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Time Varying Electric and Magnetic Fields from Lightning Discharge

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Abstract: On comparing the overall length of the lightning discharge, it should be very thin channel. Due to the high speed of the light, the electric and magnetic fields calculations require careful consideration of the retardation phenomena. Therefore, to calculate electric and magnetic fields from lightning, it is modeled as a linear antenna that has some current distribution or line charge density distribution that changes with time. In this paper, the analytical expressions for calculating the electric and magnetic fields of the source distribution that vary with time from lightning are presented.

Keywords: Electric and Magnetic Fields, Retardation Effects, Lightning Discharge, Return Stroke Channel

1. Introduction

The return stroke channel, among the lightning, is generally idealized in the cloud-to-ground lightning, vertically extending line in which one end fixed at the ground. The return stroke wave-front moving with a velocity v, vertically. At each and every point, the current verses time is specified along the return stroke channel. The general expressions of the electric and magnetic fields were derived previously [1] and are now commonly found in the literature. [2-10] The speed of a lightning return stroke wave-front changes from thirty to fifty percent of the speed of the light. The return stroke current below the extending wave-front vary rapidly whereas the current above the extending wave-front is considered to be zero. Even the speed is very high, as the comparable to the velocity of light; the finite travel time from the channel of the return stroke to the field measuring point cannot be ignored. The observer at the field measuring point observes the current on the return stroke channel at an earlier time and cannot observe the true length of the channel. Therefore, the retarded channel lengths are to be used for the calculation of the electromagnetic fields due to return stroke channel. The electric field expression was also explained in the form of electro static, induction, and radiation fields. [2, 6, 11] These components are not only important from the lightning discharge point of view but also due to individual effects created by them in the surroundings. It was described that static component is dependent on the term containing r^{-3}, the induction component is dependent on the term containing r^{-2}, and the radiation component is dependent on the term containing r^{-1}. [2] The interference problem produced by lightning discharges is due to the radiation components of electric field of lightning. The radiated electric field from lightning channel contains horizontal as well as vertical components of the electric fields that is responsible for producing transients in power lines, telecommunication lines etc. With the help of retardation effects on lightning return stroke channel, the components of the electric field have been described in this paper.

2. Theory

Let us consider L'(t) of the apparent length of the return stroke channel with a fixed point A on the ground as shown in figure 1. The observer at the point P, observes the channel emerging from the point A at the time r/c. Let L (t) be the actual length of the return stroke wave-front which moves with speed v. There is some difference between the two lengths L (t), the real length and L'(t), the apparent length or retarded length.

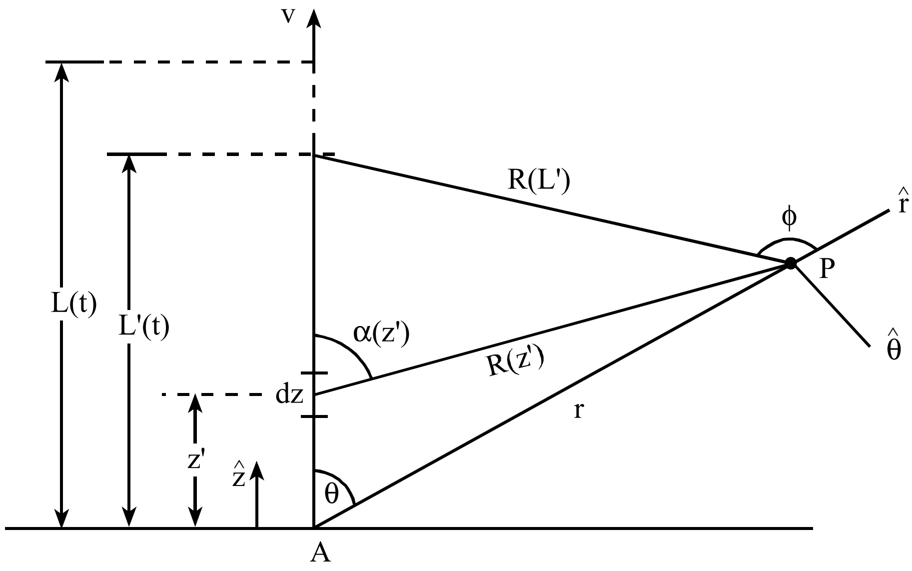

Figure 1. *Geometry of problem in the treatment of retardation effects.*

The time (t) is the sum of the time required for the return stroke wave-front to reach a height L'(t) to the observer at P.

