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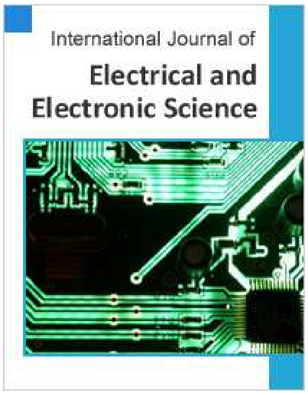

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The Relationship Between Feedback Controller Numbers and Speed of Convergent in Control and Synchronization via Nonlinear Control

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Abstract
The significance of this paper is to find the optimal control through studying the relationship between numbers of feedback controllers and the speed of convergent (time control and synchronization) in control and synchronization, where most of the previous works achieved control and synchronization without taking into consideration with time cofactor. In this paper we achieve control and synchronization of a dynamical system via nonlinear control by designing more than one novel suitable feedback controllers with consider time control and time synchronization. The proposed methods have certain significance for increasing the speed of convergent for controller implementation. Finally, numerical simulations are given to illustrate and verify the results.

1. Introduction

In the last two decades extensive studies have been done on the properties of nonlinear dynamical system. One of the most important properties of nonlinear dynamical systems is that of chaos [13]. Chaos phenomenon was firstly observed by Lorenz in 1963[2, 3, 9, 10], chaos control is one of the chaos phenomena, which contains two aspects, namely, chaos control and chaos synchronization [5]. Chaos control and chaos synchronization were once believed to be impossible until the 1990s when Ott et al. developed the OGY method to suppress chaos, Pecora and Carroll introduced a method to synchronize two identical chaotic systems with different initial conditions [1, 3, 4, 5, 6, 7, 14].

Many different techniques for chaos control and synchronization have been developed, such as linear feedback method, active control approach, adaptive technique, time delay feedback approach, and backsteeping method and so on. Among them, nonlinear control is an effective method to control chaos [1, 3, 4, 7, 8, 9, 10, 11, 12].

In some systems, the time cofactor is very important than and effective on system's activity and it makes the performance of this system poorer and poorer or any delay or late may cause a fault (as example a late command to stop a train may cause a collision). However, most of the previous works achieved control and synchronization by using the techniques mentioned above, via choice only one suitable feedback controllers without interest with time cofactor . In this paper we achieve control and synchronization of a modified hpyerchaotic pan system via nonlinear control by designing more than one suitable feedback controllers with considering time control and time synchronization and each control contains different numbers of feedback controllers. we found the effective of this different feedback controller numbers on speed convergent. Consequently, in order to obtain fast convergence speed, we will reduce the numbers of feedback controllers. The proposed methods have certain significance for increasing the speed of convergent for controller implementation.

The rest of this paper is organized as follow. Section 2 presents the system descriptions and Section 3 introduces controlling system (1) via three a novel nonlinear control. In section 4, we propose two nonlinear control to synchroniz between two identical hyperchaotic systems with unknown parameters, which is followed by the conclusion in section 5.

2. System Descriptions

The hyperchaotic system [3] is described by the four-dimensional dynamics:

$$\begin{cases} \dot{x} = a(y - x) \\ \dot{y} = cx - xz + w \\ \dot{z} = xy - bz \\ \dot{w} = -dy \end{cases} \tag{1}$$

here $(x, y, z, u) \in R^4$, and $a, b, c, d \in R$ are constant parameters, When parameters $a = 10$, $b = 8/3$ $c = 28$ and $d = 10$, system (1) is hyperchaotic and has two Lyapunov exponents, i.e. $LE_1 = 0.38352,$ $LE_2 = 0.12714$, and hyperchaotic attractors are shown in Fig. 1. The system (1) has only one equilibrium $O(0, 0, 0, 0)$, and the equilibrium is an unstable under these parameters.

