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Life predictions based on calculable materials constants from micro to macro fatigue damage processes

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Abstract
To use the theoretical approach, by means of the traditional and the modern material constants; study their compositions of conventional mathematical models and modern models; research damage growth behaviors for some steels containing pre-micro or pre-macro-flaws; thereby discover and establish lot of new calculation models in all damage growth process, which are the equations of the driving forces and the various life predictions. In addition, propose yet many calculating expressions under different loading conditions. For key parameters inside formulas, define their physical and geometrical meanings. For relationship between the damage variable D and the crack variable a, between the dimensions-units and ones inside different equations, explain in detail the conversion methods. For the transition damage value D_{tr} from micro to macro damage growth process, expound concretely the calculation processes. The purpose is to try to make the modern fatigue-damage discipline become a calculable subject as the conventional material mechanics, such that will be having practical significances for promoting applying and development for relevant disciplines.

1. Introduction

As everyone knows for the conventional material mechanics, that is a calculable subject, and it has made valuable contributions for every industrial engineering designs and calculations. But it cannot accurately calculate the life problems for some structures when it is pre-existing flaws and undergone fatigue damage under repeated loading. In that it has no to contain such calculable parameters in its calculating models as the damage variable D or as crack variable a. But, for the damage mechanics and the fracture mechanics, due to there are these variables, they can all calculate above problems. However, nowadays latter these disciplines are all subjects mainly depended on fatigue and damage tests.

Author thinks, in the mechanics and the engineering fields, in which are also to exist such scientific principles of similar to genetic and clone technology in life science. Author has done some of works used the theoretical approach as above the similar principles [1-7]. For example, for some strength calculation models from micro to macro are provided by reference [1], for some rate calculation models from micro to macro damage growth are proposed by references [2-7] which are models in each stage even in whole process, under different loading conditions. Two years ago, in order to do the lifetime calculations in whole process on fatigue-damage-fracture for an engineering structure, author was by means of Google Scholar to search the lifetime prediction models, as had been no found for this kind of calculation equations. After then, author continues to research this item, and bases on was provided and now is complemented on the calculation figure 1 of material behaviors [1-2]; still applies above genetic principles, to study and analyze data in references, thereby to provide some new calculable models for the damage growth driving force and for the lifetime predictions. Try to make the damage mechanics, step by step become calculable disciplines as the material mechanics. That way, may be having practical significances for decrease experiments, stint man powers and funds, for promoting engineering applying and developing to relevant disciplines.

2. About some Viewpoints of Existing "Genetic Gene" and "Clone Technology" in the Mechanics and Engineering Fields

As is well-known, in the conventional material mechanics, on describing material behaviors and its strength problems, its main calculating parameters are the stress σ, the strain ε and relevant material constants, e.g. yield stress σ_s, elasticity modulus E and reduction of area ψ, etc. In the damage mechanics it is based on the damage parameter D as its variable, to adopt the fatigue strength coefficient σ'_f and the fatigue ductility coefficient ε'_f etc. as its material constants. And in the fracture mechanics, describing material behaviors on the strength and the life prediction problems, it is based on the crack size a as its variable, to use the fracture toughness K_{1c} and the critical crack tip open displacement δ_c as its material constants.

Author thinks the gene and clone technology in life science, for which its traits consist in: it has both him-self genetic properties, and has transferable and recombination properties. In fact, in fracture mechanics, in the stress intensity factor $K_1 = \sigma\sqrt{\pi a}$, in the crack tip open displacement δ_t and in their critical value $K_{1c} = \sigma\sqrt{\pi a_c}$ and the δ_c, which are all including the parameters σ, ε, π and their material constants σ_s, ε_s and σ_f etc. Author thinks for the stress σ, the strain ε and its relevant material constants σ_s and E etc in the material mechanics, for which can be considered as genetic elements; for the D and σ'_f etc in the damage mechanics can also be considered as genetic elements; and for the crack size a and K_{1c} etc in the fracture mechanics can also be considered as genetic elements. If can make a link among the material mechanics, the damage mechanics and the fracture mechanics, and provide respectively some conversion methods, then they can be converted each other for their relations between the variables, between the material constants, and between the dimensional units in the equations. For example, we can consider as gene for the stress σ and its σ_s, E, ψ, to make them combination with the variable D_1 are transferred into micro-damage-mechanics, and combination with the variable D_2 are transferred into macro-damage-mechanics; In the same way, we can also consider as gene for the stress σ and its σ_s, E, ψ, to make them combination with the variable a_1 are transferred into micro-fracture-mechanics, and combination with the variable a_2 are transferred into macro-fracture-mechanics. Then we are able by these stress σ, ε, $\sigma'_f, \varepsilon'_f$, etc, to establish their the driving force models, the damage growth rate and its life equations or the crack growth rate and its life equations. Even we can also adopt the variable D or a to describe material behaviors in overall process.

Above the properties of those parameters and material constants, even though as compared to those genes the life science, due to they are in different disciplines. But, for which have both own inheritable properties (similar to genetic elements), and have the transferable, and the recombination properties, for these---on the epistemology and methodology, in practice they are all very similar.

Based on the cognitions and concepts mentioned above, author makes a link among the material mechanics, the damage mechanics and the fracture mechanics, for relationship between their parameters are analyzed, for their equations are derived, for their dimensional units are converted each other, then for new made models are calculated, checked and validated again and again, finally, to provide the equations (1-59) in following text. Thereby try to make communications for among the conventional material mechanics, the damage mechanics and the fracture mechanics, then to make such new mathematical models become calculable ones as those equations inside the material mechanics, which are the new driving force ones and the life calculation equations. Author thinks, if can realize the goals, it will all have practical significance for the engineering designs, the computational analysis for safe operation and assessment of machineries and structures.

3. Calculating Figure of Materials Behaviors

Among branch disciplines on fatigue-damage-fracture, among the conventional material mechanics and the modern mechanics, for communications and connecting their relations each other, it must study and find their correlations between the equations, even to be the relations between variables, between the material constants, between the curves. Because which are all the significant factors to research and to describe for material behaviors at each stage even in whole process, and are also all to have significant significations for the engineering calculations and designs. Therefore it should research and find an effective tool used for analyzing problems above mentioned. Here author provides the "calculating figure of materials behaviours" as Figure1 (or called bidirectional combined coordinate system and simplified schematic curves in the whole process, or called combined cross figure).

