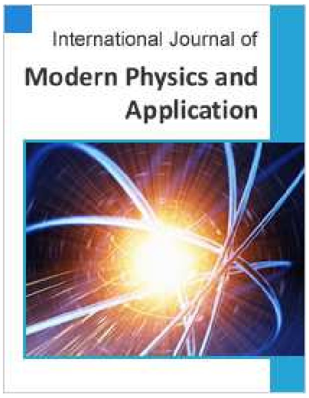
International Journal of Modern Physics and Application
2015; 2(6): 110-115
Published online January 6, 2016 (http://www.aascit.org/journal/ijmpa)
ISSN: 2375-3870

American Association for Science and Technology



Keywords
A Novel 5-D Hyperchaotic Lorenz System,
Control,
Synchronization,
Lyapunov Stability Theory,
Nonlinear Control Technique

Received: December 8, 2015
Revised: December 17, 2015
Accepted: December 19, 2015

Chaos Control and Synchronization of a Novel 5-D Hyperchaotic Lorenz System via Nonlinear Control

Maysoon M. Aziz, Saad F. AL-Azzawi

Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq

Email address
saad_fawzi78@yahoo.com (S. F. AL-Azzawi)

Citation
Maysoon M. Aziz, Saad F. AL-Azzawi. Chaos Control and Synchronization of a Novel 5-D Hyperchaotic Lorenz System via Nonlinear Control. *International Journal of Modern Physics and Application*. Vol. 2, No. 6, 2015, pp. 110-115.

Abstract
This paper is concerned with the problem of chaos control and synchronization for a novel 5-D hyperchaotic system, which is constructed by adding a feedback control to a 4-D hyperchaotic Lorenz system. Based on the Lyapunov stability theory and using nonlinear control technique with two different kinds of parameters (known and unknown parameters), and we designed control for each kind to perform control and synchronization of this system. However, in unknown parameters, we assume that Lyapunov function is always formed as $V(x) = X^T P X$, where $P = I_5$ is the identity matrix. While we focused on selecting a suitable Lyapunov functions candidate that ensured asymptotically global stability in known parameters. Then, nonlinear control technique is better than adaptive and active nonlinear control because it deals with known and unknown parameters as well as to the design of more than control compared with the rest of the controls. Moreover, numerical simulations are offered to show the strength of the proposed theoretical results.

1. Introduction

The 3-D Lorenz system is discovered in 1963 which consider the first mathematical and physical model of chaos, thereby getting the starting point and foundation rock for later research on chaos theory [7]. Number of chaotic systems and their applications in mathematics, physics, chemistry, engineering, computer science and biology is increasing, so much research has been done to introduce them. Most research in this field has been focused on the hyperchaotic system [14]. Hyperchaotic system was first reported by Rossler in 1979 which contain four dimension [7, 10, 11, 13, 14]. Since then, some other hyperchaotic systems have also been found [7]. As we know, chaos is an important topic in nonlinear science [5, 8]. But in sometimes, chaos effect is undesirable in practice, and it restricts the operating range of many electronic and mechanical devices [15]. In this case, therefore, it is necessary that the chaotic behavior should be controlled [15]. But, control methods were once believed to be impossible until the 1990s when Ott et al. developed the OGY method to suppress chaos [3, 9], Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions [3, 4, 6, 8, 9, 12, 15]. Different control strategies for stabilizing chaos also have been proposed, such as adaptive control, time delay control and fuzzy control [3, 7, 9, 12, 16].

Every day, the number of articles that relates to this topic is increasing, and numerous articles devoted to explaining the new high-dimensional chaotic systems and more complicated topological structure [13, 14]. And the dynamics of the hyperchaotic systems have not been completely understood by mathematicians until now [13]. Thus, it is necessary to get a novel study for hyperchaotic systems. Hence some of articles motivated to further study the properties of chaos and hyperchaos and some subtle characteristics of the new hyperchaotic system, so as to benefit more systematic studies of 5D system, and to reveal the true geometrical structures of lower dimensional chaotic and hyperchaotic attractors [13].

