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# K Meson Decays in Quark Model

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## Abstract

In this paper, the decay of *K* and  $\overline{K}$  mesons have been studied by means of the Effective Hamiltonian Theory. Decay rates of the constituent S and  $\overline{S}$  quarks, in the tree and penguin levels are calculated via effective Lagrangian density of the weak interaction and then by using these obtained decay rates, the decay rates and branching ratios of K and  $\overline{K}$  are calculated and compared with the experimental results. Findings from this study are in a good agreement with the experimental values.

# **1. Introduction**

One of the successful models in particle phenomenology is the quark model which is applied to calculate the decay rates of various particles. The particles called kaons, were first observed in late 1940s in cosmic-ray experiments. By today's standards, they are common, easily produced, and well understood. Over the last four decades, studies on how kaons decay have played a major role in development of the Standard Model. Yet, after all this time, kaon decays may still prove to be a valuable source of new information on some of the remaining fundamental questions in particle physics.

When first observed, kaons seemed quite mysterious. Experiments showed that they were produced in reactions involving the strong force, or strong interaction – the most powerful of the four fundamental forces in nature – but they did not decay (that is, transform into two or more less massive particles) through the strong interaction. This is due to kaons retaining a feature which ultimately labeled "strangeness," that is conserved in the strong interaction [1].

One of the most interesting and unique observed particles in the nature is kaon. There are two neutral kaons which are, in fact, strange mesons.

$$K^{\circ} = d\overline{s} \quad (s = -1)$$
  

$$\overline{K}^{\circ} = s\overline{d} \quad (s = +1)$$
(1-1)

*s* is the eigenvalue of the strange state. Since each kaon under CP effect turns into another kaon, neither of these kaons have determined CP number.  $K^{\circ}$  and  $\overline{K}^{\circ}$  are not eigenstate of CP. However, when CP acts on them, they are conjugate of each other.

$$CP \left| K^{\circ} \right\rangle = - \left| \overline{K}^{\circ} \right\rangle$$

$$CP \left| \overline{K}^{\circ} \right\rangle = - \left| K^{\circ} \right\rangle$$
(1-2)

But theorists can make a pair kaon with determined CP from combination of wave

function  $K^{\circ}$  and  $\overline{K}^{\circ}$ . According to Quantum Mechanics rules, these combinations correspond with real particles and have a mass and determined lifetime. Therefore normalized eigenstate CP are [2, 14]:

$$\begin{vmatrix} K_1 \end{pmatrix} = \frac{1}{\sqrt{2}} \left( \begin{vmatrix} K^{\circ} \end{pmatrix} - \begin{vmatrix} \overline{K}^{\circ} \\ \end{pmatrix} \right) \begin{vmatrix} K_2 \end{pmatrix} = \frac{1}{\sqrt{2}} \left( \begin{vmatrix} K^{\circ} \end{pmatrix} + \begin{vmatrix} \overline{K}^{\circ} \\ \end{pmatrix} \right)$$
(1-3)

So,

$$CP | K_1 \rangle = | K_1 \rangle \quad (CP = +1)$$
  

$$CP | K_2 \rangle = - | K_2 \rangle \quad (CP = -1)$$
(1-4)

 $K_1$  can just decays to CP = +1 state, while  $K_2$  should go to CP = -1 state. Neutral kaons usually decay to two or three pions. Arrangement of two pions has +1 parity and three pions system has -1 parity and both of them have a C = +1. As a result,  $K_1$  decays to two pions and  $K_2$ decays to three pions [3].

$$\begin{split} K_1 &\to 2\pi \\ K_2 &\to 3\pi \end{split} \tag{1-5}$$

Since a kaon has hardly enough mass to produce three pions, two pion decays are fast but three pion decays are longer. Observed lifetimes are about  $10^{-10}s$  and  $10^{-7}s$ , respectively [2, 10].

