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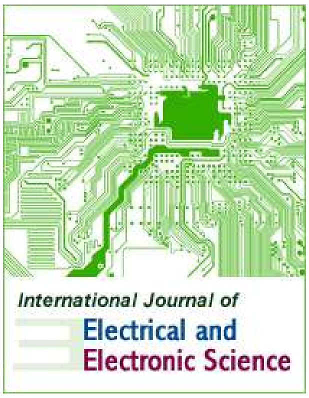

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New types of chaotic maps on topological spaces

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Abstract
In this paper, we will define a new class of chaotic maps on locally compact Hausdorff spaces called λ-type chaotic maps defined by λ-type transitive maps. This new definition implies John Tylar definition which coincides with Devanney's definition for chaos when the topological space happens to be a metric space. Relationships with some other type of chaotic maps are given. Further, we have proved that topological λ-type transitivity implies λ-dense orbits in a space X where X is a non-empty locally λ-compact Hausdorff topological space.

1. Introduction

We introduced and studied new types of topological transitivity called topologically λ-type transitive [1]. This is intended as a survey article on topological transitivity of a discrete system given by a λ-irresolute self-map of a compact topological space. On one hand it introduces postgraduate students to the study of new types of topological transitivity and gives an overview of results on the topic, but, on the other hand, it covers some of the recent developments. We introduced and defined a new type of chaotic map called λ-type chaotic map and investigate some of its properties. Relationships with some other type of chaotic maps are given. We list some relevant properties of the λ-type chaotic map. Further, We have proved that every λ-type chaotic map is chaotic map but the converse not necessarily true, . In 1986, Maki [2] continued the work of Levine and Dunham on generalized closed sets and closure operators by introducing the notion of Λ -sets in topological spaces. A subset A of a space X is called a Λ -set if it coincides with its kernel (saturated set), i.e. to the intersection of all open supersets of A. A subset A of a space X is called λ-closed [3] if A=L∩C, where L is a Λ-set and C is a closed set. The complement of a λ-closed set is called λ-open set. We denote the collection of all λ-open (resp. λ-closed) sets by λO(X) (resp. λC(X)).A point x ∈ X is called λ-cluster point of a subset A ⊂ X [4,5] if for every λ-open set U of X containing x $A \cap U \neq \emptyset$. The set of all λ-cluster points is called the λ-closure of A and is denoted by $Cl_{\lambda}(A)$. A point x ∈ X is said to be a λ-interior point of a subset A ⊂ X if there exists a λ-open set U containing x such that U ⊂ A. The set of all λ-interior points of A is said to be the λ-interior of A and is denoted by $Int_{\lambda}(A)$

Maps and of course irresolute maps stand among the most important notions in the whole of pure and applied mathematical science. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. In 1972, Crossley and Hildebrand [6] introduced the notion of irresoluteness. Many different forms of irresolute maps have been introduced over the years. Andrijevic [7] introduced a new class of generalized open sets in a topological space, the so-called b-open sets. A subset A of a topological space X is called regular open if A=Int(Cl(A)), and regular closed if X\A is regular open, or equivalently if A=Cl(Int(A)).. Throughout this work, the word "space "will mean topological space.

In this paper, we studied a new class of topological λ-type chaotic maps. We also studied some of their properties. Relationships with some other type of chaotic maps are given. We will list some relevant properties of λ-type chaotic map.

2. Preliminaries and Definitions

Definition 2.1. By a topological system we mean a pair (X, f), where X is a locally compact Hausdorff topological space (the phase space), and $f: X \to X$ is a continuous function. The dynamics of the system is given by $x_{n+1} = f(x_n)$, $x_0 \in X$, $n \in \mathbf{N}$ *and the solution passing through x is the sequence* $\{f(x_n)\}$ *where* $n \in \mathbf{N}$.

Definition 2.2 Let $x \in X$, then the set $\{x, f(x), f^2(x), \ldots\}$ is called an orbit of x under f and is denoted by $O_f(x)$.

