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Strategy and Methods for Solving Combinatorial Problems in Initial Instruction of Mathematics

Valerija Krekić-Pinter, Josip Ivanović, Žolt Namestovski, Lenke Major

University of Novi Sad, Hungarian Language Teacher Training Faculty, Subotica, Serbia

Email address
valeria.krekic@magister.uns.ac.rs (V. Krekić-Pinter),
josip.ivanovic@magister.uns.ac.rs (J. Ivanović),
zsolt.nameszto vszki@magister.uns.ac.rs (Ž. Namestovski),
major.lenke@magister.uns.ac.rs (L. Major)

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Abstract
The development of logical and combinatorial thinking begins in the earliest activities of children. Ideas of combinatorics may appear in many forms, above all in solving certain situations that occur during play, in everyday life, in certain school subjects, and in other areas. Initial instruction of mathematics introduces students to different relations, but it is not particularly concerned with the grouping and distribution of elements, namely with combinatorial problems. This paper is an attempt to highlight some relevant aspects as the basis for teaching combinatorics in the initial instruction of mathematics, as well as to examine the needs and the possibilities of its development. The aim of the paper is to contribute to the realization of the contents of the combinatorial nature, by defining strategy and adequate approach to methodological elements of teaching combinatorics and logical-combinatorial tasks in initial instruction of mathematics. Another aim of the paper is to examine the educational effects of defined methodological transformation. The result of the research is an original creative teaching strategy and methodological transformation of combinatorial elements in initial instruction of mathematics. This strategy has proved, during the experimental test, that its effects are remarkable and that in comparison to the existing low level of solving combinatorial problems, it highly on tributes to the quality of initial instruction of mathematics. This research has systematized methods and teaching and learning models of combinatorics in initial instruction of mathematics and showed that significant results in this area can be achieved by systematic methodological transformation.

1. Introduction

Compulsory primary education in some developed countries, among other general objectives and desired outcomes, lay special stress on the acquisition of basic mathematical knowledge necessary for understanding the phenomena and laws of nature and society, on providing modern mathematical culture in line with current trends in mathematics as a science, and similar. Logical – combinatorial knowledge is certainly one of the desired outcomes, of which some of them are explicitely defined in school programmes (Croatia, Hungary, Netherlands, Slovakia, Slovenia), and some implicitely (England and Wales).

In programmes for initial mathematics teaching, from the very first grade of elementary school, we can commonly find themes such as: solving simple logical-combinatorial problems related to counting objects, detecting patterns of arrays of objects and numbers, different possibilities of arrangement, grouping or selection of objects and even abstract elements later, determining total number of possibilities, ideas and combinatorics models in various applications of mathematics, etc. (Osnovno i obavezno obrazovanje u svetu, 1995).

In initial mathematics instruction, the early introduction and successful implementation of contemporary topics such as logic, sets, combinatorics, probability, and statistics require appropriate methodical transformation of these contents (Author, 2006). However, the methodics of teaching initial mathematics lacks systematic research into modelling of combinatorics problems, as well as research that could verify the existing teacher practice in this field.

This paper aims to offer a possible way of developing students' combinatorial thinking in initial instruction of mathematics and a research into effects of the defined methodical transformation. The paper deals with the problem related to propaedeutics of combinatorics and methodological representation of combinatorial elements. Today this is considered to be an important and up-to-date issue, although it has been less studied both in methodological theory and in the practice of teaching mathematics in lower grades of elementary school.

The goal of this study is to define the strategy and corresponding approaches to methodological representation of elements of combinatorics and logical-combinatorial assignments in the initial instruction of mathematics and in this way to contribute to the realization of contents that have combinatorial nature, as well as to determine educational effects from suggested methodical transformation. It is expected that the developed method will help achieve better effects in the initial mathematics instruction.

2. Theoretical Basis – Previous Research

The theoretical basis of this paper is Bruner's theory of cognitive development. This theory represents a modification of a theory established by a psychologist and epistemiologist Jean Piaget. In Bruner's opinion, the development of cognitive process does not go through a series of age-related stages, as popularized by Piaget and Inhelder, but the development coexists within different integrated and interdependant levels of mind representations (Bruner, 1974; Piaget, & Inhelder, 1951). According to him, there are three stages, or three *modes ofrepresentation,* on the basis of which human beings draw conclusions about their environment:

- *Enactive mode* involves knowledge based on one's own actions (with actual materials), without using imagination or words.
- *Iconic mode* – certain relations are based on pictures and graphic images that replace objects. The image representation is based on imagination (“internal imagery“).
- *Symbolic mode* – instead of objects and pictures, a person uses concepts and symbols to determine a hierarchical structure according to certain criteria, and to consider alternative possibilities in a combinatorial fashion. This stage implies the use of language, words, or other symbol systems.