*K* meson decay, as a weak decay, in the presence of strong interactions requires a special approach. The main tool to investigate these decays is the effective Hamiltonian Theory. Beginning of any phenomenological weak decay of hadrons is the effective weak Hamiltonian. Its structure is as follows:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) Q_i \tag{1-6}$$

Where  $G_F$  is the Fermi constant represented in terms of the  $g_w$  weak coupling constant and W boson mass is defined as follows:

$$=\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_w^2}$$
(1-7)

And  $Q_i$  are the local operators which control the decay.  $V_{CKM}^i$ , the Cabibbo–Kobayashi–Maskawa factors and  $C_i$ , the Wilson Coefficients are described by the force with which an operator enters the Hamiltonian. In fact, the effective point-like vertices are represented by local operators that can correct the picture of the decay of hadrons with a mass of the order of  $O(m_b, m_c)$  a better way to provide.  $C_i$  The Wilson coefficients to be used as coupling constants (depending on scale) corresponding to the vertices are taken into account. Selection of the  $\mu$  scale is optional, but it is customary to choose  $\mu$  from the order of the mass of hadrons decaying, e.g for *B* and *D* mesons decay, the value of  $\mu$  are in the order of  $m_b$  and  $m_c$ , respectively. For kaon decays, the common choice of  $\mu$  is in the order of 1-2 Gev instead of  $m_K$  order [4].

# 2. Calculation of $s \rightarrow q_i q_k \overline{q}_j$ Decay Rates in the Tree Level

The standard model can be used for those particle decays in which their non- perturbative aspects, mainly do not affect the decay process. In fact here, the modes of S quark decays are studied which have not entered the QCD area and take place by bosons. This area is called the tree level [2].

Effective Lagrangian density related to the weak interaction could be expressed in terms of the coupling of the weak currents. Since the  $s \rightarrow q_i q_k \overline{q}_j$  decays through W particle in tree level is being investigated here, the weak currents in Lagrangian, the charged currents and Effective Lagrangian density, could be described as following [5, 6]:

$$L_{eff} = -2\sqrt{2}G_F J^\mu J^\dagger_\mu \tag{2-1}$$

Assuming that the wave functions of particle and antiparticle are normalized in V volume, with the coefficient of normalization of  $1/\sqrt{V}$ , and assuming that the *s* quark is at rest ( $p_s = 0$ ), the currents in above equation is given as follow:

$$J^{\mu} = \frac{1}{2V} \sqrt{1 + v_i} \left\langle - \Big|_i \overline{\sigma}^{\mu} \Big| + \right\rangle_s V_{is}$$

$$J^{?}_{\mu} = \frac{1}{2V} \sqrt{1 + v_k} \sqrt{1 + v_j} \left\langle - \Big|_k \overline{\sigma}_{\mu} \Big| - \right\rangle_j V^*_{jk}$$
(2-2)

Here  $\frac{p}{E} = \frac{m \sinh \theta}{m \cosh \theta} = \frac{\gamma m v}{\gamma m}$ , so  $v = \frac{p}{E}$ ,  $\bar{\sigma}^{\mu} \equiv (\sigma^0, -\sigma^1, -\sigma^2, -\sigma^3)$  that  $\sigma^i$  are the Pauli 2 × 2 matrices, also  $\sigma^0$  is the identity matrix. Furthermore,  $V_{is}$  and  $V_{jk}$  are the elements of the *CKM* matrix. Assuming  $\phi = 0$  (quark's momentum in x-z plane) the positive and negative helicity of the quarks could be expressed as follows:

$$|+\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ \sin\left(\frac{\theta}{2}\right) \\ \\ |-\rangle = \begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{\theta}{2}\right) \\ \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix}$$
(2-3)

element in the lowest-order is [7]:

$$M \equiv i \left\langle f \left| \int d^4 x L_{eff} \right| i \right\rangle \tag{2-4}$$

However, substituting  $L_{eff}$  in Eq. (2-4), then:

$$M = -i \frac{2\sqrt{2}G_F}{V^2} V_{is} V_{jk}^* \times (2\pi)^4 \delta^4 (p_f - p_i) \times \left(\tilde{\sigma}^{\mu} \tilde{\sigma}_{\mu}\right)$$
(2-5)

