Energy flow and the speed of electric field in DC circuit

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Citation

Abstract
In the case of DC circuit, electrons are driven by the electric potential difference in the wire; the electric potential energy is then converted to kinetic energy. Experiments show that Poynting vector is just a mathematical definition in the DC case. Since it does not form real energy flow, there is no transmission of electromagnetic energy flow from the exterior to the interior of metal wire. Therefore, DC power is transmitted entirely within the metal wire. In this paper, it also shows a preliminary experimental result of the redistribution speed of the electric field.

1. Introduction

According to Coulomb's law, the electrostatic field is longitudinal field. In the DC circuit, the electrical field in the metal wire is also longitudinal field. Although these two electric fields have certain similarities [1-3], in fact, there are some differences between them. To simplify the discussion, we refer the steady current to the direct current (DC), and we use a simple DC circuit to make analysis in this paper. A typical DC circuit has a battery, and the battery connects to the load resistors by metal wires.

From textbooks of electromagnetism, the tangential (longitudinal) components of electric field strength along the external and internal surfaces of the metal wire are equal to each other. Even in a DC circuit, textbooks claim that electric energy is transmitted via the space around the metal wire, and then input to the load resistors. [3] (See Figure 1)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{Electric field and energy flow around the metal wire}
\end{figure}

In Figure 1, $E$ is the electric field and $S$ is the energy flow.

The above viewpoints may be referred from Feynman’s Lectures on Physics Vol. II as follows [4].

As another example, we ask what happens in a piece of resistance wire when it is
carrying a current. Since the wire has resistance, there is an electric field along it, driving the current. Because there is a potential drop along the wire, there is also an electric field just outside the wire, parallel to the surface. (See Fig. 27-5.) There is, in addition, a magnetic field which goes around the wire because of the current. The E and B are at right angles; therefore there is a Poynting vector directed radially inward, as shown in the figure. There is a flow of energy into the wire all around. It is, of course, equal to the energy being lost in the wire in the form of heat. So our "crazy" theory says that the electrons are getting their energy to generate heat because of the energy flowing into the wire from the field outside. Intuition would seem to tell us that the electrons get their energy from being pushed along the wire, so the energy should be flowing down (or up) along the wire. But the theory says that the electrons are really being pushed by an electric field, which has come from some charges very far away, and that the electrons get their energy for generating heat from these fields. The energy somehow flows from the distant charges into a wide area of space and then inward to the wire.

In order to verify the above viewpoints, we have conducted a few simple experiments to prove that in a DC circuit, the above boundary condition for the electric fields at the wire boundary may be not valid. Since the electrostatic field can be shielded with a metal cover, we have made a few tests. In the DC circuit, we connect a current meter in series. a), we shield the battery with a metal cover; b), we shield the resistor with a metal cover; c), we use a coaxial cable to shield the inner wire. The results are as follows:

The reading of the current meter is essentially the same (with accuracy of 4 digits). These tests show that in the DC circuit, the distribution of outside electric field (including the external electric field near the interface of the metal wire) has essentially no effects to the electric field inside the metal wire.

The DC electric field is associated with the conduction current, which has an important difference from the electrostatic field. In this paper, it shows that Poynting vector is just a mathematical definition in the DC case. There is no real energy flow formed, i.e., no flow of electromagnetic energy is transferred from outside wire to inside wire. This deduction is quite different from the conclusion in the textbooks of electromagnetism.

2. The Electric Field in the Wire of a DC Circuit

We consider that the metal wire is a good conductor. Under normal circumstances, metal wires are electrically neutral. The total positive charge of the crystal lattice is equal to the negative charges of free electrons inside the wire. For a steady current, there is a ring external magnetic field \( H \), which obeys the right-handed theorem.

