Non-linear Propagation of Ion-Acoustic Solitary and Shock Waves in Three Component Plasmas with Nonthermal and Trapped Electrons


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Citation

Abstract
An attempt have been taken to study ion-acoustic (IA) solitary and shock wave in theoretically in three component electron-ion plasmas. To do this, a non-linear propagation of IA solitary wave have been considere d in unmagnetized plasmas containing mobile positively charged cold inertial ions, negatively charged Maxwellian ions with non-thermal and trapped electrons respectively. The shock wave have also been studies for above system. The well-established reductive perturbation method has been employed to derived standard solitary and shock wave equation. The solutions was also derived to study their characteristic behaviour with parametric regims.

1. Introduction

Ion acoustic solitary waves in unmagnetized plasma have been studied by a number of authors both experimentally and theoretically. Washimi and Taniuti [1] have studied the propagation of ion-acoustic solitary waves of small amplitude. Kalita and Kalita [2] have studied mK-dV solitons in a warm plasma with negative ions. Propagation of ion-acoustic solitary waves in a warm plasma with negative ions under the drifting effect of electrons are considered by Kalita and Devi [3]. Mishra et al. [4] studied the obliquely propagating ion-acoustic solitons in a multicomponent magnetized plasma consisting of warm adiabatic positive and negative ion species and hot isothermal electrons. Haider et al. [5] have studied the nonlinear propagation of multi-ion acoustic solitary waves Maxwellian, [6, 7] trapped [8, 9] and nonthermal [10, 11] distributed electrons. In recent few years, the study of Korteweg-de Vries (K-dV) and modified K-dV (mK-dV) ion-acoustic solitons in a multispecies plasma consisting of positive ions, electrons and negative ions is a field of current research investigation. Nakamura and Tsukabayashi [12] have studied experimentally the propagation of ion-acoustic solitons in a plasma with negative ions. Experiments on the propagation of ion acoustic solitons that propagate in a positive ion-negative ion plasma are described by Cooney et al. [13]. At a certain critical negative ion concentration, the coefficient of the nonlinear term in the K-dV equation vanishes.
Therefore, to discuss the soliton solution at the critical concentration, by considering the higher order nonlinearity, the mK-dV equation has been derived for this case.

Recently, Mamun et al. [14, 15] and Duha [16] have considered ion-acoustic shock waves associated with the dynamics of negative ions in a multi-ion dusty plasma containing electrons, light positive ions, heavy negative ions, and extremely massive charge fluctuating stationary dust. Haider have studied the soliton and shock profiles in degenerate plasmas [17] and multi-dimensional instability of solitary structure with opposite polarity ions and non-thermal electrons [18]. Rahman [19] has studied the effect of super-thermal electrons in solitary and shock waves in four component unmagnetised plasmas considering positive ions as mobile and negative ions as Maxwellian with static positive dust. On the other hand, Haider and Nahar [20] studied the solitary and shock structures in multi-ion plasmas with super-thermal electrons.

In the present work, the propagation of IA solitary and shock structures have been studied in unmagnetized plasma consisting of mobile positive ions, Maxwellian distributed negative ions with nonthermal and trapped electron. The reductive perturbation method [1] has been employed to derive the solitary and shock wave structures.

The manuscript is organized as follows. The basic equations are given in Sec. 2. The solitary waves are studied for nonthermal and trapped electrons by deriving K-dV and mK-dV equations using reductive perturbation method in Sec. 3. The shock waves also studied for nonthermal and trapped electrons by deriving K-dV Burger and mK-dV Burger equations in Sec. 4. In Sec. 5 numerically studied the parametric regimes of above findings and a brief discussion has been given in Sec. 6.

2. Basic Equations

The non-linear propagation of IA solitary and shock waves have been considered in a one-dimensional, collisionless, unmagnetized electron-ion. It is assumed that

1) Positive ions are mobile.
2) Negative ions follow the Maxwellian distribution.
3) Electrons are nonthermal and trapped.