In developing *abstract* concepts significant role play various *examples* and a corresponding *mental image* which is based on personal and individual projections of the *objective* outside world. Play, simulations, and case studies appear both separately and simultaneously, and in a variety of combinations. The point is that in building mathematical concepts we should apply a spiral approach, starting from manipulation with objects and didactic materials to case studies and symbolic simulations. In this sense, the early introduction of combinatorics is considered one of the best possible means to develop creativity in students, and as an excellent basis for the formation of mental images, or later mathematical concepts in this area.

First research on combinatorial thinking appeared in Piaget's works (Inhelder, Piaget, 1958; Piaget, Inhelder, 1951), to be followed by many authors who were concerned with the structure of this ability (Barratt, 1975; Fischbein, Pampu, & Minzat, 1970; Kishta, 1979; Roberge, 1976; Scardamalia, 1977).

Modernization of mathematics curriculum for lower grades of elementary school and some aspects of combinatorics teaching methodology were in the focus of interest of several authors such as: Author (2006, 2009); Ching-Kuch Chang (2009); Cotič, Felda (2007); Cotič, Hodnik (1993); Varga (1967); Yu-Ling Tsai, (2009) etc.

However, besides the above mentioned studies, certain strategic and didactic-methodological questions highlighting the methodical transformation of combinatorial contents in the initial teaching of mathematics have not been sufficiently explored.

3. Strategies and Methods for Solving Combinatorial Problems in Initial Teaching of Mathematics

Combinatorial ideas are generally common in children play. Manipulative games of combinatorial nature, forming groups or couples in various games, different ways to arrange a bouquet from few flowers, different ways to dress a doll, arranging sitting for students in classroom or cinema, possible arrangements around one table, socializing or grouping of students, and the like occur as natural situations in everyday life of children (Dejić et al., 2008).

In many countries the goal and tasks, as well as mathematics curricula in primary education have been refreshed by the introduction of combinatorial contents to enable students acquire elementary ideas and methods of combinatorics, to enable them apply the acquired knowledge in solving different life problems, to prepare students for further learning of combinatorics, and to develop students' mental abilities particularly in the field of logical and combinatorial thinking and application of combinatorial concepts and models in various areas.

Teaching combinatorics at this age requires from young students to be able to recognize combinatorial substance in various problems and to solve them intuitively with a lot of freedom and creativity, but not to memorize and apply ready-made formulas in solving combinatorial problems.

Operational tasks in initial teaching of combinatorics in primary education may be defined, in terms of outcomes and students' performance, as follows:

- On the basis of given criteria, by manipulating objects, using diagrams or tables students should be able to form different concrete sets and subsets of objects, images and symbols;
- They should be able to compare given sets and subsets of objects, numbers, letters, and so on;
- Students should be able to examine different possibilities in forming subsets of a given set, as well as possible ways in arranging subsets;
- In simple concrete examples students should be able to determine ordered pairs of given sets, and to determine their cardinal number;
- Students should be able to produce a systematic procedure for different formation and arrangement of concrete sets, and in more simple examples to determine the number of all possibilities;
- Students should be able to recognize and successfully apply combinatorial ideas and methods into solving problems from real everyday life and other areas, in cases sensitive to elementary combinatorial modelling.

According to Skemp (1971) it is not useful to mix up logical and psychological approaches in initial teaching mathematics, since the main goal in the logical approach is "to convince a sceptic", while the main goal in psychological approach is to facilitate understanding. On the other hand, logical approach represents final learning outcomes and therefore denies students opportunity to discover methods for understanding mathematical contents (Skemp, 1971). Students learn mathematical concepts instead to develop mathematical reasoning. Freudenthal, the creator of the contemporary reform of mathematics education in the Netherlands, also emphasizes the primacy of the development of mathematical thinking over mathematical thoughts (Freudenthal, 1974). The basic ideas of his *realistic* conception of initial instruction of mathematics are: students should not be the recipients of ready-made mathematics, but should rather create a variety of real *problem situations* as the basis for discovering different mathematical ideas, concepts and perceptions. Therefore, instead of a passive recipient of knowledge, a student will become an active creator of his/her own knowledge.

In a majority of countries the approach to initial instruction of mathematics has been grounded in creative activities, such as: learning through play and manipulation with objects, abundance of teaching materials, creative approach to teaching, spiral formation of concepts, in phases, differentiation and individualization.

In this way, the defined *creative energy* enables children learn subtle contents such as sets, logic, combinatorics and probability.

The basic paradigm of modern initial mathematics teaching is a creative approach to matematization of reality, rather than algorithmic solving of mathematical problems, which emphasizes the development of computation techniques and solving a variety of ready-made mathematical models (Author, 2013).

