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# Heavy Quark Thermalization in the Expanding Deconfined Medium

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**Abstract:** The strong interactions and the deconfined properties have been hot research topics in the high energy nuclear theory. When two heavy nuclei are accelerated to nearly the speed of light and collide with each other, a lot of kinetic energy can be transformed into partons. The partons in a small volume can form an extremely hot medium, where quarks and gluons are not constrained in a small volume size of the nucleon. Instead, they evolve in a relatively larger volume. The small deconfined medium is just like our early universe after the big bang. The extremely hot medium will expand outside, and the lifetime of this medium is very small. It cannot be measured directly in experiments. The coupling strength of this new state of the medium is a difficult and important problem. The medium coupling strength is closely connected with the heavy quark energy loss. The heavy quarks including charm or bottom quarks carry large mass. They are coupled with the medium through color interactions. Due to the large mass, their evolutions in the thermal medium can be described by semi-classical theoretical models instead of quantum field theories. The acceleration and deceleration of heavy quarks in the expanding deconfined medium are studied in details in this work. The change of heavy quarks depends on the velocities and the lifetime of the hot medium. With the strong coupling between charm quarks and the quark gluon plasma (QGP), charm quarks can reach kinetic equilibrium. This work builds the theoretical models for charm accelerations in the expanding QGP. It can explain the connections between charm quark coupling strength and its momentum thermalizations. From the calculations, heavy quarks are strongly coupled with the medium, and mainly suffer energy loss in the early stage of the medium evolutions where the temperature is much higher. This means that heavy quarks can research kinetic thermalization in the early stage of two nuclear collisions.

**Keywords:** Heavy Ion Collisions, Heavy Quark Thermalization, Quark Gluon Plasma

## 1. Introduction

Strong interactions have been widely studied in particle physics and nuclear collisions [1-9]. The thermalization of heavy quarks in the hot deconfined medium consists of partons is an ideal probe for strong coupling constant. In the relativistic heavy ion collisions, a strong expanding quark gluon plasma (QGP) can be produced, which is believed to be the state of the early universe after the Big Bang. The kinetic thermalization of heavy quarks and quarkonium dissociation [10-13] have been one of hot topics in relativistic heavy ion collisions, and can be well described by non-relativistic approaches due to its large mass [14, 15]. As the heavy quark interactions with the thermal partons depends sensitively on the temperature and also their momentum, heavy quark accelerations/decelerations in the cooling and expanding systems produced in nucleus-nucleus (AA) collisions are nontrivial questions [16-20]. In this work, a toy model to simulate the expansion of quark gluon plasma is proposed, which employs a differential equation to describe the heavy quark velocity evolutions. The formulas in the model have captured the basic features of heavy quark evolutions in the experiments of nuclear collisions. It can help to understand the heavy quark energy loss in nuclear experiments and the strong interactions.

In the nuclear reactions, the longitudinal expansion of produced QGP, defined as the direction z of nucleus accelerations, is pronounced compared with the transverse expansion (x,y). Therefore, the Bjorken model with only longitudinal expansions have been proposed and widely studied many years ago [17]. This research extends the

Bjorken expansion with only longitudinal expansion to the full 3-dimensional expansions, and the lifetime of QGP evolutions is constrained by the conservation of total energy of the entire system.

## 2. Theoretical Models and the Applications

The particle distribution in the longitudinal directions is uniform in the central rapidity, which results in the longitudinal expansion with constant speed $v_z$. However, the transverse expansion is an accelerating process. Both accelerations make the QGP volume of static situation (described by Eq.(1)) evolve as Eq.(2).

$$\mathrm{V} = \pi \times r^2 \times \mathrm{h} \tag{1}$$

$$\mathrm{V} = \pi \times (\mathrm{r}_0 + \tfrac{1}{2}\mathrm{a}_{xy}\mathrm{t}^2)^2 \times (h_0 + v_z\mathrm{t}) \tag{2}$$

The initial radius of quark gluon plasma produced in nuclear collisions is taken as the nuclear radius $r_0$=7fm. Due to the gradients of pressure, QGP transverse acceleration is described by $a_{xy}$ =0.1 ($\mathrm{fm}^{-1}$). As the quark gluon plasma produced from scatterings needs some time to reach local equilibrium, where pre-equilibrium stage is a tough question and described by other models, The model starts from the time point around 1 fm/c [21]. This gives the initial longitudinal length of QGP to be $h_0$ =1fm. With the parameters of Eq. (2) fixed by the experiments, the QGP expansion becomes

$$\mathrm{V} = \pi \times (7 + \tfrac{1}{2} \times 0.1 \times \mathrm{t}^2)^2 \times (1 + 0.8 \times \mathrm{t}) \tag{3}$$

where height is on z-axis, and radii are on x,y-axis.