The dynamics of the ion-acoustic waves in one dimensional normalized form whose phase speed is between ion thermal speed \((v_p)\) and electron thermal speed \((v_e)\) \((i.e., v_e < V_p < a_n)\); is governed by

\[
\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p u_p) = 0 \tag{1}
\]

\[
\frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial x} = -\frac{\partial \varphi}{\partial x} + \eta \frac{\partial^2 u_p}{\partial x^2} \tag{2}
\]

\[
\frac{\partial^2 \varphi}{\partial x^2} = \mu_e n_e + \mu_p n_p - n_p \tag{3}
\]

Where \(n_p\) is the positive ion number density normalized by its equilibrium value \(n_{p0}\), \(u_p\) is the positive ion fluid speed normalized by \(C_p = (K_B T_e / m_p)^{1/2}\) with \(K_B\) is the Boltzmann constant, \(T_e\) is the temperature of electrons and \(m_p\) is the rest mass of positive ions. \(\varphi\) is the IA wave potential normalized by \(K_B T_e / e\), with \(e\) being the magnitude of the charge of the electron. The time variable \((t)\) is normalized by \(\omega_{pe}^{-1} = (4\pi n_0 \epsilon^2 / m_e)^{1/2}\) with \(c\) being the speed of light. The space variables are normalized by Debye radius \(\lambda_D = (K_B T_e) / 4\pi n_{p0} e^2\). The viscous term, i.e. coefficients of viscosity \((\eta)\) has been considered zero at the time of studying solitary waves.

Now, using equilibrium charge neutrality condition \(n_e + n_0 = n_{p0}\). One can write \(\mu_e = 1 - \mu_n\) where, \(\mu_e = n_e / n_{p0}\) and \(\mu_n = n_0 / n_{p0}\).

3. Solitary Waves

3.1. Nonthermal Electrons

The nonthermal electron distribution of Cairns et al. [21] is a more general class of the electron distribution including a population of fast or energetic electrons. The nonthermal electron \(n_e\) can be written as

\[
n_e = [1 - \alpha \varphi + \alpha (\varphi)^2] e^{\sigma_p} \tag{4}
\]

where \(\alpha = \frac{4\gamma}{1 + 3\gamma}\) with, \(\gamma\) is a parameter determining the fast particles present in this plasma model.

Maxwellian electron distribution can be express as

\[
n_e = e^{(\epsilon_\sigma_p \varphi)} \tag{5}
\]

where, \(\sigma_p\) is the temperature ratio of electron to negative ions.

Introducing independent variable through the stretched coordinates [22, 23, 24, 25], to follow the reductive perturbation technique to construct a weakly non-linear theory for the electrostatic waves with a small but finite amplitude, as

\[
x = e^{1/2} (x - v_p t) \tag{6}
\]

\[
t = e^{3/2} t \tag{7}
\]

where \(\epsilon\) is a small parameter measuring the weakness of the dispersion and \(v_p\) is the unknown wave phase speed (to be determined later) is normalised by the ion-acoustic speed \((C_p)\).
Figure 1. $A=0$ surface plot for nonthermal distributed electrons. Variation of $\alpha$ with respect to $\mu_e$ and $\mu_n$ for $\sigma_p=1$ and $u_0=0.1$.

The perturbed quantities can be expanded about their equilibrium values in powers of $\varepsilon$ as

\[
\begin{align*}
n_p^{(1)} &= 1 + \varepsilon n_p^{(1)} + \varepsilon^2 n_p^{(2)} + \ldots \\
u_p^{(1)} &= 0 + \varepsilon u_p^{(1)} + \varepsilon^2 u_p^{(2)} + \ldots \\
\phi &= 0 + \varepsilon \phi^{(1)} + \varepsilon^2 \phi^{(2)} + \ldots
\end{align*}
\]