Recall that a subset *S* is a □-set (resp. a V-set) if and only if it is an Intersection (resp. a union) of open (resp. closed) sets and that a subset *A* of a topological space X is called a *λ-open set* (or *λ*-closed) if *A* = *L* ∩*F*, where *L* is a □ -set and *F* is closed. Complements of *λ*-closed sets will be called λ-*open*.

Definition 2.3. [5]A function $f: X \to X$ is called λ-irresolute if the inverse image of each λ-open set is a λ-open set in X.

Example 2.4[1] Let (X, τ) be a topological space such that X={a, b, c,} and τ ={□, {a} ,{a, b},X}. We have λO(X, τ)={□, X, {a}, {c}, {a, b}, {a, c}, {b, c}.If I define the map f : X→X as follows f(c)= a, f(b)= b, f(a)= c. Then f is λ-irresolute.

Definition 2.5. A topological space (X, τ) is irreducible if every pair of nonempty open subsets of the space X has a nonempty intersection

In the study of dynamics on a topological space, it is natural and convenient to break the topological space into its irreducible parts and investigate the dynamics on each part. The topological property that precludes such decomposition is called topological transitivity.

In [1], we introduced the definitions of topological λ-type transitive and topologically λ-mixing maps as follows

Definition 2.6. Let (X, τ) be a topological space, $f: X \to X$ be λ-irresolute map, then the map f is called λ-type transitive if for every pair of non-empty λ-open sets U and V in X there is a positive integer n such that $f^n(U) \cap V \neq \phi$.

Definition 2.7. Let (X, τ) be a topological space, $f: X \to X$ be λ-irresolute map, then the map f is called topologically λ-mixing if, given any nonempty λ-open subsets $U, V \subseteq X$ $\exists N \geq 1$ such that $f^n(U) \cap V \neq \phi$ for all n>N. Clearly if *f* is topologically λ-mixing then it is also λ-transitive but not conversely

Definition 2.8[1] Two topological systems (*X, f*) and (*Y, g*) are said to be *conjugate* if there is a homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$

First of all, any property of topological systems must face the obvious question: Is it preserved under topological conjugation? That is to say, if f has property P and if we have a commutative diagram

$$\begin{array}{ccc} X & \xrightarrow{f} & X \\ {\scriptstyle h}\downarrow & & \downarrow{\scriptstyle h} \\ Y & \xrightarrow[g]{} & Y \end{array}$$

where (X, f) and (Y, g) are topological systems and h is a homeomorphism, then, is g necessarily has the same property P? Certainly transitivity and the existence of dense periodic points are preserved as they are purely topological conditions.

3. Λ-Type Transitive Maps and Topological ΛR-Conjugacy

In section 2, we introduced and defined λ-type transitive maps [1]. We will study some of their properties and prove some results associated with these new definitions. we investigated some properties and characterizations of such maps.

Definition 3.1 Recall that a subset A of a space X is called λ-dense in X if $Cl_\lambda(A) = X$, we can define equivalent definition that a subset A is said to be λ-dense if for any x in X either x in A or it is a λ-limit point for A.

Remark 3.2 any λ-dense subset in X intersects any λ-open set in X.

Definition 3.3 A subset A of a topological space (X, τ) is said to be nowhere λ-dense, if its λ-closure has an empty λ-interior, that is, $\operatorname{int}_\lambda(Cl_\lambda(A)) = \phi$.

Definition 3.4 if for $x \in X$ the set $\{f^n(x) : n \in \mathbf{N}\}$ is dense in X then x is said to have a dense orbit. If there exists such an $x \in X$, then f is said to have a dense orbit.

Definition 3.5. A map $f: X \to X$ is called λr-homeomorphism if f is λ-irresolute bijective and $f^{-1}: X \to X$ is λ-irresolute.