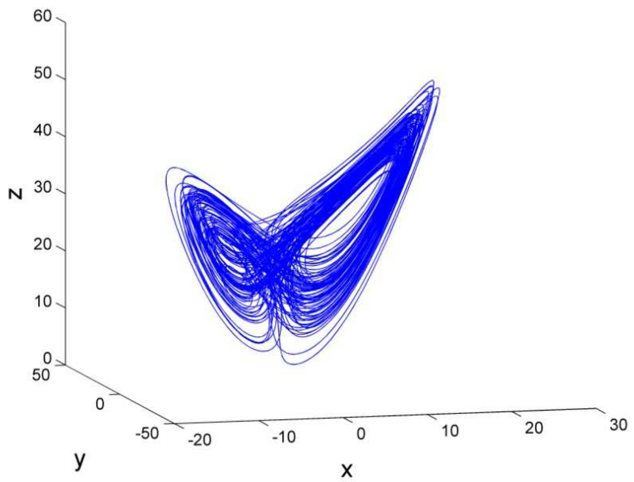


Fig. 1. *The attractor of system (1) in x-y-z space.*

3. Chaos Control

The main focus of this section is to investigate the controlling problem of system (1), by designing three suitable feedback controllers, and studying the relationship between these three controllers with speed, to achieve the control for each design. also we found those results analytically based on the Lyapunov stability theory as well as graphically by using MATLAB.

In order to control the modified hyperchaotic pan system to zero, the feedback controllers of u_1, u_2, u_3 and u_4 are added to the hyperchaotic system (1). Then the controlled hyperchaotic system is given by:

$$\begin{cases} \dot{x} = a(y - x) + u_1 \\ \dot{y} = cx - xz + w + u_2 \\ \dot{z} = xy - bz + u_3 \\ \dot{w} = -dy + u_4 \end{cases} \tag{2}$$

in which a, b, c and d are unknown parameters, and u_1, u_2, u_3, u_4 are feedback controllers to be designed.

3.1. Controlling Hyperchaotic System (2) with Four Feedback Controllers

In the following theorem, we design nonlinear control with four feedback controllers to control system (2)

Theorem 1. If the nonlinear controllers are proposed as:

$$\begin{cases} u_1 = (a - 1)x \\ u_2 = -(a + c)x - y \\ u_3 = (b - 1)z \\ u_4 = (d - 1)y - w \end{cases} \tag{3}$$

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

Proof. According to the Lyapunov stability theory, we construct the following Lyapunov candidate function

$$V = 1/2(x^2 + y^2 + z^2 + w^2)$$

and the time derivation of the Lyapunov candidate function is:

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} \\ &= x\left[a(y - x) + u_1\right] + y\left[cx - xz + w + u_2\right] \\ &\quad + z\left[xy - bz + u_3\right] + w\left[-dy + u_4\right] \\ &= -ax^2 - bz^2 + (a + c)xy + (1 - d)yw \end{aligned} \tag{4}$$

Substituting the Controller (3) into (4), yields:

$$\dot{V} = -x^2 - y^2 - z^2 - w^2 < 0$$

It is clear that V is positive definite and $\dot{V}$ is a negative definite. Therefore, based on the Lyapunov stability theory, the controlled system (2) can asymptotically converge to the unstable equilibrium with the controllers (3). Consequently, the controlling of system (2) is achieved via nonlinear control , This completes the proof.

3.2. Controlling Hyperchaotic System (2) with Three Feedback Controllers

In this theorem we propose nonlinear control which contains number of feedback controllers less than in theorem 1.

Theorem 2. If the nonlinear controllers are proposed as:

$$\begin{cases} u_1 = (a-1)x - cy \\ u_2 = -ax - y + dw \\ u_3 = 0 \\ u_4 = -y - w \end{cases} \tag{5}$$

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

Proof. Construct a Lyapunov function:

$$V = 1/2(x^2 + y^2 + z^2 + w^2)$$

and the time derivation of the Lyapunov function is

$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} \\ &= x\left[a(y-x) + u_1\right] + y\left[cx - xz + w + u_2\right] \\ &+ z\left[xy - bz + u_3\right] + w\left[-dy + u_4\right] \end{aligned} \tag{6}$$

Substituting the Controller (5) into (6) gives that:

$$\dot{V} = -x^2 - y^2 - bz^2 - w^2 < 0$$

which gives asymptotic stability of the system (2) by Lyapunov stability theory. This means that the controller propose is achieved the suppressed of system (2).This completes our proof.