Using the stretched coordinates and (6) in (1)-(3) and equating the coefficient of $\varepsilon^2$ from the continuity and momentum equation and coefficients of $\varepsilon$ from Poisson's equation, one can obtain the first order continuity, momentum and Poisson's equation as

\[
\begin{align*}
u_p^{(1)} &= \frac{\phi^{(1)}}{v_p} \\
n_p^{(1)} &= \frac{\phi^{(1)}}{v_p} \\
\phi^{(1)} &= [\mu_n(1-\alpha) + \mu_p\sigma_p] \phi^{(1)}
\end{align*}
\]

Comparing (8) and (9), the linear dispersion relation can be written as

\[
v_p = \frac{1}{\sqrt{\mu_n(1-\alpha) + \mu_p\sigma_p}}
\]

Figure 2. Variation of the amplitude of solitary waves ($\phi_m$) for nonthermal distributed electrons with respect to $\sigma_p$ and $\alpha$ considering $\mu_n=2.5$, $\mu_p=2$ and $u_0=0.1$. 

To the next higher order of $\epsilon$, i.e. equating the coefficients of $\epsilon^2$ from continuity and momentum equation and coefficients of $\epsilon^2$ from Poisson's equation, one can write respectively,

\begin{align}
-v_p^2 \frac{\partial n_p^{(2)}}{\partial \xi} + \frac{\partial u_p^{(1)}}{\partial \tau} + \frac{\partial u_p^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi} (n_p^{(1)} u_p^{(1)}) &= 0 \quad (11) \\
-\frac{\partial u_p^{(1)}}{\partial \tau} - v_p \frac{\partial u_p^{(2)}}{\partial \xi} + u_p^{(1)} \frac{\partial u_p^{(1)}}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} &= 0 \quad (12) \\
\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} &= \frac{1}{v_p^2} \phi^{(2)} - n_p^{(2)} + \frac{1}{2} [\phi^{(1)}]^2 [\mu_e + \sigma_p^2 \mu_i] \quad (13)
\end{align}

Now using (11)-(13), K-dV equation can be readily obtained as

\begin{align}
\frac{\partial \phi^{(1)}}{\partial \tau} + A \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial \xi} + B \frac{\partial^3 \phi^{(1)}}{\partial \xi^3} &= 0 \quad (14)
\end{align}

where, nonlinear and dissipation coefficients respectively are

\begin{align}
A &= \frac{3}{2v_p^2} \left( \mu_e + \sigma_p^2 \mu_i \right) v_p^3 \\
B &= \frac{v_p^2}{2}
\end{align}

Transforming the independent variables $\zeta$ and $\tau'$ to $\zeta' = \xi - u_0 \tau'$, $\tau' = \tau$ (where $u_0$ is the constant SW velocity), to obtain a stationary localized solitary wave solution of this K-dV equation, and making some mathematical calculation under appropriate boundary conditions, viz. $\phi \to 0$ and $\frac{d^2 \phi}{d \xi^2} \to 0$ at $\xi \to \pm \infty$, the stationary solitary wave solution of the K-dV equation can be found out as

\begin{align}
\phi = \phi_m \text{sech}^2 \left[ \frac{\zeta}{\Delta} \right] \quad (17)
\end{align}

where, amplitude of the solitary waves

\begin{align}
\phi_m = \left( \frac{3u_0}{A} \right) \quad (18)
\end{align}

and width of the solitary waves

\begin{align}
\Delta = \frac{1}{2} \left( \frac{4B}{u_0} \right) \quad (19)
\end{align}

**Figure 3.** Variation of the width of solitary wave ($\Delta$) for nonthermal distributed electrons with respect to $\sigma_p$ and $\alpha$ considering $\mu_i = 2.5$, $\mu_e = 2$ and $u_0 = 0.1$.

