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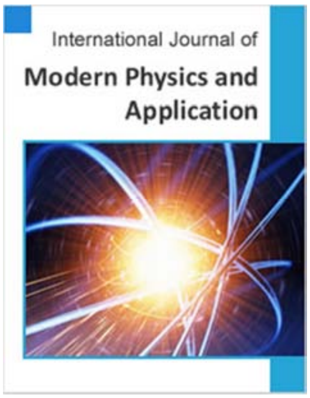

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Possibility of Forming Bistable Optical Vortex in Graded-Index Nonlinear Kerr Medium

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Abstract
We study the vortex spatiotemporal optical solitons in graded-index kerr medium. We consider the propagation of a pulse in inhomogeneous dispersive nonlinear optical fibers and we investigate the possibility of forming bistable spatiotemporal vortex solitons. We solve the multidimensional nonlinear Schrödinger equation by using a variational approach. The results show that vortex bistable spatiotemporal solitons can be stabilized under certain conditions.

1. Introduction

The existence of soliton waves for the first time in 1834 by an engineer named Scott Russell was reported. He was the designer of small ships. He observed waves in the Union shallow channel that the long journey had maintained that were later called solitons [1]. The first mathematical answer for solitons waves from solving of Kortweg-de vries (KDV) equation was obtained in 1964 by Zabusky and Kruskal [2]. It should be noted that this equation has a major application in plasma. Later in 1973, Tappert discovered nonlinear Schrödinger equation (NLS) [3]. Hasagawa and Tappert can be find answers for this equation. Later, Mollenauer observed solitons in the Optical fiber for the first time in 1980 [4].

Studies shown that the solitons as information carriers in optical fibers cause data is transferred with better quality. Therefore the communication optical systems based on studying of solitons are important. Today, we try to have the communication systems based on optics and the science of photonics searching for ways to replace electronic circuits to all optical circuits [5]. So it is the importance of studying solitons more than ever and in this regard optical vortex solitons for all digital logic have been useful [6-7].

In a light wave, the phase singularity is known to form an optical vortex: The energy flow rotates around the vortex core in a given direction; at the center, the velocity of this rotation would be infinite and thus the light intensity must vanish. The study of optical vortex is important from the viewpoint of the fundamental and applied physics. The unique nature of vortex fields is expected to lead to applications in many areas that include optical data storage, distribution, and processing and also optical trapping of particles in a vortex field [8].

In physics, wave propagation is traditionally analyzed by means of regular solutions of wave equations. However, solutions of wave equations in two and three dimensions often possess singularities, that is, points or lines in space at which or change abruptly [8]. For example, at the point of phase singularity, the phase of the wave is undefined and wave intensity vanishes. Waves that possess a phase singularity and a rotational flow around the singular point are called vortices. They can be found in the physical systems of different nature and scale, ranging from water whirlpools and atmospheric tornadoes to quantized vortices in super fluids and quantized lines of magnetic flux in superconductors [9]. They may be useful for creating an optical rotary switch based on the concept of vortex solitons [10-11].

When a pulse traveling from a medium the dispersion and diffraction effects affect the pulse propagation. These phenomena may be compensated with the self-focusing and self-phase modulation respectively. In this case the pulse will propagate without change. The pulses that their shape remains intact in time or in one space dimension so called temporal or spatial solitons respectively [12]. If the shape of the pulses simultaneously in time and space remain constant well known as spatiotemporal solitons [13]. If the input pulse contains vortex we will also have the vortex solitons [10]. Vortex solitons are extending of a special type of soliton (dark solitons) in two-dimensional space. Optical vortex solitons in a nonlinear medium, firstly in 1992, using self-defocusing nonlinearity were experimentally observed. Few years later were studied by Prof. Yuri. Kivshar and Luther. Davies again [10].