It is due to the age of children involved in this stage of education that the processes and achievements of their cognition are limited to the concrete, material, and obvious. Contents of combinatorial nature have been taught in a way that instruction starts with the formation of sets and subsets, order of objects, persons or some other didactic material, which is *permutation*, then the following step is to form subsets with less elements from more elements thus creating *combinations*, and later comes the formation of ordered subsets, *variations,* and also ordered pairs from two elements of sets.

The methodical transformation of these concepts implies creative approach and the use of new methods and techniques such as discovery method, problem method, supposition method, cybernetic methods, graphs, various diagrams, tables and sets.

Solving combinatorial problems takes place in stages as follows:

- Solving through experiments – games, or manipulation with objects. Then, in more complex problems, it is necessary to draw a tree of events, use tables and series, etc. At the beginning it is not necessary to determine the total number of possibilities (permutations, combinations, variations or ordered pairs), but it is rather important to find as many examples.
- Later on, in simple cases it is possible to start from the iconic representation of the problem, or even from the proper symbolic representation, which should be followed by setting requirements to create all possibilities, or determine or estimate the number of requested possibilities without using a predefined formula. For symbolic representation one of most suitable devices are computers with appropriate hardware and software (Author, 2009).
- Applying combinatorics to different teaching areas, such as: forming words of determined length from given letters, forming sentences with a determined number of words fro given words, alphabetical schedules: libraries, dictionaries, lexicons, etc., making melodies from given notes, possibilities of painting various pictures with multiple fields, possibilities to combine colours and make shades, possibility to realign students, possibilities to form sports teams from a certain number of students, possibilities to choose a few (2-3) leaders or representatives from a larger number of students,

possibilities of making timetables and other schedules, in mathematics, in writing numbers from given digits; in understanding different possibilities in forming sets and subsets of numbers or dots (that determine direction, length, planes, triangles, and alike); in the formation of ordered pairs (point coordinates); in the possibility to make combinations with elements of addition and multiplication (commutativity).

The absence of subtle contents from many programmes of mathematics could be roughly explained by the lack of modern methodical transformation of combinatorial elements in the initial teaching of mathematics. Despite all this, the implicit problems of logical and combinatorial nature have been increasingly present not only in the initial teaching of mathematics but in initial teaching generally, although their solutions are highly problematic (Author, 2010).

Modern methodical transformation of combinatorial elements includes: teaching sets and logic, teaching permutations, teaching combinations, teaching variations, and teaching sets of ordered pairs.

4. Teaching Sets and Logic

The idea to single out a group of objects and then give a collective name to this group is immanent to natural language (a flock of birds, a mob of sheep, a bunch of keys, etc.). However, in mathematics this would be inappropriate and not very useful; therefore an abstract term *set* has been introduced to denote a group of separated objects.

Thus, the concept of a set is, in some way, introduced in a natural way as the plural of congeaerous objects, or elements with some common features.

Instructional contents from logic and sets are delivered in the way that lesson begins with the classification of objects that are then grouped into collections, plurals, or sets according to a common property they have. The lesson requires from children to use appropriate statements, saying that a certain element belongs or does not belong to a particular set, which statements may be true or false. Then, sets are being formed on the basis of connecting two attributes by conjunctions: "and", "or", or by words: "no", "each", "some", etc. This is to say that sets or subsets are formed by the use of logic operations. What comes first is that children get familiar with natural and didactic materials (different objects, fruits, logic blocks, etc.). Then, through play, and manipulation they learn the properties of these materials, they group them and arrange them, change their order, conct them and perform similar activities.

While mathematics operates with abstract concepts, on the other hand, methodical transformation in initial instruction of mathematics requires the use of concretization of sets and corresponding relations between elements of the sets. In the beginning, students should be impelled to understand how the order of elements in a set does not matter. Subsequently, new sets-*subsets* are being created between the elements of the given set on the basis of some specific properties. In this way children understand the process of forming subsets by repeating the algorithm that determines the set, as the application of an old procedure to a new situation, because the elements of the set are now grouped according to the new common property.

Ordering of sets is also conducted on concrete examples from everyday life and the world of play. Ordering a set is understood as a higher form of grouping. It is possible to order a set of boys according their height, or a set of sticks according their length. It can be very interesting to watch how students use different strategies in solving the assigned problems. For example, in the case of sticks, it may be interesting to see whether they would look for the longest or the shortest stick and then the second next, or whether they would choose a stick by chance and put it in the right place, or would they sort sticks into several subgroups, and then arrange subgroups and finally arrange sticks within the subgroups. Another assignment that could be very interesting is to differently arrange logic blocks or coloured sticks, which may all be different, or there may be the same ones. The distribution can be linear, or in a square or some other scheme. In case of a larger number of set elements, the total number of possible arrangements would be extremely high, which will allow each student to find more or less possible arrangements by his/her own. Children can compete in discovering a large number of solutions, new solutions, or they can check each other's solutions to see whether they are original or old, and in doing so the children are willing to invest great effort of their mind and their creative abilities.

