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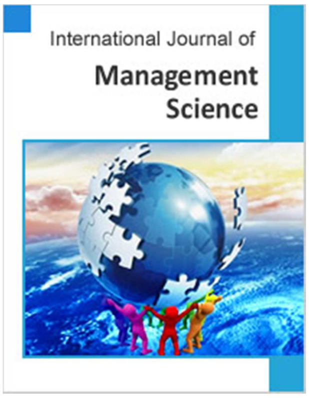

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Supply Chain Commitment Contract Model Based on Uncertainty Theory with Information Updating and Effort Level

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Abstract
Based on the uncertainty theory, market demand information updating as the background, this paper study the coordination and optimization problem of a two-stage three-phase supply chain system with two wholesalers and a manufacturer. With assumption that semi-symmetric market information and risk-neutral participants, in the situation that the manufacturer and two wholesalers have two pre-season decision-making opportunities and retailers can replenish in the sales season, this paper brings in the supply contract commitments. To exchange updating information of the market that the manufacturer itself can't obtain directly, the manufacturer will commit to the wholesaler the minimum supply in the pre-season. However, to have more replenishment inventory, wholesalers will exaggerate market updates to a certain extent. And they have competition relationship or mutually reinforcing relationship with the level of effort. According to this contract, we establish the optimization model, and get the optimal strategy of supply chain members by analyzing the supply chain system. Finally, by giving a numerical example, the conclusions are more reasonable.

1. Introduction

Supply chain issues play a very important role in the logistics field. It is made up of a range of essential activities from product manufactured by manufacturing enterprises to the final product sold to consumers. Whether enterprises in the supply chain cooperate or not will directly affect the profits of enterprises, but also affect the efficiency of the entire supply chain. Usually in the study of supply chain problems, the more study is on the case of a manufacturer and a wholesaler, but the reality is often a cooperation problem between one manufacturer and wholesalers, multiple manufacturers and a wholesaler or a number of manufacturers and wholesalers. For the cooperation problem in the supply chain with one manufacturer and multiple wholesalers, the relationship between multiple wholesalers mainly divided into three categories: 1) competitive relationship; 2) complementary relationship; 3) no effect or relationship between them. Competitive relationship among wholesalers is relatively common. They sell the same product in different areas of the same city or neighboring cities, which will inevitably lead to the existence of competition between them. Although consumers in different regions or cities have different income level, the price level, cultural differences and other factors, the competition relationship among wholesalers is the major one. Of course, their sales are also affected by other factors such as effort and price factors. Complementary relationship, mutual support relationship or mutual promotion relationship among wholesalers is also relatively common. Publicity efforts and advertising payment of a wholesaler in a region will affect not only sales in the region but also sales of another wholesaler in other regions. The more effort the wholesaler pay in a certain region, the better their products sales and it also affect sales of the other retailers in other regions; on the contrary, some issues, such as its service and quality, affect both sales volume of the retailer and that of other retailers in adjacent regions. Since there is no mutual influence between the two wholesalers, it has not much difference with a supply chain contract problem of one manufacturer and one wholesaler. Therefore, the paper focused on the first two relationships and then simplified the problem to the supply chain contract problem of one manufacturer and two wholesalers.

The uncertainty theory [1] has developed and improved constantly for more than ten years, since professor Liu Bao, Tsinghua university and his team put forward it. Its theoretical system was recognized and accepted gradually by the majority of scholars and experts, and part of them have put themselves in studying this field [2] [3]. At present, axiom system of the theory has been established on the whole, and its theoretical system has been extended to planning theory, risk analysis, reliability analysis, logical reasoning, differential equations, statistics and differential and integral calculus and other fields. Its scope of application is expanded gradually, it has many examples of applications and the potential for further development in the fields of business, social, economic and financial. This paper considers to applying the uncertainty theory to problems of the supply chain, to convert problem of random demand in the supply chain to that of uncertainty demand. At the beginning of entering the market, we do not know the actual needs of some items, producers want to know it only by experience and predict of some experts. Therefore, from this perspective, uncertain demand is more tally than random demand with the actual.

In the supply chain problems with random demand, it is indeed effective strategy that using such Revenue Sharing Contract, Quantity Flexibility Contract, the Wholesale Price Flexibility Contract, the Commitment Contract and the Option Contract in order to achieve supply chain system coordinated, no matter analyzing from either the perspective of the manufacturer and wholesaler or the perspective of a multi-stage supply chain or multi-level one. And with more and more fiercely competition for consumers, the relationship between supply chain members turn to the direction of mutual benefit and win-win from the original simple splitting profit. The members can gain more profits only by improving the competitiveness of the entire supply chain, so it has become hot spots of researches that the relational contract that concerned relationship between supply chain members. Relational contract is a convention, more accurately, a trust that could not specified by stringent legal provisions. Therefore the commitment itself may be unlike the general traditional contract that can put into effect through a simple contract. Wang Yingjun [4] define the relational contract: the relevant provisions that ensure buyers and sellers coordinate and optimize the sales channel performance by providing the proper information and incentive measures. Baker [5] studied the interaction between clear contracts and subtle ones. Tunay I. Tunca [6] analyzed the difference and gained the interaction mechanism between the two kinds of contracts based on Baker's research. Dyrer and Chu [7] indicated Relational Contract can effectively reduce the possibility of supply chain members adopting opportunistic behavior, enhance mutual trust between the members, thereby improve information sharing and reduce asymmetric information by analyzing the drawbacks of opportunistic decision behavior. Sun Yuanxin and in Mao recommended [8] summarized the main features of relational contract, including embedded relation, self-compliance, long-term time, the openness of terms, and noted that the safeguards of Relational Contract implementation: value of future cooperation, reputation and relationship rules. In addition, existing researches will also be summarized as follows: Relational Contract as an alternative of formal ones, Governance of Relational Contract, the use of relational contract in incentives. There are more researches on Relational Contracts, such as Zhao Pinghan [9], Li Ying [10], Diana Yan Wu [11], Spengler [12], Kannan Govindan [13] and so on.

The commitment contract is a typical relational contract. Helper [14], Dyers and Ouchi [15], Helper and Sako [16] et al compared business strategies between the local enterprises and foreign-funded ones in the study and found that the biggest difference is the different commitment quantity and information sharing amount. Obviously they both are related to the trust degree between each other. Studies have shown that the larger commitment quantity and information shared between supply chain members, the lower uncertainties and costs of the entire supply chain will be. Applying its conclusion to the real world, the manufacturer will reduce the supply quantity to wholesalers to some extent in order to increase its commitment amount for wholesalers as much as possible without exceeding its production capacity. On the other hand, the wholesaler will improve its degree of information sharing to exchange. Durango-Cohen and Yano [17] studied a commitment contract between ASIC manufacturers and customers, in which the manufacturer will commit a minimum supply quantity to a customer, when he provide a demand forecast and commit to buy some of them at least. Similar commitment contracts are also described in Tsay [18], Tsay and Lovejoy [19]. Eppen and Iyer [20] have proposed a commitment contract on poor market, in which wholesalers have the right to pay only a part of its order quantity in advance when ordering and then choose to buy the remainder or not. Bassok [21], Bassok and Anupindi [22], Anupindi and Bassok [23] put forward some other commitment contracts in which the wholesalers updating the order quantity once again is allowed. The application avoids the possibility of wholesalers exaggerating the market demand deliberately to a certain extent when sharing information and too low production of manufacturers.

Based on the uncertainty theory, market demand information updating as the background, this paper study the coordination and optimization problem of a two-stage three-phase supply chain system with two wholesalers and a manufacturer. With assumption that semi-symmetric market information and risk-neutral participants, in the situation that the manufacturer and two wholesalers have two pre-season decision-making opportunities and retailers can replenish in the sales season, this paper brings in the supply contract commitments. To exchange updating information of the market that the manufacturer itself can't obtain directly, the manufacturer will commit to the wholesalers the minimum supply in the pre-season. However, to have more replenishment inventory, wholesalers will exaggerate market updates to a certain extent. And they have competition relationship or mutually reinforcing relationship. Different effort levels lead to different sales results, and then different order quantity. According to this contract, we establish the optimization model respectively, and get the optimal strategy of supply chain members by analyzing the supply chain system. Finally, by giving a numerical example, the conclusions are more reasonable.

