = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix}$ there is spin wave function (spinor) [8], $\psi_\uparrow$ - the component of spinor if the projection of the magnetic moment is directed along axis *Z*, $\psi_\downarrow$ - the component of spinor if the projection of the magnetic moment is directed against axis *Z*. A direction of the precession magnetic moment vector μ we believe equilibrium with constant angle of precession θ.

Using the representation of interaction [10] for Hamiltonian interactions, we shall carry out transformations:

$$\widehat{H}_{\text{int}}(t) = e^{i\widehat{H}_0 t}\widehat{H}_{\text{int}}e^{-i\widehat{H}_0 t} = e^{i\begin{pmatrix} \mu B_0\cos\theta & 0 \\ 0 & -\mu B_0\cos\theta \end{pmatrix}t}\widehat{H}_{\text{int}}e^{-i\begin{pmatrix} \mu B_0\cos\theta & 0 \\ 0 & -\mu B_0\cos\theta \end{pmatrix}t}$$
$$= \begin{pmatrix} e^{i\mu B_0 t\cos\theta} & 0 \\ 0 & e^{-i\mu B_0 t\cos\theta} \end{pmatrix}\begin{pmatrix} 0 & \mu B_1 e^{-i\omega t}\sin\theta \\ \mu B_1 e^{i\omega t}\sin\theta & 0 \end{pmatrix}\begin{pmatrix} e^{-i\mu B_0 t\cos\theta} & 0 \\ 0 & e^{i\mu B_0 t\cos\theta} \end{pmatrix} = \mu B_1\sin\theta\begin{pmatrix} 0 & e^{-2i\Omega t} \\ e^{2i\Omega t} & 0 \end{pmatrix}, \quad (6)$$

where it is designated:

$$\Omega = \frac{\omega}{2} - \mu B_0\cos\theta. \quad (7)$$

Thus, the spinor equation of Schrodinger (5) looks like:

$$i\frac{\partial}{\partial t}\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \widehat{H}_{\text{int}}(t)\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \mu B_1\sin\theta\begin{pmatrix} 0 & e^{-2i\Omega t} \\ e^{2i\Omega t} & 0 \end{pmatrix}\begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \mu B_1\sin\theta\begin{pmatrix} e^{-2i\Omega t}\psi_\downarrow \\ e^{2i\Omega t}\psi_\uparrow \end{pmatrix}. \quad (8)$$

For the solving of the equation (8) we use new variables:

$$\phi_\uparrow = e^{i\Omega t}\psi_\uparrow \text{ and } \phi_\downarrow = e^{-i\Omega t}\psi_\downarrow . \quad (9)$$

In new variables the equation (8) can be written down as:

$$i\frac{\partial \phi}{\partial t} = \begin{pmatrix} -\Omega & \mu B_1 \sin\theta \\ \mu B_1 \sin\theta & \Omega \end{pmatrix}\phi , \quad (10)$$

where auxiliary spinor is $\phi(t) = \begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix}$, or:

$$i\frac{\partial}{\partial t}\begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix} = \begin{pmatrix} -\Omega & \mu B_1 \sin\theta \\ \mu B_1 \sin\theta & \Omega \end{pmatrix}\begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix} = \begin{pmatrix} -\Omega\phi_\uparrow + \mu B_1 \sin\theta\phi_\downarrow \\ \mu B_1 \sin\theta\phi_\uparrow + \Omega\phi_\downarrow \end{pmatrix}. \quad (11)$$

It is possible to be convinced by direct substitution that the general solution (10) or (11) looks like:

$$\phi(t) = C_+\begin{pmatrix} (1+\lambda)^{\frac{1}{2}} \\ -(1-\lambda)^{\frac{1}{2}} \end{pmatrix} e^{i\tilde{\omega}t} + C_-\begin{pmatrix} (1-\lambda)^{\frac{1}{2}} \\ (1+\lambda)^{\frac{1}{2}} \end{pmatrix} e^{-i\tilde{\omega}t}, \quad (12)$$

where for convenience of transformations it is designated $\tilde{\omega} = \sqrt{\Omega^2 + (\mu B_1 \sin\theta)^2}$ and $\lambda = \frac{\Omega}{\tilde{\omega}}$.