Then $t = \frac{L'(t)}{v} + \frac{R(L')}{c}$, where R(L') = $\sqrt{r^2 + L'^2 - 2L'r\cos\theta}$ from the cosine law and θ be the angle between r and L'.

Then $t = \frac{L'(t)}{v} + \frac{\sqrt{r^2 + L'^2(t) - 2L'(t)r\cos\theta}}{c}$

On simplifying we get,

$$L'(t) = \frac{r}{1 - v^2/c^2}\left\{-\cos\theta\frac{v^2}{c^2} + \frac{vt}{r} \pm \frac{v}{c}\sqrt{\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2}\cos^2\theta - \frac{2vt}{r}\cos\theta}\right\}$$

If the ground is treated as being perfectly conducting then the angle θ be replaced by $(180 - \theta)$. Then we get,

$$L''(t) = \frac{r}{1 - v^2/c^2}\left\{\frac{v^2}{c^2}\cos\theta + \frac{vt}{r} \pm \frac{v}{c}\sqrt{\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2}\cos^2\theta + \frac{2vt}{r}\cos\theta}\right\}$$

If all the channel sections were equidistant from the observer at a distance r, the discharge should be in circular arc. Then $\theta = 0$, so,

$$L'(t) = \frac{r}{1 - v^2/c^2}\left\{\frac{-v^2}{c^2} + \frac{vt}{r} \pm \frac{v}{c}\sqrt{\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2t^2}{r^2} + \frac{v^2}{c^2} - \frac{2vt}{r}}\right\}$$

$$= \frac{r}{1 - v^2/c^2}\left\{\frac{-v^2}{c^2} + \frac{vt}{r} \pm \frac{v}{c}\left(1 - \frac{vt}{r}\right)\right\}$$

Taking negative sign,

$$L'(t) = \frac{r}{1 - v^2/c^2}\left\{\frac{-v^2}{c^2} + \frac{vt}{r} - \frac{v}{c}\left(1 - \frac{vt}{r}\right)\right\} = \frac{v}{1 - v/c}\left(t - \frac{r}{c}\right)$$

Taking positive sign,

$$L'(t) = \frac{r}{1 - v^2/c^2}\left\{-\frac{v^2}{c^2} + \frac{vt}{r} + \frac{v}{c}\left(1 - \frac{vt}{r}\right)\right\} = \frac{v}{1 + v/c}\left(t + \frac{r}{c}\right)$$

The same condition is applied for image channel length L"(t) then we get

$$\text{L"(t)} = \frac{r}{1-v^2/c^2}\left\{\frac{v^2}{c^2}+\frac{vt}{r}\pm\frac{v}{c}\sqrt{\left(1-\frac{v^2}{c^2}\right)+\frac{v^2t^2}{r^2}+\frac{v^2}{c^2}+\frac{2vt}{r}}\right\}$$

$$\text{L"(t)} = \frac{r}{1-v^2/c^2}\left\{\frac{v^2}{c^2}+\frac{vt}{r}\pm\frac{v}{c}\left(1+\frac{vt}{r}\right)\right\}$$

Taking positive sign,

$$\text{L"(t)} = \frac{r}{1-v^2/c^2}\left\{\frac{v^2}{c^2}+\frac{vt}{r}+\frac{v}{c}\left(1+\frac{vt}{r}\right)\right\} = \frac{r}{1-v/c}\left(\frac{v}{c}+\frac{vt}{r}\right) = \frac{v}{1-v/c}\left(t+\frac{r}{c}\right)$$

Taking negative sign,

$$\text{L"(t)} = \frac{r}{1-v^2/c^2}\left\{\frac{v^2}{c^2}+\frac{vt}{r}-\frac{v}{c}\left(1+\frac{vt}{r}\right)\right\} = \frac{r}{1+v/c}\left(-\frac{v}{c}+\frac{vt}{r}\right) = \frac{v}{1+v/c}\left(t-\frac{r}{c}\right)$$