2. Model Description

Definition 1 [Liu1]: The uncertainty distribution Φ of an uncertain variable ξ is defined by $\Phi(x) = \mathrm{M}\{\xi \le x\}$ for any real number x.

Definition 2 [Liu1]: An uncertain variable ξ is called normal if it has a normal uncertainty distribution $\Phi(x) = (1+\exp(\frac{\pi(e-x)}{\sqrt{3}\sigma}))^{-1}$ $x \in R$ .Denoted by $N(e,\sigma)$ where e and σ are real numbers with $\sigma \succ 0$.

Definition 3 [Liu1]: Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} M\{\xi \ge x\}dx - \int_{-\infty}^{0} M\{\xi \le x\}dx \tag{1}$$

provided that at least one of the two integrals is finite.

Theorem 1 [Liu1]: Let ξ be an uncertain variable with uncertainty distribution Φ . Then

$$E[\xi] = \int_0^{+\infty} (1-\Phi(x))dx - \int_{-\infty}^{0} \Phi(x)dx \tag{2}$$

The newsboy model, a production enterprise with unit production costs C , unit retail price P (P>C), D is random market demands that policy makers facing. $F(x)$ and $f(x)$ note that uncertain distribution function and uncertain density function of the uncertain demand respectively. So the optimization model that makers use to choose the optimal inventory Q is

$$Y(Q) = P\cdot E\min(D,Q) - CQ = (P-C)Q - P\int_0^Q F(x)dx \tag{3}$$

Calculating its derivative, we get

$$\frac{\partial Y(Q)}{\partial Q} = (P-C) - P\cdot F(Q) \tag{4}$$

$$\frac{\partial^2 Y(Q)}{\partial Q^2} = -f(Q) < 0$$

It means that $Y(Q)$ is concave function of Q . Let $\frac{\partial Y(Q)}{\partial Q} = 0$, we get $Q_0 = F^{-1}\left({(P-C)}/{P}\right)$ is the optimal order quantity that decision makers choose, and the final optimal profits of decision makers is

$$Y(Q_0) = (P-C)Q_0 - P\int_0^{Q_0} F(x)dx \tag{5}$$

In the situation that information of market demand is uncertain and updating, as manufacturers can't contact with the market, they get the updated information only through information sharing with wholesalers. Note x_{e1} and x_{e2} as the market updates wholesalers 1, 2 get. $\overline{x_{e1}}$ and $\overline{x_{e2}}$ are that they provide to manufacturers, so we have $\overline{x_{e1}} \ge x_{e1}$, $\overline{x_{e2}} \ge x_{e2}$.

Phase 1: Manufacturers, wholesaler1 and wholesaler2 confirm uncertainty distribution of market demand with years of experience in sales and expert assessment, denoting as $f_s(x)$, $f_{r1}(x)$ and $f_{r2}(x)$ respectively, and we have $f_s(x) = f_{r1}(x) + f_{r2}(x)$. The two wholesalers issued the first order to the manufacturer, noting the order quantity as d_1^1 and d_1^2 respectively. Manufacturers receive this order to determine the first time production q_1 .

Phase 2: With the sales season approaching, wholesaler 1 and wholesaler 2 get the market updates x_e^1 and x_e^2 .They revise the order quantity $d^1\left(d^1 \ge d_1^1\right)$ and $d^2\left(d^2 \ge d_1^2\right)$ according to $f_{r1}\left(x \middle| x_e^1\right)$, $f_{r2}\left(x \middle| x_e^2\right)$, d_1^1 and d_1^2 , and provide the sharing information $\overline{x_{e1}}$ and $\overline{x_{e2}}$ to manufacturers at the same time. Manufacturers revise the production quantity $q(q \ge q_1)$ ($q = q^1 + q^2$), according to $f_s\left(x \middle| \overline{x_{e1}}, \overline{x_{e2}}\right)$, d^1 , d^2 and q_1 ($q_1 = q_1^1 + q_1^2$). And as the permutation for information sharing, they provide the lowest supply quantity C_1 and C_2 to wholesaler 1 and wholesaler 2.If the quantity that committed $C_1 < \overline{x_{e1}}$ or $C_2 < \overline{x_{e2}}$, give producers punishment π_1 or π_2 , that is called loyalty punishment. Complementary, if manufacturers and wholesalers have cooperated for many years and have a good foundation of trust and loyalty, that is, manufacturers trust $\overline{x_{e1}}$ and $\overline{x_{e2}}$ in a high level and wholesalers don't exaggerate x_e^1 and x_e^2 (Differences between x_e^1, x_e^2 and $\overline{x_{e1}}$, $\overline{x_{e2}}$ are small respectively.), which satisfy $d^1 \geq \alpha 1 \cdot \overline{x_{e1}} (\alpha 1 > 0)$ and $d^2 \geq \alpha 2 \cdot \overline{x_{e2}} (\alpha 2 > 0)$.

Phase 3: With the sales season starting, we can get the market demand x_1 and x_2, wholesaler 1 and wholesaler 2 selectively issue replenishment orders to manufacturers to receive the final order quantity according to the total order quantity d^1, d^2 and demand x_1, x_2. As a manufacturer ,they can only mechanically do their ability to distribute the surplus goods to wholesalers 1, 2 as complement goods. If the market needs are not met, give manufacturers and wholesalers 1, 2 s punishment considering loss from the customer, loss of reputation and so on. It is called shortage punishment.

Known from the above analysis, for a manufacturer, the quantity he committed must be greater than the wholesaler 1 and wholesaler 2' order amount in the first two stages, that is, $C1 \geq d^1$ and $C2 \geq d^2$. Taking 3 order opportunities of two wholesalers into consideration, after the sales seasons beginning, the order quantity wholesaler 1 and wholesaler 2 expected are $d^{1*} = \max\left(d^1, x_1\right)$ and $d^{2*} = \max\left(d^2, x_2\right)$ respectively. However, because of manufacturers' limited production, the quantity is $\min\left(d^{1*}, q^1\right)$ and $\min\left(d^{2*}, q^2\right)$ in fact, in which we have $q^1 + q^2 = q$. In addition, to be fair, shortage punishment of the contract allocates the responsibility in detail. Wholesaler 1 and wholesaler 2 shoulder the responsibility $s \cdot \left[\left(x_1 - q^1\right)^+ - \left(\min\left(x_1, C_1\right) - q^1\right)^+\right]$ and $s \cdot \left[\left(x_2 - q^2\right)^+ - \left(\min\left(x_2, C_2\right) - q^2\right)^+\right]$ respectively that caused by understanding updates insufficiently. That caused by limited production are $s \cdot \left(\min\left(x1, C1\right) - q^1\right)^+$ and $s \cdot \left(\min\left(x2, C2\right) - q^2\right)^+$ which is assumed by manufacturers.

Wholesalers take initiative relatively in the supply chain system and manufacturers make decisions due to the behavior of wholesalers. We assume that wholesalers know the manufacturers' different responses to different $\overline{x_{1e}}$ and $\overline{x_{2e}}$, although manufacturers do not know what market information x_{1e} and x_{2e} wholesaler 1 and wholesaler 2 obtained are and how wholesalers 1, 2 choose $\overline{x_{1e}}$ and $\overline{x_{2e}}$ finally, in the context of asymmetric information. That is, $f_S\left(x1 \middle| \overline{x_{1e}}\right)$ and $f_S\left(x2 \middle| \overline{x_{2e}}\right)$ are known for wholesalers 1, 2.

