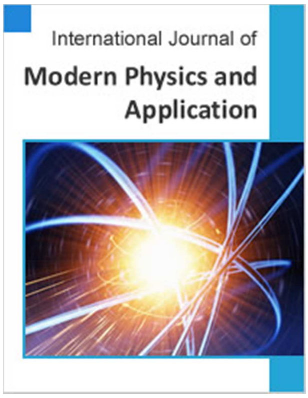
International Journal of Modern Physics and Application
2015; 2(6): 116-121
Published online January 21, 2016 (http://www.aascit.org/journal/ijmpa)
ISSN: 2375-3870

American Association for Science and Technology



Keywords
Multiple Quantum Wells,
Transfer Matrix Method,
Photonics Devices,
Quantum Tunneling,
Resonant Tunneling,
Nanophotonics

Received: December 6, 2015
Revised: December 23, 2015
Accepted: December 25, 2015

Resonant Tunneling in Fibonacci Series Multiple Quantum Wells Nanostructures

Jatindranath Gain

Department of Physics, Derozio Memorial College (WBSU), Kolkata, India

Email address
gainelc@gmail.com

Citation
Jatindranath Gain. Resonant Tunneling in Fibonacci Series Multiple Quantum Wells Nanostructures. *International Journal of Modern Physics and Application*. Vol. 2, No. 6, 2015, pp. 116-121.

Abstract
Tunneling of electrons through the Fibonacci series multiple quantum wells (FMQWs) has been studied theoretically within unified transfer matrix approach. The characterisation of light-emitting one-dimensional photonic quasicrystals based on excitonic resonances is reported. The structures consist of GaAs/AlGaAs multiple quantum wells satisfying a Fibonacci sequence. The resulting band structure causes photons to become confined within the wells, where they occupy discrete quantized states. We have obtained an expression for the transmission coefficient of the Fibonacci series MQW nanostructures using analytical Transfer matrix method (ATMM) and found the resonance state within the photonic wells. These resonant states occur due to split pairs and coupling between degenerate states. The active photonic quasicrystals are good light emitters. The lack of periodicity in the Fibonacci series MQW results in resonant tunneling and strong emission. The resonant state describe here can be used to develop new types of optical devices, photonic- switching devices, detectors and optoelectronic devices.

1. Introduction

Low dimensional carrier systems in the semiconductor heterostructures are gaining much importance in recent times due to the potential use of their unique properties in applications ranging from optoeltronics to high speed devices [1-4]. In this connection perpendicular transport of the carriers in semiconductor heterostructures has attracted much attention [5-8]. The MQW structures in particular, are becoming very important due to their potential use in the design and fabrication of quantum cascade lasers, resonant photo detectors, resonant tunneling diodes, single electron tunneling transistors [4] etc. Moreover, with the decrease in the dimensions of the CMOS devices the effect of tunneling of carriers becomes very important in order for estimating the various leakage currents flowing through the devices present in the VLSI chips.

Photonic quasicrystals [5], [8] have attracted a great deal of attention recently due to their ability to localize and control the flow of light within their structure. Considerable effort has been placed into finding ways to harness their potential for developing new photonic devices. Photonic crystal heterostructures have provided a promising means for turning raw photonic crystals into functional devices [3], [4]. The Fibonacci sequence is a fundamental and a well-known example of a 1D quasiperiodic structure exhibiting aperiodic long range order [9-11].

Periodic crystals are formed by a periodic repetition of a single building block the so-called unit cell exhibiting a long range translational and orientational symmetry [9]. Only

2-, 3-, 4-, and 6-fold non-trivial rotational symmetries are allowed in the periodic crystal and their direction patterns give sharp brag peaks reacting the symmetry and long range order. In contrast to periodic crystals, quasicrystals exhibit a long range order in spite of their lack of translational symmetry and often possess n-fold (n = 5 and > 6) rotational symmetries. Most of the quasicrystalline structures can be described by using aperiodic order where two or more different unit cells are used in the building block of the structure [12].