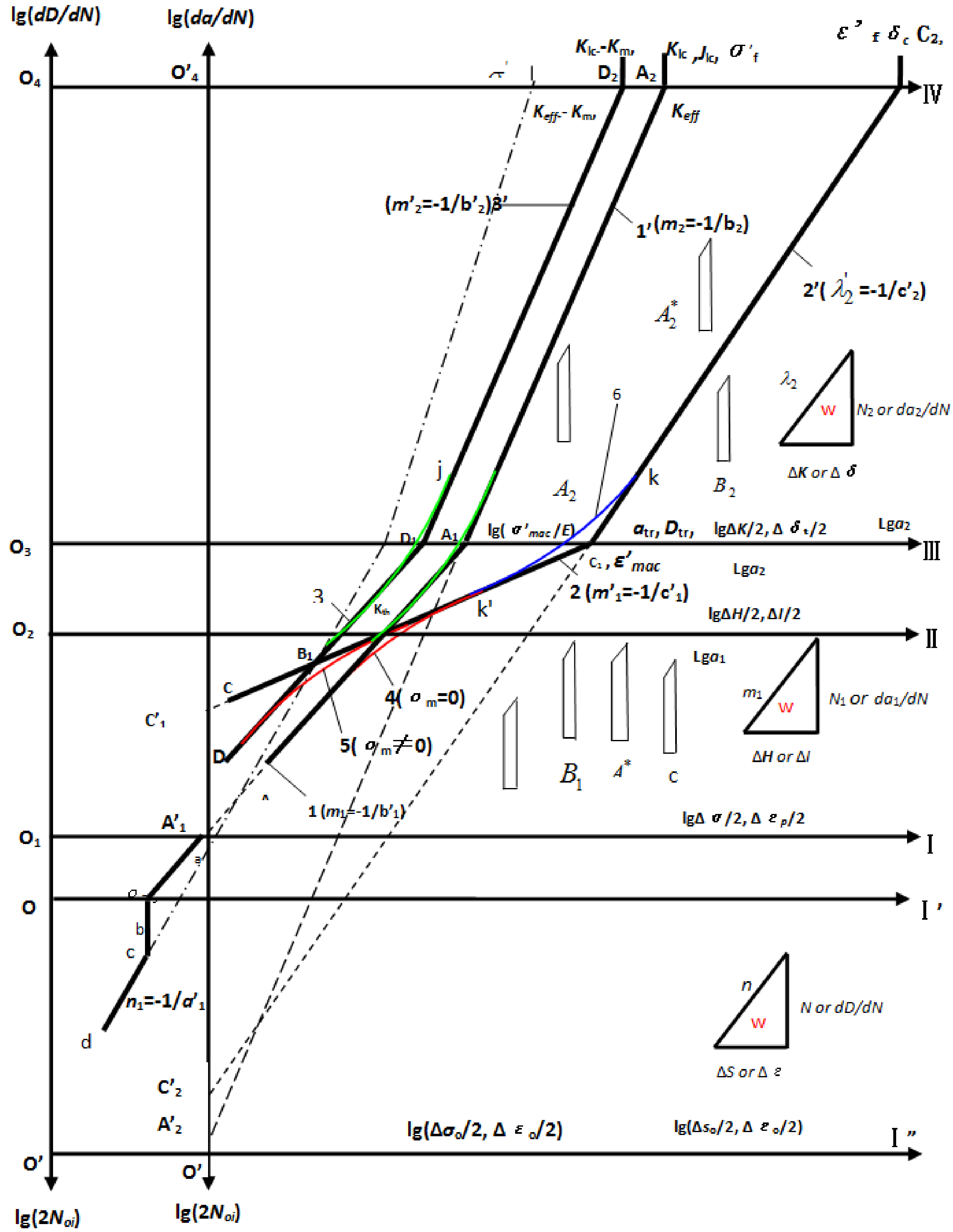

Figure 1. Calculating figure of material behaviors (Bidirectional combined coordinate system and simplified schematic curves in the whole process).

In the figure 1, it had been provided by present author [1-2]. At this time it has been corrected and complemented, that is shown diagrammatically for the damage growth process or crack propagation process of material behavior at each stage and in whole course. It is to consist of six abscissa axes O' I", O I', O_1 I, O_2 II, O_3 III, O_4 IV and two bidirectional ordinate axis O_1 O_4 and O'_1 O'_4. Between the axes O' I" and O_1 I, it was an area applied by the conventional material mechanics, currently it can be applied as micro-damage area by the very-high cycle fatigue. Among the axes O I', O_1 I and O_2 II, they are calculation areas applied by the micro-damage mechanics and the micro-fracture mechanics. Between the axes O_3 III and O_4 IV, it is calculation area applied by the macro-damage mechanics and the macro-fracture mechanics. Between the axes O_2 II and O_3 III, it is both calculation area applied for the micro-fracture mechanics and macro-fracture mechanics, or it is both calculation area applied for the micro-damage mechanics and macro damage mechanics.

On the abscissa axes O' I", O' I" and O_1 I, they are shown with stress σ and the strain ε parameters as variable, and on the abscissa axes O I' there is a fatigue limit σ_{-1} at point b. On the abscissa axes O_2 II, it is represented with the short crack stress intensity factor range ΔH or the short crack strain intensity facto ΔI as variable, and here there are the threshold stress intensity factor ΔK_{th} and the damage threshold values $\Delta K'_{th}$. On the abscissa axes O_3 III, it is shown with long crack stress intensity factor ΔK range (or $\Delta\delta_t$) as variable, that it is also a boundary between short crack and long crack growth behaviors (or between micro damage and macro damage growth), and it is also a boundary of transfer values ($a_{tr} or D_{tr}$) between the first stage and the second stage. On abscissa O_4 IV, the point A_2 is corresponding to fatigue strength coefficient σ'_f and critical values K_{1c} ,etc; the point C_2 corresponding to fatigue ductility coefficient ε'_f and critical value δ_c ; the point F corresponding to very-high cycle fatigue strength coefficient σ'_{vhf} , i.e. on same the abscissa axes O_4 IV, there are also the critical values $K'_{1c}(K_{1c}), \delta'_c(\delta_c), J'_{1c}(J_{1c})$ to fracture in long crack propagation process.

Upward direction along the ordinate axis is represented as crack growth rate da/dN or damage growth rate dD/dN in each stage and the whole process. But downward direction, it is represented as life N_{oi}, N_{oj} in each stage and lifetime N.

In area between axis O' I" and O_2 II it is a fatigue history from un-crack to micro-crack initiation. In area between axes O_2 II and O_3 III, it is a fatigue history relative to life $N_{oi}^{mic-mac}$ from micro-crack growth to macro-crack forming. Consequently, the distance $O_3 - O'$ on ordinate axis is as a history relating to life N_{mac} from grains size to micro-crack initiation until macro-crack forming; the distance O_4 - O' is as a history relating to the lifetime life $\sum N$ from micro-crack initiation until fracture.

In crack forming stage, the partial coordinate system made up with the upward the ordinate axis O O_4 and the abscissa axes O I', O_1 I and O_2 II is represented to be as relationship between the damage evolving rate dD_1 / dN_1 (or the short crack growth rate da_1/dN_1) and the damage stress factor amplitude $\Delta H_1/2$ (or damage strain factor amplitude $\Delta I_1 / 2$). In macro-crack growth stage, the partial coordinate system made up with the ordinate axis O_3 O_4 and abscissa O_3 III (O_4 IV) at same direction is represented to be the relationship between macro-crack growth rate and the stress intensity factor amplitude $\Delta K / 2$, J -integral amplitude $\Delta J/2$ and crack tip displacement amplitude $\Delta\delta_t / 2$ (da_2 / dN_2 - $\Delta K / 2$, $\Delta J / 2$ and $\Delta\delta_t / 2$). Inversely the coordinate systems made up with downward ordinate axis O_4 O_1 and abscissa axes O_4 IV, O_3 III, O_2 II, O_1 I , and O I' are represented respectively as the relationship between the $\Delta H/2$-, $\Delta K/2$-amplitude and the each stage life N_{oi} , N_{oj} , the lifetime $\sum N$ even (or between the $\Delta\varepsilon_p/2$-, $\Delta\delta_t/2$-amplitude and the life $\sum N$).

The curve ABA_1 shows the varying law as elastic material behaviors or as elastic-plastic material ones under high cycle loading at macro-crack-forming stage (the first stage): positive direction ABA_1 shows the relation between dD_1/dN_1 (or da_1/dN_1)- $\Delta H/2$; inverted A_1BA, between the $\Delta H_1/2 - 2N_{oi}$. The curve CBC_1 shows the varying law of elastic-plastic material behaviors, it is under low-cycle loading at macro-crack forming stage: positive direction CBC_1 shows the relation between da_1/dN_1- $\Delta I_1/2$; inverted C_1BC, it shows the relation between the $\Delta\varepsilon_p/2 - 2N_{oi}$. And the curve A_1A_2 at crack growth stage (the second stage) is showed as under high cycle loading: positive direction A_1A_2, shows da_2/dN_2- $\Delta K/2$ ($\Delta J / 2$); inverted A_2A_1, shows between the $\Delta K_2/2$, $\Delta J/2 - 2N_{oj}$. The C_1C_2 shows: the positive direction relation between the da_2/dN_2- $\Delta\delta_t/2$ under low-cycle loading, inverted C_2C_1, between $\Delta\delta_t / 2$ ($\Delta J/2$)- $2N_{oj}$. By the way, the curve abcd is the very-high cycle fatigue one corresponded to stress below fatigue limit.