3. The Optimization Model of Manufacturers Based on That Commitment Contract with Demand Information Updating

We assume that q^i is the production and supply quantity of the manufacturer, d^i is the order quantity of wholesalers, w_i is the wholesale price and c_i is the product cost. Known from a series of analysis above, we should set about from manufacturers. Obviously the order quantities were known in the two production decisions, so $q_1^1 \geq d_1^1, q_1^2 \geq d_1^2$ and $q \geq d$ ($q = q^1 + q^2$ and $d = d^1 + d^2$). The total profit of manufacturers is noted as Π_s :

$$\begin{aligned}\Pi_S = {} & w_1(d_1^1 + d_1^2) + w_2(d_2^1 + d_2^2) + w_3\left[\min\left(q^1, \max\left(x1, d^1\right)\right) - d^1\right]^+ - c_1(q_1^1 + q_1^2) - c_2\left(q - q_1^1 - q_1^2\right) \\ & -\pi_1\left(\overline{x_{e1}} - C1\right)^+ - s\left(\min\left(x1, C1\right) - q^1\right)^+ + w_3\left[\min\left(q^2, \max\left(x2, d^2\right)\right) - d^2\right]^+ - \pi_2\left(\overline{x_{e2}} - C2\right)^+ - s\left(\min\left(x2, C2\right) - q^2\right)^+\end{aligned} \tag{6}$$

It should be noted that manufacturers need not make any decisions in phase III considering the collaboration process in the entire supply chain. Decomposing the expression by the stages of production decision and combining stage II and stage III, we can get the optimization model of manufacturer's expected profits in this situation:

$$\Pi_S^*\left(q_1^1, q_1^2\right) = \left(c_2 - c_1\right)(q_1^1 + q_1^2) + \int_0^{+\infty} \Pi_{s1}^{**}\left(q_1^1, \overline{x_{e1}}\right)\phi_0\left(\overline{x_{e1}}\right) d\overline{x_{e1}} + \int_0^{+\infty} \Pi_{s2}^{**}\left(q_1^2, \overline{x_{e2}}\right)\phi_0\left(\overline{x_{e2}}\right) d\overline{x_{e2}} \tag{7}$$

In which,

$$\Pi_{s1}^{**}\left(q_1^1, \overline{x_{e1}}\right) = \max_{q^1 \geq \max\left(q_1^1, d^1\right), C1 \geq d^1} \Psi_{s1}\left(C1, q^1, q_1^1, \overline{x_{e1}}\right) \tag{8}$$

$$\Pi_{s2}^{**}\left(q_1^2,\overline{x_{e2}}\right)=\max_{q^2\ge\max\left(q_1^2,d^2\right),C2\ge d^2}\Psi_{s2}\left(C2,q^2,q_1^2,\overline{x_{e2}}\right) \tag{9}$$

$$\Psi_{s1}\left(C1,q^1,q_1^1,\overline{x_{e1}}\right)=w_1d_1^1+w_2d_2^1+w_3\left[\min\left(q^1,\max\left(x1,d^1\right)\right)-d^1\right]^+-c_2q^1-\pi_1\left(\overline{x_{e1}}-C1\right)^+-s\left(\min\left(x1,C1\right)-q^1\right)^+ \tag{10}$$

$$\begin{aligned}\Psi_{s2}\left(C2,q^2,q_1^2,\overline{x_{e2}}\right)=&w_1d_1^2+w_2d_2^2+w_3\left[\min\left(q^2,\max\left(x2,d^2\right)\right)-d^2\right]^+-c_2q^2\\&-\pi_2\left(\overline{x_{e2}}-C2\right)^+-s\left(\min\left(x2,C2\right)-q^2\right)^+\end{aligned} \tag{11}$$

We can analyze the decision problem in view of the stage II and solve the profit of the manufacturer from the first wholesaler $\Psi_{s1}\left(C1,q^1,q_1^1,\overline{x_{e1}}\right)$ (here Ψ_{s1} is abbreviated as the nesting issue on C_1 and q^1) and that from the other wholesaler $\Psi_{s2}\left(C2,q^2,q_1^2,\overline{x_{e2}}\right)$ from it. Therefore, assuming that q^1 and q^2 are certain when solving, we analyze the optimal amount $C1$ and $C2$ committed.

Theorem 2: In the decision problem $\Psi_{s1}\left(C1,q^1,q_1^1,\overline{x_{e1}}\right)$ of stage II, the manufacturer optimum and minimum commitment supply quantity to the first wholesaler C is same with either the production or the amount of updating information. That is, $C_1^*=q^1$ or $C_1^*=\overline{x_{e1}}$.

Proof: Reducing decision problem:

In the decision problem Ψ_{s1}, because $w_1d_1^1+w_2d_2^1+w_3\left[\min\left(q^1,\max\left(x_1,d^1\right)\right)-d^1\right]^+-c_2q^1$ has nothing to do with C_1, we note it as $K1$ and get $\Psi_{s1}=K1-\pi_1\left(\overline{x_{e1}}-C1\right)^+-s\left(\min\left(x1,C1\right)-q^1\right)^+$.

Discussing different circumstances:

a) When $q^1\le\overline{x_{e1}}$, that is $\alpha\overline{x_{e1}}\le d^1\le q^1\le\overline{x_{e1}}$.

(i) When $d^1\le C1\le q^1$, from Ψ_{s1} we can get that manufacturers do not need to bear shortage punishment. That is, the third term in the expression of Ψ_{s1} is zero. So we have $\Psi_{s1}=K1-\pi_1\left(\overline{x_{e1}}-C1\right)=K1-\pi_1\overline{x_{e1}}+\pi_1C1$. Obviously Ψ_{s1} increases monotonously with committed amount $C1$. So we have $C_1^*=q^1$.

(ii)When $q^1\le C1\le\overline{x_{e1}}$, $\Psi_{s1}=K1-\pi_1\overline{x_{e1}}+\left(\pi_1-s\right)C1+s\int_{q^1}^{C1}F_s\left(x1\middle|\overline{x_{e1}}\right)dx1+sq^1$. Known from nature of the condition uncertain distribution function, $F_s(x1|\overline{x_{e1}})=0$. Therefore, in order to maximize Ψ_{s1}, $C1$ should be taken at the endpoints: $C_1^*=q^1$ ($\pi_1<s$) or $C_1^*=\overline{x_{e1}}$ ($\pi_1\ge s$).

(iii) When $\overline{x_{e1}}\le C_1$, from Ψ_{s1} we can get that manufacturers do not need to bear loyalty punishment. That is, the second term in the expression of Ψ_{s1} is zero. So we have $\Psi_{s1}=K_1-sC_1+s\int_{q^1}^{C_1}F_s\left(x_1\middle|\overline{x_{e1}}\right)dx_1+sq^1$. In the same way, we take the derivative of C_1. Then we get

$$\frac{\partial\Psi_{s1}}{\partial C1}=\begin{cases}-s+s\left(\dfrac{F_s(C1)}{1-F_s(\overline{x_{e1}})}\wedge 0.5\right), if\ F_s(\overline{x_{e1}})\prec F_s(C1)\le\dfrac{(1+F_s(\overline{x_{e1}}))}{2}\\-s+s\left(\dfrac{F_s(C1)-F_s(\overline{x_{e1}})}{1-F_s(\overline{x_{e1}})}\right), if\ \dfrac{(1+F_s(\overline{x_{e1}}))}{2}\le F_s(C1)\end{cases}\le 0 \tag{12}$$

That is, Ψ_s decreases monotonously with C_1, so $C_1^*=\overline{x_{e1}}$.

b) When $q^1\ge\overline{x_{e1}}$, we have $\alpha\overline{x_{e1}}\le d^1\le\overline{x_{e1}}\le q^1$.

In this case, since the total production has exceeded the amount of updating information $\overline{x_{e1}}$, for manufacturers it is not necessary to increase the loyalty punishment. That is, $C_1\ge\overline{x_{e1}}$ and the second term in the expression of Ψ_s is zero.

(i) When $\overline{x_{e1}}\le C_1\le q^1$, the manufacturer does not need to bear shortage punishment, so $\Psi_{s1}=K_1$ is a constant about C_1. So C_1 can take any value in the interval $\left(\overline{x_{e1}},q^1\right)$.

(ii) When $q^1\le C_1$, it is same with (iii) of a) and we get $C_1^*=\overline{x_{e1}}$.

To sum up, the manufacturers' the most optimal minimum commitment supply quantity satisfies $C_1^*=q^1$ or $C_1^*=\overline{x_{e1}}$.

Similarly, in the decision problem $\Psi_{s2}\left(C2,q^2,q_1^2,\overline{x_{e2}}\right)$

of stage II, the manufacturer optimum and minimum commitment supply quantity to the second wholesaler C_2 is same with either the production or the amount of updating information. That is, $C_2^* = q^2$ or $C_2^* = \overline{x_{e2}}$.

Thus in the decision problem of stage II, the manufacturer optimum and minimum commitment supply quantity to the two wholesalers C ($C = C1 + C2$) is $C^* = q$ ($q = q^1 + q^2$), $C^* = \overline{x_e}$ ($\overline{x_e} = \overline{x_{e1}} + \overline{x_{e2}}$), $C^* = q^1 + \overline{x_{e2}}$ or $C^* = q^2 + \overline{x_{e1}}$.