2. Theory and Formalism

We consider the propagation of a pulse in the medium with Kerr nonlinearity unlike the studied cases in order to study the optical vortex solitons in a nonlinear medium such as fiber. We analyze the possibility of forming the vortex spatiotemporal optical solitons in graded index Kerr medium, for which the refractive index is of the form

$$n(r.\omega) = n_0(\omega) + n_1(x^2 + y^2) + n_2|E|^2, \quad (1)$$

Where the homogeneous part $n_0(\omega)$ takes into account chromatic dispersion, n_2 is the nonlinear parameter responsible for self-focusing or self-defocusing, and n_1 governs the change in refractive index in the transverse dimensions x and y. Light is assumed to propagate along the z axis. The medium can be guiding $n_1 > 0$ or anti-guiding $n_1 < 0$.

Our analysis begins with Maxwell's equation, supplemented by equation (1). We introduce the envelope $A'(r,t)$ of the electric field oscillating at the frequency ω_0 as

$$E(r,t) = \frac{1}{2}\hat{e}A'(r,t)\exp[\mathrm{i}(\beta_0 z - \omega_0 t)] + c.c. \quad (2)$$

Using a standard procedure and making the paraxial and the slowly varying envelope approximations, we obtain the following equation for the envelope of the electric field, $A'(r,t)$:

$$i(\frac{\partial A'}{\partial z} + \beta_1\frac{\partial A'}{\partial t}) - \frac{\beta_2}{2}\frac{\partial^2 A'}{\partial t^2} + \frac{1}{2\beta_0}\nabla_\perp{}^2 A' + n_1 k_0 r^2 A' + n_2 k_0|A'|^2 A' = 0, \quad (3)$$

where $\beta_0 = n_0(\omega_0)k_0$, $\beta_1 = (\mathrm{d}\beta'/\mathrm{d}\omega)_{\omega=\omega_0}$, and $\beta_2 = (\mathrm{d}^2\beta'/\mathrm{d}\omega^2)_{\omega=\omega_0}$, with $\beta' = n_0(\omega_0)\omega/c$, and $k_0 = \omega_0/c$. The gradient operator $\nabla_\perp$, also acts on transverse coordinates. To normalize equation (3), we introduce a transverse length scale $h_0 = (2k_0|n_1|\beta_0)^{-1/4}$ and scale the transverse coordinates as $(X,Y) = (x,y)/h_0$. Similarly, we introduce a longitudinal length scale using the diffraction length, $L_d = \beta_0 h_0{}^2$, and scale longitudinal coordinate as $Z = z/L_d$. We also introduce a scaled local time $\tau = (t - \beta_1 z)/T_0$ where $T_0 = \sqrt{|\beta_2|L_d}$. In terms of these normalized variables, equation (3) takes the form

$$i\frac{\partial U}{\partial Z} + \frac{1}{2}(\nabla^2 U) + \frac{\delta}{2}\frac{\partial^2 U}{\partial\tau^2} - \frac{s}{2}r^2 U + \upsilon|U|^2 U = 0, \quad (4)$$

where $U(\mathrm{X,Y,Z},\tau) = \sqrt{k_0 n_2 L_d}\,A'(x,y,z,t)$, and the parameters $\delta = sign(\beta_2) = \pm 1$, $s = sign(n_1) = \pm 1$, and $\upsilon = sign(n_2) = \pm 1$, according as whether the medium has anomalous or normal group velocity dispersion (GVD), is guiding or anti-guiding, and is self-focusing or self-defocusing respectively. Corresponding lagrangian [10]

$$L = -\mathrm{Im} < U^*\frac{\partial U}{\partial z} > + < \frac{1}{2}|\nabla U|^2 + \frac{\delta}{2}\left|\frac{\partial U}{\partial\tau}\right|^2 + \frac{s}{2}r^2|U|^2 - \frac{\upsilon}{2}|U|^4 \quad (5)$$

To describe the stationary solutions carrying orbital momentum or vortex optical solitons, we adopt the following ansatz.

$$U(X,Y,Z,\tau) = \sqrt{\varepsilon/2\pi m!a^{2m+2}w}\,\mathrm{sech}(\tau/w)\,\mathrm{r}^{\mathrm{m}}\exp(-\frac{\mathrm{r}^2}{2a^2} + i(m\theta + \alpha r + \beta\tau^2 + \phi)), \quad (6)$$

here $\varepsilon \equiv <|U|^2>$, $\alpha(Z), \beta(Z)$ is the wave front curvature and $\phi(Z)$ is the free phase. We introduce the cylindrical coordinates in the transverse plane, $r = \sqrt{X^2 + Y}^2$ and $\theta = \tan^{-1}\mathrm{Y/X}$, we also introduce m defined by the circulation of the phase gradient around the singularity is an integer so called topological charge.

