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# Formalized Estimation of $M_{\max}$ and of Strong Earthquake Recurrence

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**Abstract:** In estimating seismic hazard based on the Gutenberg-Richter law, the particular attention shall be paid to nonlinearity of the curve and justification of its restriction on the right or to unrestricted extrapolation to the region of rare events [1, 2]. These issues have been addressed based on the benchmarking study of recurrence curves of the strongest earthquakes, with the forecasting limits of the recurrence curves calculated based on the model that reflects the discrete properties of the block hierarchical structure of the earth crust, with the account of deformation conditions, possible changes in the deformation velocity, elastic limit and type of fracture. The study allowed to determine the geodynamic, geotechnical and seismotectonic factors, which influence on the curve's shape and level, and to specify their parameters, which estimate the value and the recurrence of  $M_{\max}$ .

**Keywords:** Earth, Model, Elastic Limit, Deformation Velocity, Recurrence Earthquakes

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## 1. Introduction

The Gutenberg-Richter law [3] expresses the linear dependency between the number of minor and strong earthquakes by the formula:  $\lg N_c = a - bM$ , where  $\lg N_c$  is the number of earthquakes with the magnitude  $M < M_i$ ,  $a$  is the seismic activity per year;  $b$  is the slope of the recurrence curve. This law is the basis for estimation of the seismic hazard in the course of strong earthquakes according to the results of the short-term instrumental seismologic observations of minor earthquakes. One of the problems of this approach is the lack of any physical restrictions for the maximum possible earthquake, therefore, the data extrapolation to the region of rare events can be infinite, and the maximum of the credible earthquake in this case depends on the considered time period [1, 2]. This determines actuality for establishment of preprocessing conditions for disastrous earthquakes and for searching of the physical restriction for the value  $M_{\max}$ .

For subactive and platform territories this problem worsens by the lack of the representative statistics of instrumental and historical data about the earthquakes of the region. This constituted the ground for development of a formalized estimation approach with regard to the shape and

level of the recurrence curve based on geodynamic data, as well as geotechnical and seismotectonic assumptions. As the basis for this approach there was accepted a model, which reflects the discrete properties of the block hierarchical structure of the earth crust, which allows formalization of the scale and the number of the activated structural elements with the account of the deformation conditions, possible changes in the deformation velocity and the elastic limit in the process of preprocessing of the earthquake and type of fracture. The model shall be treated as the thin outmost layer of the earth crust divided by the interblock boundaries into the structural blocks of different ranks. The structural elements shall be treated as interblock boundaries activated at neotectonic and quaternary stages of geological development. The activated interblock boundaries shall be considered further on as the zones of potential earthquake foci (hereinafter to be referred to as the PEF zones) and include seismic faults of different ranks, also the active ones.

Taking into account the genetic relationship of the geodynamic and seismic processes, it is assumed, that the model allows, on the basis of the structure and characteristics of the PEF zones, deformation conditions and deformation velocity, as well as a number of geotechnical and seismotectonic assumptions, to formalize the estimations of

the forecasting limits, which specify the shape and the level of recurrence curves for various conditions of deformation and type of fracture. The credibility of the forecasting limits, in its turn, was observed by coinciding of the strongest earthquakes recurrence curves, calculated according to [3, 4], with the data about the strongest earthquakes in the world within the period from 1985 to 2012.

The joint analysis of the obtained results provides for making a judgment about the preprocessing conditions and the occurrence of the strong earthquakes, responsible for nonlinearity of the recurrence curve, forming of the left relatively low-sloped and of the right relatively steep section of the recurrence curve, dependency of the curve level and slope on the potential changes in the deformation conditions, deformation velocity, and elastic limit and type of fracture.

## 2. Input Data

The analysis was performed for the input data about strong earthquakes, including: [4], the Earthquake Catalog over a period from 1904 to 1952 developed by B. Gutenberg and Ch. Richter [3], and data on the strongest earthquakes in the world for the period from 1985 to 2012. On the ground of these data, the magnitude-recurrence curves were calculated for the earthquakes, which were reduced to one and the same year for the sake of benchmarking convenience and are given in Figure 1.

The recurrence curve according to the data of [4] (see “rhombi” in Figure 1) perfectly reflects the linear dependency of the relationship between the numbers of the minor and strong earthquakes within the range of magnitudes from  $M = -\infty$  up to  $M_{max} \leq 8.5$  and meets the Gutenberg-Richter law with the slope of  $b \sim -0.9$ .