Theorem 3: For the first wholesaler, in the second stage of the decision problem $\Psi_{s1}\left(C1, q^1, q_1^1, \overline{x_{e1}}\right)$, the optimal production q^{1*} that the manufacturer selected has the following expression

$$q^{1*} = \arg\max_{\left(C_1, q^1\right) \in \mathrm{I}_s} \Psi_{s1}\left(C1, q^1\right) \tag{13}$$

In which $\mathrm{I}_S = \left\{(C1, q^1) \middle| 0 \le C1 < +\infty, 0 \le q^1 < +\infty\right\}$, so we have $q^{1*} = \overline{x_{e1}}$, $q^{1*} = Ex_1$ or q_A^{1*}. Here q_A^{1*} is decided by $F_S(q^1 | \overline{x_{e1}}) = \dfrac{w_3 - c_2}{w_3}$.

Proof: Compute the expectation of $\Psi_{s1}\left(C1, q^1\right)$ and we get

$$\begin{aligned} E\Psi_{s1}\left(C1, q^1, q_1^1, \overline{x_{e1}}\right) &= w_1 d_1^1 + w_2 d_2^1 + w_3 E\left[\min\left(q^1, \max\left(x1, d^1\right)\right) - d^1\right]^+ - c_2 q^1 - \pi_1\left(\overline{x_{e1}} - C1\right)^+ - sE\left(\min\left(x1, C1\right) - q^1\right)^+ \\ &= w_1 d_1^1 + w_2 d_2^1 - w_3 d^1 + \left(w_3 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 - \pi_1\left(\overline{x_{e1}} - C1\right)^+ - sE\left(\min\left(x1, C1\right) - q^1\right)^+ \end{aligned} \tag{14}$$

Here $w_1 d_1^1 + w_2 d_2^1 - w_3 d^1$ is independent of $C1$ and q^1. Write it as L_1. So we obtain

$$E\Psi_{s1} = L1 + \left(w_3 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 - \pi_1\left(\overline{x_{e1}} - C1\right)^+ - sE\left(\min\left(x1, C1\right) - q^1\right)^+ \tag{15}$$

Discussing different circumstances:

a. When $C^{1*} = q^1$, the manufacturer choosing to avoid shortage punishment, we have

$$\Psi_{s1} = L1 + \left(w_3 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 - \pi_1\left(\overline{x_{e1}} - q^1\right)^+ \tag{16}$$

(i) When $d^1 \le q^1 \le \overline{x_{e1}}$, we obtain

$$\Psi_{s1} = L1 + \left(w_3 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 - \pi_1\left(\overline{x_{e1}} - q^1\right)^+ = L1 + \left(w_3 + \pi_1 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 - \pi_1 \overline{x_{e1}} \tag{17}$$

From the nature of condition uncertain distributed function, we know $F_S\left(x1 \middle| \overline{x_{e1}}\right) = 0$. Therefore, only let $q^{1*} = \overline{x_{e1}}$ in order to maximize Ψ_{s1}. So $q^{1*} = \overline{x_{e1}}$.

(ii) When $\overline{x_{e1}} \le q^1$, the manufacturer also choosing to avoid loyalty punishment, at this moment we have

$$\Psi_{s1} = L1 + \left(w_3 - c_2\right) q^1 - w_3 \int_{d^1}^{q^1} F_S\left(x1 \middle| \overline{x_{e1}}\right) dx1 \tag{18}$$

As

$$F_S\left(x1 \middle| \overline{x_{e1}}\right) = \begin{cases} 0, if\ F_S(x1) \le F_S(\overline{x_{e1}}) \\ \dfrac{F_S(x1)}{1 - F_S(\overline{x_{e1}})} \wedge 0.5, if\ F_S(\overline{x_{e1}}) \prec F_S(x1) \le {(1 + F_S(\overline{x_{e1}}))}/{2} \\ \dfrac{F_S(x1) - F_S(\overline{x_{e1}})}{1 - F_S(\overline{x_{e1}})}, if\ {(1 + F_S(\overline{x_{e1}}))}/{2} \le F_S(x1) \end{cases} \tag{19}$$

So only let $F_s(q^1|\overline{x_{e1}})=\frac{w3-c2}{w3}$, we can maximize Ψ_{s1}. Thus we can determine the value of q_A^{1*}.

When $C^{1*}=q^1$, the optimal strategy are $\left(\overline{x_{e1}},\overline{x_{e1}}\right)$ and (q_A^{1*},q_A^{1*}).

b. When $C^{1*}=\overline{x_{e1}}$, the manufacturer choosing to avoid loyalty punishment, at this time we have

$$\Psi_{s1}=L1+\left(w_3-c_2\right)q^1-w_3\int_{d^1}^{q^1}F_s\left(x1\middle|\overline{x_{e1}}\right)dx1-s\left(\min(x1,C1)-q^1\right)^+ \quad (20)$$

(i) When $d^1\le q^1\le\overline{x_{e1}}$, as $F_s\left(x_1\middle|\overline{x_{e1}}\right)=0$, we obtain $q^{1*}=\overline{x_{e1}}$ or $q^{1*}=Ex_1$ in order to maximize Ψ_s;

(ii) When $\overline{x_{e1}}\le q^1$, we get $F_s(q^1|\overline{x_{e1}})=\frac{w3-c2}{w3}$ to maximize Ψ_s, at this time the production quantity is q_A^{1*}.

To sun up, for the first wholesaler, the decision problem in the second stage has 4 possible optimal strategies, noting as $I_{s1}^*=\left\{(\overline{x_{e1}},\overline{x_{e1}}),(\overline{x_{e1}},q_A^{1*}),(q_A^{1*},q_A^{1*}),(\overline{x_{e1}},Ex1)\right\}$.

Similarly, for the second wholesaler, the optimal production quantity that the manufacturer selected is q^{2*} and we obtain $q^{2*}=\overline{x_{e2}}$, $q^{2*}=Ex_2$ or q_A^{2*}. Here q_A^{2*} is decided by $F_s(q^2|\overline{x_{e2}})=\frac{w3-c2}{w3}$. On the other hand, for the second wholesaler, the decision problem in the second stage has 4 possible optimal strategy, noting as $I_{s2}^*=\left\{(\overline{x_{e2}},\overline{x_{e2}}),(\overline{x_{e2}},q_A^{2*}),(q_A^{2*},q_A^{2*}),(\overline{x_{e2}},Ex2)\right\}$.

Integrating above analysis, theorem 2 and theorem 3,at the beginning of stage II, the manufacturer obtain two decision programs with combining updating information $\overline{x_{e1}}$ and $\overline{x_{e2}}$ and the wholesaler's total order quantity d^1 and d^2 to the two wholesalers. Finally, compare the two decisions to obtain the more optimal one. Here we must pay attention that the second revised total production q^1 and q^2 must satisfy $q^1\ge q_1^1$ and $q^2\ge q_1^2$. We use q_1^1 instead of q^{1*} or use q_1^2 instead of q^{2*}, if the two decision program we choose don't satisfy $q^{1*}\ge q_1^1$ or $q^{2*}\ge q_1^2$, and then compare them in the end.