Manipulative games are followed by various drawings and colouring, games with numbers and letters, points and other geometric objects. A very interesting and popular letter game is, for example, to compose a meaningful word with as many letters from randomly given letters. Very similar operations can be performed with numbers, geometric forms, and other objects. For example, you should paint a square field divided in three times three boxes with blue, red and green colour, so that boxes in each column and in each row are painted with different colour.

5. Teaching Permutations

Introduction to the idea and techniques of ordering elements in a set begins with:

Manipulation with objects: build different "towers", compositions, or "trains" from three, four or more different elements (eg. blocks of different colour);try as many sitting arrangements of students in a school desk or at some event; thread beads of different colour, and in different ways; make various "mats" from different sticks or some other elements.

Drawing, colouring, symbolic games: colour drawings, flags, etc. in different ways; write words of given length from given letters; write different numbers from given digits; "compose" a song from several sounds from given sounds (eg. mi, sol, la) by using one sound several times in a song.

Many tasks can be solved by manipulating objects, as well as by thinking about the problem and writing down identified possible solutions. As they gain more experience, children will increasingly opt for graphical, and even later for purely mental solving of *similar* problems. Thus, through adequate guidance and discovery of similarities in different problems, children will, sooner or later, be able to recognize and mentally solve combinatorial problems.

Determination of as many possible solutions will be followed by finding out all possible solutions or identifying the number of these solutions. During the process in which children try to find out if there is a system that works for creating different solutions, children may figure out certain experience-based rules that can be applied to determine a total number of solutions even without generating all possibilities, and also to test these possibilities or identified rules.

How to model this system and discover these rules?

Suppose the assignment is to write down all (or at least as many) three-digit numbers from given digits, for example: 1, 2, and 3, so as to use each given digit in one number.

5.1. Object Manipulation Method

There are several boxes (from eight to ten) each containing three chips numbered with 1, 2, 3.

Chips are picket out from the first box and placed next to one another, thus the first three-digit number is defined.

Then, the same procedure is repeated with chips from the next box, taking care not to create a three-digit number which has already been determined in previous attempts.

In every case the following question is asked: is there other possible arrangement, any other three-digit number that can be written down from the given digits, so as to use the given digits in each number once and only once?

In the end, the answer is provided – how many three-digit numbers have been found.

5.2. The "Box" Method

In the first place (in the first box) any of the three numbers may be selected.

In the second place (in the second box) one or another from the two remaining numbers may be selected, so there are two choices.

In the third place (in the third box) there is only one number left.

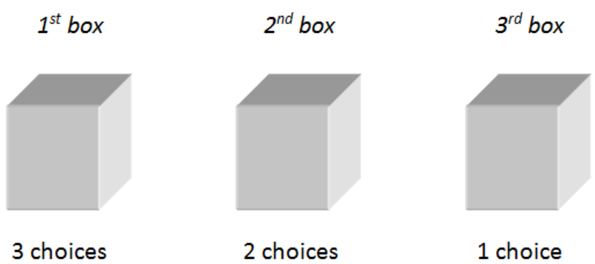

Figure 1. *The Box Method.*

Therefore, as presented in picture 1, the total number of three-digit numbers is: 3 + 2 + 1 = 6 three-digit numbers.

5.3. The Diagram Method

In a three-digit number, the digit 1 can be followed by 2 or 3, then the digit 2 can be followed only by 3, namely the digit 3 can be followed only by 2.

It is similar if it is a three-digit number beginning with 2 or 3.

This can be represented by a diagram, or a graph:

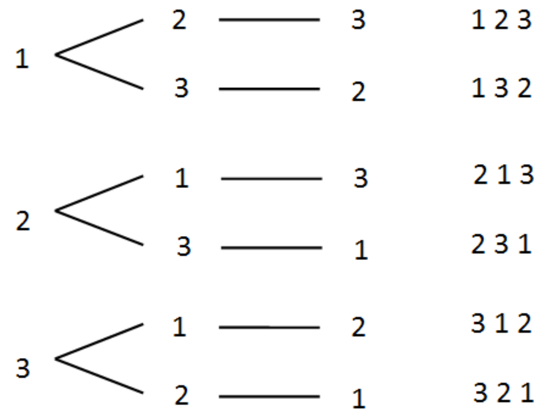

Figure 2. *Diagram Method.*

Therefore, as seen in picture 2, the total number of three digit numbers is: 3 + 2 + 1 = 6 three-digit numbers.