With the conservation of total energy, QGP energy density keeps dropping down with its expansion. Lattice QCD calculations indicate that the freedom of the systems decreases suddenly at the critical temperature, indicating that the partons with larger number of freedoms can become hadrons. There is a phase transition when the energy density drops down to a certain value. Due to the different equation of states of QGP and hadron gas, model evolutions stop at the phase transitions, and focus on the heavy quark accelerations in the deconfined medium. Note that in the hadron gas, charm quarks have become D mesons, and the interactions between D mesons and other light hadrons are much weaker compared with QGP situation, and therefore can be neglected in the calculations.

With the final charge multiplicity measured by experiments, one can obtain the QGP initial energy or the temperature in heavy ion collisions. With the fix of total energy to be $\mathrm{E}_t$, and the energy density $E_d$ evolution is connected with the QGP volume, see Eq. (4) below. In the massless situation, the energy density of thermal bath is proportional to the temperature, describe with Eq. (5)

$$\mathrm{E}_t = E_d \times \mathrm{V} \tag{4}$$

$$\mathrm{E}_d = T^4 \tag{5}$$

Where T is the QGP temperature, and V is the QGP volume. Experimental data of light hadrons suggest the initial temperature of QGP at ~TeV nuclear colliding energies to be around 400 MeV. With the setting of initial temperature (energy density) and expansion formula for QGP dynamical evolutions, This work presents the volume evolution as a function of time, and temperature evolution is easy to be obtained.

## 3. Numerical Results and Analysis

With the dynamical evolutions of the bulk medium, now one can get the charm quark energy loss which is the main motivation of this work. The charm quark acceleration or deceleration depends sensitively on the coupling constant between them and the thermal medium. The parton scatterings and emissions from heavy quarks, which contribute to the heavy quark momentum change, can be simplified as drag coefficients in the heavy quark velocity differential equations, such as the widely studied Langevin equation [17-20].

As heavy quark velocity evolution depends on not only the coupling constant but also the relative velocities between heavy quarks and the thermal medium. For simplicity, at the first step, QGP expansion is with a constant velocity acceleration. And the QGP velocity changes with time as Eq. (6). Charm quark velocity $v_c$ is controlled by the interaction strength, and is connected with the difference between its velocity and bulk medium velocity,

$$\mathrm{v}_{qgp} = v_z \times t \tag{6}$$

$$\frac{dv_c}{dt} = \mathrm{A} \times (v_{qgp} - v_c) \tag{7}$$

$v_{qgp}$ is the velocity of evolution of QGP. A is the coupling constant.

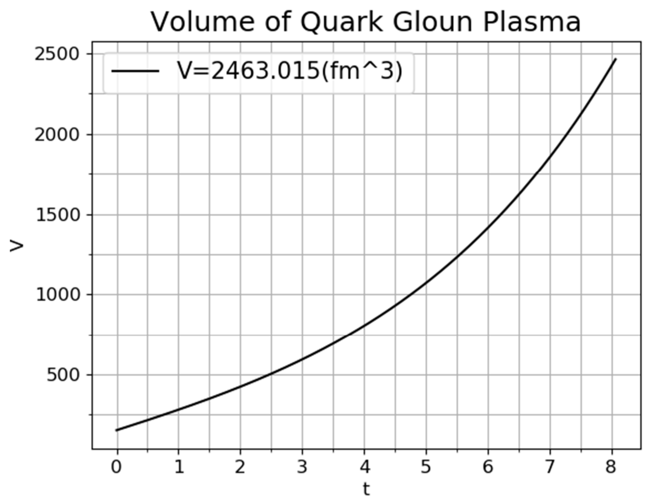


***Figure 1.*** *QGP volume evolutions as a function of time.*

Charm quarks are produced by perturbative QCD. Their velocities satisfy a distribution. Meanwhile, the magnitude of coupling constant is still under debate within many theoretical models. At the first step, put a static charm quark in the expanding QGP, and employ two different coupling constants in the equation of charm accelerations. The basic picture is that charm quarks can reach kinetic thermalization faster if taking larger coupling constant (larger interactions). In the figure below, QGP velocity increases with time (solid line), charm quark is heavy and their acceleration is delayed compared with the QGP (dashed and dotted lines). After a period of time, the heavy quark velocity is smaller than the QGP velocity which is natural. With larger A, charm quark velocity increases faster with time, see the dashed line and dotted line in the figure below.