[*Note: Hence for retarded time, we take* $\left(t-\frac{r}{c}\right)$ *so for both apparent channel length L'(t) =* $\frac{v}{1-v/c}\left(t-\frac{r}{c}\right)$ *and image channel length L"(t) =* $\frac{v}{1+v/c}\left(t-\frac{r}{c}\right)$ *obtained by taking negative sign. Similarly, for advanced time when we take positive sign, then we get L'(t) =* $\frac{v}{1+v/c}\left(t+\frac{r}{c}\right)$ *and L"(t) =* $\frac{v}{1-v/c}\left(t+\frac{r}{c}\right)$.]

Hence, apparent channel length L'(t) for retarded potential= $\frac{v}{1-v/c}\left(t-\frac{r}{c}\right)$

And image channel length L"(t) for retarded potential = $\frac{v}{1+v/c}\left(t-\frac{r}{c}\right)$

If the channel is very small compared with the distance of the observer, that is if L'(t) << r, then R (L') can be approximated as R(L') = $\sqrt{r^2+L'^2(t)-2L'(t)r\cos\theta}$ =r – L'(t) cosθ

Then, t = $\frac{L'(t)}{v}+\frac{R(L')}{c}$ SoL'(t) = $\dfrac{v\left(t-\frac{r}{c}\right)}{1-\frac{v}{c}\cos\theta}$

When θ = 0, for the apparent channel length, L'(t) = $\frac{v}{1-v/c}\left(t-\frac{r}{c}\right)$

And θ = 180° for the image channel length, L"(t) = $\frac{v}{1+v/c}\left(t-\frac{r}{c}\right)$

Hence the general form for apparent channel length L'(t) = $\dfrac{v\left(t-\frac{r}{c}\right)}{1-\left(\frac{v}{c}\right)\cos\theta}$

3. Discussion

In the computation of electromagnetic field due to lightning discharges, a pair of time dependent electro-magnetic field equation for a one dimensional current element was derived which is given by: [1]

$$dE(r,t) = \frac{dz'}{4\pi\varepsilon_\circ}\left\{cos\theta\left[\frac{2}{r^3}\int_0^t I\left(z',\tau-\frac{r}{c}\right)d\tau + \frac{2}{cr^2}I\left(z',\tau-\frac{r}{c}\right)\right]\boldsymbol{a_r}\right.$$
$$\left. + \ sin\theta\left[\frac{1}{r^3}\int_0^t I\left(z',\tau-\frac{r}{c}\right)d\tau + \frac{1}{cr^2}I\left(z',\tau-\frac{r}{c}\right) + \frac{1}{c^2r}\frac{\partial I\left(z',\tau-\frac{r}{c}\right)}{\partial t}\right]\boldsymbol{a_\theta}\right\}$$

And,

$$d\boldsymbol{B}(\boldsymbol{r},t) = \frac{\mu_\circ dz'}{4\pi}sin\theta\left[\frac{1}{r^2}I(z',\tau-r/c) + \frac{1}{cr}\frac{\partial I(z',\tau-r/c)}{\partial t}\right]\boldsymbol{a_\varphi}$$

Where dz' is the one dimensional line element of current, randr are the vector and distance from the distance dz' to the observer, t is the time in the observer's frame of reference, θ is the angle measured from dz' to R, I is the current magnitude along z' anda_R,$a_\theta$anda_φ are the unit vectors in R, θ and φ directions in a spherical coordinate frame, respectively.

Let us assume that the ground to be infinitely conducting then the effect of the ground plane as image channel carrying an image current. In the above equation, the two terms (of the above the ground and the image) are in opposite vector of $\hat{r}$ and so cancelled each other. Only the field expression containing the vector $\hat{\theta}$added up and we get the total field expression as:

$$\boldsymbol{E}(\boldsymbol{r}, \theta, t) = \frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha(z')}{R^3(z')}\int_{t_b}^{t} i\left(z',\tau-\frac{R(z')}{c}\right)\mathrm{d}\tau\mathrm{d}z'$$
$$+\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha(z')}{cR^2(z')}\, i\left(z',t-\frac{R(z')}{c}\right)\mathrm{d}z' - \frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{\sin^2\alpha(z')}{cR^2(z')}\,\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}\mathrm{d}z' -$$
$$\frac{1}{2\pi\epsilon_0}\frac{\sin^2\alpha(L')}{c^2R(L')}\, i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