Recently, based on the Lorenz system and state feedback control, a new 5D hyperchaotic system was reported by Hu in 2009, and Yang et al. 2013 study the dynamical properties of this system [13], In 2014 Vaidyanathan et al. generating another a new 5D hyperchaotic system based on the Lorenz system and perform chaos control and synchronization via adaptive control when the parameters for this system are unknown [10]. In this paper, we achieved chaos control and synchronization of this system via nonlinear control strategy with known and unknown parameters and numerical simulations are presented to demonstrate the effectiveness of the proposed controllers.

2. System Description

Lately, on studying control and synchronization of chaos, Vaidyanathan et al. [10], 2014 introduced a novel 5-D hyperchaotic system, which is described by the following nonlinear differential equation.

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 + x_5 \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 \\ \dot{x}_3 = x_1x_2 - bx_3 \\ \dot{x}_4 = -x_1x_3 + px_4 \\ \dot{x}_5 = qx_1 \end{cases} \tag{1}$$

Where $(x_1, x_2, x_3, x_4, x_5) \in R^5$, and $a, b, c, p, q \in R$ are positive, constant parameters. When parameters $a = 10$, $b = 8/3$, $c = 28$, $p = 1.3$ and $q = 2.5$ system (1) is hyperchaotic and has three positive Lyapunov exponents, i.e. $LE_1 = 0.4195$, $LE_2 = 0.2430$, $LE_3 = 0.0145$ and the other Lyapunov exponents are $LE_4 = 0$, $LE_5 = -13.0405$. System (1) contains about twelve-term with three quadratic nonlinearities and has only one equilibrium $O(0,0,0,0,0)$, and the equilibrium is an unstable under these parameters. Hyperchaotic attractors are shown in Fig. 1 and Fig. 2.

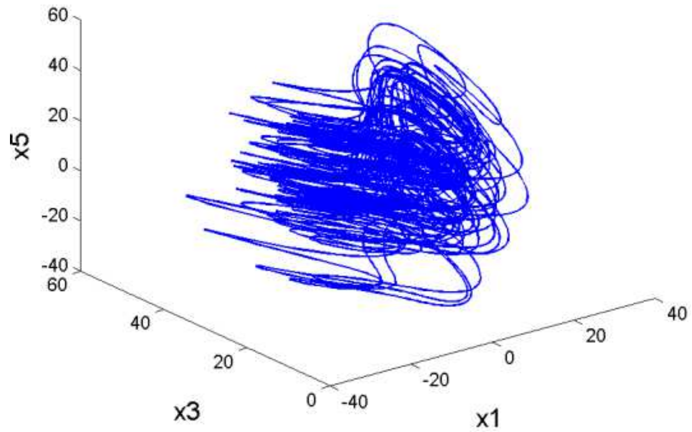


Fig. 1. *The attractor of the system (1) in x1-x3-x5 space.*

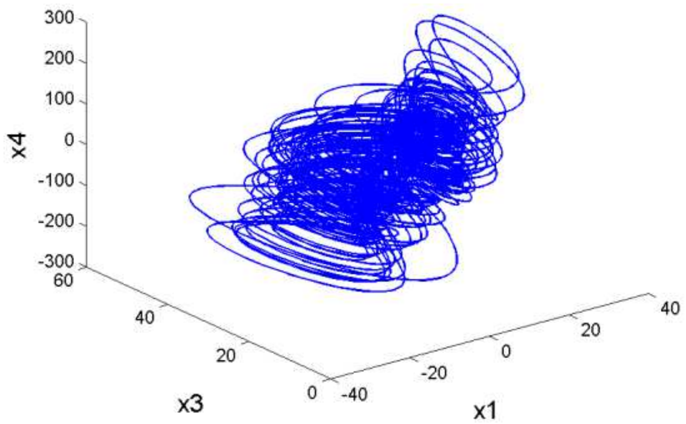


Fig. 2. *The attractor of the system (1) in x1-x3-x4 space.*

3. Chaos Control

The this section, we perform the controlling problem of system (1) via nonlinear control technique by two classes: in the first case, when the parameters are unknown, while in the second case, the parameters are known.

In order to control the system (1) to zero, the feedback controllers of u_1, u_2, u_3, u_4 and u_5 are added to the hyperchaotic system (1). Then the controlled hyperchaotic system is given by:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_4 + x_5 + u_1 \\ \dot{x}_2 = cx_1 - x_1x_3 - x_2 + u_2 \\ \dot{x}_3 = x_1x_2 - bx_3 + u_3 \\ \dot{x}_4 = -x_1x_3 + px_4 + u_4 \\ \dot{x}_5 = qx_1 + u_5 \end{cases} \tag{2}$$

3.1. Controlling 5-D Hyperchaotic System (2) with Unknown Parameters

In the next theorem, we design nonlinear control without we know the values of these parameters.