According to the Perturbation Theory, the decay matrix

In which with considering relations of quarks speed,  $\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}$  is defined:

$$\begin{split} \tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu} &= \frac{1}{4}\sqrt{1+v_{i}}\sqrt{1+v_{k}}\sqrt{1+v_{j}} \times \left[ \left( -\sin\left(\frac{\theta_{i}}{2}\right) - \cos\left(\frac{\theta_{i}}{2}\right) \right) \overline{\sigma}^{\mu} \begin{pmatrix} 1\\ 0 \end{pmatrix} \right] \\ & \times \left[ \left( -\sin\left(\frac{\theta_{k}}{2}\right) - \cos\left(\frac{\theta_{k}}{2}\right) \right) \overline{\sigma}_{\mu} \begin{pmatrix} -\sin\left(\frac{\theta_{j}}{2}\right) \\ \cos\left(\frac{\theta_{j}}{2}\right) \end{pmatrix} \right] \end{split}$$
(2-6)

Now in order to calculate the decay amplitude, the  $\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}$  components for  $\mu = 0, 1, 2, 3$  and over the quark spin states +1/2 and -1/2, are found. The spin average sentence is:

$$[\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}]_{sp-av}^{2} = \frac{1}{16}(1+v_{i})(1+v_{k})(1+v_{j})(1-\cos(\theta_{k}-\theta_{i}))$$
(2-7)

The above relation is obtained for final quarks in  $(s \rightarrow --+)$  helicity state. There are 8 helicity states. All of the helicity states are summed.

$$[\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}]_{sp-av}^{2} = \frac{1}{2}(1 - v_{i}v_{k}\cos(\theta_{k} - \theta_{i}))$$
(2-8)

The decay rate is:

$$d\Gamma = V^4 \left| M \right|^2 \frac{d^3 \vec{p}_i}{(2\pi)^3} \frac{d^3 \vec{p}_k}{(2\pi)^3} \frac{d^3 \vec{p}_j}{(2\pi)^3} (2\pi)^4 \delta^4(p_i - p_f)$$
(2-9)

Where,

$$\begin{split} \delta^4(p_i - p_f) &= \delta(E_i - E_f) \delta^3(\vec{p}_i - \vec{p}_f) & \text{and} \\ E_j &= \sqrt{m_j^2 + p_i^2 + p_k^2 + 2p_i p_k \cos \theta} = M_s - (E_i + E_k). \end{split}$$

With considering that the angle between velocity of particles must be physical,  $-1 \le \cos \theta \le 1$ , in that result:

If Eq. (2-5) is substituted in Eq. (2-9) and perform the calculations, then:

$$\Gamma_{s \to q_i q_k \overline{q}_j} = \Gamma_{0s} \left| V_{is} V_{jk}^* \right|^2 I$$
(2-11)

Where I is the phase space integral and  $\Gamma_{0s} = \frac{G_F^2 M_s^5}{192 \pi^3}$ .

The important point here is that it is assumed, the color factor to be 3 for Hadron decays. The decay rate for semilepton decays (assuming that the matrix elements of the lepton vertices are considered to be equal to 1) and the Hadron decays are:

$$\Gamma_{s \to q l \overline{\nu}_l}^{se-lep} = \Gamma_{0s} \left| V_{qs} \right|^2 I \tag{2-12}$$

$$\Gamma^{had}_{s \to q_i q_k \overline{q}_j} = 3\Gamma_{0s} \left| V_{is} V^*_{jk} \right|^2 I \qquad (2-13)$$

Table 1 presents the decay rate with corresponding errors in tree level for several numbers of semi-lepton and hadron decays of S quark, respectively, by using Eqs (2-12) and (2-13). The sources of errors correspond to quark mass data taken from [9].

$$\begin{array}{ll} m_{_{b}}=4.68^{+0.17}_{_{-0.07}} & m_{_{c}}=1.27^{+0.07}_{_{-0.11}} \\ m_{_{u}}=0.00255^{+0.00075}_{_{-0.00105}} & m_{_{d}}=0.00504^{+0.0096}_{_{-0.00154}} \\ m_{_{s}}=0.104^{+0.026}_{_{-0.034}} & m_{_{e}}=0.0005 \end{array}$$

Table 1. S Quark Decay Rate in Tree level.

