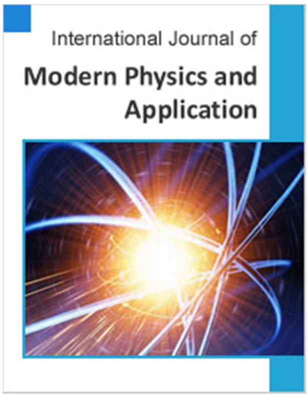
International Journal of Modern Physics and Application
2017; 4(6): 49-54
http://www.aascit.org/journal/ijmpa
ISSN: 2375-3870

American Association for Science and Technology



Thermomagnetic Effects in an External Magnetic Field in the Logarithmic-Quark Sigma Model

Mohamed Abu-Shady

Department of Mathematics, Faculty of Science, Menoufia University, Shebin El-Koom, Egypt

Email address

Dr.abushady@gmail.com

Citation

Mohamed Abu-Shady. Thermomagnetic Effects in an External Magnetic Field in the Logarithmic-Quark Sigma Model. *International Journal of Modern Physics and Application.* Vol. 4, No. 6, 2017, pp. 49-54.

Keywords

Chiral Lagrangian Density,
Magnetic Catalysis,
Mean-Field Approximation

Received: October 16, 2017
Accepted: November 1, 2017
Published: November 16, 2017

Abstract

The phenomenon of magnetic catalysis of chiral symmetry breaking in the quantum chromodynamic theory in the framework of logarithmic quark sigma model is studied. Thermodynamic properties are calculated in the mean-field approximation such as the pressure, the entropy density, the energy density, and the measure interaction. The pressure, the entropy density, and the energy density increase with increasing temperature and/or an external magnetic field. The critical temperature increases with increasing an external magnetic field. In addition, the chiral phase transition is crossover in the presence of an external magnetic field at absent of baryonic chemical potential when explicit symmetry breaking is included. A comparison is presented with the original sigma model and other works. A conclusion indicates that the logarithmic quark model enhances the magnetic catalysis phenomenon.

1. Introduction

The study of the influence of an external magnetic fields on the fundamental properties of quantum chromodynamic (QCD) theory such as the confinement of quark and gluons at low energy and asymptotic freedom at high energy is still a matter of great theoretical and experimental [1]. The transition from composite objects to colored quarks and gluons has several characteristics of both theoretical and phenomenological relevance. One such characteristic is the equation of state which is the fundamental relation encoding the thermodynamic properties of the system [1−4]. So far, most estimates have been carried out at vanishing chemical potential with the aid of effective theories such as the linear sigma model (LSM [5] and the Numbu-Jona-Lasinio (NJL) model [6,7] in the mean field approximation.

The linear sigma model exhibits many of global symmetries of QCD theory. The model was originally introduced by Gell-Mann and Levy [8] with the purpose of describing pion-nucleon interactions. During the last years an impressive amount of work has been done with this model. The idea is to consider it as an effective low energy approach for QCD theory that the model has some aspects of QCD theory such as the chiral symmetry. Thus, the model is successfully to describe the most of hadron properties at low energy such as in Refs. [5,9,10]. Some observables that are calculated in this model are conflict with experimental data. So researchers interest to modify this model to provide a good description of hadron properties such as in Refs. [11−15].

In Ref. [16], the authors have modified the linear sigma model by including the logarithmic mesonic potential and study its effect on the phase transition at finite temperature. In addition, the comparison with other models is done. On the same hand, the logarithmic sigma model successfully describes nucleon properties at finite temperature and chemical potential [17].

To continue the investigation that started in Ref. [18]. In this paper, we investigate the effect of external magnetic field on the thermodynamic properties in the framework of logarithmic quark sigma model at finite temperature and chemical potential. So for no attempts have done to investigate the thermodynamic properties in the framework of the logarithmic sigma model.

This paper is organized as follows: The original sigma model is briefly presented in Sec. 2. Next, the effective logarithmic mesonic potential in the presence of external magnetic field is presented in Sec. 3. The results are discussed and are compared with other works in Secs. 4 and 5, respectively. Finally, the summary and conclusion are presented in Sec. 6.