The speed of free electrons in the metal wire is very slow, only less than 1mm/s. However, the speed of electrical energy in the DC circuit is very fast. Many people think that the transmission speed of electricity is almost identical to the speed of light \( c \) in vacuum. For example, when the switch is closed, the electricity turns on a lamp light almost instantly. Thus, the transmission of electric power is dominated by the electric field, but not by the movement of free electrons in the DC circuits.

The direct electric field in a metal wire is steady field. This electric field does not change over time. On the other hand, this direct electric field has certain difference with the electrostatic field outside of the metal wire. The electrostatic field obeys loop theorem:

\[
\oint E \cdot dl = 0
\]

Its differential form can be written as:

\[
\nabla \times E = 0
\]

This means that the electrostatic field is a conservative field. However, in the DC circuit, there exists non-electrostatic source because of the battery. The formula (1) and (2) are not valid for the non-electrostatic source. In the DC circuit, Ohm's law plays an important role. Its differential form can be written as:

\[
J = \sigma E
\]

Here \( J \) is the vector of current density, i.e., a current intensity per unit area. Its direction is consistent with the electric field intensity \( E \); \( \sigma \) is the conductivity of the wire, which is the conductive property of the metal. Therefore, the electric field intensity \( E \) in the wire is determined by the conductivity of the wire and the current density.

3. The Process of Power Transmission in the DC Circuit

In the DC circuit, the power \( P = IV \), \( I \) is the DC circuit intensity, and \( V \) is the voltage. The power density can be written as:

\[
p = j \cdot E
\]

From the differential form of Ohm's law, Eq. (5) can be rewritten as \( p = \sigma E^2 \).

It is generally accepted that the Poynting vector transmits electromagnetic energy in whole space. Indeed, this statement has certain limitations. As we know, the Poynting vector is expressed by \( S = E \times H \). \( E \) is the electric field intensity; \( H \) is the magnetic field intensity. Poynting vector \( S \) represents energy flow density in a unit time. It is perpendicular to the electric field and a magnetic field.

In the process to introduce magnetic field intensity, Ampere's law is applied.[1-3] When a direct current flows in the wire, there exist stable longitudinal electric field and a ring magnetic field (not changing with time) inside the wire as well. Assuming that a small piece of metal wire is cylindrical, it is
shown in Figure 2. The Poynting vector points toward the center of the cylindrical conductor.

Inside the metal wire, according to Ampere’s law, the value of the magnetic field intensity $H$ is reduced with the decreased radius.

Here we need to emphasize that because these two kinds of fields are "static" in space, the Poynting vector is just a mathematical definition in the DC case. In fact, no real energy flows from the outer of the metal wires into the wire.

As mentioned in the introduction, we have conducted a few experimental tests in different conditions: to shield resistors; to shield the metal wires; or to shield both. It is found that the electric current keep almost unchanged in 3 different conditions. These simple experimental tests demonstrate that the outside tangential component of the electric field intensity may be different along the interface of the metal wire. It means that no real energy flow goes into the metal wire in the DC circuit.

4. A Preliminary Experiment to Measure the Redistribution Speed of the Electric Field in a DC Circuit

When the switch of a DC circuit turns on, the electric field inside the wire redistributes almost instantly. In order to measure the redistribution speed of the electric field, a few preliminary experiments are conducted in the Lab of Nanyang Institute of Technology.

In Fig.3, L denotes a metal wire about 1.8 meters long. R1 and R2 are taken as 50 Ohm. We take the voltages at the ends of R1 and R2 as signals inputs into an oscilloscope. When the switch is closed, the oscilloscope shows the waveform from the output voltages of R1 and R2.

The wire’s diameter is about 1mm. Total resistance of the wire in the DC circuit is about 0.6 Ohm. 3 AA batteries with a switch are used in the experiment. The oscilloscope data are input into the software Matlab. It makes the fitting by using least squares curve. The fitting equation is:

$$V_R(t) = R I_0 (1 - e^{-t (R/L)})$$

Here $I_0$ is the maximum electrical current after the switch is turned on.