4. Optimization Model of Wholesalers with Demand Information Updates and Effort Level Based on the Commitment Contract

Relative to the manufacturers, wholesalers 1, 2 have been in a dominant position throughout the entire supply chain operation process. They take the initiate to carry on two pre-season orders, obtain the market information directly, selectively share it with the manufacturer, and understand the manufacturer's response to update information. That is, they know how the manufacturer corrects the demand distribution function $f\left(x_1\middle|\overline{x_{e1}}\right)$ and $f\left(x_2\middle|\overline{x_{e2}}\right)$ according to updated information. At the same time, these two wholesalers are affected by both their own efforts and competition relationship or mutual promoting relationship with each. Though further analysis, we can get

$$\Pi_{r1}=p\min\left(x1,q^1\right)-w_1d_1^1-w_2d_2^1-w_3\left[\min\left(q^1,\max\left(x1,d^1\right)\right)-d^1\right]^+-s\left[\left(x1-q^1\right)^+-\left(\min(x1,C1)-q^1\right)^+\right] \quad (21)$$

$$\Pi_{r2}=p\min\left(x2,q^2\right)-w_1d_1^2-w_2d_2^2-w_3\left[\min\left(q^2,\max\left(x2,d^2\right)\right)-d^2\right]^+-s\left[\left(x2-q^2\right)^+-\left(\min(x2,C2)-q^2\right)^+\right] \quad (22)$$

Here Π_{r1} and Π_{r2} notes gross profit of the two wholesalers respectively and p notes the selling price. As a wholesaler, while having a larger initiative, but he has no room to make a decision in the third stage replenishment again and can only compensate the shortage possible mechanically. So explode the expression by the production phase, we get the following optimization model of wholesaler 1 and wholesaler 2:

$$\Pi_{r1}^1\left(d_1^1,e_1,e_2\right)=\left(w_2-w_1\right)d_1^1+\int_0^{+\infty}\Pi_{r1}^2\left(d_1^1,x_{e1},e_1,e_2\right)\phi_0\left(x_{e1}\right)dx_{e1} \quad (23)$$

in which

$$\Pi_{r1}^2\left(d_1^1,x_{e1},e1,e2\right)=\max_{d^1\ge d_1^1,\overline{x_{e1}}\ge x_{e1}}\Psi_{r1}\left(d_1^1,d^1,x_{e1},\overline{x_{e1}},e1,e2\right). \quad (24)$$

$$\Psi_{r1}\left(d_1^1,d^1,x_{e1},\overline{x_{e1}},e1,e2\right)=p\min\left(x1,q^1\right)-w_2d^1-w_3\left[\min\left(q^1,\max\left(x1,d^1\right)\right)-d^1\right]^+-s\left[\left(x1-q^1\right)^+-\left(\min\left(x1,C1\right)-q^1\right)^+\right] \tag{25}$$

$$\Pi_{r2}^1\left(d_1^2,e1,e2\right)=\left(w_2-w_1\right)d_1^2+\int_0^{+\infty}\Pi_{r2}^2\left(d_1^2,x_{e2},e1,e2\right)\phi_0\left(x_{e2}\right)dx_{e2} \tag{26}$$

Here

$$\Pi_{r2}^2\left(d_1^2,x_{e2},e_1,e_2\right)=\max_{d^2\ge d_1^2,\overline{x_{e2}}\ge x_{e2}}\Psi_{r2}\left(d_1^2,d^2,x_{e2},\overline{x_{e2}},e_1,e_2\right) \tag{27}$$

$$\Psi_{r2}\left(d_1^2,d^2,x_{e2},\overline{x_{e2}},e1,e2\right)=p\min\left(x2,q^2\right)-w_2d^2-w_3\left[\min\left(q^2,\max\left(x1,d^2\right)\right)-d^2\right]^+-s\left[\left(x2-q^2\right)^+-\left(\min\left(x2,C2\right)-q^2\right)^+\right] \tag{28}$$

Here $\Psi_{r1}\left(d_1^1,d^1,x_{e1},\overline{x_{e1}},e_1,e_2\right)$ and $\Psi_{r2}\left(d_1^2,d^2,x_{e2},\overline{x_{e2}},e_1,e_2\right)$ note the decision model of the wholesaler 1 and wholesaler 2 in the second phase, whose decision variables are the pre-season order quantity d^1,d^2 and the sharing information $\overline{x_{e1}}$, $\overline{x_{e2}}$ ($\overline{x_{e1}}>d^1$ and $\overline{x_{e2}}>d^2$ are obvious.). e_1 and e_2 are effort level of the wholesalers 1, 2. Here it is emphasized that wholesalers fully understand manufacturers' reaction and information. That is, the manufacturer's decision variables and C^{1*}, C^{2*}, q^{1*} and q^{2*} can be expected by the wholesaler while sharing the information and deciding the order quantity.

Theorem 4: In the second stage, the optimal pre-season order quantity d^{1*} that the first wholesaler selected is

$d^{1*}=\arg\max_{d^1\in I_{1r}}\Psi_{r1}\left(d^1,\overline{x_{e1}},e_1,e_2\right)$, whose

$$I_{1r}=\{d_A^1,d_B^1,d_C^1,x_{e1}\}$$

$$d_A^1:F_{r1}\left(d_A^1|x_{e1},e_1,e_2\right)=\frac{p+s-w_2}{p+s},$$

$$d_B^1:F_{r1}\left(d_B^1|x_{e1},e_1,e_2\right)=\frac{w_3-w_2}{w_3},$$

$$d_C^1:F_{r1}\left(d_C^1|x_{e1},e_1,e_2\right)=\frac{p-w_2}{p}$$

Proof: Simplify the decision problem and get

$$\begin{aligned}\Psi_{r1}\left(d^1,\overline{x_{e1}}\right)&=p\min\left(x1,q^1\right)-w_2d^1-w_3\left[\min\left(q^1,\max\left(x1,d^1\right)\right)-d^1\right]^+-s\left[\left(x1-q^1\right)^+-\left(\min\left(x1,C1\right)-q^1\right)^+\right]\\&=\left(w_3-w_2\right)d^1+\left(p-w_3\right)\left(q^1-\int_0^{q^1}F_{r1}\left(x1|x_{e1},e1,e2\right)dx1\right)-w_3\int_0^{d^1}F_{r1}\left(x1|x_{e1},e1,e2\right)dx1-s\left[\left(x1-q^1\right)^+-\left(\min\left(x1,C1\right)-q^1\right)^+\right]\end{aligned} \tag{29}$$

It is necessary to pay attention to that Ψ_{r1} is related to d^1 directly and explicitly, but generate indirect hidden relationship with $\overline{x_{e1}}$ by influencing C_1 and q^1. Here we assume that wholesaler 1 have the ability to find the completion in theorem 2 and theorem 3, $C^{1*}=q^1$ or $C^{1*}=\overline{x_{e1}}$, through considering its strategy from the perspective of the manufacturer.

Discussing different circumstances:

When $C_1=q^1$,

(i) When $q^1=d^1$, we have

$$\begin{aligned}\Psi_{r1}&=\left(w_3-w_2\right)d^1+\left(p-w_3\right)\left(d^1-\int_0^{d^1}F_{r1}\left(x_1|x_{e1},e_1,e_2\right)dx_1\right)-w_3\int_0^{d^1}F_{r1}\left(x_1|x_{e1},e_1,e_2\right)dx_1-s\left[\left(x_1-d^1\right)^+-\left(\min\left(x_1,d^1\right)-d^1\right)^+\right]\\&=\left(p+s-w_2\right)d^1-\left(p+s\right)\int_0^{d^1}F_{r1}\left(x_1|x_{e1},e_1,e_2\right)dx_1-s\mathrm{E}\left(x_1|x_{e1},e_1,e_2\right)\end{aligned} \tag{30}$$

if $x_{e1}\le d^1$, we obtain,

$$\begin{aligned}\Psi_{r1}&=\left(p+s-w_2\right)d^1-\left(p+s\right)\int_0^{d^1}F_{r1}\left(x_1|x_{e1},e_1,e_2\right)dx_1-s\mathrm{E}\left(x_1|x_{e1},e_1,e_2\right)\\&=(p+s-w_2)d^1-(p+s)\int_0^{x_{e1}}F_{r1}(x_1|x_{e1},e_1,e_2)dx_1-(p+s)\int_{x_{e1}}^{d^1}F_{r1}(x_1|x_{e1},e_1,e_2)dx_1-sE(x_1|x_{e1},e_1,e_2)\end{aligned} \tag{31}$$

Computing its derivation to d^1, we can get

$$\frac{\partial \Psi_{r1}}{\partial d^1} = (p+s-w_2) - (p+s) F_{r1}\left(d^1 \middle| x_{e1}, e_1, e_2\right) \tag{32}$$

$$\frac{\partial^2 \Psi_{r1}}{\partial (d^1)^2} = -(p+s) f_{r1}\left(d^1 \middle| x_{e1}, e_1, e_2\right) \le 0 \tag{33}$$

In this case, Ψ_{r1} is a concave function on d^1. So at this moment wholesaler's optimal order quantity is $d_A^1 : F_{r1}\left(d_A^1 \middle| x_{e1}, e_1, e_2\right) = \dfrac{p+s-w_2}{p+s}$.

if $x_{e1} > d^1$,we obtain

$$\Psi_{r1} = \left(p+s-w_2\right) d^1 - (p+s) \int_0^{d^1} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1 - s\mathrm{E}\left(x_1 \middle| x_{e1}, e_1, e_2\right) = (p+s-w_2) d^1 - sE(x_1 | x_{e1}, e_1, e_2) \tag{34}$$

In this case, in order to maximize Ψ_{r1}, the more d^1 is the better. So we get $d^1 \to x_{e1}$.