2. The Chiral Sigma Model with Original Effective Potential

The interactions of quarks via the exchange of $\sigma-$ and $\boldsymbol{\pi}$ - meson fields are given by the Lagrangian density [5] as follows:

$$L(r)=i\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi+\frac{1}{2}\left(\partial_{\mu}\sigma\partial^{\mu}\sigma+\partial_{\mu}\boldsymbol{\pi}.\partial^{\mu}\boldsymbol{\pi}\right)+g\overline{\Psi}\left(\sigma+i\gamma_{5}\boldsymbol{\tau}.\boldsymbol{\pi}\right)\Psi-U_{1}(\sigma,\pi), \tag{1}$$

with

$$U_1(\sigma,\pi)=\frac{\lambda^2}{4}\left(\sigma^2+\boldsymbol{\pi}^2-\nu^2\right)^2+m_\pi^2 f_\pi\sigma, \tag{2}$$

where $U(\sigma,\boldsymbol{\pi})$ is the meson-meson interaction potential and Ψ,σ and $\boldsymbol{\pi}$ are the quark, sigma, and pion fields, respectively. In the mean-field approximation, the meson fields are treated as time-independent classical fields. This means that we replace the power and products of the meson fields by corresponding powers and the products of their expectation values. The meson-meson interactions in Eq. (2) lead to hidden chiral $SU(2)\times SU(2)$ symmetry with $\sigma(r)$ taking on a vacuum expectation value

$$\langle\sigma\rangle=-f_\pi, \tag{3}$$

where $f_\pi=93\,\text{MeV}$ is the pion decay constant. The final term in Eq. (2) included to break the chiral symmetry explicitly. It leads to the partial conservation of axial-vector isospin current (PCAC). The parameters λ^2 and ν^2 can be expressed in terms of f_π, sigma and pion masses as,

$$\lambda^2=\frac{m_\sigma^2-m_\pi^2}{2f_\pi^2}, \tag{4}$$

$$\nu^2=f_\pi^2-\frac{m_\pi^2}{\lambda^2}. \tag{5}$$

3. The Effective Logarithmic Potential in the Presence of Magnetic Field

In this section, the logarithmic mesonic potential $U_2(\sigma,\pi)$ is applied. In Eq. (6), the logarithmic potential is included with the external magnetic field at finite temperature and baryonic chemical potential [2] as follows,

$$U_{eff}(\sigma,\pi)=U_2(\sigma,\pi)+U_{Vaccum}+U_{Matter}+U_{Medium}, \tag{6}$$

where

$$U_2(\sigma,\boldsymbol{\pi})=-\lambda_1^2\left(\sigma^2+\boldsymbol{\pi}^2\right)+\lambda_2^2\left(\sigma^2+\boldsymbol{\pi}^2\right)^2\log\left(\frac{\sigma^2+\boldsymbol{\pi}^2}{f_\pi^2}\right)+m_\pi^2 f_\pi\sigma, \tag{7}$$

In Eq. 7, the logarithmic potential satisfies the chiral symmetry when $m_\pi\to 0$ as well as in the original potential in Eq. 2. Spontaneous chiral symmetry breaking gives a nonzero vacuum expectation for σ and the explicit chiral symmetry breaking term in Eq. 7 gives the pion its mass.

$$\langle\sigma\rangle=-f_\pi \tag{8}$$

where

$$\lambda_1^2=\frac{m_\sigma^2-7m_\pi^2}{12}, \tag{9}$$

$$\lambda_2^2=\frac{m_\sigma^2-m_\pi^2}{12f_\pi^2}. \tag{10}$$

For details, see Refs. [17,18]. To include the external magnetic field in the present model, we follow Ref. [2] by including the pure fermionic vacuum contribution in the free potential energy. Since this model is renormalizable the usual procedure is to regularize divergent integrals using dimensional regularization and to subtract the ultra violet divergences. This procedure gives the following result

$$U_{Vacum} = \frac{N_c N_f\, g^4}{(2\pi)^2}(\sigma^2+\pi^2)^2\left(\frac{3}{2}-\ln\left(\frac{g^2(\sigma^2+\pi^2)}{\Lambda^2}\right)\right), \quad (11)$$

where $N_c = 3$ and $N_f = 2$ are color and flavor degrees of freedom, respectively and Λ is mass scale,

$$U_{Matter} = \frac{N_c}{2\pi^2}\sum_{f=u}^{d}(|q_f|B)^2[\zeta^{(1,0)}(-1,x_f) - \frac{1}{2}(x_f^2 - x_f)\ln x_f + \frac{x_f^2}{4}] \quad (12)$$