Leonardo Bonacci (1170-1250) known as Fibonacci was an Italian mathematician, considered to be the most talented western mathematician of middle age. He has been immortalized in the famous sequence – 0, 1, 1, 2, 3, 5, 8, 13, …. rather than for what is considered his far greater mathematical achievement–helping to popularize our modern number system. He did not know about quasicrystals or the impact that quasicrystals would one day have a new form of matter. Quasicrystals are aperiodic but it exhibit long range order which underlying construction principal. Man made photonic quasicrystals are available in recent years with inter atomic spacing comparable to the wave length of light. One dimensional (1D) Fibonacci series photonic quasicrystals are active and can be directly translated into layered quasicrystals which are considered as aperiodic multiple quantum wells slab. Three dimensional (3D) photonic quasicrystals are also available but. They do not emit light, so these structures are passive quasicrystals [13-15].

Fibonacci sequence is the most well-known example of 1D aperiodic structures, along with Thue-Morse structures and the Cantor structure [13] ,[16], [25]. So we choose to study Fibonacci series MQW structure. Another interesting property of Fibonacci structures is their direct connection with the 2D and 3D quasicrystals, the Penrose lattices [17]. The Fibonacci numbers can be obtained from the recurrence relation $F_n = F_{n-1} + F_{n-2}$ which is also true for $n < 1$. Here $F_0 = 0$, $F_1=1$, $F_2=1$ and so on. The *n*th element in the Fibonacci series as an analytic function of n and s obtained from the relation

$$F_n=\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n-\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n \tag{1}$$

The one –dimensional Fibonacci lattice, being one of the most studied quasicrystals, is determined by the substitution rule: L→LS, S→L where L(large) and S(small) are two elements.

The parameters of this structure are given by $\tau = (\sqrt{5}+1)/2 \approx 1.618$, which is known as golden mean.

$\Delta=S_p- L_p$ and $d=S_p+ (L_p-S_p)/\tau$, where d is the mean period of the lattice structure. When Lp = Sp the structure becomes periodic [24] and when Lp/Sp= τ (golden mean), it becomes the Fibonacci chain multiple quantum wells structures [9], [19].

Quantum mechanical tunneling through aperiodic and asymmetric MQWs is a long-standing problem in this context. Here we developed a model under most generalized assumptions and is based on Analytical Transfer Matrix Method (ATMM), which has been applied to any arbitrary potential well and barrier sequence successfully [28], [30]. This is a most general mathematical model that helps in computing the energy Eigen values inside asymmetric and aperiodic MQW structures and predicts the carrier distribution for each of the energy states in terms of the eigen function. This model also helps in studying the extent of carrier tunneling through the quantum barrier and also Fibonacci chain MQWS. Tunneling depends significantly on the barrier width. Scaling of structure dimension affects this variation very sharply [20-21], [31].

Photonic multiple quantum wells (MQWs) are a class of photonic crystal heterostructures that possess a distinct band structure. A MQW consists of a photonic well imbedded between two photonic barriers. The barriers are photonic crystals with band gaps that may be regarded as potential barriers for photons, whereas the photonic well consists of either a uniform dielectric material or another photonic crystal with a different band gap than that of the barrier. Due to the photonic band mismatch between the well and the barrier, photons become confined within the well and occupy quantized states. This so-called photonic confinement effect has been observed in both theoretical and experimental [22-23] studies and is analogous to the electronic confinement effect that occurs in semiconductor quantum wells. It has been shown that the phenomenon of resonant photonic tunneling can occur for a MQW with sufficiently thin photonic barriers [24]. Resonant tunneling occurs when a photon with an energy corresponding to a bound state of the MQW tunnels through one of the barriers, where it occupies this bound state within the well for a finite period of time before escaping by tunneling back out; thus the photon is said to have occupied a quasibound or resonant state within the MQW [18]. As a consequence of this phenomenon, an incident photon with an energy matching a resonant state of the MQW will undergo perfect transmission through barriers. In the transmission spectrum of a MQW, resonant states appear as sharp peaks approaching unity [11–18]. Here, we study resonant photonic states in photonic multiple quantum well (MQW) heterostructures in Fibonacci series consisting of two different photonic crystals. Using the transfer matrix method [4], [29], we have obtained an expression for the transmission coefficient and hence tunneling probabilities of the MQWs Fibonacci chain heterostructures. From this, we have performed numerical simulations of the transmission spectra for GaAs/AlGaAs MQW heterostructures in Fibonacci sequence. In our simulations, we vary the thicknesses of the photonic barriers in order to study their effect on the resonant states of the system. The resonant states described here will be useful for developing new types of photonic-switching devices, optical filters, and other optoelectronic and photonic devices [25-28].