Applying standard variational approach [14] we obtain the first pair of equations, $d\omega/dZ = 2\beta\delta\omega$ and $da/dZ = 2a\alpha$. Using this result, the other variational equations (not shown), can be reduced, after some algebra, to the set of two coupled equations for the transverse width a and the pulse duration w,

$$\frac{d^2 a}{dZ^2} = -sa + \frac{1}{a^3} - \frac{2}{3}\frac{\upsilon\tilde{\varepsilon}}{(m+1)a^3\omega} \equiv I(a,\omega), \quad (7)$$

$$\frac{d^2\omega}{dZ^2} = \frac{4}{\pi^2}\left(\frac{1}{\omega^3} - \frac{\upsilon\delta\tilde{\varepsilon}}{\omega^2 a^2}\right) \equiv J(a,\omega), \quad (8)$$

here $\tilde{\varepsilon} = \varepsilon(2m)!/2^{2m+2}\pi(m!)^2$. The first term on the right hand sides of equations (7) and (8) represent the diffraction and dispersion, respectively, while the last term, proportional to the pulse energy, show the contribution of nonlinearity.

3. Discussion and Results

Soliton solutions a_0 and w_0 can be found in the stationary limit of equations (7) and (8) with left-hand sides vanishing. When s=0 in equation (4), the transversely localized solutions, $a_0 < \infty$, exit for $\upsilon = +1$ only, while for s=1 the field is trapped in the waveguide irrespectively of the sign of υ. In particular, for $\varepsilon = 0$, the linear guided mode with a_0=1 appears as the cw solution with $\omega_0 = \infty$. In contrast, the soliton solution pulses of final width, $\omega_0 < \infty$, exist only if $\delta\upsilon = +1$. We obtain $\omega_0 = a_0{}^2/\delta\upsilon\tilde{\varepsilon}$ and the solution is given by the positive roots of

$$sa'^3 - a' + \gamma = 0, \quad (9)$$

here $a' = a_0{}^2$ and $\gamma = 2\delta\tilde{\varepsilon}^2/3(m+1)$. For s=0, the so-called "spinning light bullets" [15] appear as the only stationary with $\delta = \upsilon = +1$ and $a_0{}^2 = 2\tilde{\varepsilon}^2/3(m+1)$. For s=1 the cubic polynomial equation (14) has no positive solutions for $\gamma > 2/3\sqrt{3}$, and the first solution, $a_0{}^2 = 1/\sqrt{3}$, appears at $\gamma = 2/3\sqrt{3}$. Further decreasing the pulse energy, $0 < \gamma < 2/3\sqrt{3}$, bring two solitons, i.e., the solliton became bistable. Then, passing through the linear guided mode at $\gamma = 0$ and inverting the sign of nonlinearity, $\gamma < 0$, we obtain single solution.

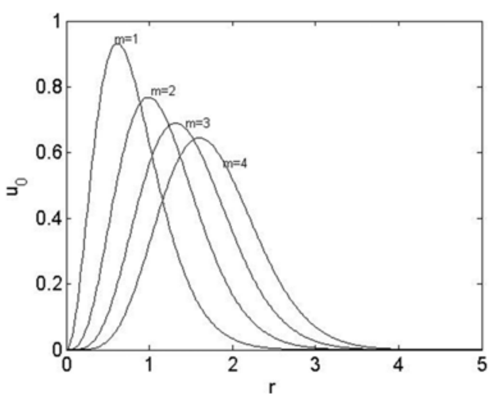

Figure 1. *The profile of the vortex spatiotemporal optical solitons versus radius coordinate for different topological charges.*