In Eq. 12, we have used $x_f = \frac{g^2(\sigma^2+\pi^2)}{(2|q_f|B)}$ and $\zeta^{(1,0)}(-1,x_f) = \frac{d\,\zeta(z,x_f)}{dz}|_{z=-1}$ that represents the Riemann-Hurwitz function, and also $|q_f|$ is the absolute value of quark electric charge in the external magnetic field with intense B.

$$U_{Medium} = \frac{N_c}{(2\pi)^2}\sum_{f=u}^{d}\sum_{k=0}^{\infty}\alpha_k(|q_f|B)\int_{\infty}^{-\infty}dp_z[T\ln\left[1+e^{-\frac{[E_{P,k}(B)+\mu]}{T}}\right] + T\ln\left[1+e^{-\frac{[E_{P,k}(B)-\mu]}{T}}\right]], \quad (13)$$

where $E_{P,k}(B) = \sqrt{P_z^2 + 2k|q_f|B + M^2}$, M is the effective self-consistent quark mass and μ is baryonic chemical potential.

4. Discussion of Results

In this section, the effective potential of the logarithmic sigma model is studied. For this purpose, the effective potential is numerically calculated in Eq. (6). The parameters of the present model are the coupling constant g and the sigma mass m_σ. The choice of free parameters of g and m_σ based on Ref. [2]. The parameters are usually chosen so that the chiral symmetry is spontaneously broken in the vacuum and the expectation values of the meson fields. In this work, two different sets of parameters are considered in order to get high and a low value for sigma mass. The first set is given by $\Lambda = 16.48$ MeV which yields $m_\pi = 138$ MeV and $m_\sigma = 600$ MeV. The second set as the first, yielding $m_\sigma = 400$ MeV. The quantities such as the dimensionless of pressure $\frac{P}{T^4}$, the dimensionless energy density, $\frac{E_n}{T^4}$ and the dimensionless entropy density $\frac{S}{T^3}$ are calculated. These quantities can be readily obtained by the effective potential that defines in Eq. (6), gives the negative pressure $U_{eff}(\sigma,\pi) = -P_n$. Then the net quark number density is obtained from $\rho = \frac{dP_n}{d\mu}$, and the entropy density from $s = \frac{dP_n}{dT}$ while the energy density is $E_n = -P_n + TS + \mu\rho$. These quantities are displayed in Figures (1, 2, 3, 4, 5, and 6). First of all, one observes that the quantities are calculated at zero and finite strong magnetic field in the two cases at vanishing of chemical potential and non-vanishing of chemical potential.

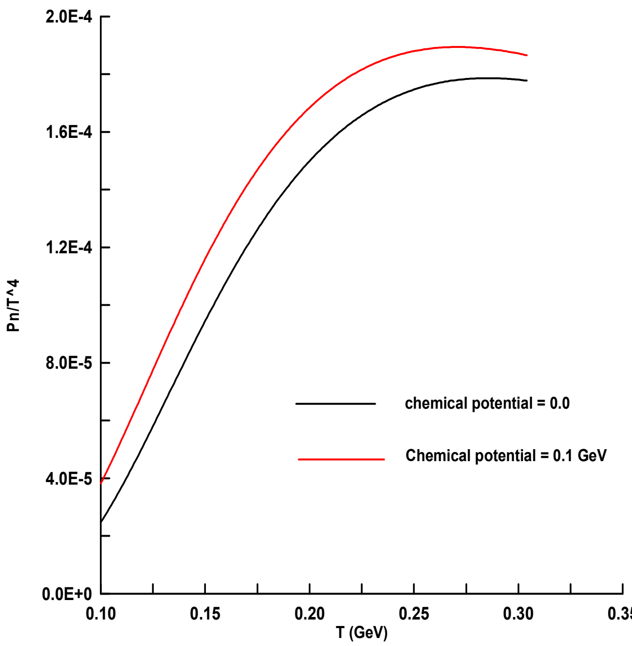


Figure 1. *The dimensionless pressure density is plotted as a function of temperature at μ=0 and μ=0.1 GeV at vanishing magnetic field.*

In Figure 1, the dimensionless pressure density is plotted as a function of temperature at vanishing of magnetic field (B). At vanishing of chemical potential ($\mu = 0$), the pressure increases with increasing temperature. The behavior is good qualitative agreement with recent lattice calculation in Ref. [1], in which the pressure increases with increasing temperature. Also, the behavior of pressure is in qualitative agreement with the original sigma model and the NJL model [2]. On the same hand, the behavior of pressure is qualitative agreement work of Tawfik et al. [3], in which the hadron gas and the Polyakov linear sigma models are applied in their calculations. At $\mu \neq 0$, we note that the curve shifts to higher values, in particular, at higher-values of temperature.