2. Theory

Here we consider the most generalized MQW structures where the well and barrier widths are all unequal as shown in

Fig.1. The well and barrier materials are also different with different barrier heights and effective electron masses. The barrier widths are narrow enough so that the adjacent wells are coupled through the intervening barrier. Here we have to deal with five wave functions; those that represent the electrons inside the wells and those that describe the electrons in the barrier regions on either side of these wells. We have considered the general quantum structure as shown in Fig. 1 where the widths of the three barriers are 2d, 2b and 2f (d≠ b≠ f.) The two wells having widths 2a and 2c (a ≠ c) lie between the barriers 2d, 2b

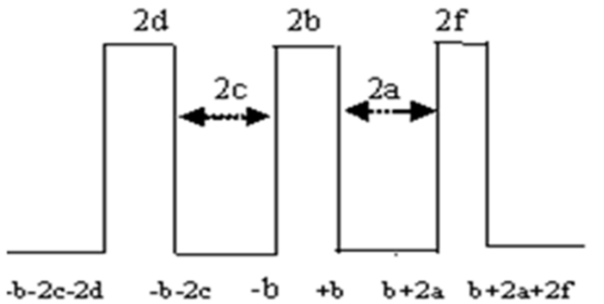


Figure 1. *Array of Aperiodic Multiple Quantum Wells.*

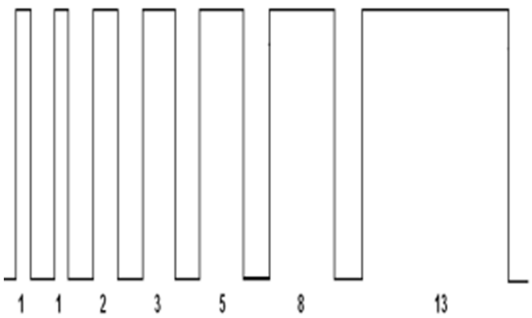


Figure 2. *Array of Fibonacci series Multiple Quantum Wells.*

The Schrödinger wave equation for this combination is of the form

$$(\hbar^2/2m^*)\ d/dx\{d\Psi/dx\} + (E-V)\ \Psi = 0 \quad (2)$$

where m* is the effective mass, h, is the Planck's constant E, is the energy Eigen value and V is the potential energy for the region where the equation is defined. The effective mass $m^* = m_B$ inside the barrier region with potential energy is V_o and $m^* = m_w$ outside the barrier and potential energy is zero.

The solution of the Schrödinger equation for the Quantum well and barrier region obtained by putting the appropriate values of the potential energy V, which may differ from region to region. The solutions take the following form as given by Equations (3) to (7).

$$\Psi_{B1}(x) = A_{B1} \exp(ik_{B1}x) + B_{B1} \exp(-ik_{B1}x) \quad (3)$$

for - (b-2c-2d) <x< -(b+2c) in the first barrier

$$\Psi_{W1}(x) = A_{W1} \exp(ikw_1 x) + B_{W1} \exp(-ik_{W1}x) \quad (4)$$

for - (b+2c)<x<-b inside the first well

$$\Psi_{B2}(x) = A_{B2} \exp(ik_{B2}x) + B_{B2} \exp(-ik_{B2}x) \quad (5)$$

for -b <x<+b inside the second barrier.

$$\Psi_{W2}(x) = A_{W2} \exp(ikw_2 x) + BW_2 \exp(-ik_{W2}x) \quad (6)$$

for b<x< b+2a inside the second well

$$\Psi_{B3}(x) = A_{B3} \exp(ik_{B3}x) \quad (7)$$

For (b+2a) < x<(b+2a+2f) inside the third barrier
Where $k_{Bn} = \sqrt{\left[\frac{2m_{Bn}(V_{0n} - E)}{\hbar^2}\right]}$ and $k_{wi} = \sqrt{\left(\frac{2m_{wi}E}{\hbar^2}\right)}$