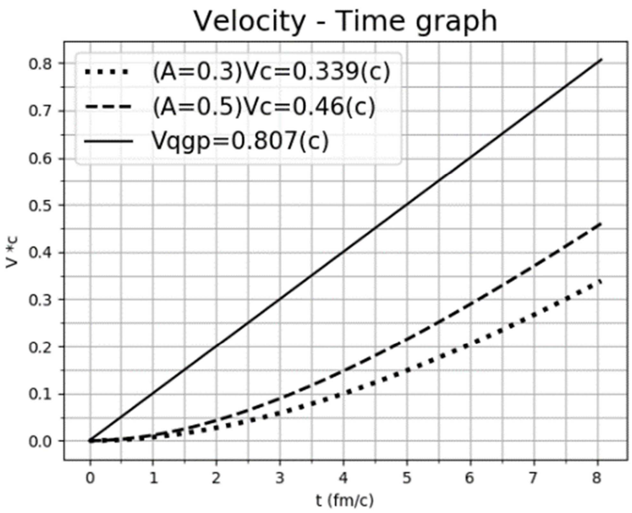


***Figure 2.*** *QGP velocity (solid line) and charm quark velocities with different coupling constant (dashed line for A=0.5, dotted line for A=0.3) as a function of time.*

As heavy quarks produced in heavy ion collisions carry many velocities, the heavy quarks with large velocity may suffer energy loss at first in the hot medium, and then pick up more energy from the expanding QGP. This makes charm quark velocity decreases at first and then increases with time, and the accurate tendency depends on both heavy quark initial velocity and the coupling constant A.

The initial velocity of QGP is zero. Charm quark velocity is larger than the velocity of thermal medium at the beginning of evolutions. This makes charm quarks suffer energy loss. At the later stage, QGP develop larger velocity due to strong acceleration in the transverse plane, pushes charm quarks to a larger velocity at the time scale of t>4 fm/c. Note that the lifetime of QGP in heavy ion collisions is around 5~10 fm/c, which indicates that in the realistic experiments, charm quarks should experience deceleration at first and then acceleration in the later stage.

In the above cases, coupling constant is taken as a constant value, but actually A will depend on the energy density of QGP, and therefore coupling constant changes with time in the cooling system. With the basic feature of coupling constant proportional to the temperature and the inverse of charm quark momentum, one can take the form of coupling constant to be

$$\mathrm{P}_c = m \times v_c \tag{8}$$

$$\mathrm{E}_c = \sqrt{m^2 + {P_c}^2} \tag{9}$$

$$\mathrm{A} = {}^{\mathrm{T}}\!/_{\mathrm{E_c}} \tag{10}$$

and the temperature can be extracted from the energy density Eq. (5). With the expansion of QGP and the decreases of temperature, the coupling constant A evolves with time.

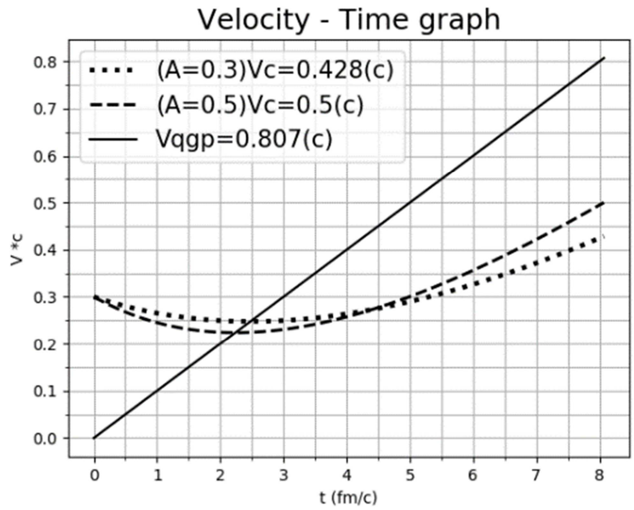


***Figure 3.*** *QGP velocity increases with time (solid line). Charm quark initial velocity is taken as 0.3c, coupling constant is taken as A=0.5 (dashed line) and A=0.3 (dotted line).*

In Figure 4, it shows time dependence of coupling constant in the cooling system. Apparently, the value of A keep decreasing until the phase transition. The slopes of charm quark velocities in Figure 5 are much smaller than the slopes in Figure 2 and Figure 3. It means charm quark acceleration in the expanding QGP becomes smaller when employing more realistic time-dependent coupling constant, which decreases with time.