3.3. Controlling Hyperchaotic System (2) with Two Feedback Controllers

The following theorem, we design nonlinear control with two feedback controllers for controlling system(2)

Theorem 3. If the nonlinear controllers are proposed as:

$$\begin{cases} u_1 = 0 \\ u_2 = -(a+c)x - y \\ u_3 = 0 \\ u_4 = (d-1)y - w \end{cases} \tag{7}$$

Then the zero solution of the controlled hyperchaotic system (2) is globally asymptotically stable.

Proof. Let us consider the Lyapunov function as follows:

$$V = 1/2(x^2 + y^2 + z^2 + w^2)$$

and the time derivation of the Lyapunov candidate function is:

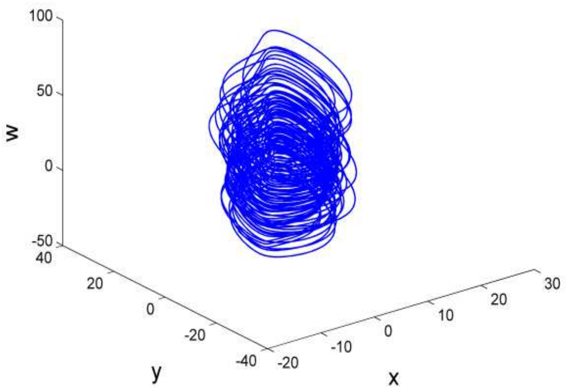
$$\begin{aligned} \dot{V} &= x\dot{x} + y\dot{y} + z\dot{z} + w\dot{w} \\ &= x\left[a(y-x) + u_1\right] + y\left[cx - xz + w + u_2\right] \\ &+ z\left[xy - bz + u_3\right] + w\left[-dy + u_4\right] \end{aligned} \tag{8}$$

Substituting the Controller (7) into (8), yields:

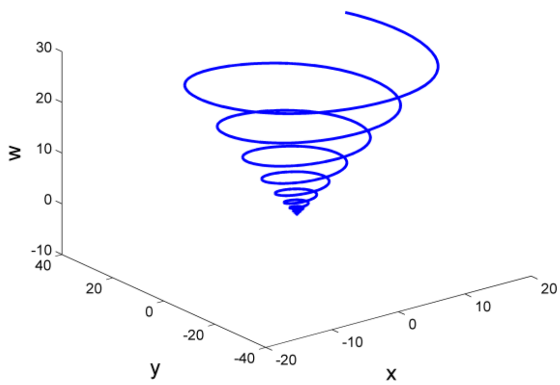
$$\dot{V} = -ax^2 - y^2 - bz^2 - w^2 < 0$$

then $\dot{V}$ is negative definite and the Lyapunov function V is positive definite. From Lyapunov stability theory it follows that the system (2) is asymptotically stable with control (7), this completes the proof.

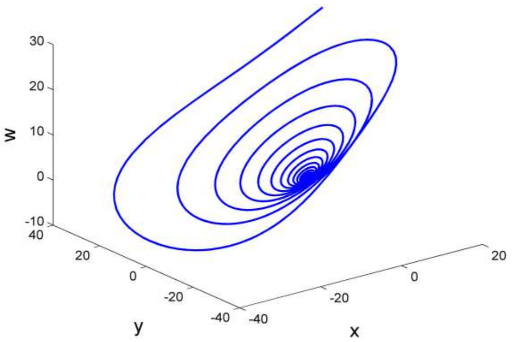
Also the following figures describe the attractor of system (2) before and after add the feedback controllers. where it is obvious the control of system (2) is achieve successfully with control (3),(5) and (7).