(ii) When $q^1 \ne d^1$, we have

$$\begin{aligned}\Psi_{r1} = (w_3 - w_2) d^1 + (p - w_3)\left(q^1 - \int_0^{q^1} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1\right) - w_3 \int_0^{d^1} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1 \\ - s\left[\left(x_1 - q^1\right)^+ - \left(\min\left(x_1, q^1\right) - q^1\right)^+\right]\end{aligned} \tag{35}$$

Here the second term and the fourth one are independent of d^1.

if $x_{e1} \le d^1$, we have

$$\begin{aligned}\Psi_{r1} = \left(w_3 - w_2\right) d^1 + \left(p - w_3\right)\left(q^1 - \int_0^{q^1} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1\right) - w_3 \int_0^{x_{e1}} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1 \\ - w_3 \int_{x_{e1}}^{d^1} F_{r1}(x_1 | x_{e1}, e_1, e_2) dx_1 - s\left[\left(x_1 - q^1\right)^+ - \left(\min\left(x_1, q^1\right) - q^1\right)^+\right]\end{aligned} \tag{36}$$

And we can get further

$$\frac{\partial \Psi_{r1}}{\partial d^1} = (w_3 - w_2) - w_3 F_{r1}\left(d^1 \middle| x_{e1}, e_1, e_2\right) \tag{37}$$

and

$$\frac{\partial^2 \Psi_{r1}}{\partial (d^1)^2} = -w_3 f_{r1}\left(d^1 \middle| x_{e1}, e_1, e_2\right) \le 0 \tag{38}$$

In the similar way, we obtain the optimal decision is $d_B^1 : F_{r1}\left(d_B^1 \middle| x_{e1}, e_1, e_2\right) = \dfrac{w_3 - w_2}{w_3}$.

if $x_{e1} > d^1$, we have

$$\Psi_{r1} = \left(w_3 - w_2\right) d^1 + \left(p - w_3\right)\left(q^1 - \int_0^{q^1} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right) dx_1\right) - s\left[\left(x_1 - q^1\right)^+ - \left(\min\left(x_1, q^1\right) - q^1\right)^+\right] \tag{39}$$

At this moment, in order to maximize Ψ_{r1}, the more d^1 is the better. So we get $d^1 \to x_{e1}$.

When $C^1 = \overline{x_{e1}}$,

(i) When $q^1 = d^1$, that the manufacturer's decision policy is $\left(\overline{x_{e1}}, d^1\right)$, we have

$$\begin{aligned}\Psi_{r1} &= (w_3 - w_2)d^1 + (p - w_3)\left(d^1 - \int_0^{d^1} F_{r1}\left(x_1 | x_{e1}, e_1, e_2\right)dx_1\right) - w_3\int_0^{d^1} F_{r1}\left(x_1 | x_{e1}, e_1, e_2\right)dx_1 - s\left(x_1 - \overline{x_{e1}}\right)^+ \\ &= (p - w_2)d^1 - p\int_0^{d^1} F_{r1}\left(x_1 | x_{e1}, e_1, e_2\right)dx_1 - s\left(x_1 - \overline{x_{e1}}\right)^+\end{aligned} \tag{40}$$

whose the last term is independent of d^1.

(1) if $x_{e1} \le d^1$, we obtain

$$\Psi_{r1} = \left(p - w_2\right)d^1 - p\int_0^{x_{e1}} F_{r1}\left(x_1 \middle| x_{e1}, e_1, e_2\right)dx_1 - p\int_{x_{e1}}^{d^1} F_{r1}(x_1 | x_{e1}, e_1, e_2)dx_1 - s\left(x_1 - \overline{x_{e1}}\right)^+ \tag{41}$$

So we have $\dfrac{\partial \Psi_{r1}}{\partial d^1} = (p - w_2) - pF_{r1}\left(d^1 | x_{e1}, e_1, e_2\right)$ and $\dfrac{\partial^2 \Psi_{r1}}{\partial (d^1)^2} = -pf_{r1}\left(d^1 | x_{e1}, e_1, e_2\right) \le 0$. That is, Ψ_{r1} is a concave function on d^1 in this case. So at this moment we get $d_C^1 : F_{r1}\left(d_C^1 | x_{e1}, e_1, e_2\right) = \dfrac{p - w_2}{p}$ is the optimal decision of the wholesaler.

(2)if $x_{e1} > d^1$, we obtain

$$\Psi_{r1} = \left(p - w_2\right)d^1 - s\left(x_1 - \overline{x_{e1}}\right)^+ \tag{42}$$

At this moment, in order to maximize Ψ_{r1}, the more d^1 is the better. So we get $d^1 \to x_{e1}$.

(ii) When $q^1 \ne d^1$, it is similar with case (ii) of a). So we have the optimal decision is $d_B^1 : F_{r1}\left(d_B^1 | x_{e1}, e_1, e_2\right) = \dfrac{w_3 - w_2}{w_3}$ or $d^1 \to x_{e1}$.

Instruction: For the wholesaler 1, when choosing the optimal pre-season order quantity d^1 from $\mathrm{I}_{r1} = \{d_A^1, d_B^1, d_C^1, x_{e1}\}$ in the second stage, if d^1 we want to choose doesn't satisfy the pre-assumption $d^1 \ge d_1^1$,we use d_1^1 to compare and chose the optimal decision instead of d^1.

Similarly, in the second stage, the optimal pre-season order quantity d^{2*} that the second wholesaler selected is

$$d^{2*} = \arg\max_{d^2 \in \mathrm{I}_{2r}} \Psi_{r2}\left(d^2, \overline{x_{e2}}, e_1, e_2\right), \text{ whose}$$

$$\mathrm{I}_{2r} = \{d_A^2, d_B^2, d_C^2, x_{e2}\}$$

$$d_A^2 : F_{r2}\left(d_A^2 | x_{e2}, e_1, e_2\right) = \frac{p + s - w_2}{p + s},$$

$$d_B^2 : F_{r2}\left(d_B^2 | x_{e2}, e_1, e_2\right) = \frac{w_3 - w_2}{w_3},$$

$$d_C^2 : F_{r2}\left(d_C^2 | x_{e2}, e_1, e_2\right) = \frac{p - w_2}{p}$$

Uncertain distribution function: $F_{r1}\left(d_A^1 | x_{e1}, e_1, e_2\right)$, $F_{r1}\left(d_B^1 | x_{e1}, e_1, e_2\right)$, $F_{r1}\left(d_C^1 | x_{e1}, e_1, e_2\right)$, $F_{r2}\left(d_A^2 | x_{e2}, e_1, e_2\right)$, $F_{r2}\left(d_B^2 | x_{e2}, e_1, e_2\right)$, $F_{r2}\left(d_C^2 | x_{e2}, e_1, e_2\right)$ are concerned with both x_{e1}, x_{e2} and e_1, e_2. If the wholesalers 1 and wholesaler 2 are competition relationship, the uncertainty distribution function F_{r1} should be proportional to their own efforts e_1 and inversely proportional to the opponent's efforts e_2. Let's assume the function that $k_1 e^{\alpha 1 e 1} e^{-\alpha 2 e 2}$; similarly, the uncertainty distribution function F_{r2} should be proportional to their own efforts e_2 and inversely proportional to the opponent's efforts e_1 .Let's assume the function that $k_1 e^{\alpha 1 e 1} e^{-\alpha 2 e 2}$ .If the wholesalers 1 and wholesaler 2 are mutually reinforcing relationship, the uncertainty distribution function F_{r1} should be proportional to their own efforts e_1 and also proportional to the opponent's efforts e_2. Let's assume the function that $k_1 e^{\alpha 1 e 1} e^{\alpha 2 e 2}$; similarly, the uncertainty distribution function F_{r2} should be proportional to its own efforts e_2 and also proportional to the opponent's efforts e_1. Let's assume the function that $k_2 e^{\beta_2 e_2} e^{\beta_1 e_1}$. Therefore, the optimal order quantity of the wholesalers1,2 d_A^1, d_B^1, d_C^1 and d_A^2, d_B^2, d_C^2 are functions of effort level e_1 and e_2.