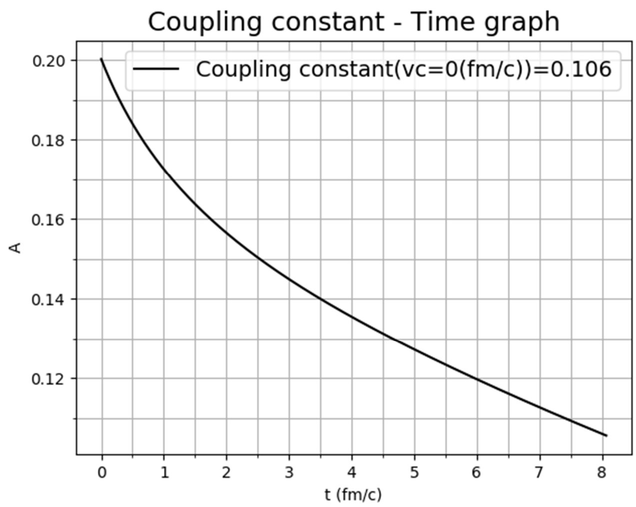


***Figure 4.*** *Coupling constant evolves with time in the expanding QGP.*

Further, this work also extends the model to a more general situation. In the low colliding energies, the final charge multiplicity is not uniform in the central rapidity region, which means the QGP is not expand

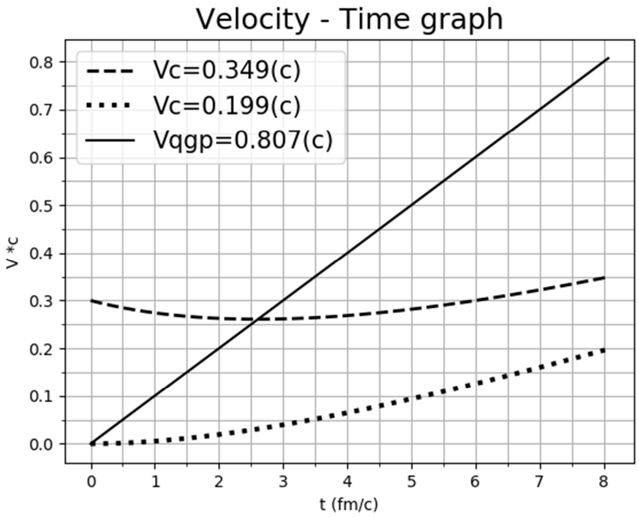


***Figure 5.*** *Charm velocity evolves with time, and the coupling constant depends on time. Solid line is for QGP velocity, dashed and dotted lines are for charm velocities, with different initialization of 0 and 0.3.*

as a “cylinder”, instead, its shape in each time step looks like “rugby”. The transverse radius of this “rugby” at each longitudinal coordinate z is specified as Eq. (11). The parameter σ in Eq. (12) is determined by the final light hadron distributions measured by experiments. In the lower colliding energies, σ becomes larger. The QGP shape as a function of longitudinal coordinate z is plotted in Figure 6.

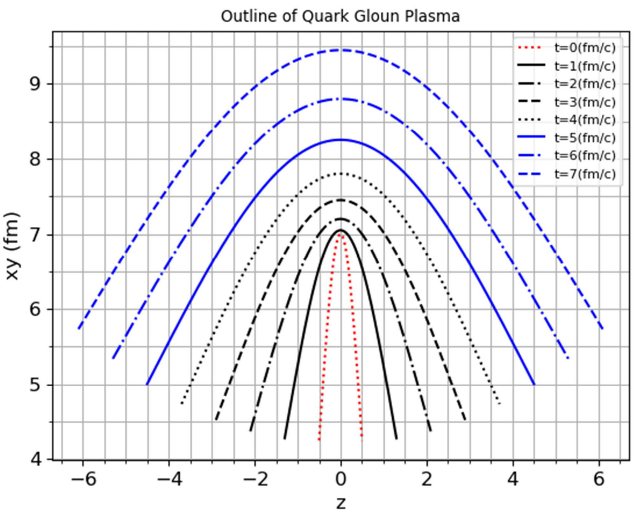


***Figure 6.*** *QGP shape as a function of longitudinal coordinate z at different time step, calculated with Eq. (11-12).*