It should point that the curve AA_1A_2 (1-1') is depicted as the rate curve in whole process under symmetrical and high cycle loading (i.e. zero mean stress); the curve DD_1D_2 (3-3'), as the rate curve under unsymmetrical cycle loading (i.e. non-zero mean stress). The curves $dcbaBA_1A_2$ and $dcaB_1FG$ are depicted as the rate curve in whole process under very high cycle loading. The curve CC_1C_2 (2-2') is depicted as the rate curve under low cycle loading. Inverse，the curve A_2A_1A is depicted as the lifetime curve under symmetrical cycle loading, the curve D_2D_1D, as the lifetime curve under unsymmetrical cycle loading. The curve C_2C_1C is depicted as the lifetime curve under low cycle loading. The curves A_2A_1Babcd and GFB_1acd are depicted as the lifetime ones in whole process under very high cycle loading. And should yet point that the calculating figure 1 of materials behaviors may be a complement as a basis that it is to design and calculate for different structures and materials under different loading conditions, and it is also a tool and bridge, that is to communicate and link the conventional material mechanics and the modern mechanics.

4. The Life Prediction Calculations for Elastic-Plastic Materials Containing Pre-Flaws

4.1. The Life Prediction Calculations in Micro Damage Process (Called the First Stage)

The life curves of micro damage D_1 as in the first stage are just described with curves 1 ($\sigma < \sigma_s, \sigma_m = 0$), 2 ($\sigma > \sigma_s$) and 3 ($(\sigma < \sigma_s, \sigma_m \neq 0)$) at reversed direction coordinate system in fig.1

(1) Under work tress $\sigma < \sigma_s$ (high cycle fatigue) condition

In fig.1, under work tress $\sigma < \sigma_s$ condition, the life prediction equation corresponding reversed curves 1 and 3 can be calculated as following form

$$N_1 = \int_{D_{01}}^{D_{tr}} \frac{dD_1}{A'_1 \times (\Delta H'_1)^{m_1}}, (cycle) \tag{1-1}$$

or

$$N_1 = \int_{D_{01}}^{D_{tr}} \frac{dD_1}{A'_1 \times (\Delta\sigma)^{m_1} D_1}, (\text{cycle}) \tag{1-2}$$

Where

$$H'_1 = \sigma \cdot D_1^{1/m_1}, \ (MPa \cdot \%^{1/m_1}, orMPa) \tag{2}$$

$$\Delta H'_1 = \Delta\sigma \cdot D_1^{1/m_1} \ (MPa \cdot \%^{1/m_1}, orMPa) \tag{3}$$

Here the H'_1 in eqn (2) is defined as damage stress intensity factor, it is driving force of micro-damage under monotonic loading, and the $\Delta H'_1$ in eqn (3) is defined as damage stress intensity factor range, it is driving force under fatigue loading. The damage variable D_1 (or below D_2 and D) is a non-dimensional value, it is equivalent to short crack a_1 discussed as reference [1-2]. Here must put up conversion for dimensions and units, and must be defined in 1mm (1 millimeter) of crack length equivalent to one of damage-unite (1 damage unit), in 1m (1 meter) equivalent to 1000 of damage-unit (1000 damage units). The A_1' is defined as the damage comprehensive material constant. Author researches and thinks, physical meaning of the A_1' is a concept of the power, is to give out an energy that is a material to resist outside force, just is a maximal increment value to give out energy in one cycle before to cause failure. Its geometrical meaning is a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1 ($\sigma_m = 0$) or 3 $\sigma_m \neq 0$ on the y-axis, also is an intercept between $O_1 - O_3$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m_1 of the formula (4-5).The comprehensive material constant A_1' in formulas (4-5) is a calculable one, it has function relation with other parameters m_1 and σ_f', etc. the σ_f' is a fatigue strength coefficient.

$$A'_1 = 2(2\sigma'_f)^{-m_1} \times (v'_{eff})^{-1}, \quad (\text{MPa}^{m_1} \text{ damage-unit}/cycle), (\sigma_m = 0) \tag{4}$$

$$A'_1 = 2[2\sigma'_f(1 - \sigma_m/\sigma'_f)]^{-m_1} \times (v'_{eff})^{-1}, \quad (\text{MPa}^{m_1} \text{ damage-unit}/cycle)\ (\sigma_m \neq 0) \quad (damage-unit-number/cycle) \tag{5}$$

And here should yet explain the A_1' in eqn (4) is corresponding reversed curves 1, its mean stress $\sigma_m = 0$, the A_1' in eqn (5) is corresponding curves 3, its mean stress is $\sigma_m \neq 0$. And in the eqn (5) is used the correctional method for mean stress $\sigma_m \neq 0$ by reference [8].

$$v'_{eff} = \ln(D_{1f}/D_0)/N_{1fc} - N_{01} = [\ln(D_{1f}/D_0) - \ln D_1/D_{01})]/N_{1f} - N_{01}, \quad (damage\text{-}unit\text{-}number/cycle) \tag{6}$$

or

$$v'_{eff} = [D_{1f}\ln(1/1\text{-}\psi)]/N_{1f} - N_{01}, \quad (\text{damage-unit-number/cycle}) \tag{7}$$

The v'_{eff} in eqn (6-7) is defined as an effective history correction factor in first stage, its physical meaning is the damage rate of whole failure to cause specimen material in a cycle, its unit is the $damage-unit-number/cycle$. ψ is a reduction of area. D_0 is pre-micro-damage value which has no effect on fatigue damage under prior cycle loading [9]. D_{01} is an initial damage value, D_f is a critical damage value before failure, N_{01} is initial life in first stage, $N_{01} = 0$; N_{1f} is failure life, $N_{1f} = 1$. By the way, here is also to adopt those material constants $\sigma'_f, b_1', \varepsilon'_f, c_1'$ as "genes" in the fatigue damage subject.

So, for the eqn (1), its final expansion equation corresponded reversed to curves 1 (A_1A) is as below form:

$$N_1 = \frac{\ln D_{tr} - \ln D_1}{2(2\sigma'_f)^{-m_1} \times (v'_{eff})^{-1} (\Delta\sigma)^{m_1}}, (Cycle), \quad (\sigma < \sigma_s, \sigma_m = 0) \tag{8}$$

And its final expansion equation corresponded reversed to curves 3 (D_1D) should be:

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2[2\sigma'_f (1-\sigma_m/\sigma_f)]^{-m_1} \times (v'_{eff})^{-1} \times (\Delta\sigma)^{m_1}} (\sigma_m \neq 0) \quad (9)$$

Where D_{tr} is a transitional damage value transited from micro to macro damage, $D_{tr} \approx D_{mac}$, D_{mac} is a macro damage value corresponding forming macro crack. D_{oi} is a medial damage value between initial damage value and transitional damage value corresponding medial life N_{oi}.

(2) Under work stress $\sigma > \sigma_s$ (low cycle fatigue) condition.