5. Optimization Analysis of Supply Chain Based on the Commitment Contract with Information Updates and Effort Level

From the analysis in above part 2 and part 3, we get that in order to achieve coordination, supply chain should have the following conclusions, when the commitment amount C satisfy certain conditions

Completion 1: Under the condition $C = q$ and $C_1 = q^1$, when $d \le q \le \overline{x_e}$, the supply chain could achieve coordination in the case of $q = d$ and $q^1 = d^1$, while it could not achieve coordination in the case of $q \ne d$. On the other hand, when $\overline{x_e} \le q$, the supply chain could achieve coordination in the case of q＝d and $q^1 = d^1$, while it could not achieve coordination in the case of q≠d

Proof: Under the condition C＝q and $C_1 = q^1$, when $d \le q \le \overline{x_e}$, the manufacturer's optimal production quantity to the first wholesaler is $q^{1*} = \overline{x_{e1}}$ or $q^{1*} = q_A^1$ known from above part 2 and when $q^1 = d^1$ and $x_{e1} \le d^1$, the first wholesaler's optimal order quantity is d_A^1 ($d_A^1 \le q^{1*}$) known from above part 3. At this time to achieve coordination of supply chain, we should have $d_A^1 = q^{1*}$. It need satisfy $q^{1*} = \overline{x_{e1}}$ or $\dfrac{p+s-w2}{p+s} = \dfrac{w3-c2}{w3}$. What we need to do is only to adjust p. At the same time, for the second wholesaler, we should have $d_A^2 = q^{2*}$ to achieve coordination of supply chain. It need satisfy $q^{2*} = \overline{x_{e2}}$ or $\dfrac{p+s-w2}{p+s} = \dfrac{w3-c2}{w3}$. In order to achieve it, we only need to adjust p. When q＝d and $x_e > d$, we get $q^1 = d^1$ and $x_{e1} > d^1$. The first wholesaler's optimal order quantity is $d^1 \to x_{e1}$ ($x_{e1} \le \overline{x_{e1}}$). We should have $x_{e1} = \overline{x_{e1}}$ to achieve coordination of supply chain. This condition can be achieved, so we can achieve supply chain coordination. In the case of $q \ne d$ and $x_e \le d$, the wholesalers' optimal order quantity are d_B^1, d_B^2 ($d_B^1 < q^{1*}$ and $d_B^2 < q^{2*}$). The wholesalers' optimal order quantity is less than optimal production of the manufacturer, so they can't be equal. Therefore the supply chain coordination can't be achieved. Similarly, in the case of q≠d and $x_e > d$, the optimal order quantity of the wholesaler 1 and wholesalers 2 are d^1 and d^2 ($d^1 < q^{1*}$ and $d^2 < q^{2*}$). The wholesaler's optimal order quantity is less than optimal production of the manufacturer, so they can't be equal. Therefore the supply chain coordination can't be achieved.

Completion 2: Under the condition $C^* = \overline{x_e}$ and $C^{1*} = \overline{x_{e1}}$, when $d \le q \le \overline{x_e}$, the supply chain could achieve coordination in the case of q＝d and $q^1 = d^1$, while it could not achieve coordination in the case of q≠d On the other hand, when $\overline{x_e} \le q$, the supply chain could achieve coordination in the case of q＝d and $q^1 = d^1$, while it could not achieve coordination in the case of q≠d

Proof: Under the condition $C^* = \overline{x_e}$ and $C^{1*} = \overline{x_{e1}}$ when $d \le q \le \overline{x_e}$, the manufacturer's optimal production quantity to the first wholesaler is $q^{1*} = \overline{x_{e1}}$, $q^{1*} = Ex_1$ or $q^{1*} = q_A^1$ known from above part 2 and when q＝d, $q^1 = d^1$ and $x_{e1} \le d^1$, the first wholesaler's optimal order quantity is d_C^1 ($d_C^1 \le q^{1*}$) known from above part 3. At this time to achieve coordination of supply chain, we should have $d_C^1 = q^{1*}$. It need satisfy $d^{1*} = \overline{x_{e1}}$, $d^{1*} = Ex_1$ or $\dfrac{p-w2}{p} = \dfrac{w3-c2}{w3}$. What we need to do is only to adjust p .There is a similar conclusion for the second wholesaler, and then achieve coordination of supply chain. When q＝d, $q^1 = d^1$ and $x_{e1} > d^1$, the first wholesaler's optimal order quantity is $d^1 \to x_{e1}$ ($x_{e1} \le \overline{x_{e1}}$). We should have $x_{e1} = \overline{x_{e1}}$ to achieve coordination of supply chain. This condition can be achieved, so we can achieve supply chain coordination. There is a similar conclusion for the second wholesaler, and then achieve coordination of supply chain. In the case of q≠d and $x_e \le d$, the two wholesalers' optimal order quantity are d_B^1 and d_B^2 ($d_B^1 < q^{1*}$ and $d_B^2 < q^{2*}$). The two wholesalers' optimal order quantity is less than optimal production of the manufacturer, so they can not be equal. Therefore the supply chain coordination can not be achieved. Similarly, in the case of q≠d and $x_e \succ d$, the two wholesalers' optimal order quantity are d^1 and d^2 ($d \prec q^*$). The two wholesalers' optimal order quantity is less than optimal production of the manufacturer, so they can not be equal. Therefore the supply chain coordination can not be achieved.

Known from the above two completion and above analysis, when $q = d$, the supply chain coordination can be achieved. Here because of the difference between uncertain distribution functions, the terms $w_3\left[\min\left(q^1, \max\left(x_1, d^1\right)\right) - d^1\right]$, $w_3\left[\min\left(q^2, \max\left(x_2, d^2\right)\right) - d^2\right]^+$ of the manufacturer's profit function are different from the terms $w_3\left[\min\left(q^1, \max\left(x_1, d^1\right)\right) - d^1\right]$, $w_3\left[\min\left(q^2, \max\left(x_2, d^2\right)\right) - d^2\right]^+$ of the wholesalers' profit function. To make them equal we must have $F_s(q|\overline{x_e}) = F_r(d|x_e, e_1, e_2)$. In this case, the gross profit of the supply chain system is Π_z.

In order to maximize the gross profit of the supply chain, we compute the partial derivative of Π_z for q^1 and q^2 respectively. Then we obtain

$$\Pi_z = p\min\left(x1,q^1\right)+p\min\left(x2,q^2\right)-c_1(q_1^1+q_1^2)-c_2\left(q-q_1^1-q_1^2\right)-\pi_1\left(\overline{x_{e1}}-C1\right)^+-s\left[\left(x1-q^1\right)^+\right]-s\left[\left(x2-q^2\right)^+\right]-\pi_2\left(\overline{x_{e2}}-C2\right)^+ \tag{43}$$

$$x1>q^1,q^1:F_z\left(q^1\middle|x_{e1},e1,e2\right)=\frac{p+s-c_2}{p+s}$$

$$x2>q^2,q^2:F_z\left(q^2\middle|x_{e2},e1,e2\right)=\frac{p+s-c_2}{p+s}$$

$$x1\le q^1,q^1:F_z\left(q^1\middle|x_{e1},e1,e2\right)=\frac{p-c_2}{p};\ x2\le q^2,$$
$$q^2:F_z\left(q^2\middle|x_{e2},e1,e2\right)=\frac{p-c_2}{p}$$

To make the most advantage of supply chain members to be same with that of the supply chain, we get $w_2=c_2$.

6. Exponential Analysis

We assume that before market information updates, the manufacturer and the first wholesaler analyze and estimate that the market demand obey the normal uncertainty distribution $x\sim \mathrm{N}(e,\sigma)$, in which $e=50,\sigma=5$. Let $p=200$, $c_1=40$, $c_2=60$, $c_3=160$, $w_1=130$, $w_2=140$, $w_3=190$, $\pi_1=10$, $s=70$, the first wholesaler update the information with the market demand $x_{el}=60$,which obey the uncertainty distribution $x_{r1}\sim \mathrm{N}(x_{el},\sigma)$. $k1=1/2,\alpha1=1,\alpha2=1,e1=2,e2=1$; The second wholesaler update the information with the market demand $x_{e2}=60$, which obey the uncertainty distribution $x_{r2}\sim \mathrm{N}(x_{e2},\sigma)$, $x_{e2}=60$. And the two wholesalers are competitive relationship.