Theorem 1. If the nonlinear controllers are proposed as:

$$\begin{cases} u_1 = -(a+c)x_2 + x_3x_4 - (1+q)x_5 \\ u_2 = 0 \\ u_3 = 0 \\ u_4 = -(p+1)x_4 - x_1 \\ u_5 = -x_5 \end{cases} \tag{3}$$

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

Proof. According to the Lyapunov stability theory, we construct the following Lyapunov candidate function

$$V(x) = X^T P X \tag{4}$$

and $P = I_5$ is identity matrix.

With controller (3) we have the time derivative of the Lyapunov function as:

$$\begin{aligned} \dot{V}(x) &= -2ax_1^2 - 2x_2^2 - 2bx_3^2 - 2x_4^2 - 2x_5^2 \\ &= -X^T Q X \end{aligned} \tag{5}$$

where

$$Q = \begin{bmatrix} 2a & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2b & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix} \tag{6}$$

Since a and b are positive parameters, therefore Q is a positive defined matrix, So, we have $\dot{V}(x)$ is a negative definite function. Hence, the controlled system (1) can asymptotically converge to the unstable equilibrium with the controllers (3).

3.2. Controlling 5-D Hyperchaotic System (2) with Known Parameters

Assuming that the parameters of the system (2) are known and the states system are measurable. According to Ref [10] the parameter values are taken as

$$a = 10, b = 8/3, c = 28, p = 1.3, q = 2.5.$$

Theorem 2. If the nonlinear controllers are proposed as:

$$\begin{cases} u_1 = -2ax_2 - x_4 \\ u_2 = 0 \\ u_3 = 0 \\ u_4 = -(p+1)x_4 + x_1x_3 \\ u_5 = -2qx_1 - x_5 \end{cases} \tag{7}$$

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

Proof. Construct a Lyapunov function:

$$V(x) = X^T P_1 X \tag{8}$$

and

$$P_1 = \begin{bmatrix} 2.8 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1.12 \end{bmatrix} \tag{9}$$

With controller (7), we have the time derivative of the Lyapunov function as:

$$\begin{aligned} \dot{V}(x_i) = &-5.6\,ax_1^2 - 2x_2^2 - 2bx_3^2 - 2x_4^2 - 2.24x_5^2 \\ &+ (2c - 5.6\,a)x_1x_2 + (5.6 - 2.24\,q)x_1x_5 \end{aligned} \tag{10}$$

Substituting the value of parameters in the above equation, we have

$$\begin{aligned} \dot{V}(x) &= -56\,x_1^2 - 2x_2^2 - 16/3x_3^2 - 2x_4^2 - 2.24x_5^2 \\ &= -X^T Q_1 X \end{aligned} \tag{11}$$

and

$$Q_1 = \begin{bmatrix} 56 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 16/3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2.24 \end{bmatrix} \tag{12}$$

Hence, Q_1 is a positive defined matrix,

Consequently, we have $\dot{V}(x)$ is a negative definite function. This completes our proof.

Numerical simulations are furnished to verify the effectiveness of the proposed controller based on fourth-order Runge -Kutta scheme with time step 0.5 . the parameter values are taken as; $a = 10, b = 8/3, c = 28, p = 1.3, q = 2.5$, with initial conditions $(10, 5, -5, 0, -10)$. Fig. 3 and Fig. 4 show the convergent with controllers (3) and (7) respectively.