Under $\sigma > \sigma_s$ condition, here to adopt the damage strain factor range $\Delta I'$ to express its life equation, that is corresponded to reversed direction curve C_1C in Fig1, it is as following form

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{B'_1 \times (\Delta I)^{m'_1}}, (Cycle), (\sigma > \sigma_s) \quad (10)$$

Here

$$\Delta I'_1 = (\Delta\varepsilon_p)^{m'_1} \cdot D_1 \quad (11)$$

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{B'_1 \times (\Delta\varepsilon_p)^{m'_1} D_1}, (Cycle), (\sigma > \sigma_s) \quad (12)$$

Here the B'_1 is also calculable comprehensive material constants.

$$B'_1 = 2[2\varepsilon'_f]^{-m'_1} \times (v_{eff})^{-1} \quad (13)$$

If via the damage stress factor amplitude $\Delta H'_1/2$ in eqn (10) to express it, due to plastic strain occur cyclic hysteresis loop effect it should be

$$N_1 = \int_{D_1}^{D_{tr}} \frac{dD_1}{A'_1 \times (\Delta\sigma/2)^{m_1} \times D_1}, (Cycle), (\sigma > \sigma_s) \quad (14)$$

Where the A'_1 is also calculable comprehensive material constants:

$$A'_1 = 2(2\sigma'_f)^{-m_1} (v_{eff})^{-1}, (\sigma_m = 0) \quad (15)$$

$$A'_1 = 2[2\sigma'_f (1-\sigma_m/\sigma_f)]^{-m_1} (v_{eff})^{-1}, (\sigma_m \neq 0) \quad (16)$$

Or

$$A'_1 = 2K'^{-m_1} [2\varepsilon'_f (1-\sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1} \ (\sigma_m \neq 0) \quad (17)$$

Where K' is a cyclic strength coefficient. m'_1 is defined to be damage ductility exponent, $m'_1 = -1/c'_1$, $m_1 = -1/c'_1 \times n'$, c'_1 just is a fatigue ductility exponent under low cycle fatigue, $n' = b'_1/c'_1$, n' is a strain hardening exponent. So that, its final expansion equation for (10) is as below form,

$$N_1 = \frac{\ln D_{tr} - \ln D_1}{2(2\varepsilon'_f)^{-m'_1} \times (v_{eff})^{-1} (\Delta\varepsilon_p)^{m'_1}}, (Cycle), (\sigma > \sigma_s) \quad (18)$$

Its final expansion equation for (14) is as following form,

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2[2\sigma'_f (1-\sigma_m/\sigma_f)]^{-m_1} (v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1}} \quad (19)$$
$$(\sigma > \sigma_s, \sigma_m \neq 0)$$

If take eqn (17) to supersede the A'_1 in eqn (14), its final expansion equation is as below forming

$$N_{oi} = \frac{\ln D_{oi} - \ln D_1}{2K'^{-m_1} [2\varepsilon'_f (1-\sigma_m/\sigma_f)]^{1/c'} \times (v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1}} \quad (20)$$
$$(\sigma > \sigma_s, \sigma_m \neq 0)$$

Here influence of mean stress in eqn (19-20) can be ignored.

4.2. The Calculations for Fatigue-Damage in Macro Damage Process (or Called the Second Stage)

In Fig.1, the residual life curves of macro damage in the second stage are just described with curves 1' ($\sigma < \sigma_s, \sigma_m = 0$), 2' ($\sigma > \sigma_s$) and 3' ($\sigma < \sigma_s, \sigma_m \neq 0$)) at reversed direction coordinate system. Here can adopt two of kind methods to calculate life: the factor method and the stress method.

(1) Under work tress $\sigma < \sigma_s$ condition

1) K'_2 -factor method

For macro damage growth process, its life prediction equation corresponding reversed curves A_2A_1 and D_2D_1 should be as following

$$N_2 = \int_{D_{tr}}^{D_{eff}} \frac{dD_2}{A'_2 \times [y_2(a/b)\Delta K'_2]^{m_2}}, (\text{cycle}) \quad (21)$$

Where

$$K'_2 = \sigma\sqrt{\pi D_2} \quad (22)$$

$$\Delta K'_2 = \Delta\sigma\sqrt{\pi D_2} \quad (23)$$

The K'_2 -factor is defined as the macro-damage stress intensity factor, it is drive force for macro-damage under monotonous load; and the $\Delta K'_2$ is defined as the macro-damage stress intensity factor range, it is drive force under fatigue load. The $y_2(a/b)$ is correction factor [10] related to long crack form and structure size. The A'_2 in eqn. is defined as comprehensive material constants of macro-damage, for $\sigma_m = 0$, it is corresponding curve A_1A_2, and is also calculable one as following

$$A'_2 = 2(2K'_{2eff})^{-m_2} \times v_{pv}, \quad (MPa^{m_1} \times damage-unit-number/cycle), \quad (\sigma_m = 0) \tag{24}$$

or

$$A'_2 = \frac{\frac{2}{2-m_2}(D_{2eff}^{1-\frac{m_2}{2}} - D_{02}^{1-\frac{m_2}{2}})}{(2\sigma_{2eff}\sqrt{\pi})^{m_2}(N_{2eff} - N_{02})}, \quad (MPa^{m_1} \times damage-unit-number/cycle) \quad (\sigma_m = 0) \tag{25}$$

And for $\sigma_m \neq 0$, the A'_2 is corresponded to curve D_1D_2, it should be as following form

$$A'_2 = 2\left[2K'_{2eff}(1 - K_{2m}/K_{2fc})\right]^{-m_2} \times v_{pv}, (\sigma_m \neq 0) \tag{26}$$

Where K'_{2m} is mean damage stress intensity factor, K'_{2eff} is an effective damage stress intensity factor, K'_{2fc} is a critical damage stress intensity factor, which they are parameters under cyclic loading. It should be point that the physical meaning for the A'_2 is also a concept of the power, that just is a maximal increment value to give out energy in one cycle before failure. Its geometrical meaning is also a maximal micro-trapezium area approximating to beeline (Fig1), that is a projection of corresponding to curve 1' ($\sigma_m = 0$) or 3' $\sigma_m \neq 0$ on the y-axis, also is an intercept between $O_3 - O_4$. Its slope of micro-trapezium bevel edge just is corresponding to the exponent m_2 of the formula (24~26). Here,

$$v'_{pv} = \frac{(D_{2pv} - D_{02})}{N_{2eff} - N_{02}} \approx 3\times10^{-5} \sim 3\times10^{-4} = v^*(damage-unit-number/Cycle) \tag{27}$$

Author researches and thinks, the parameter v'_{pv} is defined to be the virtual rate, it is an equivalent rate caused in precrack, it can take same value with the "v^*" ($m/cycle$) by reference [11]. But its unit is different, here unit of the v'_{pv} is "$damage-unit-number/cycle$". The damage D_{2pv} is a virtual damage value as equivalent to a precrack size a_{2pv}, D_{02} is an initial damage value as equivalent to the initial micro-crack size a_{02}. N_{02} is an initial life, $N_{02} = 0$. N_{pv} is a virtual life, $N_{2eff} = 1$. In references [12-13], all refer to effective stress intensity factor, here to propose to take equivalent values as follow

$$K'_{2eff} \approx \sqrt{K'_{th} K'_{1c}}, \tag{28}$$

or

$$K'_{2eff} \approx (0.25 - 0.4)K'_{1c} \tag{29}$$

K'_{th} is threshold damage stress intensity factor value.

$$K'_{1c} = \sigma_f \sqrt{\pi D_{2f}} \tag{30}$$

$$K'_{2fc} = \sigma'_f \sqrt{\pi D_{2f}}, (MPa\sqrt{damage-unit-number}) \tag{31}$$

$$K'_{2eff} = \sigma'_f \sqrt{\pi D_{2eff}}, (MPa\sqrt{damage-unit-number}) \tag{32}$$

$$K'_{2m} = (K'_{2max} + K'_{2min})/2 \tag{33}$$

So the effective life expanded equation corresponding reversed direction curve A_2A_1 is following forming.

$$N_{2eff} = \frac{\frac{2}{2-m_2}(D_{2eff}^{1-\frac{m_2}{2}} - D_{02}^{1-\frac{m_2}{2}})}{2[2K'_{2eff}]^{-m_2} \times v'_{pv} \times [(Ya/b)^{m_2} \Delta\sigma^{m_2} \pi^{\frac{m_2}{2}}]}, \quad (\sigma_m = 0) \tag{34}$$