### 3.2. Trapped Electrons

The trapped electron distribution [26] can be represent as

\begin{align}
n_e = n_0 [1 + \varphi - b(\varphi)^{\frac{3}{2}} + \frac{1}{2}(\varphi)^2] \quad (20)
\end{align}

Where, $b = \frac{4(1-\gamma_2)}{3\sqrt{\pi}}$ with, $\gamma_2$ is a parameter determining the number of trapped electrons. Maxwillan electron distribution express as

\begin{align}
n_m = \epsilon^{\beta \varphi} \quad
\end{align}

where, $\beta$ is the temperature ratio of positive ions to electron.
Introducing independent variable through the stretched coordinates [22, 23, 24, 25], to follow the reductive perturbation technique to construct a weakly non-linear theory for the electrostatic waves with a small but finite amplitude, as

$$\xi = \epsilon^{1/4} (x - v_\epsilon t)$$  \hspace{1cm} (21)

$$\tau = \epsilon^{3/4} t$$  \hspace{1cm} (22)

The perturbed quantities can be expanded about their equilibrium values in powers of $\epsilon$ as

$$n_p = 1 + \epsilon n_p^{(1)} + \epsilon^2 n_p^{(2)} + ...$$

$$u_p = 0 + \epsilon u_p^{(1)} + \epsilon^2 u_p^{(2)} + ...$$

$$\varphi = 0 + \epsilon \varphi^{(1)} + \epsilon^2 \varphi^{(2)} + ...$$  \hspace{1cm} (23)

Using the stretched coordinates and (23) in (1)-(3) and equating the coefficient of $\epsilon^{3/2}$ from the continuity and momentum equation one can obtain $u_p$ and $n_p$ as in (7) and (8) respectively and equating the coefficients of $\epsilon$ from Poissons equation the linear dispersion relation can be written as

$$v_0 = \sqrt{\frac{1}{(\mu_e + B \mu_i)}}$$  \hspace{1cm} (24)

To the next higher order of $\epsilon$, i.e. equating the coefficients $\frac{7}{4} \epsilon^4$ from continuity and momentum equation and coefficients $\frac{3}{4} \epsilon^4$ of $\epsilon^2$ from Poissons equation, mKdV equation can be readily obtained as

$$\frac{\partial \varphi^{(1)}}{\partial \tau} + A \sqrt{\varphi^{(1)}} \frac{\partial \varphi^{(1)}}{\partial \xi} + B \frac{\partial^3 \varphi^{(1)}}{\partial \xi^3} = 0$$  \hspace{1cm} (25)

where nonlinear coefficient

$$A = \frac{(1 - \gamma_3)}{\sqrt{\pi}} - \mu_e v_0$$  \hspace{1cm} (26)

and dissipation coefficient $B$ is the same as (16).

Under appropriate boundary conditions the stationary solitary wave solution of the mK-dV equation is

$$\varphi = \varphi_m \text{sech}^4 \left[ \frac{(\xi - u_0 \tau)}{\Delta} \right]$$  \hspace{1cm} (27)

where, amplitude of the solitary waves

$$\varphi_m = \left( \frac{15 u_0}{8 A} \right)^2$$  \hspace{1cm} (28)

and width of the solitary waves

$$\Delta = \frac{16 B}{u_0}$$  \hspace{1cm} (29)

Figure 4. Variation of the amplitude of solitary wave ($\varphi_m$) for the case of nonthermal distributed electrons with respect to $\mu_e$ and $\mu_i$ considering $\sigma_y = 1$, $\alpha = 0.5$ and $u_i = 0.1$. 
4. Shock Waves

4.1. Nonthermal Electrons

Introducing stretched co-ordinates in reductive perturbation method to obtain K-dV Burger equation, as

\[ \xi = \varepsilon (x - v_p l) \]

\[ \tau = \varepsilon^2 t \]

and expanding the perturbed quantities about their equilibrium values in powers of \( \varepsilon \) as in (23) and equating the coefficients of the lowest order of \( \varepsilon^2 \) and \( \varepsilon \) from the continuity, momentum, and Poisson’s equation, one can obtain the linear dispersion relation are found similar as solitary waves as in (10).