Definition 3.6 Two topological systems $f: X \to X$, $x_{n+1} = f(x_n)$ and $g: Y \to Y$, $y_{n+1} = g(y_n)$ are said to be topologically λr -conjugate if there is λr -homeomorphism $h: X \to Y$ such that $h \circ f = g \circ h$ *(i.e.* $h(f(x)) = g(h(x))$*)*. We will call h a topological λr -conjugacy.

Remark 3.7

If $\{x_0, x_1, x_2, \ldots\}$ denotes an orbit of $x_{n+1} = f(x_n)$ then $\{y_0 = h(x_0),\ y_1 = h(x_1),\ y_2 = h(x_2), \ldots\}$ yields an. In particular, h maps periodic orbits of f onto periodic orbits of g since $y_{n+1} = h(x_{n+1}) = h(f(x_n)) = g(h(x_n)) = g(y_n)$

We introduced and studied some new types of transitivity and minimality [1] in such a way that they are preserved under topologically λr- conjugation.

In [1], you can see the proof of the following theorem and propositions.

Theorem 3.8. For a λ-irresolute map f : X → X, where X is a topological space, the following are equivalent:.

1. f is topologically λ-type transitive;
2. Any Proper λ-closed subset $A \subset X \ni f(A) \subseteq A$ is nowhere λ-dense;
3. $\forall\ A \subseteq X \ni f(A) \subseteq A$, A is either λ-dense or nowhere λ-dense;
4. *Any subset* $A \subseteq X \ni f^{-1}(A) \subseteq A$ with non-empty λ-interior is λ-dense.

Proposition 3.9 if $f : X \to X$ *and* $g : Y \to Y$ are λr -conjugated by the λr-homeomorphism $h : Y \to X$. Then for all y ∈ Y the orbit $O_g(y)$ is λ-dense in Y if and only if the orbit $O_f(h(y))$ of h(y) is λ-dense in X.

Proposition 3.10 Let X be a λ-compact space without isolated point, if there exists a λ-dense orbit, that is there exists x0 ∈X such that the set $O_f(x_0)$ is λ- dense then the map f is λ-type transitive .

In [1] we have proved that, the new classes of λ-type transitive , λ-minimal and λ-mixing are preserved under λr-conjugation as shown in the following proposition:

Proposition3.11 if $f : X \to X$ *and* $g : Y \to Y$ are λr -conjugate. Then

(1) f is λ-type transitive if and only if g is λ-type transitive;

(2) f is λ-minimal if and only if g is λ-minimal;

(3) f is topologically λ-mixing if and only if g is topologically λ-mixing.

Proof (1)

Assume that $f : X \to X$ *and* $g : Y \to Y$ are topological systems which are topologically λr-conjugated by $h : X \to Y$. Thus, h is λr-homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are λ-irresolute) and $h \circ f = g \circ h$

Suppose f is λ-type transitive. Let A, B be λ-open subsets of Y $(to\ show\ g^n(A) \cap B \neq \varphi\ for\ some\ n > 0)$.

$U = h^{-1}(A)$ *and* $V = h^{-1}(B)$ are λ–open subsets of X since h is an $\lambda - irresolute$.

Then there exists some n>0 such that $f^n(U) \cap V \neq \varphi$ since f is λ–type transitive map. Thus $(as\ \ f \circ h^{-1} = h^{-1} \circ g\ \ implies\ \ f^n \circ h^{-1} = h^{-1} \circ g^n)$.

$\phi \neq f^n(h^{-1}(A)) \cap h^{-1}(B) = h^{-1}(g^n(A)) \cap h^{-1}(B)$

Therefore,

$h^{-1}(g^n(A) \cap B) \neq \phi$ *implies* $g^n(A) \cap B \neq \phi$ since h^{-1} *is* invertible. So g is λ-type transitive.