And the effective life expanded equation corresponding reversed direction curve D_2D_1 should be

$$N_{2eff} = \frac{\frac{2}{2-m_2}(D_{2eff}^{1-\frac{m_2}{2}} - D_{02}^{1-\frac{m_2}{2}})}{2[2K'_{2eff}(1 - K'_m/K'_{fc})]^{-m_2} \times v'_{pv} \times [(Ya/b)^{m_2} \Delta\sigma^{m_2} \pi^{\frac{m_2}{2}}]}, \quad (\sigma_m \neq 0) \tag{35}$$

Its medial life N_{2oj} in second stage is

$$N_{2oj} = \frac{\frac{2}{2-m_2}(D_{oj}^{1-\frac{m_2}{2}} - D_{tr}^{1-\frac{m_2}{2}})}{2\left[2K'_{2eff}(1 - K'_{2m}/K'_{2fc})\right]^{-m_1} \times v_{pv}[y_2(a/b)\Delta\sigma\sqrt{\pi}]^{m_2}}, \quad (cycle)(\sigma_m \neq 0) \tag{36}$$

Where D_{tr} is a transitional damage value between two stages, it is equivalent to the crack transitional size a_{tr} from short crack a_{mic} to long crack a_{mac} growth, the $D_{tr} \approx D_{mac}$ is equivalent to $a_{tr} \approx a_{mac}$, the D_{oj} is a medial damage value. $D_{o2} < D_{oj} < D_{2eff}$.

2) Stress σ -method

Due to word stress is still $\sigma/\sigma_s << 1$ ($\sigma \leq 0.5\sigma_s$), in the macro damage growth process, its residual life equation of corresponding reversed direction curve A_2A_1 and D_2D_1 in fig.1 is as following form

$$N_1 = \int_{D_{tr}}^{D_{2eff}} \frac{dD_1}{B'_2 \times (\Delta\delta'_t)^{m_2}}, (Cycle) \tag{37}$$

Where

$$\delta'_t = \pi D_2 \sigma_s \times (\sigma/\sigma_s)^2 / E, (damage-unit-number) \quad (38)$$

$$\Delta\delta'_t = \frac{\beta\Delta\sigma^2\pi D_2}{4\sigma_s E}, (damage-unit-number), \ (\sigma_m = 0) \quad (39)$$

The δ'_t is defined as damage crack tip open displacement, it is driving force of macro-damage under monotonic loading; and the $\Delta\delta'_t$ is defined as damage crack tip open displacement range, it is driving force under fatigue loading. For the coefficient β in eqn (39), it equal 1($\beta = 1$) under plane stress condition; under plane strain condition, $\beta = (1-\nu^2)/2$. ν is Poisson's ratio. E is an elasticity modulus. The B'_2 is comprehensive material constant in second stage, its physical meaning of the B'_2 is also a concept of the power. Its geometrical meaning is also a maximal micro-trapezium area approximating to beeline (Fig1). B'_2 is also calculable comprehensive material constant, for $\sigma = 0$ that is

$$B'_2 = 2\left(\frac{\beta(\sigma'^2_f \times D_{2eff} \times \pi / \sigma^2_s)\sigma_s}{E}\right)^{-m_2} \times v_{pv}, \quad (\sigma = 0) \quad (40)$$

$$(damage-unit-number/cycle)$$

And for $\sigma \neq 0$ that is

$$B'_2 = 2\left(\frac{2\beta(\sigma'^2_f \times D_{2eff} \times \pi / \sigma^2_s)\sigma_s}{E}\left[1 - \frac{D_{02}(\sigma^2_{max} + \sigma^2_{min})}{2\times D_{2eff}\sigma^2_s}\right]\right)^{-m_2} v_{pv} \quad (41)$$

$$(\sigma \neq 0)$$

Where σ_{max} and σ_{min} are maximum and minimum work stress. The D_{2eff} can be calculable effective damage value. So its final expansion form corresponded reversed direction curve A_2A_1 for eqn (37) is as below,

$$N_{2eff} = \frac{(4E\cdot\sigma_s)^{m_2} \times \frac{1}{1-m_2}(D_{2eff}^{1-m_2} - D_{tr}^{1-m_2})}{2\left(\frac{2\beta(\sigma^2_{2eff} \times a_{2eff} \times \pi / \sigma^2_s)\sigma_s}{E}\right)^{-m_2} v'_{pv}[y_2(a/b)\beta\times\Delta\sigma^2\pi]^{m_2}}, (\sigma_m = 0) \quad (42)$$

And the life equation corresponded to reversed direction curve D_2D_1 is following

$$N_{2eff} = \frac{(2E\cdot\sigma_s)^{m_2} \times \frac{1}{1-m_2}(D_{2eff}^{1-m_2} - D_{tr}^{1-m_2})}{2\left(\frac{2\beta(\sigma^2_{2eff} \times a_{2eff} \times \pi / \sigma^2_s)\sigma_s}{E}\left[1 - \frac{D_{02}(\sigma^2_{max} + \sigma^2_{min})}{2\times a_{meff} \times \sigma^2_s}\right]\right)^{-m_2} v'_{pv}[y_2(a/b)\beta\Delta\sigma^2\pi]^{m_2}}, (\sigma_m \neq 0) \quad (43)$$

(2) Under work tress $\sigma > \sigma_s$ condition

Under $\sigma > \sigma_s$ condition, its effective life models corresponding reversed curve C_2C_1 in figure 1 is as below form,

$$N_{2eff} = \int_{D_{tr}}^{D_{2eff}} \frac{dD_2}{B'_2 \times (\Delta\delta'_t / 2)^{\lambda_2}}, (Cycle), (\sigma > \sigma_s) \quad (44)$$

B'_2 is also calculable comprehensive material constant,

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)D_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}, (\sigma_m = 0) \quad (45)$$

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)(1 - \sigma_m/\sigma'_f)D_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}, \ (\sigma_m \neq 0) \quad (46)$$

Where λ_2 is defined to be ductility exponent in macro damage process, $\lambda_2 = -1/c'_2$, c'_2 is a fatigue ductility exponent in second stage.

So that, the final expansion equations is derived from above mentioned eqn. (44) as follow

For $\sigma_m = 0$,

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(D_{2eff}{}^{1-\lambda_2} - D_{02}{}^{1-\lambda_2})}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)D_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(a/b)(\Delta\sigma/2\sigma_s+1)}{E}\right]^{\lambda_2}}, (cycle) \quad (47)$$

For $\sigma_m \neq 0$, it should be

$$N_{2eff} = \frac{\frac{1}{1-\lambda_2}(D_{2eff}{}^{1-\lambda_2} - D_{02}{}^{1-\lambda_2})}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)}{E}\right]^{\lambda_2}}, (cycle) \quad (48)$$

In the eqn (48), influence to mean stress usually can be ignored.

Where, D_{2eff} are effective damage value, it can calculate from effective damage crack tip opening displacement δ'_{2eff}

$$D_{2eff} = \frac{E\times\delta'_{2eff}}{\pi\sigma_s(\sigma'_f/\sigma_s+1)}, (damage-unit-number) \quad (49)$$

And

$$\delta'_{2eff} = (0.25\sim0.4)\delta'_c, (damage-unit-number) \quad (50)$$

Where the δ'_c is critical damage crack tip displacement, it is equivalent critical crack tip displacement δ_c, both is only on the unit to be different. So the D_{2eff} in (48) can convert out by δ_c-value in "1 damage-unit" value equivalent to "1mm" by means of equations (49-50). It must be point that the life units in eqns. (42-44,47-48) are cyclic number.