Here m_{wi} and m_{Bn} are the electron effective masses in the well and barrier regions respectively where n = 1, 2, 3 and i = 1, 2. In the most general case the barrier heights Vo_n and electron effective masses m_{wi} and m_{Bn} in the different regions are all different. Now we applied boundary conditions and we got the the transfer matrix for the coefficients of the wave function at the leftmost slab to those of the right most slabs is given below:

$$\begin{bmatrix} A_{Bi} \\ B_{Bi} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} A_{Bi+1} \\ B_{Bi+1} \end{bmatrix} \quad (8)$$

Where i=1,2,3,….etc

Using equations (3-7) we obtained the coefficients of the wave function at the leftmost slab to those of the right most slabs.

$$\begin{bmatrix} A_{Bi} \\ B_{Bi} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -ik_{wi}^{-1} \\ 1 & ik_{wi}^{-1} \end{bmatrix} [M_j] \begin{bmatrix} 1 & 1 \\ ik_{wi} & -ik_{wi} \end{bmatrix} \begin{bmatrix} A_{Bi+1} \\ B_{Bi+1} \end{bmatrix} \quad (9)$$

Where M_J is the j^{th} transfer matrix corresponding to the j^{th} junction written as:

$$M_j = M_{Bn}(b_j) M_{wi}(a_j) M_{Bn}(b_{j+1}) \quad (10)$$

Where b_j and a_j are the widths of the jth barrier and jth well respectively. $M_{Bn}(b_j)$ and $M_{wi}(a_j)$ correspond to the transfer matrices for the jth barrier and jth well respectively.

Here the total transfer matrix is expressed as the cascading of a series of individual barrier and well.

From equation (9) & (10) it is readily found the transmission amplitude Q is given by

$$Q = \frac{2}{M_{11} + M_{22} + i(k_w M_{12} - k_w^{-1} M_{21})} \quad (11)$$

Where M_{ij} are the elements of the total transfer matrix.

This can be written in the Matrix form as:

$$U_1 = T^1 U_2 \quad (12)$$

The complete transfer matrix T^2 at the slice x = 2c and

$$U_2 = T^2 U_3 \text{ with } U_1 = T^1 U_2 = T^1 T^2 U_3 \quad (13)$$

Doing this for all the slices X_1……….X_n, we obtained the complete transfer matrix M that connects the wave function on the left side of the potential with the one on the right side,

$$U_1 = MU_{n+1} \text{ where } M = T^1 T^2 \ldots\ldots\ldots\ldots\ldots T^n \quad (14)$$

From the definition of transmission coefficient (T), we obtained the following expression from equation (11)

$$T = |Q|^2 \quad (15)$$

Assuming that there is no reflection of the wave in the region N and the amplitude of the incident wave is unity, we find:

$$\begin{bmatrix} A_{Bn} \\ 0 \end{bmatrix} = M \begin{bmatrix} 1 \\ B_{B1} \end{bmatrix} \quad (16)$$

The coefficient of wave functions can be derived from this equation. Here the Matrix equation is very complex and involves a large number of terms. At the coupling energy the electron wave tunnels through the barriers so that the transmission coefficients at the expected energy values are equated to unity putting $\tau = 1$.The coupling energies between adjacent wells are derived iteratively to yield the energy for which electrons may be coupled to both the wells by penetrating through the intervening barrier of the Fibonacci series MQWS.

The transmission coefficient derived by Transfer Matrix method for the most general case simplified to some extent and becomes

$$\tau=[4k_Bk_W/(m_wm_B)]^4/[T_1+2T_2+2T_3+2T_4] \quad (17)$$

Where $T_1 = \{(k_B / m_B) + (k_W /m_W)\}^8+\{(k_B / m_B) - (k_W /m_W)\}^8 + 6\{(k_B / m_B)^2 - (k_W /m_W)^2\}^4$ (18)

$T_2 = \{(k_B / m_B) + (k_W /m_W)\}^4\{(k_B / m_B)^2 – (k_W /m_W)^2\}^2$ [cos (4{bk$_B$+ck$_W$}) +cos (4{ak$_B$+bk$_W$}) –2cos (4ck$_W$) - cos (4ak$_B$)-cos(4bk$_B$)–cos(4{ak$_W$+bk$_B$+ck$_W$})] (19)