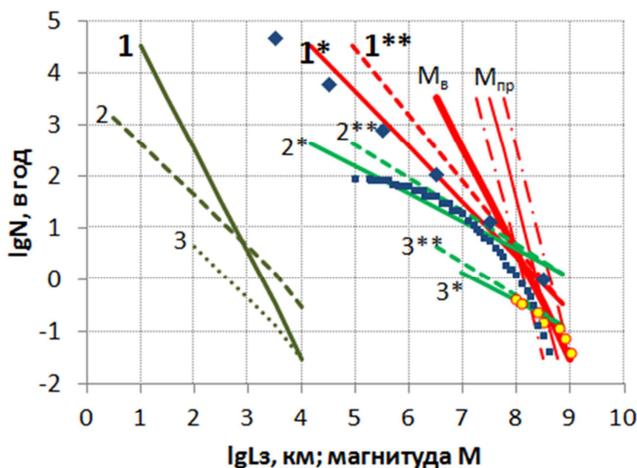


Figure 1. Magnitude-recurrence curves for strong earthquakes.

The recurrence curve according to the data of [3] (see “squares” in Figure 1) in contrast to the data of [4] is not linear and can be expressed, at least, by two linear sections having different slopes. The left one is a relatively plateau

section of the curve and has a slope of  $b \sim 0.54-0.65$ , while the slope of the right relatively steeper section is close to 2.0. These linear sections can hypothetically intersect in the point of coordinate  $M = 8.0$  and ordinate 0.5, which corresponds to the recurrence of earthquakes  $M = 8.0$  approximately once per three years.

Extrapolation of the left section of the curve, which corresponds to relatively minor earthquakes, to the region of rarer seismic events can lead to increase of seismic hazard, which was reflected in the course of general seismic zoning of the Russian Federation territory [5]. However the question of the curve restriction from the right with this criteria for the curve restriction from the right was not addressed.

It was assumed that the data about the strongest earthquakes within the period from 1985 to 2012 allow for credible estimation of the recurrence curve in the certain range of magnitudes (see “yellow circles” in Figure 1). It shall be noted that the given curve, as well as the curve according to the data of [3], is not linear. It can be expressed by two linear sections: by the left one, which is relatively more plateau with the slope of  $b \sim -0.7$ , and by the right one, which is relatively steeper with the slope of  $b \sim -2.4$ . These sections in a first approximation intersect in the points of coordinates  $M = 8.4-8.8$  and ordinates  $-0.7-1.0$ .

Up to date, the nature of the recurrence curve nonlinearity has not been determined definitely. In this regard, the benchmarking of the above-mentioned magnitude-recurrence curves with the forecasting limits of the recurrence curves, calculated according to the adopted model with the account of potential changes in the deformation conditions, deformation velocity, elastic limit, type of fracture and relation of the focus size and the extent of the PEF zone, is presented below. The performed benchmarking of the magnitude-recurrence curves and potential forecasting limits allowed for determination of factors, which influence on the shape and the level of the recurrence curves, and to justify the physical restriction of  $M_{max}$  with the account of the process immensity.

## 3. Approach and Accepted Assumptions

The approach is based on a number of assumptions regarding the structure and characteristics of the PEF zones and their relation with the size of earthquake foci, dependency of the maximum magnitude on the deformation conditions and deformation velocity, extent of the shear modulus (or elastic limit), immensity of the process and other factors [6-9].

The earth crust represents a thin outmost layer, which under the action of endogenous and exogenous processes is broken down into the system of tectonic blocks of different ranks, which are delimited by the interblock boundaries of the same rank. Activated in the neotectonic and present time the interblock boundaries are considered as the PEF zones. The PEF zones include faults, also the active ones, and

earthquake foci of different ranks.

According to the model of “lumpiness” of geophysical medium defined by the academician M. A. Sadovskii [6] the component elements of the earth crust form the discrete sequence, and their dimensions are linked in-between by the following equation:  $L_n = k_n \cdot L_{n+1}$ , where  $k_n$  is the coefficient of similarity,  $n$  is the component element rank, which varies from 1, 2, ... up to  $n$ . The element of the rank model  $n = 1$  characterizes the maximum element of the model, the size of which is equal to the extent of the maximum PEF zone. The coefficient of similarity  $k_n$  characterizes the discrete properties of the model (or the structure of PEF zones).