4.3. The damage life prediction calculations in whole process

In damage evolving process, for availing to life calculation in whole process, it can take a damage value D_{tr} of transition point between two stages from micro to macro damage growth process, and the transition point D_{tr} can be derived to make equal between the damage rate equations by two stages, for instance [2,14],

$$dD_1/dN_1 = dD_{tr}/dN_{tr} = dD_2/dN_2 \quad (51)$$

(1) Under work stress $\sigma < \sigma_s$

For $\sigma < \sigma_s, \sigma_m = 0$, its expanded rate link equation for eqn (51) corresponding to positive curve AA_1A_2 is as following form

$$\frac{dD_1}{dN} = \left\{2[2\sigma'_f]^{-m_1} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} D\right\}_{D_{01}->D_{tr}} = \frac{dD_{tr}}{dN} = \frac{dD_{2tr}}{dN_2}$$
$$= \left\{2\left(\frac{2\beta(\sigma^2{}_{2eff}\times D_{eff}\pi/\sigma^2{}_s)\sigma_s}{E}\right)^{-m_2} v_{pv}\times\left(\frac{y_2(a/b)\beta\Delta\sigma^2\pi D}{4\sigma_s E}\right)^{m_2}\right\}_{D_{tr}->D_{eff}}, \quad (52)$$
$$(\sigma_m = 0), (damage-unit-number/cycle)$$

For $\sigma < \sigma_s, \sigma_m \neq 0$, its expanded rate link equation for eqn (51) corresponded to positive curve DD_1D_2 is as following form

$$\frac{dD_1}{dN} = \left\{2[2\sigma'_f(1-\sigma_m/\sigma'_f)]^{-m_1} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} D\right\}_{D_{01}->D_{tr}} = \frac{dD_{tr}}{dN} = \frac{dD_{2tr}}{dN_2}$$
$$= \left\{2\left(\frac{2\beta(\sigma^2{}_{eff}\times D_{eff}\times\pi/\sigma^2{}_s)\sigma_s}{E}\left[1-\frac{a_{01}(\sigma^2_{\max}+\sigma^2_{\min})}{2a_{meff}\sigma^2_s}\right]\right)^{-m_2} v_{pv}\times\left(\frac{y_2(a/b)\beta\Delta\sigma^2\pi D}{2\sigma_s E}\right)^{m_2}\right\}_{D_{tr}->D_{eff}}, \quad (53)$$
$$damage-unit-number/cycle, (\sigma_m \neq 0)$$

And the life equations in whole process corresponding to reversed direction curves A_2A_1A and D_2D_1D should be as below

$$\sum N = N_1 + N_2 = \int_{D_{01}}^{D_{tr}} \frac{dD}{A'_1 \times (\Delta\sigma)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{A_2 (\Delta\delta'_t)^{m_2}}, \quad (54)$$

Its expanded equation corresponding to reversed direction curves A_2A_1A is as following form

$$\sum N = N_1 + N_2 = \int_{D_{01}}^{D_{tr}} \frac{dD}{2[2\sigma'_f]^{-m_1} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{2\left(\frac{2\beta(\sigma^2_{eff} \times \pi \times D_{eff} / \sigma^2_s)\sigma_s}{E}\right)^{-m_2} v_{pv}[2y_2\beta\Delta\sigma^2\pi D)/4E\sigma_s]^{m_2}}, \ (\sigma_m = 0) \quad (55)$$

But for $\sigma_m \neq 0$, its expanded equation corresponding to reversed direction curves D_2D_1D should be

$$\sum N = N_1 + N_2 = \int_{D_{01}}^{D_{tr}} \frac{dD}{2[2\sigma'_f(1-\sigma_m/\sigma'_f)]^{-m_1} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{2\left(\frac{2\beta(\sigma^2_{eff} \times D_{eff}\pi / \sigma^2_s)\sigma_s}{E}\left[1 - \frac{D_{01}(\sigma^2_{\max} + \sigma^2_{\min})}{2D_{meff}\sigma^2_s}\right]\right)^{-m_2} v_{pv}[2y_2\beta\Delta\sigma^2\pi D)/2E\sigma_s]^{m_2}}, \ (\sigma_m \neq 0) \quad (56)$$

(2) Under work stress $\sigma > \sigma_s$

Under work stress $\sigma > \sigma_s$, its expanded rate link equation for eqn (51) corresponding to positive curve CC_1C_2 is as following form

$$\frac{dD_1}{dN} = \left\{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (v_f \times D_{tr})^{-1} \times (\Delta\sigma/2)^{m_1} \times D\right\}_{D_{01}->D_{tr}} = \frac{dD_{tr}}{dN} = \frac{dD_{2tr}}{dN_2}$$
$$= \left\{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)D_{eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)D}{E}\right]^{\lambda_2}\right\}_{D_{tr} \ ->D_{eff}}, \quad (57)$$
$$damage - unit - number / cycle, (\sigma \neq 0)$$

The life equations in whole process corresponding to reversed direction curve C_2C_1C should be as following

$$\sum N = N_1 + N_2 = \int_{D_{01}}^{D_{tr}} \frac{dD}{B'_1 \times (\Delta\sigma/2)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{B'_2(\Delta\delta'_t/2)^{\lambda_2}}, \quad (58)$$

(curve C_2C_1C)

And the expanded life prediction expression in whole process corresponded reversed curve C_2C_1C, it should be

$$\sum N = \int_{D_{01}}^{D_{tr}} \frac{dD}{2K'^{-m_1}[2\varepsilon'_f]^{1/c'} \times (D_f \cdot v_{eff})^{-1} \times (\Delta\sigma/2)^{m_1} \times D} + \int_{D_{tr}}^{D_{2eff}} \frac{dD}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s + 1)D_{2eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s + 1)D}{E}\right]^{\lambda_2}}, \quad (59)$$

$(cycle)$

It should point that the calculations for rate and life in whole process should be according to different stress level, to select appropriate calculable equation. And must explain that its meaning of the eqns (51-53, 57) is to make link between the first stage rate and the second stage rate, that should be calculated by the micro damage rate equation before the transition point D_{tr}; it should be calculated by the macro damage rate equation after the transition point D_{tr}, which they not been added together by the rates for two stages. But the life calculations for two stages can be added together. About calculation method, it can calculate by means of computer doing computing by different damage value [15].

5. Calculating Example

5.1. Contents of Example Calculations

To suppose a pressure vessel is made with elastic-plastic steel 16MnR[16], its strength limit of material $\sigma_b = 573MPa$, yield limit $\sigma_s = 361MPa$, fatigue limit $\sigma_{-1} = 267.2MPa$, reduction of area is $\psi = 0.51$, modulus of elasticity $E = 200000MPa$; Cyclic strength coefficient $K' = 1165MPa$, strain-hardening exponent $n' = 0.187$; Fatigue strength coefficient $\sigma'_f = 947.1MPa$, fatigue strength exponent $b'_1 = -0.111$, $m_1 = 9.009$; Fatigue ductility coefficient $\varepsilon'_f = 0.464$, fatigue ductility exponent $c'_1 = -0.5395$, $m'_1 = 1.8536$. Threshold value $\Delta K_{th} = 8.6\text{MPa}\sqrt{\text{m}}$, critical stress intensity factor $K_{2c} = K_{1c} = 92.7\text{MPa}\sqrt{\text{m}}$, critical damage stress intensity factor $K'_{2c} = 92.7MPa\sqrt{damage-unit-number}$ of equivalent to the $K_{1c}(K_{2c})$. Its working stress $\sigma_{\max} = 450MPa$, $\sigma_{\min} = 0$ in pressure vessel. And suppose that for long crack shape has been simplified via treatment become an equivalent through-crack, the correction coefficient $y_2(a/b)$ of crack shapes and sizes equal 1, i.e. $y_2(a/b) = 1$. Other computing data are all included in table 1.