5.4. Symbolic Manipulation

The strategy is to write three-digit numbers by size without repeating digits in the same number:1 2 3, 1 3 2, 2 1 3, 2 3 1, 3 1 2, 3 2 1

Again, the result is 6 three-digit numbers.

Very similar are the problems of arrangements, such as, for example, the arrangement of logic blocks of different colour, or the arrangement of beads of different shape and different colour, or even colouring areas with different colours, and the like. They are similar (they have the same mathematical model), although they differ from the methodical aspect. In solving these problems, the knowledge about multi-digit numbers or about other mathematical concepts is not needed, since arrangements are not "abstract" constructs, but it is much harder to create and compare possible solutions, particularly if the number of elements is greater.

6. Teaching Combinations

From three, four, or more elements select a smaller number of elements. How do you do this, and in how many possible ways can you do this?

Examples:

- Choosing pairs (of two) from three students.
- Choosing two or three toys out of four different toys in all possible ways.
- Making as many different bouquets containing three flowers out of six different flowers.
- How many such bouquets can be made?
- How is it possible to choose two or three points in a

plane?

In other words, how many directions or lines, or triangles determine these points?

Additional example:

Let's solve a problem: A group of five students shake hands to say goodbye to friends.

Present possible handshakes and determine their number by the use of different methods.

6.1. Method of Graphs, Diagrams, Tree

Some persons are denoted by letters: A, B, C, D, and E. The following diagram represents the problem:

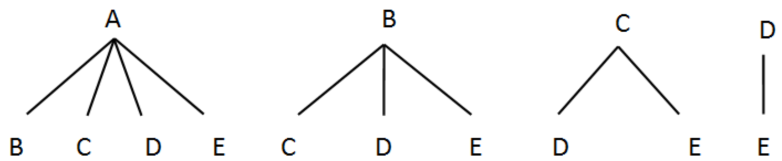

Figure 3. *Method of Graphs, Diagrams, Tree.*

Therefore, as presented in picture 3 the total number of handshakes is: 4 + 3 + 2 + 1 = 10.

6.2. Logical Analysis, Graphs

Each student will shake hands with the rest four students.

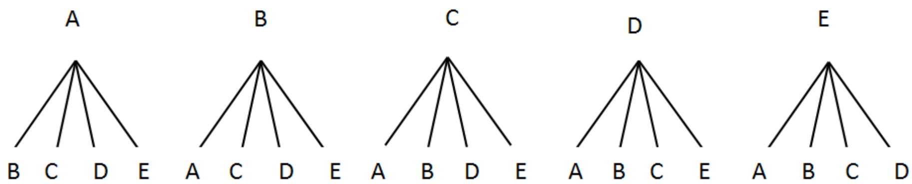

Figure 4. *Method of logical analysis.*

Therefore, as can be seen in picture 4, there is 5 x 4 = 20 handshakes in total. But, due to the fact that each handshake is counted twice (shaking hands occurs in pairs and is symmetrical), this number must be divided by 2.

If we generalize this to n persons we will get x (n – 1): 2 handshakes.

This may represent a kind of mathematical model or "a formula" for determining the total number of handshakes, or similar cases (events with the same mathematical model, such as number of lines and directions – if points are given).

Naturally, this formula can be tested by a simple induction experiment.

In any case this can help to evaluate results in similar examples.

Tables

Students' initials are appropriately entered into columns and rows. Pairs who shook hands are indicated in intersecting cells (some cells have remained empty because no one shakes his own hand, and shaking hands is a symmetrical operation).

In combinatorics assignments, there may be repetition of elements in some cases, which can mean that the elements may repeat several times.

Table 1. *Table Method.*

	A	**B**	**C**	**D**	**E**
A		A B	A C	A D	A E
B			B C	B D	B E
C				C D	C E
D					D E
E					

For example:One student has two marks in a school subject. What could be the marks?

How many different solutions to the problem are there?

1. Simply, we may predict, or write down all possible solutions:

1 1 2 2 3 3 4 4 5 5
1 2 2 3 3 4 4 5
1 3 2 4 3 5
1 4 2 5
1 5

2. This may be presented by the means of graphs as follows:

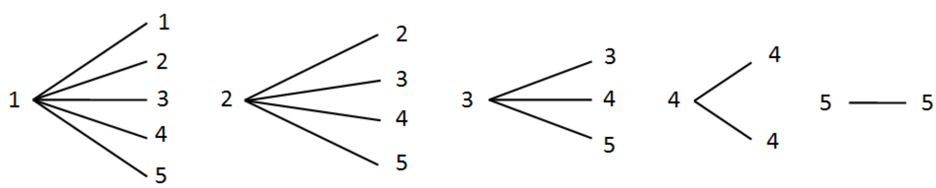

Figure 5. *Graph Method.*

In every case, the total of possible solutions is: 15 combinations.