In Figure 2, the effect of strong magnetic field is studied on the behavior of pressure, then one chooses magnetic field $eB = 0.2$ GeV ^{2} as in Refs. $[1,2,3]$. The qualitative agreement is noted between Figure 1 represents the case of vanishing magnetic field and Figure 2 represents the presence of strong magnetic field. The effect of magnetic field appears on the pressure value that the pressure increases strongly at higher-values of temperature. This behavior is qualitative agreement with Refs. $[1,2,3]$.

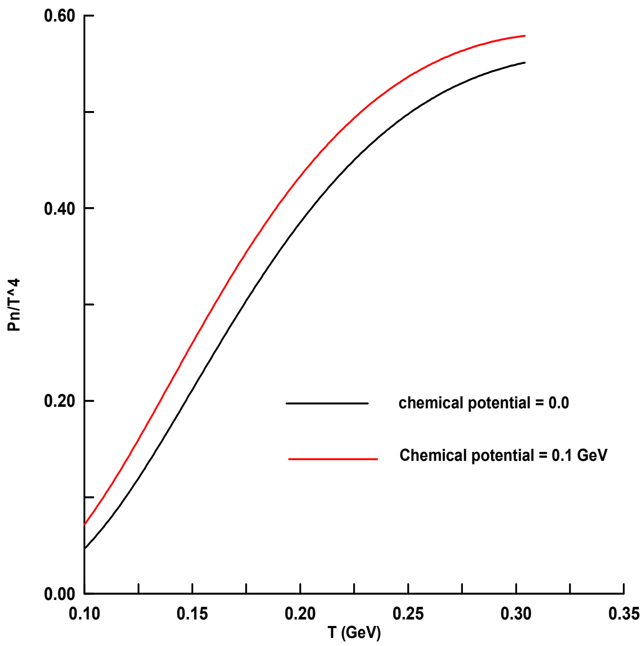


Figure 2. *The dimensionless pressure density is plotted as a function of temperature for at μ=0 and μ=0.1 GeV at strong magnetic field eB = 0.4 GeV².*

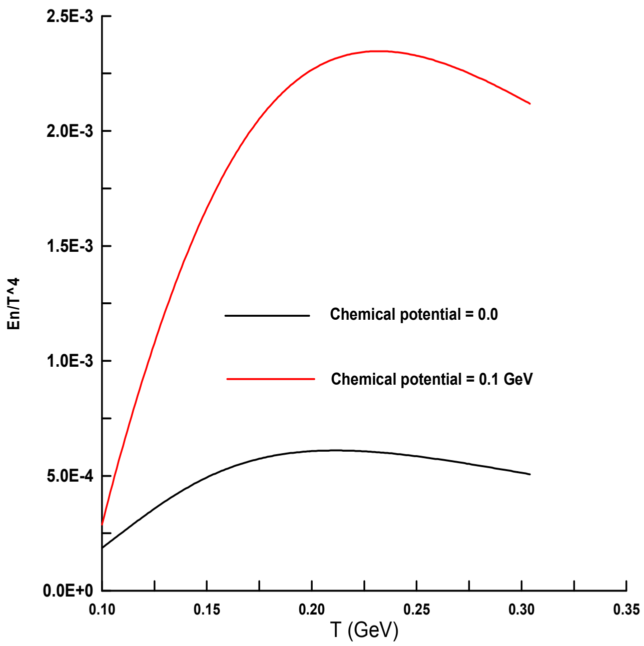


Figure 3. *The dimensionless energy density is plotted as a function of temperature for at μ=0 and μ=0.2 GeV at vanishing magnetic field.*