Table 1. *Computing data*

$K_{1c}, MPa\sqrt{m}$	$K_{eff}, MPa\sqrt{m}$	$K_{th}, MPa\sqrt{m}$	v_{pv}	m_2	δ_c, mm	λ_2	$y_2(a/b)$	a'_{th}, mm
92.7	28.23	8.6	2×10^{-4}	3.91	0.18	2.9	1.0	0.07

5.2. Required Calculating Data

Try to calculate respectively as following different data and depicting their curves:

(1) To calculate the transitional point damage value D_{tr} between two stages;

(2) To calculate the damage rate at transitional point (at damage value D_{tr})

(3) To calculate the life N_1 in first stage from micro damage value $D_1 = 0.02damage-unit$ growth to transitional point D_{tr}

(4) To calculate the life N_2 in second stage N_2 from transitional point D_{tr} to macro damage value $D_{2\text{eff}} = 5 - damage-unit$;

(5) Calculating the whole service life $\sum N$.

(6) Depicting their damage life curves in whole process.

5.3. Calculating Processes and Methods

The concrete calculation methods and processes are as follows:

5.3.1. Dimensions and Units Conversions

Table 2. *Computing data*

$K'_{1c}, MPa\sqrt{1000damage-nuit}$	$K'_{eff}, MPa\sqrt{1000damage-nuit}$	$K'_{th}, MPaMPa\sqrt{1000damage-nuit}$
92.7	28.23	8.6

Table 3. *Computing data*

v'_{pv}	m_2	$\delta'_c, damage-unit$	λ_2	$y_2(a/b)$	$D_{eff}, damage-unit$
2×10^{-4}	3.91	0.18	2.9	1.0	2

5.3.2. Calculations for Relevant Parameters

1) Stress calculation

Stress range calculation:

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min} = 450 - 0 = 450(MPa)$$

Mean stress calculation:

$$\sigma_m = (\sigma_{\max} + \sigma_{\min})/2 = (450-0)/2 = 225MPa$$

2) According to formulas (7), calculation for correction coefficient v'_{eff} in first stage

$$\begin{aligned} v'_{eff} &= D_{eff}\ln[1/(1-\psi)] \\ &= 2\times\ln[1/(1-0.51)] \\ &= 1.43,(\text{damage-unit/cycle}) \end{aligned}$$

3) By eqn (27), to select virtual rate v'_{pv} in second stage, here take:

$$v_{pv} = \frac{D_{2eff} - D_{02}}{N_{2f} - N_{02}} \qquad N_{2f} = 1, \quad N_{02} = 0$$
$$\approx 2.0 \times 10^{-4} (damage - unit / Cycle),$$

4) According to formulas (49), Calculating effective size a_{eff}

$$D_{eff} = \frac{E \times \delta'_{eff}}{\pi\sigma_s(\sigma_f / \sigma_s + 1)}$$
$$= \frac{200000 \times 0.25 \times 0.18}{\pi 361(947.1/361+1)}$$
$$= 2.1(damage - unit),$$

Take $D_{eff} = 2.0mm$

Here take effective crack size:

$D_{1eff} = D_{2eff} = 2damage - unit$ in first and the second stage

5.3.3. To Calculate the Transitional Point Damage Value D_{tr} Between Two Stages

1) Select damage rate calculating equation, calculating for micro damage process in first stage

At first, calculation for comprehensive material constant B_1 by eqn (17)

$$A'_1 = 2K'^{-m_1}[2\varepsilon'_f(1 - \sigma_m / \sigma'_f)]^{1/c'} \times (D_{ef} \times v_f)^{-1}$$
$$= 2 \times 1165^{-9.01} \times [2 \times 0.464(1 - 225/947.1)]^{1/-0.5395}(2 \times 0.713)^{-1}$$
$$= 6.28 \times 10^{-28}, (MPa^{m_1}\sqrt{damage - unit})^{-m_1}$$

Here select the damage rate linking equation (57), calculation for damage rate in first stage, and make to simplify calculations as follow form:

$$dD_1 / dN_1 = A'_1 \times (\Delta\sigma / 2)^{m_1} \times D_1$$
$$= 3.193 \times 10^{-28} \times (450/2)^{9.01} \times D_1$$
$$= 6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times D_1$$
$$= 9.8 \times 10^{-7} \times D_1$$

2) By linking equation (57), calculating for macro damage process in second stage

Calculation for comprehensive material constant B_2 by eqn (46)

$$B'_2 = 2\left[(\pi\sigma_s(\sigma'_f / \sigma_s + 1)(1 - \sigma_m / \sigma'_f)D_{eff} / E)\right]^{-\lambda_2} \times v_{pv}$$
$$= 2\left[2(3.1416 \times 361(947.1/361+1)(1 - 225/947.1) \times 2/200000)\right]^{-2.9}$$
$$\times 2 \times 10^{-4} = 9.1988, (damage - unit)^{-\lambda_2} \times damage - unit / Cycle$$

Calculations for the damage rate in second stage, and to take brief calculations as follow form,

$$dD_2 / dN_2 = B_2\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma / 2\sigma_s + 1)D_2}{E}\right]^{\lambda_2}$$
$$= 9.1988 \times \left[\frac{0.5\pi 361(450/(2 \times 361)+1)D_2}{E}\right]^{2.9}$$
$$= 9.1988 \times 1.6698 \times 10^{-7} D_2^{2.9}$$
$$= 1.5384 \times 10^{-6} D_2^{2.9}$$

3) Calculation for transitional point damage value D_{tr}

According to the equations (51) and (57), for the transitional point damage value D_{tr}, it can be calculated to make equal between brief rate expansion in two stages:

$$6.28 \times 10^{-28} \times 1.56 \times 10^{21} \times D_{tr}$$
$$= 9.1988 \times 1.6698 \times 10^{-7} \times D_{tr}^{2.9}$$

$$D_{tr} = (0.638)^{\frac{1}{1.9}}$$
$$= (0.638)^{0.5263}$$
$$= 0.789(damage - unit)$$

So to obtain the transitional point damage value $D_{tr} = 0.789(damage - unit)$ between two stages.

5.3.4. To calculate the Damage Rate at Transitional Point (Damage Value D_{tr})

$$dD_1 / dN_1 = dD_{tr} / dN_{tr}$$
$$= 9.8 \times 10^{-7} D_1 = 9.8 \times 10^{-7} \times 0.789$$
$$= 7.74 \times 10^{-7} (damage - unit / cycle)$$

$$dD_2 / dN_2 = dD_{tr} / dN_{tr} = 1.5384 \times 10^{-6} D_{tr}^{2.9}$$
$$= 1.5384 \times 10^{-6} \times (0.79)^{2.9}$$
$$= 7.74 \times 10^{-7} (damage - unit / cycle)$$

Here can be seen, the damage rate at the transition point $(D_{tr} = 0.789 damage - unit)$ is same.

And above damage rate value equivalent to the crack growth rate at the transition point of crack size $a_{tr} = 0.789(mm)$, it is $da_{tr} / dN_{tr} = 7.74 \times 10^{-7}(mm/cycle)$.

5.3.5. Life Prediction Calculations in Whole Process

1) To select eqn (20), the life N_1 from micro-damage $D_1 = 0.02 damage - unit$ to transitional point (damage value $D_{tr} = 0.789 damage - unit$) is as follow,

$$N_1 = \frac{\ln D_{tr} - \ln D_{01}}{2K'^{-m_1}[2\varepsilon'_f(1-\sigma_m/\sigma'_f)]^{1/c'} \times (D_{eff} \times v_f)^{-1} \times (\Delta\sigma/2)^{m_1} \times D}$$

$$= \frac{\ln 0.789 - \ln 0.02}{2\times 1165^{-9.01}\times[2\times 0.464(1-225/947.1)]^{1/-0.5395}(2\times 0.713)^{-1}\times(450/2)^{9.01}}$$

$$= \frac{3.675}{6.28\times 10^{-28}\times 1.56\times 10^{21}} = \frac{3.675}{9.8\times 10^{-7}}$$

$$= 3751260(Cycle)$$

So predicting life in first stage $N_1 = 3751260(Cycle)$

And for above formulas, we can derive simplified life equation in first stage corresponded to different damage value as follow form