Proof (2)

Assume that $f : X \to X$ *and* $g : Y \to Y$ are topological systems which are topologically λr-conjugated by $h : Y \to X$. Thus, h is λr-homeomorphism (that is, h is bijective and thus invertible and both h and h^{-1} are λ-irresolute) and $h \circ g = f \circ h.$

We show that if g is λ-minimal, then f is λ-minimal. We want to show that for any $x \in X,\ O_f(x)\ is\ \lambda - dense$. Since h is surjective, there exists x □X such that $y = h^{-1}(x)$. Since g is λ-minimal, $O_g(y)\ is\ \lambda - dense$. For any non-empty λ– open subset U of X, $h^{-1}(U)$ is an λ-open set in X since h^{-1} is λ-irresolute because h is an λr-homeomorphism and it is non-empty since h is invertible. By λ -density of $O_g(y)$ there exist k in N such that $g^k(y) \in h^{-1}(U) \Leftrightarrow h(g^k(y)) \in U$

Since h is λr-conjugacy; as $f \circ h = h \circ g$ implies $f^k \circ h = h \circ g^k$ *so* $f^k(h(y)) = h(g^k(y)) \in U$ thus $O_f(h(y))$ intersects U. This holds for any non-empty λ-open set U and thus shows that $O_f(x) = O_f(h(y))$ is λ-dense

Proof (3)

We only prove that if g is topologically λ-mixing then f is also topologically λ-mixing. Let U,V be two λ-open subsets of X. We have to show that there is N>0 such that for any n>N, $f^n(U) \cap V \neq \phi.$ $h^{-1}(U)$ and $h^{-1}(V)$ are two λ-open sets since h is λ-irresolute. If the map g is topologically λ-mixing then there is N > 0 such that for any n>N, $g^n(h^{-1}(U)) \cap h^{-1}(V) \neq \phi.$ So $\exists\ x \in g^n(h^{-1}(U)) \cap h^{-1}(V)$. That is $x \in g^n(h^{-1}(U))$ *and* $x \in h^{-1}(V) \Leftrightarrow x = g^n(y)\ \ for\ \ y \in h^{-1}(U)$. h(x) ε V. Thus, since $h \circ g^n = f^n \circ h$, so that, $h(x) = h(g^n(y) = f^n(h(y)) \in f^n(U)$ and we have $h(x) \in V$ that is $f^n(U) \cap V \neq \phi.$ So, f is λ-mixing.

Definition 3.12 (i) A space X is said to be 2nd countable if it has a countable basis.

(ii) X is said to be of First Category if it is a countable union of nowhere dense subsets of X. It is of second Category if it is not of First Category.

Theorem 3.13 Let X be a non-empty locally λ-compact Hausdorff space. Then the intersection of a countable collection of λ-open λ-dense subsets of X is λ-dense in X. Moreover, X is of second Category.

Definition 3.14 A space (X, τ) is said to be second countable iff the λ-topology of X has a countable basis.

Definition 3.15 A space X is said to be λ-type separable if X contains a countable λ-dense subset.

Corollary 3.16 A subset A of a space (X,τ) is λ-dense if and only if $A\cap U\neq\phi$ for all $U\in\tau^{\lambda}$ other than $U=\phi$

Definition 3.17 Let (X, τ) be a topological space , $f:X\rightarrow X$ be λ-irresolute map then f is said to be topologically λ-type transitive if every pair of non-empty λ-open sets U and V in X there is a positive integer n such that $f^{n}(U)\cap V\neq\phi$..

The purpose of the following theorem is to prove that topological λ-type transitivity implies λ-dense orbits in a space X where X is a non-empty locally λ-compact Hausdorff topological space.

Theorem 3.18 Let (X,f) be a topological system where X is a non-empty locally λ-compact Hausdorff topological space and that X is λ-type separable. Suppose that f is topologically λ-type transitive. Then there is an element $x\in X$ such that the orbit $O_f(x)=\{x,\ f(x),\ f^2(x),\ ...f^n(x),.....\}$ is λ-dense in X.