Decay Process	$oldsymbol{\Gamma} \Big( \!  imes \! 10^{-20} oldsymbol{GeV} \Big)$
$s \rightarrow u d \overline{u}$	$3.1456^{\scriptscriptstyle +1.4343}_{\scriptscriptstyle -0.8430}$
$\overline{s} \to \overline{u}\overline{d}u$	$3.1456^{\scriptscriptstyle +1.4343}_{\scriptscriptstyle -0.8430}$
$s \to u \mu^- \overline{\vartheta}_\mu$	$1.7480^{\scriptscriptstyle +0.0014}_{\scriptscriptstyle +0.3789}$
$\bar{s} \to \bar{u}  \mu^+ \vartheta_\mu$	$1.7480^{\scriptscriptstyle +0.0014}_{\scriptscriptstyle +0.3789}$
$s \to u e^- \overline{\vartheta}_e$	$2.0341_{\rm -0.1670}^{\rm +0.2461}$
$\bar{s} \to \bar{u} e^+ \vartheta_e$	$2.0341^{\rm +0.2461}_{\rm -0.1670}$

## 3. Calculation of the $s \rightarrow q_i q_k \overline{q}_j$ Decay Rates in Tree and Penguin Levels

In the standard model, Flavor Changing Neutral Current (*FCNC*) is prohibited. For example, there is no direct coupling between the *b* quark and the *d* and *s* quarks. In fact, at the vertices of the  $Z \cdot \gamma$  and *g* neutral boson, the quark flavors do not change. This fact indicates the absence of *FCNC* currents in the tree level.

 $W^{\pm}$  vertices could be used in building up the desired structure of the loops. In another words, *FCNC* currents are created by penguin or one-loop diagrams. A penguin diagram and the schema of the Penguin are presented in Figure 1:



Figure 1. Penguin level.

As it could be observed in Figure 1, one loop is created. In fact, in this loop, for example, a W boson is emitted during the  $s \rightarrow d$  decay, while s quark is converted to the t internal quark. Then the boson is absorbed and s quark is converted to d quark. So Penguin diagrams contain a W - bosons - quarks loop. It could be seen that the  $s \rightarrow t \rightarrow d$  transition in  $K^-(s\overline{u}) \rightarrow \pi^-(d\overline{u})\pi^\circ(u\overline{u})$  decay in figure 2.



**Figure 2.**  $s \rightarrow d$  Transition in  $K^- \rightarrow \pi^- \pi^\circ$  decay in penguin level.

Different types of penguin processes include Electromagnetic Penguin, Electroweak Penguin and Gluons and QCD Penguin. The gluon penguins in which from Penguin's loop a gluon is emitted and it will generate a quark and anti-quark will be examined. Since the flavor of quark remains constant in gluon coupling, the Penguins could not be produced in the  $s \rightarrow u d\overline{u}$  decay, by the  $(\overline{s}_{\alpha}u_{\beta})_{V-A}(\overline{u}_{\beta}d_{\alpha})_{V-A}$  operator (which represents  $Q_1$  and  $Q_2$ operators of the decays in the tree level). Therefore, the Hamiltonian in the presence of additional interactions in penguins becomes more complicated; consequently, the new operators enter.

The Effective Hamiltonian in the Tree and Penguin level is given as [13]:

$$H_{eff} = 2\sqrt{2}G_F \sum_{i=1}^{6} d_i(\mu)Q_i(\mu)$$
(3-1)

Where,  $Q_{1,2}$  are the current-current operators and  $Q_{3,...,6}$ are the gluon penguin. For  $s \rightarrow q_i q_k \overline{q}_j$  decay,  $d_i(\mu)$ coefficients are:

$$d_{1,2} \equiv V_{is} V_{jk}^* C_{1,2} d_{3,\dots,6} \equiv -V_{ts} V_{tk}^* C_{3,\dots,6}$$
(3-2)

And as it is already known,  $C_i$  are the Wilson coefficients. It is required to find all helicity states, for  $Q_1 \cdot \ldots \cdot Q_6$ , and then sum up.