To the next higher order of \( \varepsilon \), i.e. equating the coefficient of \( \varepsilon^3 \) from continuity and momentum equation coefficients of \( \varepsilon^2 \) from Poisson’s equation, one can write, respectively,

\[ \frac{\partial u_p^{(1)}}{\partial \tau} - v_p \frac{\partial u_p^{(2)}}{\partial \xi} + \frac{\partial}{\partial \xi} \left( n_p^{(1)} u_p^{(1)} \right) + \frac{\partial u_p^{(2)}}{\partial \xi} = 0 \]

\[ \frac{\partial u_p^{(1)}}{\partial \tau} - v_p \frac{\partial u_p^{(2)}}{\partial \xi} + u_p^{(1)} \frac{\partial u_p^{(1)}}{\partial \xi} = - \frac{\partial \varphi^{(2)}}{\partial \xi} + \eta \frac{\partial^2 u_p^{(1)}}{\partial \xi^2} \]

\[ n_p^{(2)} = \frac{1}{v_p} \varphi^{(2)} + \frac{1}{2} \left( \varphi^{(1)} \right)^2 (\mu_e + \sigma_p^2 \mu_n) \]

Now, using (32)-(34), one can really obtain the K-dV Burger equation as

\[ \frac{\partial \varphi^{(1)}}{\partial \tau} + A \varphi^{(1)} \frac{\partial \varphi^{(1)}}{\partial \xi} - C \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} = 0 \]

where, nonlinear coefficient \( A \) is the same as (15), and

\[ C = \frac{\eta}{2} \]

It can be found out from the above analysis that the nonlinear coefficient \( A \) of the solitary and shock waves are same but dissipation constants are different for the two cases.

Transforming the independent variables \( \zeta \) and \( \tau' \) to \( \zeta = \xi - u_0 \tau', \quad \tau' = \tau \); and imposing the appropriate boundary conditions as in the solitary waves, one can express the stationary solution of the K-dV Burger equation (35) as

\[ \varphi = \varphi_m \left[ 1 + \tanh\left( \frac{\zeta}{\Delta} \right) \right]^2 \]

where, amplitude of the solitary waves

\[ \varphi_m = \left( \frac{u_0}{A} \right)^2 \]

and width of the solitary waves

\[ \Delta = \frac{2C}{u_0} \]

4.2. Trapped Electrons

Introducing stretched co-ordinates in reductive perturbation method to obtain mK-dV Burger equation, as

\[ \xi = \varepsilon^{1/2} (x - v_p l) \]

\[ \tau = \varepsilon t \]

and considering the first order approximation one can find the linear dispersion relation similar as solitary waves as shown in (24).

To the next higher order of \( \varepsilon \), i.e. equating the coefficient of \( \varepsilon^2 \) from continuity and momentum equation coefficients of \( \varepsilon^2 \) from Poisson’s equation and doing some mathematical calculation one can really obtain the mK-dV Burger equation as

\[ \frac{\partial \varphi^{(1)}}{\partial \tau} + A \sqrt{\varphi^{(1)}} \frac{\partial \varphi^{(1)}}{\partial \xi} - C \frac{\partial^2 \varphi^{(1)}}{\partial \xi^2} = 0 \]

where, nonlinear coefficient \( A \) is same as solitary waves for trapped electrons as shown in (26) and \( C \) is the same as (36).

Using the same procedure one can express the stationary solution of the mK-dV Burger equation (42) as

\[ \varphi = \varphi_m \left[ 1 + \tanh\left( \frac{\zeta}{\Delta} \right) \right]^2 \]

where, amplitude of the solitary waves

\[ \varphi_m = \left( \frac{3u_0}{4A} \right)^2 \]

and width of the solitary waves

\[ \Delta = \frac{4C}{u_0} \]
Figure 5. Variation of the width of solitary wave ($\Delta$) for nonthermal distributed electrons with respect to $\mu_n$ and $\mu_e$ considering $\sigma_e = 1$, $\alpha = 0.5$ and $u_0 = 0.1$.