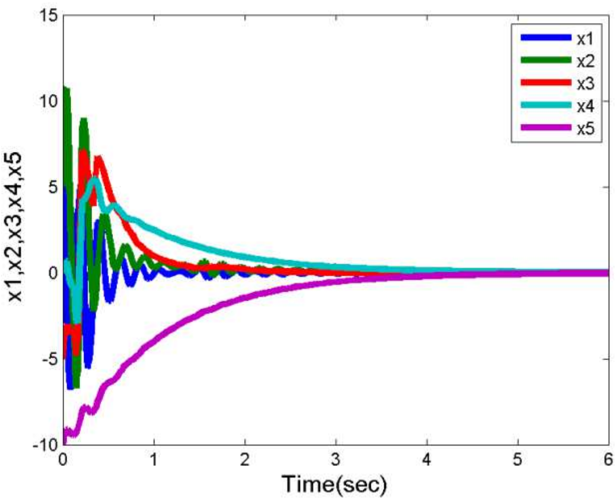


Fig. 3. *The converges of system (2) with controllers (3).*

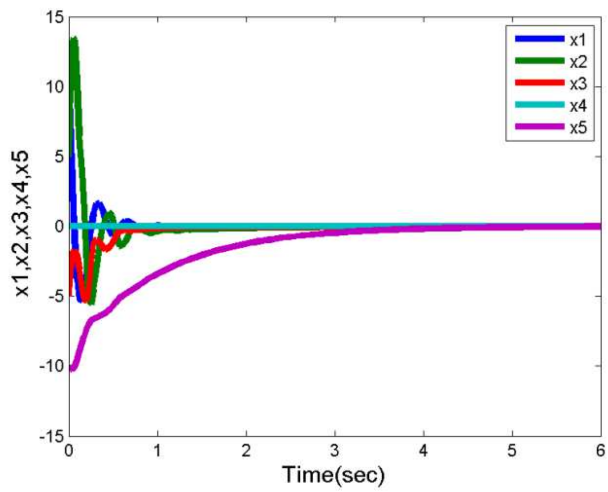


Fig. 4. *The converges of system (2) with controllers(7).*

4. Chaos Synchronization

To begin with, the definition of chaos synchronization used in this paper is given below.

Definition [15, 16] For two nonlinear hyperchaotic systems:

$$\dot{x} = f_1(x) \tag{13}$$

$$\dot{y} = f_2(y) + u(x, y) \tag{14}$$

where $x, y \in R^n$, $f_1, f_2 : R^n \to R^n$. Assume that Eq. (13) is the drive system, Eq.(14) is the response system, and $u(x, y)$ is the nonlinear control vector. The response system and drive system are said to be synchronized if for $\forall x(t_0), y(t_0) \in R^n$, $\lim_{t\to\infty} \left\| y(t) - x(t) \right\| = 0$.

Let the system (1) be the drive system and response systems are given as the following

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1) + y_4 + y_5 + u_1 \\ \dot{y}_2 = cy_1 - y_1 y_3 - y_2 + u_2 \\ \dot{y}_3 = y_1 y_2 - b y_3 + u_3 \\ \dot{y}_4 = -y_1 y_3 + p y_4 + u_4 \\ \dot{y}_5 = q y_1 + u_5 \end{cases} \tag{15}$$

Subtracting system (1) from the system (15), we obtain the error dynamical system between the drive system and the response system which is given by:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + e_4 + e_5 + u_1 \\ \dot{e}_2 = ce_1 - e_2 - e_1 e_3 - x_1 e_3 - x_3 e_1 + u_2 \\ \dot{e}_3 = -b e_3 + e_1 e_2 + x_1 e_2 + x_2 e_1 + u_3 \\ \dot{e}_4 = p e_4 - e_1 e_3 - x_1 e_3 - x_3 e_1 + u_4 \\ \dot{e}_5 = q e_1 + u_5 \end{cases} \tag{16}$$

where $e_i = y_i - x_i, \quad i = 1, 2, \ldots, 5$.

System (16) describes the error dynamics. It is clear that the synchronization problem is replaced by the equivalent problem of stabilizing the system (16) using a suitable choice of the feedback controller.

4.1. Chaos Synchronization of System (16) with Unknown Parameters

Suppose that the parameters are unknown, In light of this, we design nonlinear controllers which ensure convergence between system (1) and system (15), then we obtain the following theorem:

Theorem 3. The two hyperchaotic systems (1) and (15) will approach global and asymptotically synchronization with following control:

$$\begin{cases} u_1 = e_3 e_4 - x_2 e_3 + x_3(e_2 + e_4) \\ u_2 = -(a + c) e_1 \\ u_3 = x_1 e_4 \\ u_4 = -(p + 1) e_4 - e_1 \\ u_5 = -e_5 - (q + 1) e_1 \end{cases} \tag{17}$$

Proof. Let us consider the Lyapunov function is:

$$V(e) = e^T P e \tag{18}$$

and $P = I_5$ is the identity matrix.