The discrete properties of the model are described by the characteristics of geometric progression. This renders it possible to formalize estimation of the dimension  $L_i$  of the component elements of the model of rank  $n$  in the form of an equation:  $\lg L_n = \lg L_1 - (n-1) \cdot \lg k_n$  and of the total number of the model activated elements  $N$ , starting from the rank  $i$  up to the maximum element ( $i = 1$ ), inclusively, in the form of an equation:  $N = (3^i - 1)/(3 - 1)$ , where  $3$  is the ratio of a geometric progression, the value of which depends on the deformation conditions. In case of an omnidirectional deformation, all elements of the model are involved into the process, and their number shall be determined with the account of  $3 = k_n^2$ . In the event of a uniaxial deformation, normally oriented to the maximum element of the model, only a part of the model elements is involved into the deformation process, in estimation of the number of which the account shall be taken of  $3 = k_n$ .

The maximum period  $T_{\max}$  of accumulation of the ultimate strains in the maximum element of the model (or in the maximum PEF zone) shall be calculated by the relation of the elastic limit to the deformation velocity [10]. The recurrence of maximum earthquakes on the model component elements equals to the relationship of  $T_{\max}$  to the number of the component elements of the rank  $n$  and more up to  $n = 1$ , inclusively. Standardization of the total number of the activated elements within  $T_{\max}$  renders it possible to determine the rate of forming of structures of different ranks and to proceed to the fractality curve, reduced to one year.

It is assumed, that in the PEF zone the relaxation of stresses and strains in the small-scale inhomogeneities and their accumulation in the large-scale ones take place as a result of the long-term deformation. When the accumulated deformations reach the elastic limit in the maximum PEF zone, the fracture takes place, which can be accompanied by a strong earthquake.

When estimating the  $T_{\max}$ , it is recommended to take account of the deformation velocity based on the results of geodynamic studies and observations of the current movements of the earth crust. According to [11] the velocity gradient of the tectonic deformation for the large-size objects in the earth crust varies within the limits from  $10^{-9}$  up to  $10^{-5}$  per year. The velocity gradient of neotectonic movements in the active regions varies from  $5 \cdot 10^{-8}$  up to  $2 \cdot 10^{-7}$  per year, while in frames of subactive areas it is from  $1 \cdot 10^{-9}$  up to  $2 \cdot 10^{-8}$  per year. The similar results were obtained in the

course of the study of the velocity of the quaternary and current movements in the regions of NPP siting [12, 13]. In the course of assessment of the forecasting limits of magnitude-recurrence curves in conditions of omnidirectional deformation, this allows to adopt the minimum deformation velocity equal to  $G_{\min} = 3 \cdot 10^{-9}$  per year, while in case of a uniaxial deformation to address the maximum deformation velocity as  $G_{\max} = 10^{-7}$  per year. The difference in the deformation velocity in conditions of omnidirectional and uniaxial deformation shall be specified solely by the difference in sizes of deformation zones (their extent and width).

Seismotectonic assumptions for the transfer from the fractality curves to the forecasting limits were accepted based on the analysis of seismogenic dislocation parameters in the earthquake focal zones [14, 15 et al.]. It is established, that deformations in the earthquake foci vary from  $10^{-7}$  up to  $10^{-3}$ , while the effective elastic limit in the foci of strong earthquakes aims for  $e_{\text{eff}} = 3.2 \cdot 10^{-5}$ , when the maximum focus size tends towards 1000 km [16]. Moreover, it was established, that the value of the elastic limit under brittle-ductile fracture is estimated due to the brittle-ductile limit  $e_{\text{xn}}$ , the magnitude of which depends on the focus size  $L_0$ (km), expressed in the form of an equation:

$$\lg e_{\text{xn}} = -0.5 \cdot \lg L_0 - 3.0. \quad (1)$$

When making the formalized estimations of the forecasting limits of magnitude-recurrence curves, it could be useful to take account of the relation of the prevailing sizes of foci and extent of the PEF zone, which is estimated by using a coefficient of similarity  $k_n$ : the minimum focus size is by  $k_n^3$  times, the most probable one is by  $k_n^2$  times, and the maximum one is by  $k_n$  times less, than the extent of the PEF zone.