$T_3 = \{(k_B / m_B)^2-(k_W /m_W)^2\}^4$[4cos(4ak$_W$)cos(4ck$_W$) - 2cos (4bk$_B$){1+cos (4[a-c]k$_W$)}-2{cos (4ak$_W$) +Cos (4ck$_W$) {1-cos(4bk$_B$)} (20)

$T_4 = \{(k_B/m_B) - (k_W/m_W)\}^4\{(k_B/m_B)^2 – (k_W /m_W)^2\}^2$[Cos (4{bk$_B$ -ak$_W$}) +cos (4{bk$_B$ -ck$_W$})-cos (4ak - cos (4bk$_B$) - cos (4{bk$_B$ -ak$_W$-ck$_W$})] (21)

In order to compute the coupling energies the transmission coefficient is equated to 1 and energy values are obtained by iterative method.

The expression for the transmission coefficient simplifies and becomes:

$$\tau = 16(k_B k_W)^2/ [(m_B m_W)^2\{(k_B / m_B) +(k_W /m_W)\}^4 + \{(k_W/m_W)-(k_B/m_B)\}^4-2\{(k_W/m_W)^2-(k_B/m_B)^2\}^2\cos(4ck_B)\}] \quad (22)$$

The Eigen value energy equation takes the form

$$(m_W k_B) / (m_Bk_W) = \tan (ck_B) /\tan (ck_W) \quad (23)$$

where c is the well width.

The coupling energies and tunneling probabilities in Fibonacci series multiple quantum wells are obtained from the above equations (22) and (23).

Resonant tunneling MQWs are quantum devices that have been investigated in recent years, and there is a great attention focused on transmission phenomenon. The resonant tunneling across the Fibonacci multiple quantum wells (FMQW) system reached when τ =1. The incident energy of the electron for which the resonant tunneling condition is satisfied is termed as resonant tunneling energy. Here we have found the resonant tunneling energies in the MQWs system from the τ vs E curve by a computer program using the search technique. Resonant tunneling across the MQWs system occurs for definite values of the incident energy of the incident electron in the entire region of the energy spectrum i.e. for both the regions E < V0 and E > V0.

3. Results and Discussion

In this section we present our results obtained numerically by using MATLAB programming for the transmission coefficient across Fibonacci chain multiple quantum wells containing asymmetric and aperiodic heterostructures. The general model of AMQW structure is tested on the FMQW structure.

The materials chosen are AlGaAs/GaAs. The parameters used in the computation are given below. The effective mass of electrons in $Al_xGa_{1-x}As$ depends on the mole fraction of x, where x represents the concentration of Al. The values of the parameters chosen are: Electron effective mass of $Al_xGa_{1-x}As$ as is m(AlGaAs) = (0.063+0.083x) and the energy band gap is given by the expression E(AlGaAs)= (1.9+0.125x+0.143x2). The mole fraction x= 0.47. The band gaps for AlGaAs and GaAs are respectively E_g (AlGaAs) = 1.99eV and E_g (GaAs) = 1.42eV. The conduction band difference is ΔE_c =67% x= E_g (AlGaAs) – E_g (GaAs) = 0.38 eV. The electron effective mass of GaAs is m*(GaAs) = 0.067m$_o$; and that for AlGaAs is m*(AlGaAs) = 0.106m$_o$. Here we consider the wells are in Fibonacci series and barriers lengths are in same. The wells with are taken as 5nm, 5nm, 10nm, 15nm, 25nm ….etc. up to 10 wells, denoted by FIB10.These computed band structures and transmission coefficient are shown in figures (Figure. 3-Figure. 6).