7. Teaching Variations

Select and arrange a smaller number of elements from a larger number of elements in as many ways as possible.

Use three coulours (blue, white, and red) to colour the two-striped flag with different colours.

The problem shall be modelled by means of a diagram: The first line shows all possibilities for the first stripe, whereas in the second line we have provided possible solutions for the second stripe colour, which depends on the solutions for the first stripe.

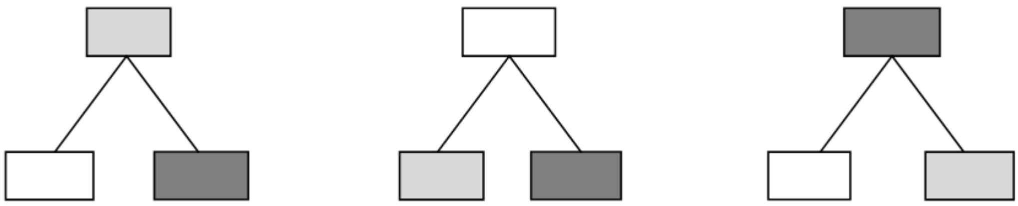

Figure 6. *Variations with colours.*

The assignment is principally solved; all possible solutions have been provided.

As could be seen, there are 6 solutions in total.

The assignment can be generalized to a larger number of colours and stripes (eg. to tricolor, etc.).

8. Problem Solving Methods

8.1. By Graphing

By the use of digits 4, 7, 8 write as many (or all) two-digit numbers without repeating digits in the same number.

In the first line there are all possible values for the tens, while in the second line, depending on the given data and the digit chosen in the first line, we have provided all possibilities for ones digits.

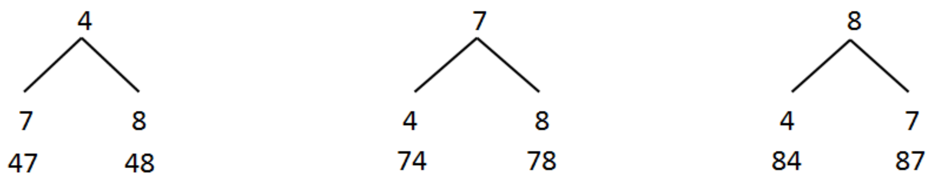

Figure 7. *Variations with numbers.*

Thus, all possible two-digit numbers in the assignment have been defined. There is a total of 3 x 2 = 6 two-digit numbers.

The assignment could be generalized (to more elements) and expanded (to other types of elements, such as letters or words), and, additionally, the number of all possible solutions can be determined in more ways.

8.2. Set of Ordered Pairs

Use elements from two or more sets to form ordered pairs, triads, etc.

Find different routes that lead from Cvijeta to Maša, if on the way there you have to drop by on Hana.

The problem is represented by the means of a graph. It is clear that there are three routes leading from Cvijeta to Hana (let them be 1, 2, and 3), and two routes that lead from Hana to Maša (let these routes be a, and e).

Possible route combinations are as follows: 1 a, 1 e, 2 a, 2 e, 3 a, 3 e.

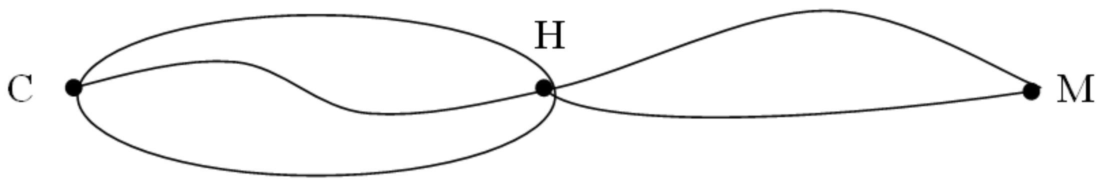

Figure 8. *Relation method.*

Therefore, as presented in picture 8 the total number of different routes is 3 x·2 = 6 different routes.

We have presented here different examples and various ideas, methods and techniques for solving combinatorial-nature problems. A larger number and greater diversity of similar examples should be introduced into the classroom practice for each type of combinatorial problems. This will allow for the formation of mental images of the observed phenomena, understanding of the basic ideas, methods and models of combinatorics.

The development of concepts, corresponding terminology and symbols from this field, as well as from other fields of mathematics, is a longlasting and a very complex process. Although many students never shall be able to lift their elementary knowledge of combinatorics to a theoretical level, this should not block their intuitive understanding and use of basic ideas and models of combinatorics.