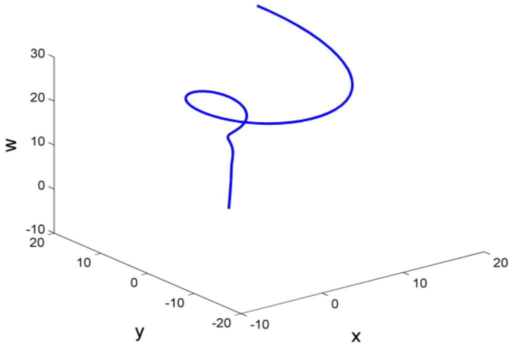
(a)



(b)



(c)



(d)

Fig. 2. *(a) the attractor of system (2) in x-y-w space before control , (b) the attractor of system (2) in x-y-w space with control (3), (c) the attractor of system (2) in x-y-w space with control (5), (d) the attractor of system (2) in x-y-w space after control (7).*

But in order to increasing to accuracy of this results, we use numerical simulation based on fourth-order Runge -Kutta scheme with time step 0.5 .we choose the parameters $a = 10$, $b = 8/3$ $c = 28$ and $d = 10$ the initial values are taken as $(-5, -3, 20, 10)$. Fig. 3 show the state of hyperchaotic system (2) with four feedback controllers, while Fig.4 show the state of hyperchaotic system (2) with three feedback controllers. finally controlling system (2) with two feedback controllers is present in Fig. 5. Obviously, the convergent of hyperchaotic system (2) in Fig. 3, Fig. 4 and Fig. 5 are the time 8, 4 and 2.5 respectively. Consequently, wherever reduce the numbers of feedback controllers then we get fast convergent.

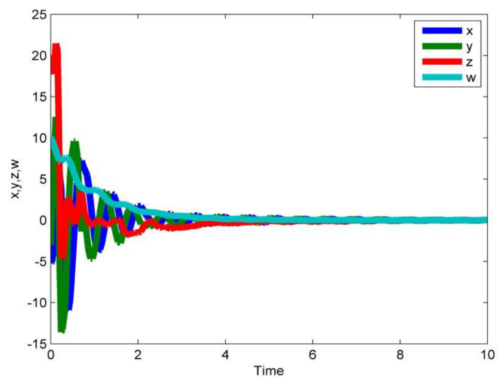


Fig. 3. *Controlling system (2) with four feedback controllers.*

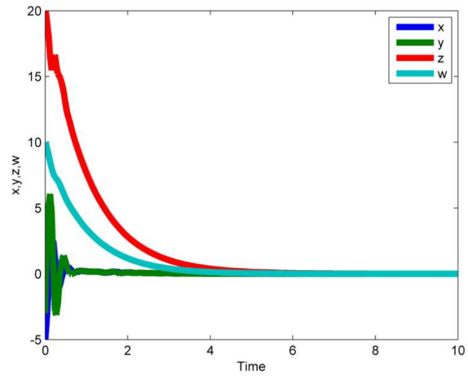


Fig. 4. *Controlling system (2) with three feedback controllers.*

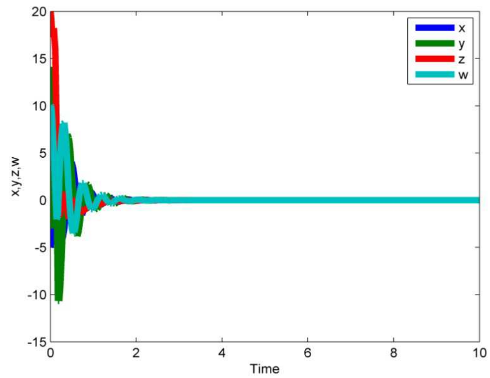


Fig. 5. *Controlling system (2) with two feedback controllers.*

4. Chaos Synchronization

The main focus of this section is to investigate the synchronization problem of system (1) by designing two suitable feedback controllers and studying the effective numbers of feedback controllers with speed to achieve the synchronize for each design. also we found those results analytically based on the Lyapunov stability theory as well as graphically by using MATLAB.