$$Q_{1} = (\overline{q}_{i\alpha}s_{\beta})_{V-A}(\overline{q}_{k\beta}q_{j\alpha})_{V-A}$$

$$Q_{2} = (\overline{q}_{i}s)_{V-A}(\overline{q}_{k}q_{j})_{V-A}$$

$$Q_{3} = (\overline{q}_{k}s)_{V-A}\sum_{q}(\overline{q}q)_{V-A}$$

$$Q_{4} = (\overline{q}_{k\alpha}s_{\beta})_{V-A}\sum_{q}(\overline{q}_{\beta}q_{\alpha})_{V-A}$$

$$Q_{5} = (\overline{q}_{k}s)_{V-A}\sum_{q}(\overline{q}q)_{V+A}$$

$$Q_{6} = (\overline{q}_{k\alpha}s_{\beta})_{V-A}\sum_{q}(\overline{q}_{\beta}q_{\alpha})_{V+A}$$
(3-3)

It is necessary for calculating all helecity states,  $Q_1 : \ldots : Q_6$  . The helicity states include:

$$\begin{array}{l} Q_1(--+), Q_2(--+), Q_3(--+), \\ Q_4(--+), Q_5(+--), Q_6(+--) \end{array} \tag{3-4}$$

We know that the helicity states  $Q_1 \cdot \ldots \cdot Q_4$  a

 $\left[\left(\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}\right)_{LL}+\left(\tilde{\sigma}^{\mu}\tilde{\sigma}_{\mu}\right)_{LR}\right]_{sp-av}^{2}=$ 

+), (3-4) with 
$$(--+) \propto \sqrt{1+v_i} \sqrt{1+v_k} \sqrt{1+v_j}$$
 terms and the helicity states  $Q_5$ ,  $Q_6$  are proportional with reproportional  $(+--) \propto \sqrt{1-v_i} \sqrt{1+v_k} \sqrt{1-v_j}$  terms. Therefore,  
 $\alpha_1(\frac{1}{16})(1+v_i)(1+v_k)(1+v_j)[1-\cos(\theta_k-\theta_i)]$ 

$$+ \alpha_1(\frac{1}{16})(1-v_j)(1+v_j)(1-v_j)[1+\cos(\theta_k-\theta_j)]$$

$$+ \alpha_{2} (\frac{1}{16})(1 - v_{i})(1 + v_{k})(1 - v_{j})[1 + \cos(\theta_{k} - \theta_{j})]$$

$$+ \alpha_{3} (\frac{1}{16})\sqrt{1 - v_{i}^{2}}(1 + v_{k})\sqrt{1 - v_{j}^{2}}[1 + \cos(\theta_{j} - \theta_{i})$$

$$- \cos(\theta_{k} - \theta_{j})\cos(\theta_{k} - \theta_{i})]$$

$$(3-5)$$

Where,  $\alpha_1$ ,  $\alpha_2$  and  $\alpha_3$  are defined as follows:

$$\begin{aligned} \alpha_{1} &= \left| d_{1} + d_{2} + d_{3} + d_{4} \right|^{2} + 2 \left| d_{1} + d_{4} \right|^{2} + 2 \left| d_{2} + d_{3} \right|^{2} \\ \alpha_{2} &= \left| d_{5} + d_{6} \right|^{2} + 2 \left| d_{5} \right|^{2} + 2 \left| d_{6} \right|^{2} \\ \alpha_{3} &= \operatorname{Re}\{ (3d_{1} + d_{2} + d_{3} + 3d_{4})d_{6}^{*} + (d_{1} + 3d_{2} + 3d_{3} + d_{4})d_{5}^{*} \} \end{aligned}$$
(3-6)

Decay rate is:

$$\Gamma = \int \frac{d^3 \vec{p}_i}{(2\pi)^3} \frac{d^3 \vec{p}_k}{(2\pi)^3} \frac{d^3 \vec{p}_j}{(2\pi)^3} (2\pi)^4 \delta^4 (p_s - p_i - p_k - p_j) \left| M \right|^2$$
(3-7)

$$\Gamma_{s \to q, q_k \overline{q}_j} = \Gamma_1 + \Gamma_2 + \Gamma_3 \tag{3-8}$$

$$\begin{split} &\Gamma_1 = \Gamma_{0s}(\alpha_1 I_1) \\ &\Gamma_2 = \Gamma_{0s}(\alpha_2 I_2) \\ &\Gamma_3 = \Gamma_{0s}(\alpha_3 I_3) \end{split} \tag{3-9}$$

Where,  $I_1$ ,  $I_2$  and  $I_3$  are the phase space integrals.