9. Research Methodology

The experimental research was carried out in Vojvodina to examine the level of solving combinatorial problems in initial instruction of mathematics and the effects that modern methodical transformation may have on spontaneous understanding and application of ideas, concepts and models of combinatorics. The research sample involved 125 fourth grade primary school students both in experimental and in control group.

Both the experimental and control groups were formed in two city schools with similar working conditions and socio-economic status of parents. Equalling of the control and experimental group was done according to: descriptive general success of students from the previous year, achieved success in mathematics, gender.

The common characteristics or creative abilities of students were not directly inspected. However, students' marks in mathematics and general success indirectly speak about these characteristics and provide certain pieces of information that correlate them well with working habits and other psychological and pedagogical traits relevant for this study.

The basic hypothesis of the research is that students in lower grades of primary school understand and solve combinatorial problems very poorly, but the systematic application of appropriate methodical transformations in this area can significantly improve the situation.

During the formation of experimental and control group, namely prior to the introduction of an experimental factor, we have made the initial measurements and tests of students.

IBM Statistics 22 has been used for the purpose of data analysis. Groups have been compared via F-test and paired-sample t-test, in-group variables were analysed via paired-sample t-test.

Table 2. *The structure of research sample according to the gender of students was the following.*

Groups		N	Valid Percent
Experimental	Male	30	46,2
	Female	35	63,8
	Total	85	100
Control	Male	29	48,3
	Female	31	51,7
	Total	60	100

Table 3. *Student success.*

	Experimental group	Control group
according to overall school success	4.47	4,43
according to average students success in mathematics	4.22	4,15

The initial test included 6 assignments of objective type, and the solving of the assigned problems served to examine elementary abilities, knowledge and skills in the following fields:

- logical and combinatorial reasoning in the field of quantitative sizes,
- combinations in playing games in pairs, forming sets of concrete objects,
- combinations of elements of the two sets in the case of children dressing in various ways,
- permutations in tricolor painting,
- variations with or without repeating the problem of writing two-digit numbers,
- other problems of combinatorial and logical type.

In the beginning of the research control and experimental groups were equalized. The control group worked in a traditional way, but teachers in the experimental group carried out a special three months programme in teaching elements of combinatorics that followed the elaborated design of methodical transformation – experimental factor.

In the end of the experiment students in both control and experimental group were tested. The test of six assignments of objective type demanded from students to solve problems and thus examined the abilities, knowledge and skills in the following areas:

- arrangements of concrete objects – permutations,
- formation of subsets from a given set of elements – combinations,
- formation of ordered subsets with concrete elements – variations,
- formation of ordered subsets with repetition – variations,
- combinatorial geometry,
- complex combinatorial geometry assignments.

Most of the problems have been taken from the textbooks or collection of mathematical assignments for lower grades of primary school (Miljković and Marinković, 2005).

10. Results

Table 4. *Performance of the experimental and control group at the first and second assessment.*

Groups		N	Mean	SD	Minimum score	Maximum score	Cronbach's Alpha
Experimental	Initial	65	31.68	17.08	0	69	0.75
	Final	65	46.42	16.94	15	78	0.81
Control	Initial	60	30.83	19.37	0	65	0.75
	Final	60	16.35	15.23	0	53	0.81

Table 5. *Difference between the control and experimental groups at the first assessment.*

F	Sig.	t	df	Sig. (2-tailed)
1.81	0.181	0.26	123	0.79

No significant difference has been detected between the mean scores of the various groups in the course of the first assessment, according to the two-sample t-test group performance is statistically similar, thus they are eligible for comparison.

Table 6. *Difference between the control and experimental groups at the second assessment.*

F	Sig.	t	df	Sig. (2-tailed)
0.98	0.32	10.39	123	0.001

In the course of the second assessment, significant differences were detected between the performances of the groups, to be specific higher performance level of the experimental group was detected in comparison to the control group.

Table 7. *Control group performance at the first and second assessment.*

t	df	Sig. (2-tailed)
-4.96	64	0.001

The paired-sample t-test has demonstrates that significantly higher values of performance can be noticed within the control group at the second assessment procedure when comparing the data to the first assessment procedure.

Sex-based performance analysis

The Pearson correlation coefficient does not show any linear association between neither sexes, nor the two assessment procedures (Pearson Correlation=-0.1 p=0.93).

At the final test, students of the control group appear in large numbers up to 17 points achieved, and they dominate at all levels up to 32 points. Students of the experimental group start with 17 points achieved and dominate from 32 points. The control group students appear only up to 52 points achieved, while a part of (5% - 12%) experimental group students achieve excellent results (52 - 77 points).

The level to which combinatorial elements have been acquired and combinatorial ideas applied in the experimental group is as follows:

Arrangement of concrete objects: The starting point is a concrete problem that concerns time-table – four classes. Different ways of arrangement imply permutation of four elements without repetition. The students were very successful in solving this problem (81.54 %).