Now the time derivative of the Lyapunov error function is:

$$\dot{V}(e) = -2ae_1^2 - 2e_2^2 - 2be_3^2 - 2e_4^2 - 2e_5^2 \tag{19}$$

Therefore, $\dot{V}(e) = -e^T Q e$ and Q is a positive definite matrix(Q is defined as Eq. 6).

Hence, the two hyperchaotic systems (1) and (15) are asymptotically globally synchronized.

4.2. Chaos Synchronization of System (16) with Known Parameters

In this subsection, supposed that all the variables and parameters of the drive and response systems are available and measurable.

Theorem 4. The zero solution of the error system (16) is asymptotically stable if a nonlinear control is designed as following:

$$\begin{cases} u_1 = -2ae_2 - e_4 \\ u_2 = x_3 e_1 \\ u_3 = -x_2 e_1 \\ u_4 = -(p+1)e_4 + e_1 e_3 + x_1 e_3 + x_3 e_1 \\ u_5 = -e_5 - 2qe_1 \end{cases} \tag{20}$$

Proof. Let us consider the Lyapunov function is:

$$V(e) = e^T P_2 \ e \tag{21}$$

and

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 10/28 & 0 & 0 & 0 \\ 0 & 0 & 10/28 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1/2.5 \end{bmatrix} \tag{22}$$

With controller (20), we have the time derivative of the Lyapunov function as:

$$\begin{aligned} \dot{V}(e_i) = & -2ae_1^2 - 10/14e_2^2 - 10/14be_3^2 - 2e_4^2 \\ & -2/2.5e_5^2 + [10/14c - 2a)e_1 e_2 \\ & + 2(1 - 1/2.5q)e_1 e_5 \end{aligned} \tag{23}$$

Substituting the value of parameters in the above equation, we have the time derivative of the Lyapunov function is:

$$\begin{aligned} \dot{V}(e) = & -20e_1^2 - 10/14e_2^2 - 40/21e_3^2 - 2e_4^2 - 2/2.5e_5^2 \\ = & -e^T Q_2 \ e \end{aligned} \tag{24}$$

and

$$Q_2 = \begin{bmatrix} 20 & 0 & 0 & 0 & 0 \\ 0 & 10/14 & 0 & 0 & 0 \\ 0 & 0 & 40/21 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2/2.5 \end{bmatrix} \tag{25}$$

which is a positive definite matrix.

Hence, based on Lyapunov stability theory, the error dynamics converge to the origin asymptotically.

Numerical simulations are used to investigate the controlled error dynamical system (16), using fourth-order Runge-Kutta scheme with time step 0.5 . We choose the parameters $a = 10, b = 8/3, c = 28, p = 1.3, q = 2.5$ and the initial values of the drive system and the response system are $(10, 5, -5, 0, -10)$ and (-10, -5, 10, 10, 10,) respectively. From Fig. 5 and Fig. 6, we can see the convergent for system (16) with controllers (17) and (20) respectively.