This is the analytic solution for the voltage across R in a L-R circuit. From the experimental data, we obtain the following curves:

In Fig. 4, the blue line and the yellow line represent the voltages input to the Channel 1 and Channel 2 of the oscilloscope respectively. The first 100 ns data was truncated because the electric current is not formed yet. The truncated
position has some influence on the fitting results. Due to the distributed capacitance of the circuit, the starting current is affected by high frequency components, which form certain current jump transition. On the other hands, after 200ns, the low-frequency component plays leading role, then two fitting curves nearly coincident. Therefore, as the preliminary results, we used the data of -50 ns to 150 ns in Figure 4.

As a preliminary result, the time interval between 2 fitting curve is about 5.0 ns, and the mean square error 2.0 ns. Considering the length of the wire between R1 and R2 is 1.8 meter, we obtain the redistribution speed of the electric field in the DC circuit is:

$$v_{loc} = (1.2 \pm 0.5)c$$ (7)

Here $c$ is the speed of light in vacuum. Because there is a big error bar in our preliminary experiment, more tests and more research are needed. The experimental details and more data will be reported in another paper. Although there is a big error bar here, this case is quite similar to the measurement of the mass square of the electron neutrino, the recent average value of $m^2(\nu_e) = -1.1 \pm 2.4$ eV$^2$.[5]

5. Discussion and Conclusion

In electrodynamics, Poynting's theorem is related to the conservation of energy for the electromagnetic field, because it relates the energy stored in the electromagnetic field to the work done on an electrically charged object through energy flux. The expression of Poynting vector physically means that the energy transfer due to time-varying electric and magnetic fields is perpendicular to the fields. However, Poynting's theorem is not needed to apply to the DC circuit.

In the DC circuit, there is a power source (battery), and the electric energy is produced from non-static electric force. Since air is a good insulator, there are very few free electrons outside the metal wire, and the electric field intensity at both sides of the boundary of the metal wire has a sudden change.

Based on the experimental results and theoretical analysis, we have obtained a few new deductions of electromagnetism as follows:

1. Electrons are driven by an electric potential difference in the metal wire, and then the electric potential energy converts to kinetic energy. In the battery, chemical effects are converted into electrical energy. It gathers the positive and negative charges at the ends of the battery and produces an electromotive force (electric potential difference, i.e. voltage). This potential difference generates an electrostatic field in the whole space. In the outer of conductor, since there are almost no free electrons, it does not form a electrical current. Inside the wire, the electrical potential difference drives electrons, and then forms a current. Thus the electric potential energy is converted into kinetic energy of electrons, and it consumes electric power. This process completes the loop of energy conservation.

2. Outside the metal wire interface, there are stable magnetic fields and electrostatic fields. However, these are the effects of the DC circuit. The Poynting vector is only a mathematical definition. In fact, no real energy flow is formed. Thus, no electromagnetic energy flow is transferred from the external wire into the internal wire.

3. The DC power from the battery is entirely transmitted inside the metal wire.

4. In principle, the redistribution speed of electric field inside the metal wire is allowed to be superluminal. In this paper, we have shown a preliminary experimental result to measure the redistribution speed of the electric field.

When the circuit switch is closed, the electric field inside the wires is redistributed almost instantly. The electric field from the positive end of the battery leads to the negative end inside the wire. The electric field drives the free electrons in the metal wire, and then the current is generated. The electric field carries the force. Furthermore, the electric field combines with the electric current at the same point, and then the combined effects produce the energy flow, and turns to Joule heat. The DC power is entirely transmitted inside the metal wire. The above description basically illustrates the transmission process of DC power in the metal wire.

There is a related issue in this paper. When the DC circuit switch turns on, the redistribution speed of electric field inside the metal wire is allowed to be superluminal. This is a microscopic process, and it is related to a research area of quantum mechanics. Some studies for the possibility of superluminal motion have been studied theoretically and experimentally in [6-11].

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References