We choose the modified hyperchaotic pan system (1) as the drive system, and the controlled modified hyperchaotic pan system (2) as the response system.

Subtracting system (1) from the system (2), we obtain the error dynamical system between the drive system and the response system which is given by:

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1) + u_1 \\ \dot{e}_2 = (c - z)e_1 + e_4 - e_1 e_3 - x e_3 + u_2 \\ \dot{e}_3 = -b e_3 + e_1 e_2 + x e_2 + y e_1 + u_3 \\ \dot{e}_4 = -d e_2 + u_4 \end{cases} \tag{9}$$

where $e_1 = x_1 - x, e_2 = y_1 - y, e_3 = z_1 - z$ and $e_4 = w_1 - w$.

System (9) describes the error dynamics. It is clear that the synchronization problem is replaced by the equivalent problem of stabilizing the system (9) using a suitable choice of the feedback controller.

4.1. Chaos Synchronization of System (9) with Four Feedback Controllers

In this subsection, the synchronization is performed through a nonlinear controller which contains four feedback controllers based on Lyapunov stability theory to stabilize the error dynamics system (9), then we obtain the following theorem:

Theorem 4. The zero solution of the error dynamical system (9) is asymptotic stable if nonlinear control is designed as following:

$$\begin{cases} u_1 = (a-1)e_1 - (a+c-z)e_2 \\ u_2 = -\,e_2 + (d-1)e_4 \\ u_3 = (b-1)e_3 - ye_1 \\ u_4 = -e_4 \end{cases} \tag{10}$$

Proof. Let us consider Lyapunov function is:

$$V = 1/2\left(e_1^{\,2} + e_2^{\,2} + e_3^{\,2} + e_4^{\,2}\right)$$

Time derivation of the Lyapunov function is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\ &= e_1\left[a(e_2 - e_1) + u_1\right] \\ &+ e_2\left[(c-z)e_1 + e_4 - e_1e_3 - xe_3 + u_2\right] \\ &+ e_3\left[-be_3 + e_1e_2 + xe_2 + ye_1 + u_3\right] \\ &+ e_4\left[-de_2 + u_4\right] \\ &= -ae_1^{\,2} - be_3^{\,2} + (a+c-z)e_1e_2 \\ &+ (1-d)e_2e_4 + ye_1e_3 \\ &+ e_1u_1 + e_2u_2 + e_3u_3 + e_4u_4 \end{aligned} \tag{11}$$

Substituting the Controller (10) into (11), yields:

$$\dot{V} = -e_1^2 - e_2^2 - e_3^2 - e_4^2 < 0 \tag{12}$$

From (12) we show that $\dot{V}$ is negative definite and Lyapunov function V is positive definite. From Lyapunov stability theory it follows that error dynamical system (9) is asymptotic stable. The proof is completed.

4.2. Chaos Synchronization of System (9) with Three Feedback Controllers

In this theorem we propose nonlinear control which contains number of feedback controllers less than in theorem 4.

Theorem 5. The zero solution of the error system (9) is asymptotic stable if nonlinear control is designed as following :

$$\begin{cases} u_1 = 0 \\ u_2 = -(a+c-z)e_1 - e_2 \\ u_3 = -ye_1 \\ u_4 = -e_4 + (d-1)e_2 \end{cases} \tag{13}$$