Let's accept the initial condition, with the account (9), that at $t = 0$ the wave function is $\psi(0) = \phi(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. We shall note that initial on time position of a vector of the magnetic moment of the proton μ precession around of the constant magnetic field B_0 with an angle θ has any character.

Hence:

$$\phi(0) = C_+\begin{pmatrix} (1+\lambda)^{\frac{1}{2}} \\ -(1-\lambda)^{\frac{1}{2}} \end{pmatrix} + C_-\begin{pmatrix} (1-\lambda)^{\frac{1}{2}} \\ (1+\lambda)^{\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \quad (13)$$

Where follows:

$$C_- = \frac{(1-\lambda)^{\frac{1}{2}}}{2},\ C_+ = \frac{(1+\lambda)^{\frac{1}{2}}}{2}. \quad (14)$$

Substituting (14) in the formula (12) we find the particular solving of the equations (10) or (11) as:

$$\begin{aligned} \phi(t) = \begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix} &= \frac{(1+\lambda)^{\frac{1}{2}}}{2}\begin{pmatrix} (1+\lambda)^{\frac{1}{2}} \\ -(1-\lambda)^{\frac{1}{2}} \end{pmatrix} e^{i\tilde{\omega}t} + \frac{(1-\lambda)^{\frac{1}{2}}}{2}\begin{pmatrix} (1-\lambda)^{\frac{1}{2}} \\ (1+\lambda)^{\frac{1}{2}} \end{pmatrix} e^{-i\tilde{\omega}t} \\ &= \frac{1}{2}\begin{pmatrix} (1+\lambda) \\ -(1-\lambda^2)^{\frac{1}{2}} \end{pmatrix} e^{i\tilde{\omega}t} + \frac{1}{2}\begin{pmatrix} (1-\lambda) \\ (1-\lambda^2)^{\frac{1}{2}} \end{pmatrix} e^{-i\tilde{\omega}t} = \begin{pmatrix} W(t) \\ -i(1-\lambda^2)^{\frac{1}{2}} \sin\tilde{\omega}t \end{pmatrix} \end{aligned}, \quad (15)$$

where it is designated $W(t) = \cos\tilde{\omega}t + i\lambda\sin\tilde{\omega}t$.

Thus, the particular solving of the equation (5) or (8) will be written down as:

$$\psi(t) = \begin{pmatrix} \psi_\uparrow \\ \psi_\downarrow \end{pmatrix} = \begin{pmatrix} \phi_\uparrow e^{-i\Omega t} \\ \phi_\downarrow e^{i\Omega t} \end{pmatrix} = \begin{pmatrix} W(t)e^{-i\Omega t} \\ -i\left(1-\lambda^2\right)^{\frac{1}{2}} e^{i\Omega t} \sin\tilde{\omega}t \end{pmatrix} \quad (16)$$

Let's check up the condition of normalization:

$$P_\uparrow(t) + P_\downarrow(t) = 1 , \quad (17)$$

where probabilities of the magnetic moment projections of the proton along axis *Z* and against axis *Z* are accordingly equal:

$$P_\uparrow(t) = |\psi_\uparrow|^2 = \left|W(t)e^{-i\Omega t}\right|^2 = |W(t)|^2 = \cos^2\tilde{\omega}t + \lambda^2\sin^2\tilde{\omega}t , \quad (18)$$

$$\begin{aligned} P_\downarrow(t) = |\psi_\downarrow(t)|^2 &= \left|-i\left(1-\lambda^2\right)^{\frac{1}{2}} e^{i\Omega t}\sin\tilde{\omega}t\right|^2 \\ &= \left|-i\left(1-\lambda^2\right)^{\frac{1}{2}}\sin\tilde{\omega}t\right|^2 = \left(1-\lambda^2\right)\sin^2\tilde{\omega}t \end{aligned} , \quad (19)$$

Hence, the condition of normalization (17) is carried out.