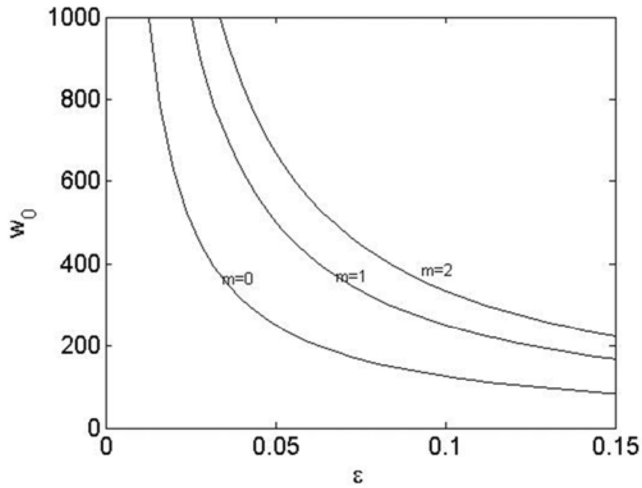

Figure 2. *The pulse duration versus the energy for different topological charges in self-focusing medium* $\delta = \upsilon = +1$.

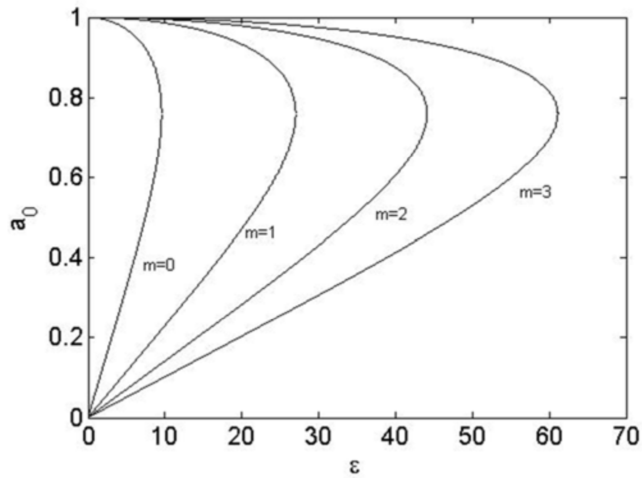

Figure 3. *The spatial width versus the energy for different topological charges in self-focusing medium* $\delta = \upsilon = +1$.

Figure 1 shows the vortex spatiotemporal optical solitons versus radius coordinate of studied medium for example an optical fiber for the different topological charges (m). For one of the topological charges, By decreasing radius coordinate *r,* the intensity of the vortex spatiotemporal optical solitons decreased and limit zero in r=0 which is corresponding to the phase singularity and the vortex spatiotemporal optical soliton core, as it is shown from figure 1. Also it is clearly shown in this figure, the intensity distribution similar to donut as same as optical vortex solitons for one of the topological charges. It can be observed that, for the higher order of *m*, the intensity of vortex spatiotemporal optical soliton increased.

We plot in figure 2 the change of the pulse duration versus the pulse energy for different order of topological charges (m). It is obvious that the threshold of the energy of pulse increasing by increasing of the order of topological charges. Also, the pulse duration decreasing with increasing to the energy of pulse for given order of topological charge. Figure 3 shows the pulse width versus the pulse energy for different order of topological charges. As in figure 3 can be seen by taking each of the topological charge, for the characteristic energy pulse two answers for pulse width. We have included the concept of bistable vortex spatiotemporal optical solitons in nonlinear self-focusing kerr medium.

4. Conclusion

We assume the propagation of a pulse in the graded index medium with Kerr nonlinearity in order to study the optical vortex solitons in a nonlinear fiber unlike the studied cases. We study the vortex spatiotemporal optical solitons in nonlinear media and analyze the possibility of forming the optical vortex solitons in such media. We see forming the optical vortex solitons from the profile of the electric field of incident pulse versus radius coordinate of the graded index fiber with Kerr nonlinearity. We consider the multi-dimensional nonlinear Schrödinger equation and we solve this equation by variational method. The result shown that formation of bistable vortex spatiotemporal optical solitons in self-focusing medium depending on the amount of pulse energy. So that, there are two values for pulse width in a specified amount of pulse energy for each of topological charge.

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