Proof. Let us consider Lyapunov function is:

$$V = 1/2\left(e_1^{\,2} + e_2^{\,2} + e_3^{\,2} + e_4^{\,2}\right)$$

Time derivation of the Lyapunov function is:

$$\begin{aligned} \dot{V} &= e_1\dot{e}_1 + e_2\dot{e}_2 + e_3\dot{e}_3 + e_4\dot{e}_4 \\ &= e_1\left[a(e_2 - e_1) + u_1\right] \\ &+ e_2\left[(c-z)e_1 + e_4 - e_1e_3 - xe_3 + u_2\right] \\ &+ e_3\left[-be_3 + e_1e_2 + xe_2 + ye_1 + u_3\right] + e_4\left[-de_2 + u_4\right] \\ &= -ae_1^{\,2} - be_3^{\,2} + (a+c-z)e_1e_2 + (1-d)e_2e_4 \\ &+ ye_1e_3 + e_1u_1 + e_2u_2 + e_3u_3 + e_4u_4 \end{aligned} \tag{14}$$

Substituting the Controller (13) into (14), yields:

$$\dot{V} = -ae_1^2 - e_2^2 - be_3^2 - e_4^2 < 0 \tag{15}$$

then $\dot{V}$ is negative definite and the Lyapunov function V is positive definite. From Lyapunov stability theory it follows that the error dynamical system (9) is asymptotically stable, Consequently, the drive system (1) is synchronous asymptotically with the response system(2) with control (13). This completes the proof.

Numerical simulations are used to investigate the controlled error dynamical system (9), and also achieved synchronization between drive system (1) and response system (2) using fourth-order Runge-Kutta scheme with time step 0.3 . We choose the parameters $a = 10$, $b = 8/3$ $c = 28$ and $d = 10$ the initial values for the drive system and the response system are $(-5, -3, 20, 10)$ and (5, 3, 35,-10) respectively. From Fig.6 and Fig.7, we can see the speed of convergent for system (9) while Fig .8 and Fig. 9 show the synchronization between two identical hyperchaotic systems with controller (10) and (13) respectively .

Obviously, from Fig.6 and Fig.7, both controller (10) and (13) are guarantee to convergent system (9) at the origin, but controller (13) is faster than from controller (10) to convergent. Also, Fig. 8 and Fig.9 show the synchronization between system (1) and system (2) with controller (10) and (13) respectively, and justified the same result in Fig. and Fig. 7. where it is obvious that the synchronization in Fig. 9 with controller (13) is better than the synchronization in Fig.8 with controller (10). Consequently, the synchronization with controller (13) is also faster than controller (10). So, wherever we reduce numbers of feedback controllers then we obtain fast convergent .

Remark 1. In the linear feedback control methods, if we use multiple feedback controllers such as enhancing feedback control then get least complexity and cost, but in the nonlinear control method we get fast convergence if we reduce the number of feedback controllers.

Remark 2. In both active and adaptive methods, we can design only one feedback controllers since those methods are depended on fixed base or law for designing while in nonlinear method don't need to the fixed base, Consequently, we can design more than one suitable feedback controllers.