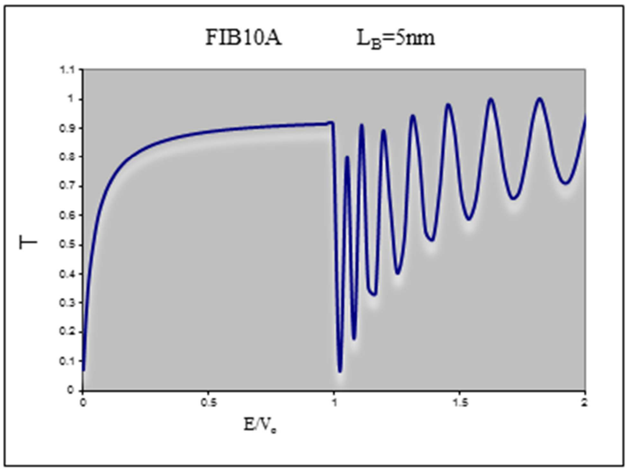


Figure 3. *Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for GaAs/AlGaAs/GaAs FMQWs for barriers with L_B= 5 nm. (FIB10A).*

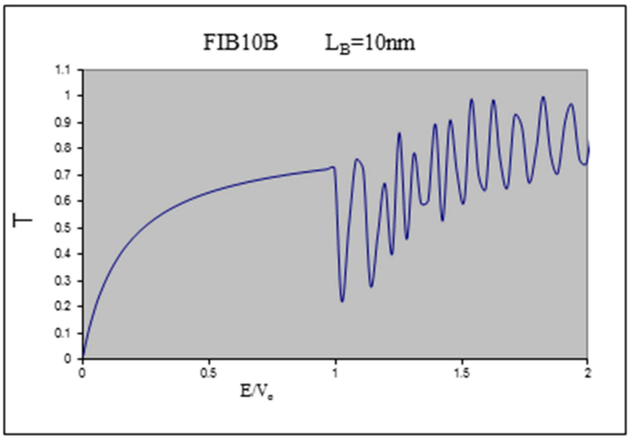


Figure 4. *Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for GaAs/AlGaAs/GaAs FMQWs for barriers with L_B= 10nm. (FIB10B).*

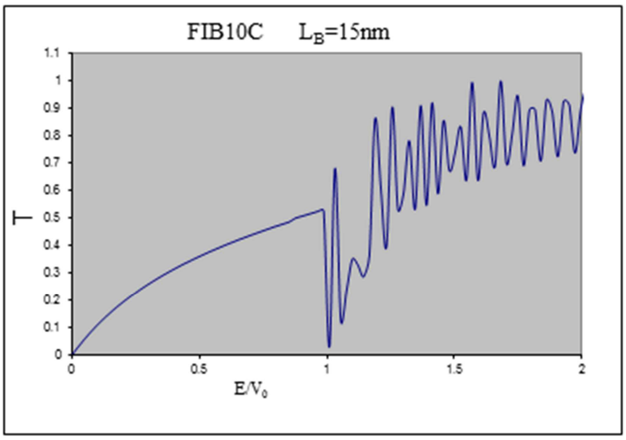


Figure 5. *Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for GaAs/AlGaAs/GaAS FMQWs for well with L= 5 nm, 5nm, 10nm, 15nm, 25nm…etc up to 10 wells and barriers with L_B=15nm. for FIB10C.*

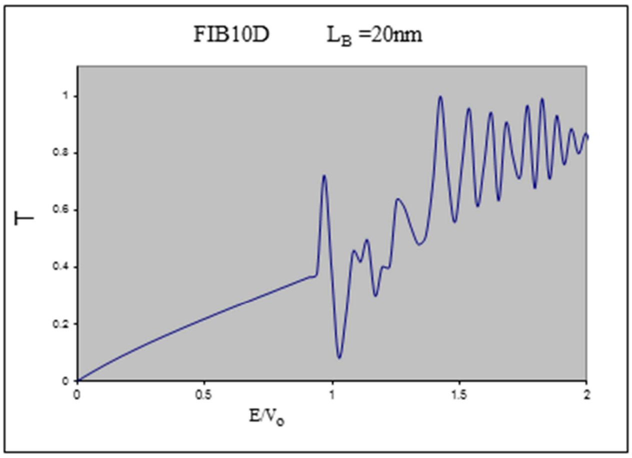


Figure 6. *Variation of transmission coefficient of electrons (τ=T) with normalized energy E/Vo for GaAs/AlGaAs/GaAS FMQWs for well with L= 5 nm, 5nm, 10nm, 15nm, 25nm…etc up to 10 wells and barriers with L_B=20nm. for FIB10D.*

Here we changed the barriers length of same Fibonacci structure FIB10 and found structures FIB10A, FIB10B FIB10C and FIB10D for barriers lengths 5nm, 10nm, 15nm and 20nm respectively and the variation of transmission coefficient of electrons with normalized Eigen energies [24] as shown above figures (Fig. 3 –Fig. 6). Here we observed that when $E/V_o<1$, the transmission coefficient T increases from 0 to1 in a non-linear fashion.