Proof: Let $B=\{U_i\}$ $i=1,2,3,....$ be a countable basis for the λ-topology of X. For each i, let $O_i=\{x\in X: f^n(x)\in U_i$ for some $n\geq 0\}$

Then, clearly O_i is λ-open and λ-dense. It is λ-open since f i s λ-irresolute, so, $O_i=\bigcup_{i=1}^{\infty}f^{-1}(U_i)$ is λ- open and λ-dense since f is topological λ-type transitive map. Further, for every λ-open set V, there is a positive integer n such that $f^n(V)\cap U_i\neq\phi,$ since f is λ-type transitive.

Now, apply theorem 3.13 to the countable λ-dense sets $\{O_i\}$ to say that $\bigcap_{i=0}^{\infty}O_i$ is λ-dense and so non-empty. Let $y\in\bigcap_{i=0}^{\infty}O_i$. This means that, for each i, there is a positive integer n such that $f^n(y)\in U_i$ for every i.

By corollary 3.16, this implies that $O_f(x)$ is λ-dense in X

4. New Types of Chaos of Topological Spaces

We will give a new definition of chaos for λ-irresolute self map $f:X\rightarrow X$ of a locally compact Hausdorff topological space X, so called λ-type chaos. This new definition implies John Tylar definition which coincides with Devanney's definition for chaos when the topological space happens to be a metric space.

Definition 4.1 Let (X,f) be a topological system, the dynamics is obtained by iterating the map. Then, f is said to be λ-type chaotic on X provided that for any nonempty λ-open sets U and V in X, there is a periodic point $p\in X$ such that $U\cap O_f(p)\neq\phi$ and $V\cap O_f(p)\neq\phi$.

Proposition 4.2 Let (X,f) be a topological system. The map f is λ-type chaotic on X if and only if f is λ-type transitive and the set of periodic points of the map f is $\lambda-dense$ in X.

Proof: ⇒) If f is λ-type chaotic on X, then for every pair of nonempty λ-open sets U and V, there is a periodic orbit intersects them; in particular, the periodic points are λ-dense in X. Then there is a periodic point p and $x,y\in O_f(p)$ with x ∈ U and y ∈ V and some positive integer n such that $f^n(x)=y$, so that $y=f^n(x)\in f^n(U)$ therefore $f^n(U)\cap V\neq\phi$, that is, f is λ-type transitive map.

⇐:) The λ-type transitivity of f on X implies that for any nonempty λ-open subsets U, V ⊂ X, there is n such that for some x ∈ U, $f^n(x)\in V$. Now define

$W=f^{-n}(V)\cap U$. Then W is λ-open and nonempty with the property that $f^n(W)\subset V$.

But since the periodic points of f are λ-dense in X, there is a p ∈ W such that $f^n(p)\in V$. Therefore, $U\cap O_f(p)\neq\phi$ and $V\cap O_f(p)\neq\phi$, so that f is λ-type chaotic.

5. Conclusion

There are the following results:

Definition 5.1 Let (X,f) be a topological system, the dynamics is obtained by iterating the map. Then, f is said to be λ-type chaotic on X provided that for any nonempty λ-open sets U and V in X, there is a periodic point $p\in X$ such that $U\cap O_f(p)\neq\phi$ and $V\cap O_f(p)\neq\phi$.

Proposition 5.2 Let (X,f) be a topological system. The map f is λ-type chaotic on X if and only if f is λ-type transitive and the periodic points of the map f are $\lambda-dense$ in X.

Theorem 5.3 Let (X,f) be a topological system where X is a non-empty locally λ-compact Hausdorff topological space and $f:X\rightarrow X$ is λ-irresolute map and that X is λ-type separable. Suppose that f is topologically λ-type transitive. Then there is an element $x\in X$ such that the orbit $O_f(x)=\{x,\ f(x),\ f^2(x),\ ...f^n(x),.....\}$ is λ-dense in X.

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