Where, $\sin\alpha\ (z') = \frac{r\sin\theta}{R(z')}$ and $\sin\alpha\ (L') = \frac{r\sin\theta}{R(L')}$

If there is no current discontinuity at the propagating wave-front, i.e. if $i\left(L',t-\frac{R(L')}{c}\right) = 0$, then the above equation becomes,

$$\boldsymbol{E}(\boldsymbol{r}, \theta, t) = \frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha}{R^3(z')}\int_{t_b}^{t} i\left(z',\tau-\frac{R(z')}{c}\right)\mathrm{d}\tau\,\mathrm{d}z' + \frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{2-3\sin^2\alpha}{cR^2(z')}\, i\left(z',t-\frac{R(z')}{c}\right)\mathrm{d}z'$$
$$-\frac{1}{2\pi\epsilon_0}\int_0^{L'(t)}\frac{\sin^2\alpha}{c^2R(z')}\,\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}\mathrm{d}z'$$

The magnetic field is given by B = ∇× A. For a vertical channel, there is only a horizontal component of the magnetic field. On taking the curl of the vector potential, the magnetic field at the ground level is

$$\boldsymbol{B} = \frac{1}{2\pi\epsilon_0 c^2}\int_0^{L'(t)}\left[\frac{\sin^2\alpha(z')}{R^2(z')}\, i\left(z',t-\frac{R(z')}{c}\right) + \frac{\sin\alpha(z')}{cR(z')}\,\frac{\partial i\left(z',t-\frac{R(z')}{c}\right)}{\partial t}\right]\mathrm{d}z'$$
$$+\frac{1}{2\pi\epsilon_0 c^2}\frac{\sin\alpha(L')}{cR(L')}\, i\left(L',t-\frac{R(L')}{c}\right)\frac{dL'}{dt}$$

If there is no current discontinuity at the propagating wave-front i.e. if $i\left(L',t-\frac{R(L')}{c}\right) = 0$, then the last term of the above equation vanishes. Hence the total magnetic field at the point on the ground level is

$$\boldsymbol{B} = \frac{1}{2\pi \in_0 c^2} \int_0^{L'(t)} \left[\frac{\sin\alpha}{R^2(z')} i\left(z', t - \frac{R(z')}{c}\right) + \frac{\sin\alpha}{cR(z')} \frac{\partial i\left(z', t - \frac{R(z')}{c}\right)}{\partial t} \right] \mathrm{d}z'$$

The first term containing $\frac{1}{R^2(z')}$ factor indicates the induction component and the term containing $\frac{1}{CR(z')}$ indicates the radiation components.

The components of lightning electric field are represented graphically shown in figure 2. The three components of the lightning electric field are described in the figure 2 a, b, c. The static component in the general expression of the electric field is dependent on the term containing R $^{-3}$. Figure 2(a), represents the static component of the electric field which increases gradually and remains constant over time. The induction component in the general expression of the electric field is dependent on the term containing C $^{-1}$ R^{-2}, which increases to its maximum amplitude and then decreases with time is shown in figure 2(b). The radiation component in the general expression of the electric field is dependent on the term containing C^{-2} R^{-1} which decreases to its minimum value and then gradually increases and reaches maximum and then decreases gradually with the increase in time that is shown in figure 2(c). The total of electric field which is the sum of static component, induction component and radiation component is given in the figure 2(d). The duration of the static component is the largest in the representations.