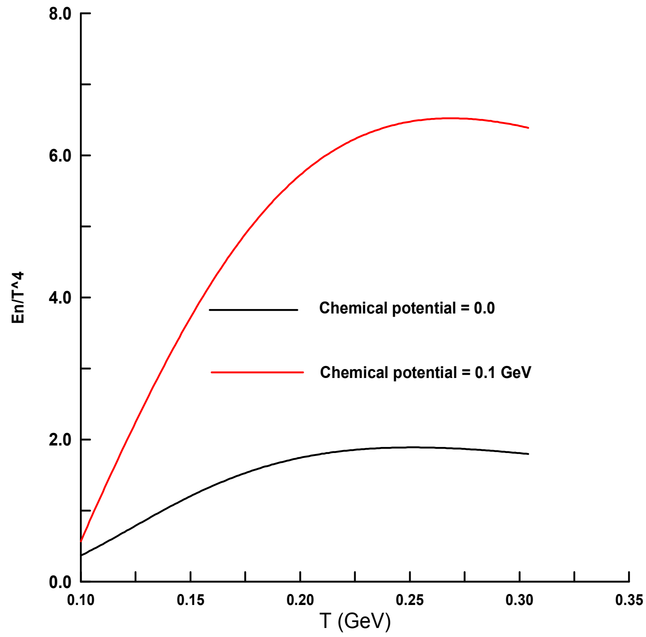


Figure 4. *The dimensionless energy density is plotted as a function of temperature at μ=0 and μ=0.2 GeV at non-vanishing magnetic field eB =0.4 GeV².*

In Figure 3, the dimensionless energy density is plotted as a function of temperature. The energy density increases with increasing temperature at vanishing and nonvanishing chemical potential. At vanishing of chemical potential $(\mu = 0)$, the energy density increases with increasing temperature up to 0.225 GeV and then slowly decreases at larger values of temperature above critical temperature due to taking ratio $\frac{E_n}{T^4}$. By increasing magnetic field as in Figure 4, the behavior of the energy density is a qualitative agreement with the behavior at vanishing magnetic field as in Figure 3. The qualitative agreement with Refs. $[1,2,3]$ is noted.

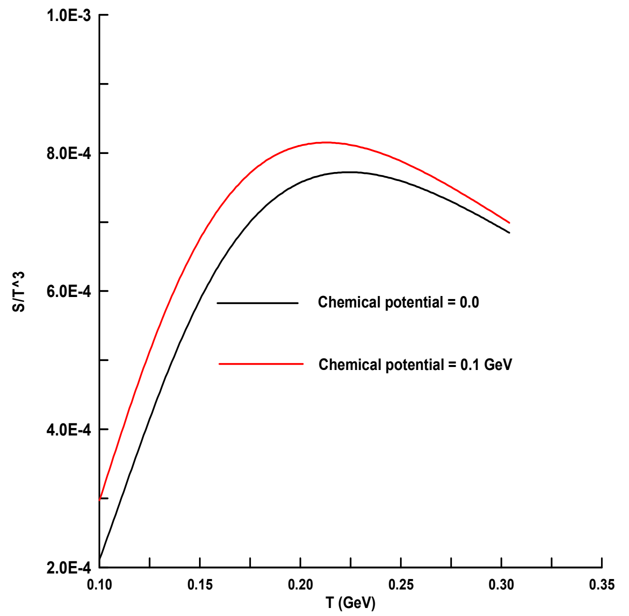


Figure 5. *The dimensionless entropy density is plotted as a function of temperature for at μ=0 and μ=0.1 GeV at vanishing magnetic field.*