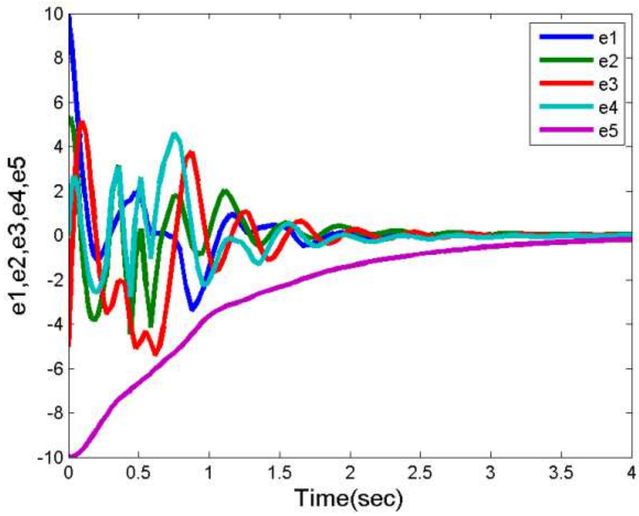


Fig. 5. *The converges of system (16) with controllers(17).*

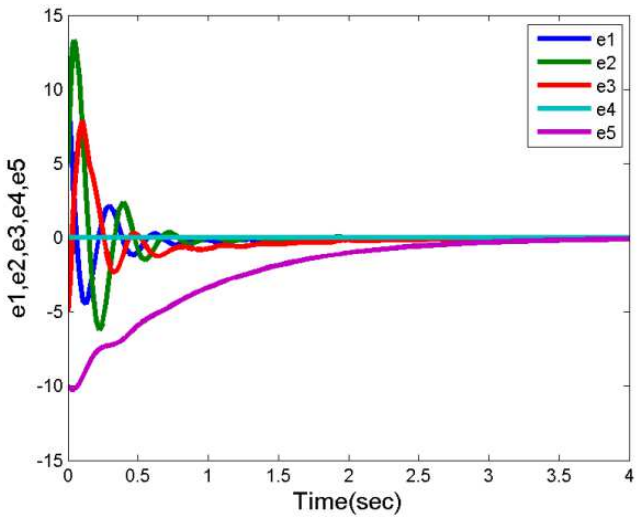


Fig. 6. *The converges of system (16) with controllers(20).*

5. Conclusions

In this paper, many controls were designed via the nonlinear control strategy for control of a new 5-D hyperchaotic system. Based on the Lyapunov stability theory and kind of parameters, obviously from these controllers, we have the controlling with unknown parameters achieved easy while in knowning parameters more complex. The effectiveness of these proposed control strategies was validated by numerical simulation results.

References

[1] Ahmad I., Saaban A. B., Ibrahim A. B., Shahzad M., A Research on synchronization of two novel chaotic system based on nonlinear active control algorithm, Engineering, Technology & Applied Science Research, 5 (2015) 739-747.

[2] AL-Azzawi S. F., Stability and bifurcation of pan chaotic system by using Routh-Hurwitz and Gardan method, Appl. Math. Comput. 219 (2012) 1144-1152.

[3] Aziz M. M., AL-Azzawi S. F., Control and synchronization of a modified hyperchaotic Pan System via active and adaptive control techniques, Comput. Appl. Math. 1 (2015) 151-155.

[4] Cai G., Tan Z., Chaos synchronize of a new chaotic system via nonlinear control, J. Uncertain Systems, 1 (2007) 235-240.

[5] Changjin X., Peiluan L., Chaos control for a 4D hyperchaotic system, Applied Mechanics and Materials, 1 (2013) 84-87.

[6] Chen HK., Global Chaos synchronization of new chaotic systems via nonlinear control, Chaos, Solitons and Fractals, 23 (2005) 1245-1251.

[7] Pang S., Liu Y., A new hyperchaotic system from the Lu system and its control, J Compu. Appl. Math. 235(2011) 2775–2789.

[8] Park J. H., Chaos synchronization of a chaotic system via nonlinear control, Chaos Solitons Fractals 25 (2005) 579-584.

[9] Lu D., Wang A., Tian X., Control and synchronization of a new hyperchaotic system with unknown parameters, International Journal of Nonlinear Science. 6 (2008) 224-229.

[10] Vaidyanatan S., Volos C., Pham VT., Hyperchaos, Adaptive control and synchronization of a novel 5-D hyperchaotic system with three positive Lyapunov exponents and its SPICE implementation, Archives of Control Sciences. 24 (2014) 409 - 446.

[11] Vaidyanatan S., Volos C., Pham VT., Analysis, Adaptive control and synchronization of a novel 4-D hyperchaotic hyperjerk system and its SPICE implementation, Archives of Control Sciences. 25 (2015) 135 - 158.

[12] Xu J., Cai G., Zheng S., Adaptive synchronization for an uncertain new hyperchaos Lorenz system, International Journal of Nonlinear Science. 8 (2009) 117-123.

[13] Yang Q. Chen C., A 5D hyperchaotic system with three positive Lyapunov exponents coined International Journal of Bifurcation and Chaos. 23 (2013) 13501091-135010923.

[14] Zarei A., Complex dynamics in a 5-D hyper-chaotic attractor with four -wing, one equilibrium and multiple chaotic attractors. Nonlinear Dyn 5 (2013).

[15] Zhu C., Control and synchronize a novel hyperchaotic system, Appl. Math. Comput. 216 (2010) 276-284.

[16] Wang Y. Guan ZH., Wang H. O., Feedback and adaptive control for the synchronization of Chen system via a single variable. Physics Letters A 312 (2003) 34-40.