Then

The wholesalers 1, 2 reflect to market information honestly. Here there are $x_{el}=\overline{x_{el}}$ and $x_{e2}=\overline{x_{e2}}$. As a manufacturer, $x_{sl}\sim \mathrm{N}(x_{el},\sigma)$, the optimal product to the first wholesaler is $q^{1*}=\overline{x_{el}}=x_{el}=60$ or $q^{1*}=q^1_A=64.1529$. In this case of competitive relationship, because the effort level of the first wholesaler is larger than that of the second one, the optimal order quantity of the first wholesaler is $d^1_A=64.299545$, $d^1_B=62.057224$, $d^1_C=62.37963$ and $d^1=60$. Corresponding optimal order quantity of the second wholesaler is $d^2_A=60.491298$, $d^2_B=60.26803$, $d^2_C=60.3056$ and $d^2=60$.

The wholesalers give larger $\overline{x_{el}}$ and $\overline{x_{e2}}$. At this moment, we assume that $x_{el}=50<\overline{x_{el}}=60$ and $x_{e2}=50<\overline{x_{e2}}=60$. As a manufacturer, $x_{s1}\sim \mathrm{N}(x_{el},\sigma)$ whose uncertain mean is 60 and uncertain variance is 5. In this case, the optimal production to the first wholesaler is $q^{1*}=\overline{x_{el}}=60$ or $q^{1*}=q^1_A=64.61529$. Corresponding optimal order quantity of the first wholesaler whose uncertain mean is 50 and uncertain variance is 5 are $d^1_A=54.299545$, $d^1_B=52.057224$, $d^1_C=52.37963$ and $d^1=50$;similarly, the optimal order quantity of the second wholesaler are $d^2_A=50.491298$, $d^2_B=50.26803$, $d^2_C=50.3056$ and $d^2=50$. In this case, some costs of the manufacturer will increase, such as the product cost and storage cost.

If the manufacturer does not consider new information the wholesaler provided, he still think $x_{e1}=50,\sigma=5$ and $x_{e2}=50,\sigma=5$.

As a manufacturer, $x_{s1}\sim \mathrm{N}(x_{el},\sigma)$ whose uncertain mean is 50 and uncertain variance is 5. In this case, the optimal production is $q^{1*}=50$ or $q^{1*}=q^1_A=54.61529$. Corresponding with the new information of competition, the uncertain distribution of the first wholesaler's optimal order quantity whose uncertain mean is 60 and uncertain variance is 5 is $d^1_A=64.299545$, $d^1_B=62.057224$, $d^1_C=62.37963$ and $d^1=60$. At this moment, shortage cost exists. Corresponding the uncertain distribution of the second wholesaler's optimal order quantity whose uncertain mean is 60 and uncertain variance is 5 is $d^2_A=60.491298$, $d^2_B=60.26803$, $d^2_C=60.3056$ and $d^2=60$. In this case, some costs will be generated, such as shortage cost.

Based on the competition relationship between the two wholesalers and assumption that the effect level of the first wholesaler is larger than that of the second one, comparing the three cases above, we get the first one is the optimal and its cost is the lowest. If the effect level of the second wholesaler is larger than that of the first one, we have similarly results. From above three cases, the first one is the most ideal with the lowest cost. The other two cases will appear larger expenses.

If the wholesaler 1 and wholesaler 2 are interdependence, mutual promotion, then for the first wholesaler $k2=3/4e^2,\alpha1=1,\alpha2=1,e1=2,e2=1$; corresponding for the second one, $k_2^*=3/4e^2,\alpha1=1,\alpha2=1,e1=2,e2=1$. Other assumptions are same with preamble, then we have

The wholesalers1,2 reflect to market information honestly. Here there are $x_{el}=\overline{x_{el}}$ and $x_{e2}=\overline{x_{e2}}$. As a manufacturer, $x_{s1}\sim \mathrm{N}(x_{el},\sigma)$, the optimal product to the first wholesaler is $q^{1*}=\overline{x_{el}}=x_{el}=60$ or $q^{1*}=q^1_A=64.1529$. In this case of mutual promotion relationship, because the effort level of the first wholesaler is larger than that of the second one, the optimal order quantity of the first wholesaler is $d^1_A=72.48606189$, $d^1_B=63.292674$, $d^1_C=63.90816$ and

$d^1 = 60$ .Corresponding optimal order quantity of the second wholesaler is $d_A^2 = 67.3503653$, $d_B^2 = 62.852141$, $d_C^2 = 63.348073$ and $d^2 = 60$.

The wholesalers give larger $\overline{x_{e1}}$ and $\overline{x_{e2}}$. At this moment, we assume that $x_{e1} = 50 < \overline{x_{e1}} = 60$ and $x_{e2} = 50 < \overline{x_{e2}} = 60$. As a manufacturer, $x_{s1} \sim N(x_{e1}, \sigma)$ whose uncertain mean is 60 and uncertain variance is 5. In this case, the optimal production to the first wholesaler is $q^{1*} = \overline{x_{e1}} = 60$ or $q^{1*} = q_A^1 = 64.61529$. Corresponding optimal order quantity of the first wholesaler in this case whose uncertain mean is 50 and uncertain variance is 5 are $d_A^1 = 62.48606189$, $d_B^1 = 53.292674$, $d_C^1 = 53.90816$ and $d^1 = 50$; similarly, the optimal order quantity of the second wholesaler are $d_A^2 = 57.3503653$, $d_B^2 = 52.852141$, $d_C^2 = 53.348073$ and $d^2 = 50$. At this moment, some costs of the manufacturer will increase, such as the product cost and storage cost.

If the manufacturer does not consider new information the wholesaler provided, he still think $x_{e1} = 50, \sigma = 5$ and $x_{e2} = 50, \sigma = 5$.

As a manufacturer, $x_{s1} \sim N(x_{e1}, \sigma)$ whose uncertain mean is 50 and uncertain variance is 5. In this case, the optimal production is $q^{1*} = 50$ or $q^{1*} = q_A^1 = 54.61529$. Corresponding with the new information of mutual promotion, the uncertain distribution of the first wholesaler's optimal order quantity whose uncertain mean is 60 and uncertain variance is 5 is $d_A^1 = 72.48606189$, $d_B^1 = 63.292674$, $d_C^1 = 63.90816$ and $d^1 = 60$. At this moment, shortage cost exists. Corresponding the uncertain distribution of the second wholesaler's optimal order quantity whose uncertain mean is 60 and uncertain variance is 5 is $d_A^2 = 67.3503653$, $d_B^2 = 62.852141$, $d_C^2 = 63.348073$ and $d^2 = 60$. In this case, some costs will be generated, such as shortage cost.

The following discussion is based on the mutual promotion relationship between the two wholesalers and assumption that the effect level of the first wholesaler is larger than that of the second one. If the effect level of the second wholesaler is larger than that of the first one, we have similarly results. From above three cases, the fourth one is the most ideal with the lowest cost. The other two cases will appear larger expenses.

In addition, if the uncertain distribution that the manufacturer obeys is in an unity form (x_{s1} and x_{s2} subject to the same uncertainty distribution), while two wholesalers subject to different uncertainty distributions, at this time there will be more storage cost and product cost or more shortage cost. Here we do not list specific examples.

7. Conclusion

Based on the uncertainty theory, this paper studied seasonal merchandise with long production cycle, relatively short marketing period, uncertain and strong volatile market demand. Setting about the perspective of market demand information updated, analyzes the coordination and optimization problem of a two-stage three-phase supply chain system with two risk-neutral wholesalers and a risk-neutral manufacturer. In the case that pure contract supply chain coordination can't be achieved, this paper brings in the commitment contract. We establish contract model aimed to the assumption that the manufacturer owns two pre-season production opportunities and wholesalers have two pre-season opportunities and an opportunity to replenish in the selling season. After analyzing and optimizing the model, we get optimal order quantity, optimal production volume, optimal promise and information sharing policy of the supply chain system when the wholesalers hold dominant position for competition or mutually reinforcing relationship between two wholesalers respectively. Of course, during the modeling process, the wholesale price of two wholesalers, cost price of the product and the selling price in the market that we consider here are same, but in fact they are different because different factors in different regions, such as income levels and consumer attitudes. It is just an attempt that applying the uncertainty theory to supply chain system in this paper. There may also be many problems, but after all, it has been applied to the practical problem. I believe it will be applied to analyze the supply chain system by more researchers.

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