For transfer from the fractality curves to the magnitude-recurrence curves, in contrast to the traditional average relations specifying the dependency of  $M_{\max}$  on the extent of the PEF zone [5, 17, 18 et al.], we used the dependency of  $M_{\max}$  on the extent of PEF zone  $L_3$  and elastic limit  $e$  in the form of an equation:

$$M_{\max} = 5.1 + 0.625 \cdot \lg e + 1.875 \cdot \lg L_3. \quad (2)$$

The dependency (2) was obtained with the account of the relation between the seismic moment  $M_0$  and magnitude  $M$  in the equation:  $\lg M_0 = 15.4 + 1.6M$  [19, 20, 21] and the results of logarithmation of the well-known equation of K. Aki [22]:  $M_0 = \mu AS$ , where  $\mu$  is the shear modulus,  $A$  is the displacement amplitude,  $S$  is the fracture plane area. The absolute term in the equation (2) corresponds to the account of the shear modulus  $\mu = 5 \cdot 10^{11}$  dine/cm<sup>2</sup>; the maximum focus size in the PEF zone and relation of the horizontal  $L$  and vertical  $W$  dimensions of the focus  $L/W = 2.5$ .

In constraint environment under the long-term elastic deformation and brittle-ductile fracture due to relaxation of strains in small-scale inhomogeneities and their accumulation in large-scale ones during estimation of  $M_{\max}$  in the equation

(2), it is recommended to take account of the brittle-ductile limit instead of the elastic limit, according to the equation (1).

Taking account of the nonrandom dependency of the maximum magnitude on the focus size and the type of fracture allows for transfer from the fractality curve to estimation of the forecasting limit of the magnitude-recurrence curve for the model component elements of various ranks (or PEF zones, corresponding to these elements) [23, 24].

#### 4. Discussion of the Results

The above-mentioned geodynamic data, geotechnical and seismotectonic assumptions allowed for adoption of the following characteristics of factors, which influence on the shape and level of magnitude-recurrence curves in various ranges of magnitudes:

1. the maximum extent of the PEF zone is 10 000 km (the size of the maximum element of the model);
2. the coefficient of similarity of the model component elements is  $k_n$ , which equals to the square root of ten;
3. the deformation conditions: an omnidirectional deformation ( $\varepsilon = k_n^2$ ) and a uniaxial deformation ( $\varepsilon = k_n$ );
4. the elastic limit in the focus zone is  $10^{-7}$ ;
5. the minimum deformation velocity is  $\sim 3.2 \cdot 10^{-9}$  per year;
6. the maximum deformation velocity is  $\sim 3.2 \cdot 10^{-8}$  per year;
7. the shear modulus is  $5 \cdot 10^{11}$  dine/cm<sup>2</sup>;
8. the relationship of the focus size and the extent of the PEF zone equals to  $1/k_n$ ;
9. the empirically established equations for estimation of the probable ( $M_b$ ) and ultimate ( $M_{np}$ ) magnitude of sub-foci are:  $M_b = \lg L_3(\text{km}) + 5.0$  and  $M_{np} = 0.5 \lg L_3 + 6.75$ ;
10. the semianalytic dependencies for estimation of  $M_{\max}$  with the account of the PEF zone extent and the elastic limit (equation (2)) and of the extent of the PEF zone and brittle-ductile limit (the joint accounting of the equation (1) and the equation (2)).

The account of characteristics of the model and geodynamic, geotechnical and seismotectonic factors allowed for determination of the fractality curves and the shape and level of the forecasting limits of the recurrence curves for various deformation conditions and type of fracture (Figure 1):

Line 1 is the fractality curve (the slope is 2.0) for the conditions of omnidirectional deformation with the account of  $T_{\max} \sim 33$  years;

Line 2 is the fractality curve (the slope is 1.0) for the conditions of uniaxial deformation with the account of  $T_{\min} \sim 3.3$  years;

Line 3 is the fractality curve (the slope is 1.0) for the conditions of uniaxial deformation with the account of  $T_{\max} \sim 33$  года;

Lines 1\* and 1\*\* are the recurrence curves, calculated

with the account of the Line 1 under the brittle ( $b = 1.08$ ) and brittle-ductile ( $b = 1.28$ ) fracture, respectively;

Line  $M_b$  is the recurrence curve ( $b = 2.0$ ), calculated with the account of the Line 1, for estimation of the most probable magnitudes of sub-foci;

Line  $M_{np}$  is the ultimate recurrence curve ( $b = 4.0$ ), calculated with the account of the Line 1, to be applied in case of disastrous earthquakes;

Lines 2\* and 2\*\* are the recurrence curves, calculated with the account of the Line 2 under the brittle ( $b = 0.54$ ) and brittle-ductile ( $b = 0.65$ ) fracture, respectively;

Lines 3\* and 3\*\* are the recurrence curves, calculated with the account of the Line 3 under the brittle ( $b = 0.54$ ) and brittle-ductile ( $b = 0.65$ ) fracture, respectively.