Since there is orienting action on magnetic moment of the constant magnetic field B_0 directed along axis *Z* it is natural to assume that maximal probability an angle of precession $\theta < 90^0$.

Let's find the angle θ at which it is reached the maximal probability from the condition:

$$\frac{\partial P_\uparrow(t)}{\partial\theta} = \frac{\partial P_\uparrow(t)}{\partial\tilde{\omega}}\frac{\partial\tilde{\omega}}{\partial\theta} = 0 . \quad (20)$$

Taking into account (18) and using $\frac{\partial\lambda}{\partial\tilde{\omega}} = \frac{\partial}{\partial\tilde{\omega}}\left(\frac{\Omega}{\tilde{\omega}}\right) = \frac{1}{\tilde{\omega}}\left(\frac{\partial\Omega}{\partial\tilde{\omega}} - \lambda\right)$, we shall find:

$$\begin{aligned} \frac{\partial P_\uparrow(t)}{\partial\theta} &= \left(t\left(\lambda^2-1\right)\sin 2\tilde{\omega}t - 2\sin^2\tilde{\omega}t\frac{\lambda^2}{\tilde{\omega}} + 2\sin^2\tilde{\omega}t\frac{\lambda}{\tilde{\omega}}\frac{\partial\Omega}{\partial\tilde{\omega}}\right)\frac{\partial\tilde{\omega}}{\partial\theta} \\ &= \left(t\left(\lambda^2-1\right)\sin 2\tilde{\omega}t - 2\sin^2\tilde{\omega}t\frac{\lambda^2}{\tilde{\omega}}\right)\frac{\partial\tilde{\omega}}{\partial\theta} + 2\sin^2\tilde{\omega}t\frac{\lambda}{\tilde{\omega}}\frac{\partial\Omega}{\partial\theta} \\ &= \left(t\left(\lambda^2-1\right)\sin 2\tilde{\omega}t - 2\sin^2\tilde{\omega}t\frac{\lambda^2}{\tilde{\omega}}\right)\frac{1}{2\tilde{\omega}} \\ &\left(\omega\mu B_0\sin\theta - \left((\mu B_0)^2 - (\mu B_1)^2\right)\sin 2\theta\right) + \\ &+ 2\sin^2\tilde{\omega}t\frac{\lambda}{\tilde{\omega}}\mu B_0\sin\theta = 0 \end{aligned} . \quad (21)$$

The equilibrium angle of precession is reached very quickly therefore the term proportional to time in (21) is neglected. Thus:

$$\frac{\omega}{2}-\left(\frac{(\mu B_0)^2-(\mu B_1)^2}{\mu B_0}\right)\cos\theta=\frac{\lambda\tilde{\omega}}{\lambda^2}=\frac{\Omega}{\lambda^2}. \quad (22)$$

About transition from the formula (21) to the formula (22) it is necessary to make some explanations. The initial condition for the wave function $\psi(0)=\phi(0)=\begin{pmatrix}1\\0\end{pmatrix}$ is accepted as for continuous size what it and is. In the formula (21) the time also is continuous size it is not quantize. However, angular speed $\tilde{\omega}$ and angle of the precession θ it is quantize. Quantization θ is determined by that the projection of the proton magnetic moment μ to the direction of the constant magnetic field B_0 can accept only discrete line of values. Therefore and the angle of precession θ also has discrete, quantum character. Thereof to take advantage at transition from (21) to (22) $\lim\frac{\sin\tilde{\omega}t}{\tilde{\omega}t}=1$ at $\tilde{\omega}t\to 0$ it is impossible. In numerator and denominator of the given limit there are discrete sizes. Therefore, at $\tilde{\omega}t=0$ division of numerator into the denominator it is impossible, and at $\tilde{\omega}t\neq 0$ result of division it is not equal to unit.

The finding of derivative $\frac{\partial P_{\uparrow}(t)}{\partial\theta}$ on quantum size has the approximated character and it is possible only at the big divisibility of the angle of precession that is correct at rather big sizes θ.