Formation of subsets from a given set: The assignment is from the field of combinations, and is presented to students in an abstract way (it is about elements of a set and subsets). The achieved results are solid (49.23 % successful problem solving), but students were mainly solving this problem in a concretized form (in the field of natural numbers).

Variation of a given set elements: The idea to apply a variation with repetition was, certainly, a big challenge for the four grade students of primary school. In a form of a concrete problem (the necessary number of dictionaries for a group of students who do not speak the same languages), students solved this assignment, due to its complex nature, at a satisfactory level (with 38.26 % success).

Classical problem of writing multi-digit numbers: The application of the variation with repetition, in a more familiar case (writing three-digit numbers) gave similar results as in previous assignment (41.03 % successful problem solving).

Combinatorial geometry: Problems that that deal with combinations in the field of geometry (lengths, squares, rectangles) were solved by the students at the obviousness level, however rather successfully (51.23 %).

Complex combinatorial geometry assignments: This is a more subtle combinatorial geometry problem which includes the relation of points, directions and lengths, and requires a creative approach ("In which way does number of directions and lengths depend on the location of given points in the plane"?), and the results in successful solving of this problem have been fairly modest (it was solved with 21.15 % of success). Y

11. Discussion

Given that all side factors in the sample were controlled and equalized, the resulting difference may be attributed to the actions of the experimental factor. This means that we confirmed the basic hypothesis that in the initial instruction of mathematics, the application of a defined methodical transformation of combinatorial elements into lectures about topics in combinatorics would provide appropriate educational outcomes and thus contribute to the modernization and the increased efficiency in teaching mathematics.

Yet, this result proves that even such assignments could be possibly solved by students in lower grades of primary school.

On the basis of this analysis of the research results, it is evident that students in 4^{th} grade grade of primary school, by applying adequate methodological transformation, which implies a creative way of solving problems, are able to successfully solve the following:

- problems from the field of permutation of concrete elements,
- simple problems in the field of combinations,
- different concrete problems which imply the ideas of variations,
- understand concrete combinatorial geometry assignments,
- solving abstract combinatorial assignments encounters more serious problems.

Therefore, the difference between the level of solving combinatorial problems in the experimental and control group is highly significant, thus the results that the experimental group students in the fourth grade of primary school have achieved in solving combinatorial assignments represent an encouraging fact, which promisses that in future in "natural situations" these results could become even better.

12. Conclusion

Besides arithmetics, geometry and measurings, modern programmes for initial mathematics teaching worldwide include topics such as sets, logic, combinatorics, probability and statistics. In the recent past, some unsuccessful attempts were made in order to modernize the initial instruction of mathematics, which showed that the realization of subtle contents in class-teaching is possible only by the means of didactical-methodical transformation of these contents.

Early introduction of the idea and elements of a set, logics and combinatorics into initial teaching of mathematics, and the corresponding methodical transformation of these contents, play a significant role which manifests as follows:

- understanding different phenomena of everyday life that occur in nature and society and in other school subjects,
- direct development of logical and combinatorial reasoning,
- development of creative mathematical reasoning,
- understanding and determining the rules of statistics – the law of probability,
- connecting play and learning – motivational activities in initial mathematics teaching.

Regardless of programmes applied in the initial instruction of mathematics, the practice can witness various logical and combinatorial problems which students solve quite unprepared and rather unsuccessfully.

The goal of this paper was to contribute to the realization of combinatorial-nature contents through defining a strategy and methodical approach in teaching elements of combinatorics and logical and combinatorial assignments in the initial instruction of mathematics, as well as to additionally explore educational effects of the described methodical transformation.

The basic hypothesis of the research was that students in lower grades of primary school hardly understand and poorly solve problems of combinatorial nature, but that this situation could be significantly improved in regard to both quantity and quality by introducing a systematic and appropriate methodical transformation to the contents in this field.

Taking Bruner's (1974) model of mathematical concept formation and other relevant theoretical findings as the starting point, we have defined an original and creative teaching strategy and developed an original methodical transformation of combinatorial elements for the initial instruction of mathematics. Thus, we have contributed to overcome the gap between the need to introduce combinatorics elements as early into teaching and the possibility to realize it in practice.

The research seems to indicate that educational effects are of high significance and, in relation to the existing low level in solving combinatorial problems; they may contribute to the quality of initial teaching.

Thus, the hypothesis is fully proven true.

Combinatorial prespedeutics, which is something new in methodical theory and practice, makes a significant contribution to the understanding and solving logical and combinatorial problems and to the development of students' creativity, and therefore it can increase the efficiency of education in general. It must therefore be recognized that we have justified scientific and practical importance of the research.

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