Figure 6. Variation of the amplitude ($\phi_m$) of solitary wave for trapped distributed electrons with respect to $\mu_n$ and $\mu_e$ considering $\beta = 1$, $\gamma_e = 0.5$ and $u_0 = 0.1$.

5. Numerical Analysis

The effects of nonthermal and trapped electrons in a three-component plasma with positive as well as negative ions have been theoretically studied. It is seen from the above analysis that the amplitude of the solitary and shock waves is proportional to the wave speed $u_0$, for both the cases where the width is inversely proportional to that. Hence the profile of the faster wave will be taller and narrower than slower one.
Equation (15) indicate that \( A \) is independent on \( v_p \), \( \mu_n \), \( \mu_e \) and \( \sigma_p \). Therefore, these parameters are responsible for the solitary and shock waves to be associate with positive and negative potentials. Figure 1 shows the variation of the \( \alpha \) with negative ion concentration \( (\mu_n) \) and electron concentration \( (\mu_e) \) keeping the values \( u_0 = 0.1 \) and \( \sigma_p = 1 \). It is found that \( \alpha \) increases with increasing \( \mu_n \) and decreases with increasing \( \mu_e \). Figure 2 shows the variation of the amplitude \( (\varphi_m) \) with temperature ratio of electron and ion \( (\sigma_p) \) and \( \alpha \) keeping the values \( \mu_e = 2.5 \) and \( \mu_n = 2 \) and \( u_0 = 0.1 \). The amplitude slightly decreases with increasing \( \sigma_p \) and increases with increasing \( \alpha \). Figure 3 shows the variation of the width \( (\Delta) \) with \( \sigma_p \) and \( \alpha \) keeping the values \( \mu_e = 2.5 \) and \( \mu_n = 2 \) and \( u_0 = 0.1 \). It is seen that \( \Delta \) decreases with increasing \( \sigma_p \) and \( \alpha \). Figure 4 shows the variation of the amplitude \( (\varphi_m) \) with \( \mu_n \) and \( \mu_e \) keeping the values \( \sigma_p = 1 \), \( \alpha = 0.5 \) and \( u_0 = 0.1 \). The amplitude \( (\varphi_m) \) decreases with increasing \( \mu_n \) and \( \mu_e \). Figure 5 shows the variation of the width \( (\Delta) \) with \( \mu_n \) and \( \mu_e \) keeping the values \( \sigma_p = 1 \), \( \alpha = 0.5 \) and \( u_0 = 0.1 \) which indicates that the width \( (\Delta) \) decreases with increasing \( \mu_n \) and \( \mu_e \).
6. Conclusion

IA solitary and shock waves has been analysed in an unmagnetized plasma containing positively charged ion fluid with nonthermal and trapped electron and Maxwellian distributed negative ions. The basic features of amplitude and width and temperature effects of electron and ions have been investigated. The results obtained from this investigation can be summarized as follows:

a) The amplitude of the faster solitary and shock waves will be taller and narrower than slower one.
b) Depending on the constant $A$, solitary and shock waves might be associated with positive or negative potentials.
c) The population of nonthermal number density of negative ions, and electron are responsible for producing narrower solitary and shock structures.
d) In the case of solitary waves having nonthermal electron the amplitude ($\varphi_m$) decreases with increasing negative ion concentration ($\mu_n$) and electron concentration ($\mu_e$) and the width ($\Delta$) decreases with increasing $\mu_n$ and $\mu_e$.
e) In the case of solitary and shock waves having trapped electron the amplitude increases with increasing the value of temperature ratio. The amplitude also increases with the increasing negative ion concentration ($\mu_n$) and
decreases with the increasing electron concentration ($\mu_e$).

f) The increasing value of $\beta$ and $\gamma_2$ for trapped electron make the solitary and shock waves more spiky but damped the amplitude.

g) Width of the shock waves is linearly proportional to $\eta$ for both the cases, so width increases with increasing $\eta$ shock waves.

h) The present investigation may helpful for understanding different astrophysical objects and can give a guideline to future researcher in the relevant field.

References