The joint analysis of the forecasting limits and recurrence curves of the strongest earthquakes makes it possible to note, that:

1. the recurrence curve, calculated as per the data of [4], in the whole range of magnitudes shall be estimated using the forecasting limit  $M_{\max}$ , calculated for the conditions of omnidirectional deformation with the account of the effective deformation in foci  $\varepsilon_{\text{eff}} \sim 3.2 \cdot 10^{-5}$ ;
2. the recurrence curve, calculated as per the data of [3], within the range of magnitudes  $M \geq 8.0$  shall be estimated using the forecasting limits, calculated with the account of  $M_b$  and  $M_{np}$ , while within the range of magnitudes from 5.5 to 7.5 it shall be estimated using the forecasting limits, calculated for the conditions of a uniaxial deformation under the brittle and brittle-ductile fracture. The increase of the slope shall be specified by the convergence of the effective elastic and brittle-ductile limits progressively, as the focus size approaches 1000 km, and when reaching it, according to the equation (1), which determines the similarity of estimations of  $M_{\max}$  under the brittle and brittle-ductile fracture for the strong earthquakes;
3. the recurrence curve, calculated with the account of data on the strongest earthquakes within the period from 1985 up to 2012, is also nonlinear and can be expressed by two linear sections. The right steep section is estimated using the forecasting limit  $M_{np}$ , and with the account of the confidence interval  $\pm 0.25$  it matches the estimations made according to the data of [3]. The left plateau section is estimated using the linear sections, corresponding to the brittle (Lines 3\*) and brittle-ductile (Lines 3\*\*) fracture, but has the lower level, in contrast to the data of [3], which depends on the size of structures, where deformations reach the elastic limit, and on the relevant increase of  $T_{\max}$ ;
4. the value  $M_{\max}$  for the certain PEF zone shall be estimated using the intersection point of forecasting limits, which were calculated with the account of the effective elastic and brittle-ductile limit, and corresponds to the maximum focus size equal to  $1/k_n$  of the maximum extent of the PEF zone. In this case, the maximum earthquake recurrence shall be determined by the relation of the effective elastic limit to the

maximum deformation velocity in the PEF zone.

Moreover, it shall be noted, that the ultimate curve of  $M_{\max}$ , calculated with the account of  $M_b$  for the conditions of an omnidirectional deformation, corresponds to the moment magnitude-recurrence curve, addressed in the reference [25] as a criterion for estimation of the value  $M_{\max}$  and maximum earthquake recurrence under extrapolation of the results of the short-term instrumental observations of the seismic regime.

## 5. Conclusions

The use of the model allowed for determination of the geodynamic, geotechnical and seismotectonic factors, which specify the characteristics of the seismic process, which make it possible to understand the nature of the nonlinearity of the curve and to introduce the physically justified restrictions for extrapolation of the earthquake magnitude-recurrence curve to the region of rare events.

Benchmarking of the forecasting limits and observed recurrence curves rendered it possible to understand the nature and to estimate the quantitative dependences between the parameters of the mentioned factors and the nonlinearity of the recurrence curve in various ranges of magnitudes.

In the process of a long-term deformation, the relaxation of strains in the small-scale inhomogeneities and their accumulation in large-scale ones takes place, which leads to changes in the strain-stress state of the medium, and in the deformation strength (elastic limit) and velocity.

When deformations reach the elastic limit, the fracture of the maximum PEF zone takes place, and the number of activated elements in this case depends on the deformation conditions, deformation velocity, elastic limit and type of fracture, which is reflected by the shape of the recurrence curve and by the level of seismic activity.

The similarity of the seismic process in various scale levels has been established. It was shown, that the parameters of the seismic regime are estimated with the account of the size (extent) of the maximum PEF zone, deformation conditions, values of elastic limit and deformation velocity, type of fracture and the relation between the focus size and the extent of the PEF zone. The value  $M_{\max}$  and the recurrence of the strongest earthquakes shall be estimated using the intersection point of the forecasting limits under the brittle and brittle-ductile fracture, calculated with the account of the extent of the PEF zone.

Nonlinearity of recurrence curves takes place in various scale levels, and is determined by the immensity of the geodynamic and seismic processes and by the relationship between a number of events, which occurred in various deformation conditions and have different type of fracture.

The physical nature of a man-induced seismicity can also be addressed using the adopted model. In this case, characteristics of the geodynamic and seismic processes shall be estimated using the size of the maximum PEF zone, involved into the process of a man-induced deformation, including the deformation conditions, the potential changes in the deformation velocity, elastic limit and type of fracture.

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