Beyond the normalized energy (E_{nor} = E/Vo) >1, there is resonance; i.e., there are quantized energy values where transmission reaches peak values sharply. When the barrier width decreases the peaks get more separated in energy and the normalized energy values vary continuously. If the widths of the lower band gap materials outside the barrier are reduced to the order of nanometers then the energy values inside the quantum wells will be quantized. This effect will be reflected in the nature of variation of the transmission coefficients with normalized energy and is expected to change significantly. This will have a crucial effect on carrier tunneling in FMQWs.

4. Conclusions

We have theoretically studied the transmission coefficients and the resonant photonic tunneling effect in Fibonacci series multiple quantum wells Nanostructures produced by AlGaAs/GaAs photonic quasicrystals (FIB10). By performing numerical simulations of effect of thickness of barriers and the transmission spectra for various FMQWs nanostructures. We have demonstrated that resonant photonic tunneling occurs in these structures, whereby photons with energies corresponding to bound states of the system undergo perfect transmission through the entire structure. Our studies have shown that these states occur in split pairs coupling energies with adjacent wells.

The resonant energy states are obtained on the basis of the resonance condition [27-29] T_N =1. The resonant energy states are found to group into allowed energy bands separated by forbidden gaps. During the resonance tunneling, the electron energy resonates at the bound states of the single quantum well [10], [30-31]. Here we see that the transmission coefficient exhibits a series of resonant peaks and valleys. The first series of resonant peaks are attributed to the resonant transmission tunneling through the fundamental quasibound state in the quantum well, while the second series is due to the tunneling through the first excited state. The width of the allowed band reduces significantly with increase in barrier width. Increase in barrier width causes decrease in the overlap interaction among the states of adjacent wells resulting in the decrease of band width. This spectral splitting is attributed to the coupling of the degenerate states in each photonic well, where the finite photonic barrier separating these wells allows for an overlap of the electromagnetic fields of the degenerate states. The degree of spectral splitting can be controlled by varying the thickness of barriers. The interwell barriers widths are determined from resonant Bragg condition which satisfying the constructive interference of the waves reflected from the FMQWS at excitonic resonance.

The energy-splitting phenomenon described here is in agreement with reported experimental results involving 1-D photonic crystals [12]. Our simulations have also shown that the total number of transmitted resonant states can be controlled by modifying the width of the photonic barriers in these nanostructures. Due to the enhanced light-matter coupling, we find that the values of transmission coefficient in the FMQWs for higher generation orders are significantly stronger than those in the PQWs under the Bragg or anti-Bragg conditions [20]. The resonant state describe here in proposed device might be useful for developing new types of photonic-switching devices, optical filters, detectors and other optoelectronic devices.

Acknowledgment

Authors are indebted to Prof. (Dr.) Sukakshina Kundu and Dr. Madhumita Dassarkar of Department of Computer Science and Engineering, West Bengal University of Technology (WBUT), India for useful discussions.

References

[1] R. Tsu and L. Esaki, "Tunneling in Finite Superlattice", Appl. Phys. Lett. 22 (1973) 562.

[2] S. John, "Strong localization of photons in certain disordered dielectric superlattices", Phys. Rev. Lett. 58, 2486(1987).

[3] B. Freedman et al, "Wave and defect dynamics in nonlinear Photonic quasicrystals" Nature 440, 1166-1169 (2006).

[4] J Gain et al, "Aperidic and asymmetric Multiple Quantum Wells photonic devices: A novel transfer matrix based Model", IJEST, Vol.6, 4954-4961(2011).

[5] Ralf Menzel, "Photonics", Springer, (2006).

[6] Joel D. Cox et al, "Resonant tunnling in Photonic double quantum well Heterostrctures", Nanoscale Res. Lett. (2010)5:484-488.