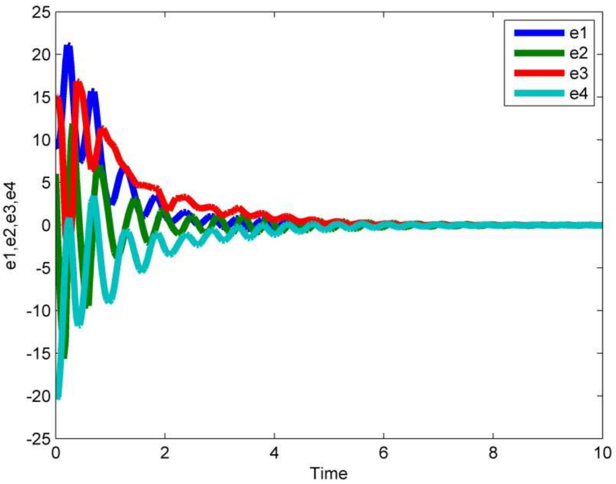


Fig. 6. *controlling error dynamical system (9) with four feedback controllers.*

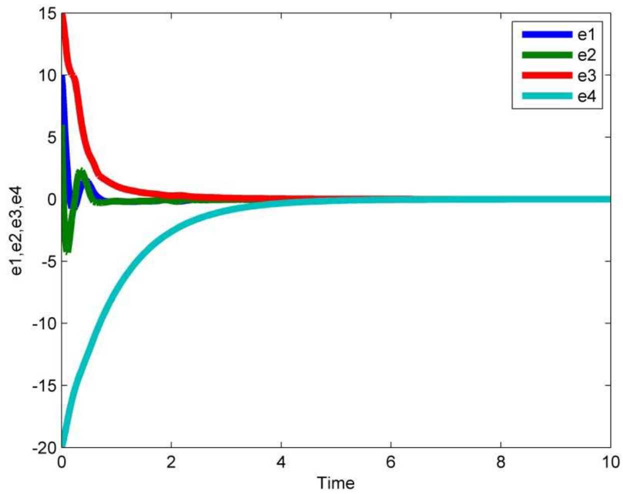


Fig. 7. *controlling error dynamical system (9) with three feedback controllers.*

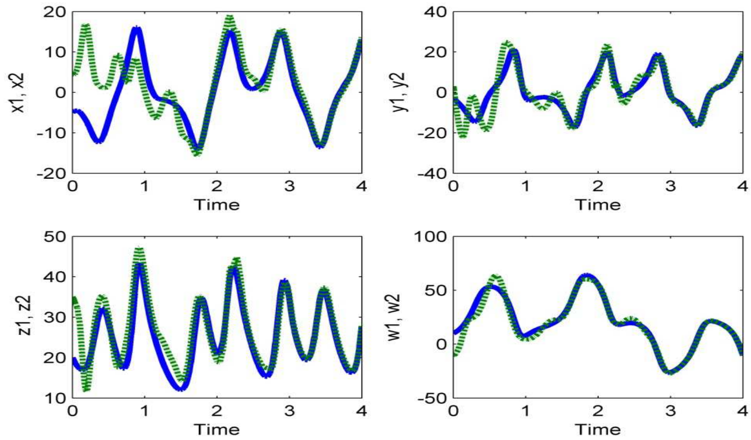


Fig. 8. *The synchronization between the drive system (1) and the response system (2) with controller (10).*

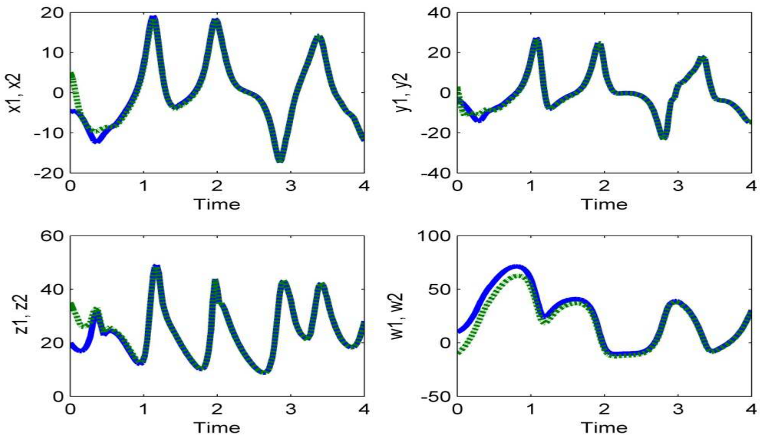


Fig. 9. *The synchronization between the drive system (1) and the response system (2) with controller (13).*

5. Conclusions

In this paper, based on the Lyapunov stability theory, nonlinear control method and numbers of feedback controllers, we consider the control and synchronization of a modified hyperchaotic pan system with unknown parameters. First, three controllers designed by nonlinear control method are used to control a modified hyperchaotic pan system to the unstable equilibrium point successfully. Second, synchronizations between two identical hyperchaotic systems are achieved via two designed controllers. Obviously from those controllers, time control and time synchronization are different according to those controllers and we can get a fast convergence speed if we reduce the numbers of feedback controller Finally, numerical simulations show the effectiveness of the proposed chaos control and synchronization schemes

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