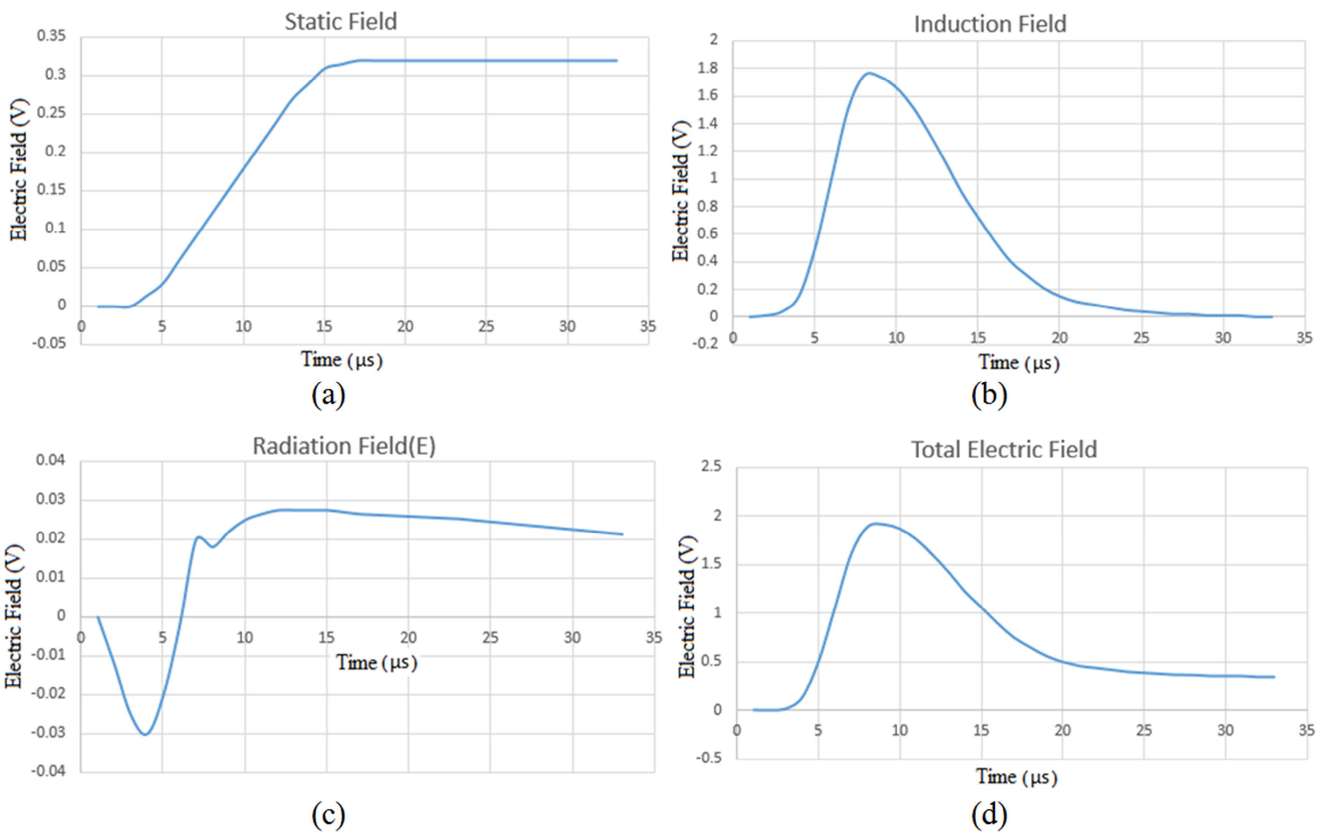

Figure 2. *The three components of the electric field due to lightning are given in the figure 2 (a), (b) and (c); and the total of these components in the figure 2(d).*

The interference problem is produced due to the radiation components of electric field of lightning. This radiated electric field contains horizontal as well as vertical components of the electric fields which resonates with the wave produced in power lines, telecommunication lines etc. With the help of retardation effects on lightning return stroke channel, the components of the electric field have been described.

4. Conclusions

The electric and magnetic fields produced by lightning discharge are the most important aspect of lightning. Due to the high speed of the light, the electric and magnetic fields calculations require careful consideration of the retardation phenomena. Analyzing its components helps in a better understanding of the electric and magnetic fields generated during lightning discharge. As we know the radiation component is opposite to the static and induction components for a closed distances, which suggests that decrease in the radiation component increases the total electric field. Hence, from these representations we can conclude that the electrostatic field component dominating for the longer duration and the sum of the electrostatic component and the induction component of the electric field has relatively greater effect to the total electric field than the radiation components in the lightning discharge.

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