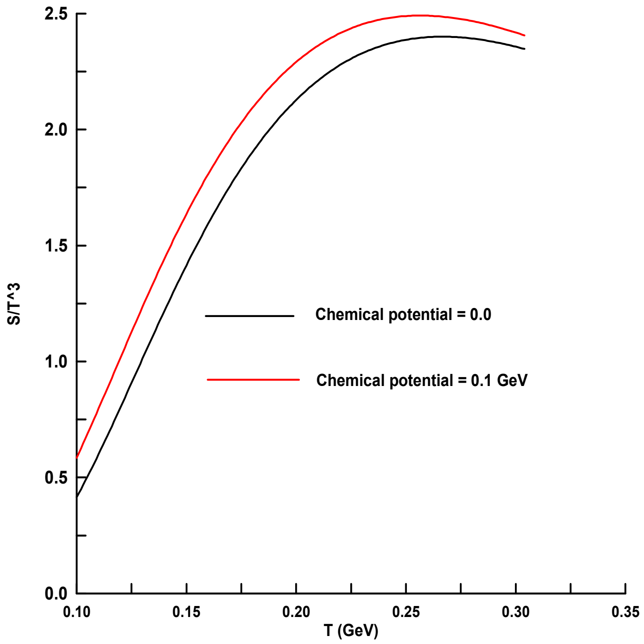


Figure 6. *The dimensionless entropy density is plotted as a function of temperature for at μ=0 and μ=0.1 GeV at magnetic field eB = 0.4 GeV².*

In Figure 5, the dimensionless entropy density is plotted as a function of temperature. One notes that the entropy density increases with increasing temperature at vanishing and nonvanishing chemical potential. The effect of temperature appears on entropy density at higher values of temperatures. By increasing magnetic field as in Figure 6, the behavior of the entropy density is a qualitative agreement with the behavior at vanishing magnetic field as in Figure 5. Also, the entropy density decreases at higher temperature above critical temperature $T_c = 0.2$ GeV. This due to taking ratio $\frac{S}{T^3}$

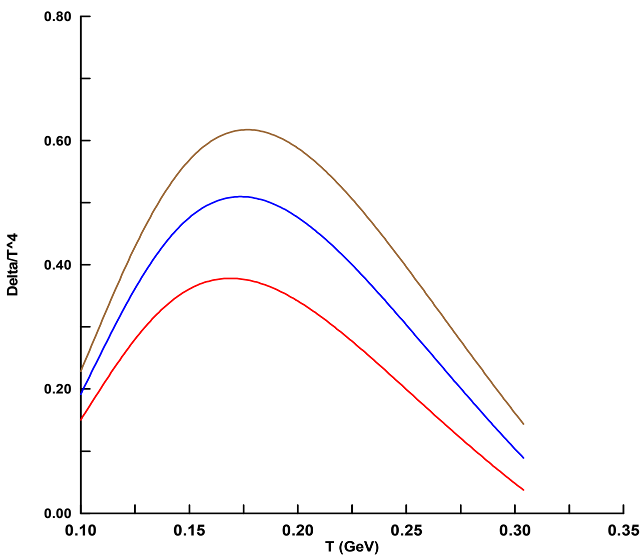


Figure 7. *The dimensionless interaction measure is plotted as a function of temperature for different values of magnetic field eB = 0.2, 0.3, and 0.4 GeV² are arranged from down to up.*

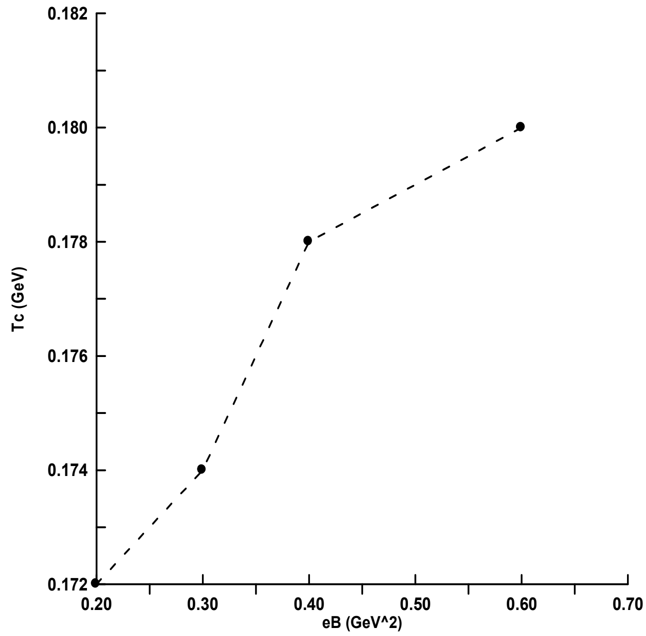


Figure 8. *Critical of temperature is plotted as a function of magnetic field at vanishing of baryonic chemical potential.*