The general framework to obtain the Wilson coefficients is similar to what was mentioned in Eq (1-6). Therefore, the effective Hamiltonian of the  $K \rightarrow \pi \pi$  transition is defined as follows [7]:

$$H_{eff}\left(\Delta S=1\right) = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left(\sum_{i=1}^{10} C_i(\mu) Q_i(\mu)\right) + h.c. \quad (3-10)$$

Where  $G_F$  is the Fermi constant and  $Q_i$  are the local operators which control the decay.  $C_i$  coefficients are the Wilson Coefficients. The overall structure of the Wilson coefficients is as follow [12]:

$$C_{i}(\mu) = Z_{i}(\mu) + \tau y_{i}(\mu)$$
(3-11)

In this equation  $\tau$  is defined as follows:

$$\tau = \frac{-V_{td} V_{ts}^*}{V_{ud} V_{us}^*}$$
(3-12)

To obtain s quark decay rate, the effective Wilson

coefficients of the tree and penguin decay is needed. The effective Wilson coefficients could be defined as follows [7]:

$$C_{i}^{eff}(\mu) = Z_{i}^{eff}(\mu) + \tau y_{i}^{eff}(\mu)$$
(3-13)

Table 2 presents the calculated values for the Wilson coefficients of the s quark and  $\overline{s}$  anti-quark decay.

Table 2. The effective Wilson Coefficients at the Renormalization Scale  $\mu = 1 GeV$ 

	$oldsymbol{C}_{i}^{e\!f\!f}\left(oldsymbol{NDR} ight)$	$oldsymbol{C}_{i}^{e\!f\!f}\left(oldsymbol{H}oldsymbol{V} ight)$
$C_1^{\rm eff}$	1.718	1.713
$C_2^{\rm eff}$	-1.113	-1.110
$C_3^{\rm eff}$	0.0320834 - 0.0000335139i	0.0320817 - 0.0000328436i
$C_4^{\rm eff}$	-0.0810884 + 0.0000355247i	-0.0840884 + 0.0000355247i
$C_{\rm 5}^{\rm eff}$	$0.024005 - 2.01083 \times 10^{^{-6}}i$	$0.0250033 - 1.34056 \times 10^{^{-6}}i$
$C_6^{e\!f\!f}$	-0.0862669 + 0.000107244 i	-0.0862302 + 0.0000924984i

s quark and  $\overline{s}$  anti-quark decay rates in tree and penguin level by the use of effective Hamiltonian theory are presented in table 3.

Table 3. s Quark and  $\overline{s}$  Anti-Quark Decay Rates in Tree and Penguin Level.

Decay Process	$oldsymbol{\Gamma}\left( imes 10^{-20} oldsymbol{GeV} ight)$
$s \rightarrow u d \overline{u}$	$8.7811^{+4.0040}_{-2.3534}$
$\overline{s} \to \overline{u}\overline{d}u$	$8.7811^{+4.0040}_{-2.3534}$

the

and

#### 4. Calculation of Branching Ratio

As it is known, most of the particles could decay in many different ways. For i decay modes, the branching ratio is defined as follows:

$$BR = \frac{\Gamma_i}{\Gamma_{tot}} \tag{4-1}$$

The branching ratio of the quark decay mode is defined as, quark decay rate of each mode to the summation rates of semi-lepton and non-lepton decays. For example, there is:

$$BR_{s \to ud\overline{u}} = \frac{\Gamma_{s \to ud\overline{u}}}{\Gamma_{SL} + \Gamma_{NL}}$$
(4-2)