$$y = \left(r_0 + \frac{1}{2} \times a_{xy} \times t^2\right) \times e^{\frac{-x^2}{2\times\sigma^2}} \tag{11}$$

$$\sigma = 0.5 \times \frac{1}{2} \times v_z \times t \tag{12}$$

In the function (11) and (12), the value of each parameter is as same as functions above.

The graph below depicts the outline of the QGP at different moment. Apply the *Trapezium Rule* to simulate the *Solid of Revolution*, one can obtain the QGP volume at each time step by the integration

$$V = \pi \int_{-z}^{z} y^2 \, dx \tag{13}$$

$$V = \pi \int_{-\frac{h_0}{2}-v_z t}^{\frac{h_0}{2}+v_z t} [r_0 + (\tfrac{1}{2} a_{xy} t^2) \times e^{\frac{-x^2}{2\sigma^2}}]^2 \, dx \tag{14}$$

This extended model approaches to the first model at high colliding energies at Large Hadron Collider, and can give more realistic simulations in the low colliding energies such as at Relativistic Heavy Ion Collider.

## 4. Conclusions

In summary, This work builds a model for quark gluon plasma expansion produced in relativistic heavy ion collisions. In the cooling system, This research studies the heavy quark velocity evolutions due to their strong interactions with the thermal partons. One can find that the charm quarks with different initial momentum or velocity, experience different acceleration/deceleration at the different stage of QGP expansions whose velocity increases with time. The charm quark interactions with thermal medium also depends on the temperature, which can make charm quarks suffer strong energy loss at the early stage but are less affected in the later stage of QGP evolutions. This can help to understand the charm kinetic thermalization in the cooling systems.

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## References

[1] Warburton A. arXiv:1509.04380 [hep-ex], 2015.

[2] Gupta D. Physical Review C, 1986, 33 (5): 127C155.

[3] Ogilvie C A. Journal of Physics G: Nuclear and Particle Physics, 1999, 25 (2): 159C169.

[4] Eskola K J. Plenary talk given at, 1999: 231C250.

[5] Kapusta J I. Journal of Physics G: Nuclear and Particle Physics, 2007, 34 (8): S295CS303.

[6] Klusek-Gawenda M, Kozik E, Rybicki A and so on. Acta Physica Polonica B Proceedings Supplement, 2013, 6 (2): 451.

[7] Xu N. Nuclear Physics A, 2014, 931: 1C12.

[8] Jena C. Trieste, Italy: Sissa Medialab, 2017, IC-PAQGP2015: 86.

[9] Beraudo A. arXiv:1610.08269 [hep-ph], 2016.

[10] B. Chen, K. Zhou and P. Zhuang, Phys. Rev. C 86, 034906 (2012).

[11] B. Chen, Y. Liu, K. Zhou and P. Zhuang, Phys. Lett. B 726, 725 (2013).

[12] B. Chen, Phys. Rev. C 93, no. 5, 054905 (2016).

[13] B. Chen, T. Guo, Y. Liu and P. Zhuang, Phys. Lett. B 765, 323 (2017).

[14] Kurkela A, Mazeliauskas A, Paquet J-F and so on. Nu-clear Physics A, 2017, 967: 289C292.

[15] Tanjin N, Venugopalan R. Trieste, Italy: Sissa Medialab, 2017, EPS-HEP2017: 197.

[16] Tanjin N, Berges J. Physical Review D, 2017, 97 (3): 34013.

[17] Brown S G, West G B. Physical Review, 1968, 174 (5): 1777C1786.

[18] B. Chen and J. Zhao, Phys. Lett. B 772, 819 (2017).

[19] B. Chen, Phys. Rev. C 95, no. 3, 034908 (2017).

[20] J. Zhao and B. Chen, Phys. Lett. B 776, 17 (2018).

[21] Chun Shen, Bjorn Schenke. Phys. Rev. C 97, 024907, 2018.