Transforming (22), we shall find:

$$\left(\frac{(\mu B_0)^2-(\mu B_1)^2}{\mu B_0}\right)\cos\theta=\mu B_0\cos\theta-\frac{(\mu B_1\sin\theta)^2}{\frac{\omega}{2}-\mu B_0\cos\theta},$$

or

$$\left(\frac{\omega\cos\theta}{2\mu B_0}-\cos^2\theta\right)=\sin^2\theta.$$

Where follows:

$$\cos\theta=\frac{2\mu B_0}{\omega}=\frac{\omega_r}{\omega}, \quad (23)$$

where $\omega_r=2\mu B_0$ there is resonant frequency of precession.

The formula (23) actually coincides with the classical analogue for the precession of the spherical-symmetric body (top) [11]. Really, angular speed of the body rotation about the own axis $\omega_3=\frac{M}{I_3}\cos\theta$, where M there is mechanical moment working on the top, I_3 - the moment of inertia around of own axis of the top rotation. Taking into account that the frequency of the top precession $\omega=\frac{M}{I_1}$ where I_1 there is the top moment of inertia around of axis perpendicular own axis of top rotation we find $\cos\theta=\frac{I_3}{I_1}\frac{\omega_3}{\omega}$. For the spherical-symmetric top is $I_1=I_3$, hence:

$$\cos\theta=\frac{\omega_3}{\omega}, \quad (24)$$

where ω_3 there is angular speed of the body rotation about the axis, analogue of resonant frequency of the precession. Indexes of sizes are used by analogy with [11]. We shall notice, that the vector ω_3 coincides on the direction with the precession magnetic moment μ, see fig. 3.

4. Discussion

Let's estimate the frequency of variable magnetic field under the formula $\omega=\frac{2\mu B_0}{\cos\theta}$. We shall assume that optimum angle of precession $\theta=45^0$. Size of the external constant magnetic field we shall find from approximately equality of energies orienting the magnetic moment action of the constant magnetic field and chaotic actions of an organism heat $\frac{\mu_0 B_0^2}{2}=\alpha N_A kT$ or $B_0=\sqrt{\frac{2\alpha N_A kT}{\mu_0}}$. In last formula $T\approx 300\,K$ there is temperature of an organism, $k=1.38\cdot 10^{-23}\frac{J}{K}$ - Boltzmann's constant, $N_A=6.02\cdot 10^{23}\frac{1}{mole}$ - Avogadro's number, $\mu_0=4\pi\cdot 10^{-7}\frac{H}{m}$ - magnetic constant, $\alpha\sim 10^{-10}\,mole$ - a fraction of hydrogen protons mole ($\sim 10^{-10}\,gram$) of hydrogen-containing tissues of an organism in the area of visualization which are exposed to influence of the variable magnetic field. Hence, the induction of constant magnetic field is $B_0\approx 0.63\,T$. Thus, frequency of the variable magnetic field is equal $\omega=\frac{2\mu B_0}{\cos\theta}\approx 2,4\cdot 10^8\,s^{-1}$ or $\nu\approx 38\,MHz$, i.e. lays in the range of ultrahigh frequencies (in the radiorange).

5. Conclusion

It is obvious that maximal MRT-signal (EMF) arises in the registering coil at the angle of the magnetic moment precession $\theta=90^0$. But according to the formula (23) there is angle of precession $\theta\to 90^0$ in case of infinite increase in frequency ω of the external variable magnetic field B_1 or reduction of the external constant magnetic field B_0 up to zero. Last circumstance is connected to reduction of the constant magnetic field orienting action on the magnetic moment μ . Therefore accomplishment of the angle of precession $\theta = 90^0$ is impossible.

Use of weak constant magnetic fields is impossible, since it is necessary to overcome a chaotic direction of the protons magnetic moments which results from thermal influence of the organism tissues. From this point of view there is the inconsistent situation when on the one hand the constant magnetic field B_0 should be powerful enough that the protons magnetic moments in hydrogen-containing tissues of an organism in researched area basically have been directed along this field. On the other hand the size constant magnetic should not interfere strongly the occurrence enough the big angle of precession of protons the magnetic moment μ according to the formula (23).

At calculation of the magnetic moment precession it is possible to use results of the classical theory of a top.

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