$$N_1 = \frac{1}{9.8\times 10^{-7} D_1}$$

2) To select eqn (48), to calculate the life N_2 in second stage from transitional point damage value $D_{tr} = 0.789(damage-unit)$ to $D_{2eff} = 5damage-unit$ is as follow,

$$N_2 = \frac{\frac{1}{1-\lambda_2}(D_{2eff}^{1-\lambda_2} - D_{tr}^{1-\lambda_2})}{2\left[(\pi\sigma_s(\sigma'_f/\sigma_s+1)(1-\sigma_m/\sigma'_f)D_{eff}/E)\right]^{-\lambda_2} \times v_{pv}\left[\frac{0.5\pi\sigma_s y_2(\Delta\sigma/2\sigma_s+1)}{E}\right]^{\lambda_2}}$$

$$= \frac{\frac{1}{1-2.9}(5^{1-2.9} - 0.789^{1-2.9})}{2\left[(3.1416\times 361(947.1/361+1)(1-225/947.1)\times 2/200000)\right]^{-2.9} \times v_{pv}} \times \frac{1}{\left[\frac{0.5\pi 361(450/(2\times 361)+1)}{E}\right]^{2.9}}$$

$$= \frac{0.8}{9.1988\times 1.6698\times 10^{-7}} = \frac{0.8}{1.5384\times 10^{-6}}$$

$$= 520625(Cycle)$$

From above formulas, we can also derive simplified life equation corresponding different damage value as follow form

$$\rightarrow N_2 = \frac{1}{1.5384\times 10^{-6} D_2^{2.9}}$$

Therefore, predicting life in whole process is

$$\sum N = N_1 + N_2 = 3751260 + 520625 = 4271885(\text{Cycle})$$

The life data corresponded to different damage values which are all converted into the relation between the crack growth sizes and the life, and are all included in table 4, 5, 6. These data are basically close with those results calculated by author with another calculated method, it will be published recently [17].

Table 4. *Crack growth life data in whole process*

Data point of number	1	2	3	4	5
Crack size (mm)	0.02	0.04	0.1	0.2	0.4
Data of the first stage	510204 08	255102 04	102040 82	510204 1	255102 0
Data of the second stage	Invalid section				

Table 5. *Crack growth life data in whole process*

Data point of number	5	6	7 transition point	8
Crack size (mm)	0.5	0.6	0.789	1.0
Data of the first stage	2040816	170068 0	1293293	102040 8
Data of the second stage	4851966	285951 3	1292431	650026

Table 6. *crack growth life data in whole process*

Data point of number	9	10	11	12	13
Crack size (mm)	1.5	2.0	3.0	4	5
Data of the first stage	680272	51020 4	Invalid section		
Data of the second stage	200570	87085	26871	11667	6108

(6) To depict the life curves in the whole process

By means of the data in tables 4-6 mentioned above have depicted the life curves for two stages and for whole course are respectively in figure 2 and 3.

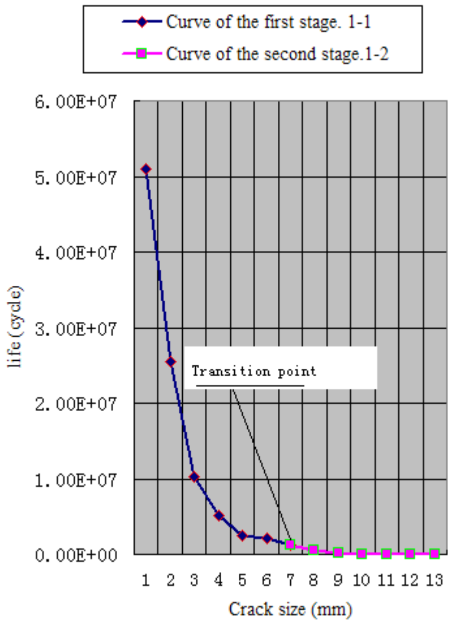

Figure 2. *life curve in whole course (in decimal coordinate system)*

(A) 1-1--- data curve in first stage obtained by single-parameter calculating method;
(B) 1-2---data curve in second stage obtained by single-parameter calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789mm).

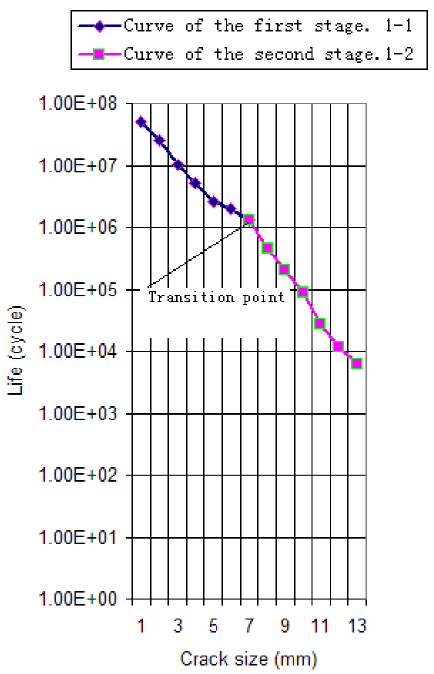

Figure 3. *life curve in whole course (in logarithmic coordinate system)*

(A) 1-1--- data curve in first stage obtained by single-parameter calculating method;

(B) 1-2---data curve in second stage obtained by single-parameter calculating method;
(C) This example transition point from micro-crack size 0.02mm to long crack size 5 is just at seventh point (crack size 0.789mm).

6. Conclusions

(1) About the significations of figure1: The calculating figure of materials behaviors may be a calculation route diagram on fatigue-damage-fracture, it is also a tool and bridge to communicate relationships among the conventional material mechanics, the modern mechanics and the engineering materials disciplines.

(2) About difference cognition for material constants: True material constants must show the inherent characters of materials, such as the σ_s and E, δ, ψ etc in the material mechanics; and for instance the σ_f and σ'_f ; ε_f, and ε'_f; b_1 and b'_1; c_1 and c'_1 and so on in the fatigue damage mechanics; which could all be checked and obtained from general handbooks; But some new material constants in the damage and fracture mechanics which are essentially all functional formulas for which can all be calculated by means of the relational expressions, e.g. eqns (4-5), (13,15-16), (24-26), (40-41), (45-46), etc. and have to combine experiments to verify. Therefore for this kind of material constants can be defined as comprehensive materials constants.

(3) About cognitions to the physical and geometrical meanings for key parameters A_1' and A_2', B_2': The parameters A_1' in the first stage and A_2', B_2' in the second stage, they are calculable comprehensive materials constants, and they are all with other parameters to have functional relations. Their physical meanings of the A_1', A_2' and B_2' are a concept of the power, just are a maximal increment value to pay energy in one cycle before to cause failure. Their geometrical meanings are a maximal micro-trapezium area approximating to beeline.

(4) About conversion rules for variables, dimensions and units: Inside mathematical models, to convert crack variable a into damage variable D, it must define "1mm-crack-length" equivalent to 'one-damage-unit', "1m-crack-length" equivalent to '1000-damage-unit'. For the sake of making link between damage mechanics and fracture mechanics, this is a key.

(5) About the regulations and methods for whole process rate and life calculations: for calculation damage transition value D_{tr} it can be calculated to make equal by between the micro-damage rate and the macro-damage rate equation; For the rate before the transition point D_{tr}, it should be calculated by the micro damage rate equation, after the transition point D_{tr} it should be calculated by the macro damage rate equation. But for the lifetime calculations it can be added together by life cycle number in two stages.

(6) Based on the traditional material mechanics is a calculable subject; in consideration of the traditional parameters there are "the hereditary characters"; In view of the relatedness and the transferability between

related parameters among each disciplines; and based on above viewpoints and cognitions (1)~(5); then nowadays for the fatigue-, the damage-, and the fracture disciplines yet mainly depended on tests, if make them become calculable subjects, that are: their required data are be given via theoretical calculation as priority method, via the experiments verify as complementary, that will be to exist the possibility.

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