In Figure 7, the measure interaction is plotted as a function of temperature. The measure interaction increases with increasing temperature up to the top curve and then gradually decreases with increasing temperature. By increasing magnetic field, we note that the top of each curve shifts to the direction of increasing temperature. This behavior is an agreement with original sigma model and the NJL model [2]. This behavior is interpreted as crossover phase transition. In Figure 8, the critical temperature increases with increasing magnetic field. Thus, the logarithmic quark enhances magnetic catalysis as well as in the original sigma model and the NJL model [2]. By including Polyakov loop to take confinement into account. The results show that T_c related to deconfinment also increases with increasing magnetic field as in [26]. On other hand, the first lattice attempt to solve the problem taken two quark flavors and high values of pion mass $(m_\pi = 200-400\,\text{MeV})$ confirming that T_c increasing with increasing magnetic field [27].

5. Summary and Conclusion

In this work, the effective logarithmic potential have been employed to study thermodynamic properties in the presence of magnetic field. So, the novelty in this work, thermodynamic properties is investigated in the framework of logarithmic quark sigma model. The present results are agreement with first lattice calculations as in Ref. [27] and other models as in Refs. [2,26]. In addition, the present calculations are carried out beyond the zero chemical potential which are not taken many recent works. The conclusion indicates that the logarithmic quark sigma model enhances magnetic catalysis at finite temperature and chemical potential.

References

[1] G. S. Bali, F. Bruckmann, G. Endrődi, S. D. Katz, A. Schafer, J. High Energy Phys. 177, 35 (2014).

[2] G. N. Ferrari, A. F. Garcia, and M. B. Pinto, Phys. Rev. D 86, 096005 (2014).

[3] A. N. Tawfik, A. M. Diab, N. Ezzelarab, A. G. Shalaby, Advances in High Energy Physics 2016, 1381479 (2016).

[4] R. L. S. Farias, V. S. Timoteo, S. S. Avancini, M. B. Pinto, G. Krein, hep-ph\1603.03847 (2016).

[5] M. Birse and M. Banerjee, Phys. Rev. D 31, 118 (1985).

[6] R. Gatto and M. Ruggier D 83, 040163 (2011).

[7] S. S. Avancini, D. P. Menezes and C. Providencia, C 83, 065805 (2011).

[8] M. Gell-Mann and M. Levy, Nuono Cinmento 16, 705 (1960).

[9] M. Abu-Shady, Inter. J. Mod. Phys. A 26, 235 (2011).

[10] T. S. T. Aly, M. Rashdan, and M. Abu-Shady, Inter. J. Theor. Phys. 45, 1645 (2006).

[11] M. Abu-Shady and M. Soleiman, Phys. Part. and Nuclei Lett. 10, 683 (2013).

[12] M. Abu-Shady, Inter. J. Theor. Phys. 48 (4), (2009).

[13] M Abu-Shady, Inter. J. of Mod. Phys. E 21 (06), 1250061 (2012).

[14] M. Abu-Shady, Inter. J. Mod. Phys. A 26, 235 (2011).

[15] M. Abu-Shady, Mod. Phys. Lett. A 29, 1450176 (2014).

[16] M Abu-Shady, Inter. J. Theor. Phys. 48, 115-126 (2009).

[17] M. Abu-Shady and A. Abu-Nab, the Euro. Phys. J. Plus 130, 248 (2015).

[18] M. Abu-Shady, Applied Math. and Information Sciences Lett. 4, 5 (2016).

[19] A. Goyal and M. Dahiya, Phys. Rev. D 62, 025022 (2011).

[20] S. P. Klevansky and R. H. Lemmar, Phys. Rev. D 39, 3478 (1989).

[21] I. A. Shushpanov and A. V. Smilga, Phys. Lett. B 16, 402 (1997).

[22] I. A. Shushpanov and A. V. Smilga, Phys. Lett. B 16, 351 (1997).

[23] H. Suganuma and T. Tastsumi, Annals. Phys. 208, 470 (1991).

[24] K. G. Klimenko and T. Mat. Fiz. 89, 211 (1991).

[25] V. P. Gusynin, V. A. Miransky, and I. A. Shovkovy, Phy. Rev. Lett. 73, 3499 (1994).

[26] A. J. Mizher, M. N. Chernoub, and E. S. Fraga, Phys. Rev. D 82, 105016 (2010).

[27] M. D. Elia, S. Mukherjee and F. Sanfilippo, Phys. Rev. D 82, 051501 (2010).