Where  $\Gamma_{SL}$  is the summation of semi-leptonic transition and  $\Gamma_{NL}$  is the non-leptonic transition. The total decay rate is given by:

$$\Gamma_{tot} = \Gamma_{SL} + \Gamma_{NL} = (31.4176) \times 10^{-20} GeV$$

We observe *s* Quark and  $\overline{s}$  anti-quark decays, branching ratios that are calculated and summarized in Table 4 in which the  $\Gamma_{sL}$  and  $\Gamma_{NL}$  values are taken from tables 1 and 3, respectively. These calculated values are compared with experimental BR values for *K* mesons decays, related to  $s \rightarrow u d\overline{u}$ ,  $\overline{s} \rightarrow \overline{u} \overline{d} u$ ,  $s \rightarrow u \mu \overline{\vartheta}_{\mu}$  and  $s \rightarrow u e \overline{\vartheta}_{e}$  transitions. Through comparison, it could be concluded that the theoretical values are in good agreement with the experimental values.

*Table 4.* Experimental Values of the Branching Ratio of Several K Meson Decays and Comparison with Theoretical Values.

Decay Process	Decay Process	$\left( \boldsymbol{B} \boldsymbol{R} \right)_{our}$	$\left( oldsymbol{BR}  ight)_{ m exp}$ [9]
$s \to u d \overline{u}$	$K^- \to \pi^- \pi^\circ$	$27.94^{-1.99}_{+1.94}\%$	$20.68 \pm 0.13\%$
$\bar{s} \to \bar{u}  \mu^+ \vartheta_\mu$	$K^{+} \rightarrow \pi^{\circ} \mu^{+} \vartheta_{\mu}$	$5.56_{-1.44}^{+1.65}\%$	$3.25\pm0.04\%$
$\bar{s}\to \bar{u}e^+\vartheta_e$	$K^+ \to \pi^\circ e^+ \vartheta_e$	$6.47^{+1.06}_{-1.14}\%$	$5.08\pm0.05\%$
$\overline{s} \to \overline{u} \overline{d} u$	$K^+ \to \pi^+ \pi^\circ$	$27.94^{\scriptscriptstyle -1.99}_{\scriptscriptstyle +1.94}\%$	$20.68\pm0.13\%$
$s \to u \mu^- \overline{\vartheta}_\mu$	$K^- \to \pi^\circ \mu^- \vartheta_\mu$	$5.56^{+1.65}_{-1.44}\%$	$3.25\pm0.04\%$
$s \to u e^- \overline{\vartheta}_e$	$K^- \to \pi^\circ e^- \vartheta_e$	$6.47^{+1.06}_{-1.14}\%$	$5.08\pm0.05\%$

<b>Table 5.</b> Theoretical Values for the CP Violation in $s$ Quark De	cays.
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Decay Process	Decay Process	$a_{_{cp}}$
$s \rightarrow u d \overline{u}$	$K^-  ightarrow \pi^- \pi^\circ$	0
$\overline{s} \to \overline{u}\overline{d}u$	$K^+ \to \pi^+ \pi^\circ$	0

Now applying the asymmetric relationship:

$$a_{cp} = \frac{\Gamma_{\overline{s} \to \overline{q}_i \overline{q}_k q_j} - \Gamma_{s \to q_i q_k \overline{q}_j}}{\Gamma_{\overline{s} \to \overline{q}_i \overline{q}_k q_j} + \Gamma_{\overline{s} \to q_i q_k \overline{q}_j}}$$
(4-3)

And by using  $\Gamma_{EH}$  values listed in Table 3, violations are calculated. Results are presented in Table 5.

#### 5. Conclusion

By using the effective Lagrangian density of the weak interaction,  $s \rightarrow q_i q_k \overline{q}_j$  decay rate is calculated in tree level. Furthermore, decay rates of S quark –anti quark are calculated in tree and penguin level by the use of the effective Hamiltonian Theory.

Comparison between  $\Gamma_{EH}$  Values in Table 3 and the results in Table 1 shows that the penguin contributions in the quark decays are small.

Comparing the S quarks decay branching ratio without non-perturbative inclusion in table 4, it could be concluded that findings from this study are in a good agreement with the experimental values.

Considering Table 5, it could be realized that, since  $K^+$  and  $K^-$  are not anti-symmetric, the *CP* violation does not happen. In  $K \to \pi\pi^\circ$  decays, *CP* conservation is observed, which is in agreement with experiment given in reference [11].

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