[7] C. Jannot et al, "Quasicrystalls "Clarendon Press, Oxford, 1994.

[8] S.H. Xu et al. "Photonics and nanostructures-fundamentals and applications," 4 17–22 (2006).

[9] Tignon, J. et al. "Unified picture of polariton propagation in bulk GaAs Semiconductors." Phys. Rev. Lett. 84, 3382–3385 (2000).

[10] M. Werchner et al, "One dimensional resonant Fibonacci quasicrystals: Noncanonical linear and canonical nonlinear effects" Optics Express, 6813 Vol. 17, No. 8 (2009).

[11] Cerne, J. et al. "Terahertz dynamics of excitons in GaAs/AlGaAs quantum wells." Phys. Rev. Lett. 77, 1131–1134 (1996).

[12] Poddubny, A. N. et al. "Photonic quasicrystalline and aperiodic Structures." Physica E, Volume-42, 1871–1895 (2010).

[13] Miller, D. A. B. "The role of optics in computing." Nature Photon. 4, 406 (2010).

[14] A. Yariv, P. Yeh, "Photonics: optical electronics in modern communications." Oxford University Press (2007).

[15] T. Matsui, A. Agrawal, A. Nahata, and Z. V. Vardeny, "Transmission Resonances through aperiodic arrays of sub wavelength apertures," Nature 446, 517–521 (2007).

[16] M. Kohmoto, B. Sutherland, and K. Iguchi, "Localization of optics: Quasiperiodic media," Phys. Rev. Lett. 58, 2436–2438 (1987).

[17] Alan L, "Crystallography and the Penrose Pattern" Physica 114A, 609- 613,(1982).

[18] L. D. Negro et al, "Photon band gap properties and Omnidirectional reflectance in Si/SiO2 Thue–Morse quasicrystals," Appl. Phys. Lett. 84, 5186–5188 (2004).

[19] R. Merlin et al, "Quasiperiodic GaAs-AlAs Heterostructures,"Phys. Rev. Lett. 55, 1768–1770 (1985).

[20] E. L. Ivchenko, A. I. Nesvizhskii, and S. Jorda, "Bragg reflection of Light from quantum-well structures," Phys. Solid State 36, 1156–1161 (1994).

[21] H. Haug and S. W. Koch, Quantum Theory of the Optical and Electronic Properties of Semiconductors (fifth ed., World Scientific Publishing, Singapore (2009).

[22] V. V. Chaldyshev et al, "Optical lattices of InGaN quantum well excitons," Appl. Phys. Lett. 99(25), 251103 (2011).

[23] R. W. Boyd, "Nonlirear Optics," 3rd edition, Academic press, Orlando, 2007.

[24] Daniel J Costinett and Theodoros P Horikis, "High-order eigenstate calculation of arbitrary quantum structures," 2009 J. Phys. A: Math. Theory. 42, 235201.

[25] M.O. Manasreh, "Optoelectronic Properties of Semiconductors and Superlattices,"Vol.8-Semiconductor Quantum Wells Intermixing, Edited by E. Herbert Li, Gordon and Breach Science Publishers.

[26] A. Hamed Majedi,"Multilayer Josephson junction as a multiple quantum well Structure", IEEE Trans. On applied Superconductivity, Vol-17, No: 2, June (2006).

[27] J. Gain et al, "Transfer Matrix method for computation of electron transmission through aperiodic MQW" IEEE explore, ELECTRO-2009, doi10.1109/ELECTRO.2009.5441050.

[28] Jatindranath Gain et al "Fibonacci Series Multiple Quantum Wells: Big Future in Quantum Computation, "International Congress on Science and Engineering Research (ICSER), Organized by GRDS in Singapore, Nov17-18, 2015.

[29] A. Mayer and J.-P. Vigneron, "Transfer-matrix quantum-mechanical theory of electronic field emission from nanotips" J. Vac. Sci. Technol. B, Vol. 17, No. 2, (1999).

[30] M. S. Vasconcelos et al, "Photonic band gaps in quasiperiodic photonic crystals with negative refractive index" Phys. Rev. B 76, 16511(2007).

[31] Michael Shur, "Physics of Semiconductor Devices" Prentice Hall Inc. New Jersey, USA, (2005).