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A semi-active isolator model for the nonlinear optimal control analysis of structures subjected to seismic excitations

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Abstract

The nonlinearity is an essential characteristic of structural materials, and there's luck indeed in the literature, when talking about nonlinear structures rested on active or semi-active base isolations. Except the electro-rheological base isolations, there are be developments of semi-active mechanical models even nonlinear with their resolution algorithms, which analyze the structures as ideally linear, these analyses make these algorithms and the results obtaining less efficiency. In this contribution, a nonlinear closed-loop optimal control algorithm and a Semi-Active Isolator (SAI) model are developed. The algorithm developed represents a discrete-time mathematical procedure for determining the optimal control matrix which allows to deducting the output optimal feedback. The combination of active, and passive variable stiffness and friction model developed, which in turn a combination of strings and frictions arranged in a certain manner, to reduce as possible the responses due to earthquakes in the two horizontal principal directions of the structures analyzed. As an application, a same seven floors uncontrolled, active-controlled and semi-active-controlled structure was studied according to the new developments. The illustrations shown below the responses and the acted forces and too, the considerable differences between them for uncontrolled, active-controlled and semi-active-controlled for the same structure studied.

1. Introduction

In fact that, the structural materials are not linearly elastic; but their behaviors in reality are nonlinear, and this essential characteristic shall being considered when analyzing these constructed with structures. The algorithms proposed in the literature [1-10,12-20] for the analyses of structures supported by active or semi-active base isolations, avoiding the nonlinearity of the real behavior of the structures, such analyses make the numerical studies made and the results obtained less effectively and efficiency.

The development of semi-active base isolations is being more and more interesting field of research in the earthquake engineering and anti-seismicity. The researchers and researches in this domain are growth since the beginning of this century, and of course that there are be tenths of semi-active base isolation models developed. Among these developments, Krishnamoorthy A. [10] had developed a

Variable Curvature Pendulum Isolator with the Viscous Fluid Damper, which represents a combination of a mechanical organ VCPI and the passive VFD. Lu L. Y. et Lin G. L. [13] proposed too, a combination of a mechanical Semi-Active Isolation System SAIS with the Variable Friction Damper VFD that combined some strings and dampers. Lu L. Y. et al. [12] also, expanded a Variable Stiffness Isolation System using a Leverage mechanism, that represent a combined leverage-string mechanical system, Nagarajaiah S. et al. [15] developed a model called the Semi-Active Independently Variable Stiffness SAIVS which is represent a combination of strings and dampers by such manner.

However, herein, a discrete-time algorithm for the resolution of the matrix differential equation of Riccati for the active closed-loop optimal control of nonlinear dynamic structures is proposed (the reader would see too, Latreche T. [11] for more details and another demonstrative way); this algorithm allows deducting numerically the optimal control output feedback for the nonlinear analysis of controlled structures according to the Non-Linear Quadratic Regulator method.

Moreover, the mechanical passive variable stiffness and friction proposed model, is composed of four strings and frictions arranged intelligently under the form of plus, such that it sturdiest more and more when the displacements and the velocities of the base increase and then by conclusion, allows to reduce more and more the semi-active structural responses in comparison with the increasing active intensities. The combination of this mechanical model with two perpendicular actuators posed according the two principal horizontal axes of the structure, which can realize in reality, to interestingly improve the resistance and reduce the drifts of the structures subjected to seismic excitations. A same seven floors-structure was analyzed with an active isolator, and with a semi-active model proposed. The results such that the responses versus time and the forces for the uncontrolled, active and semi-active controlled structure, show the great differences between the uncontrolled and controlled results, and the differences between the active and semi-active results in comparison with the results cited in the literature with the indicated proposed models. It is certain that active control is an interesting way for the structural responses reduction in comparison with the uncontrolled ones, and a good proposed variable stiffness and friction model which stands up behind of the more decreasing of the structural responses when the active ones increases.

2. The Closed-Loop Optimal Control Resolution Algorithm

In this section, we will present the algorithm that it shall follows by the computer, connected with the sensor of the ground motion accelerations, and the actuators connected by their role to the structural platform (the base of the structure) directly, or by means of the after proposed passive variable isolator according to the case, which it being active or semi-active, the mentioned computer following the steps of the present algorithm have then to compute, for every step of time, the closed-loop optimal control matrices and the output optimal forces that the actuators should be provoke to the base or platform of the structure or to the connected, to the structure, passive variable stiffness and friction organ.

The second order matrix equation of dynamic equilibrium of the structure to be analyzed is given by

$$M\ddot{U}(t) + C(t)\dot{U}(t) + K(t)U(t) = -M\Gamma a_g$$

$M, C(t), K(t)$ and Γ are respectively, the mass, damping, stiffness matrices of the structure and, the unity vector with the dimension of number of degrees of liberty of the structure, and a_g is the ground acceleration.

The state space formulation of this matrix dynamic equilibrium equation, of the controlled structure, would be expressed as

$$\dot{Z}(t) = A(t)Z(t) + Bf_e(t) + Bf_c(t) \quad (1)$$

$f_c(t)$ is the controlled force vector expressed after. $f_e(t), Z(t), A(t), B$ are respectively, the seismic vector force, the state space response vector, and state space matrices given by

$$f_e(t) = -\Gamma a_g \qquad Z(t) = \begin{Bmatrix} U(t) \\ \dot{U}(t) \end{Bmatrix}$$

$$A(t) = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B = \begin{Bmatrix} 0 \\ -M \end{Bmatrix}$$

Where $\boldsymbol{I}$ is the unity matrix.

Suppose that the optimal control matrix, for the closed-loop control, is expressed as following

$$P(t) = \lambda(t)Z^{-1}(t) \quad (2)$$

According to the mathematics, the derivatives of any inverse vector or matrix, should have the form

$$\dot{T}^{-1} = -T^{-1}\dot{T}T^{-1} \quad (3)$$

Thus, the derivative of $\boldsymbol{P(t)}$ with respect to t (Eqns. 2. and 3.) is given by the expression

$$\dot{P}(t) = \dot{\lambda}(t)Z^{-1}(t) - \lambda(t)Z^{-1}(t)\dot{Z}(t)Z^{-1}(t) \quad (4)$$

As known that, the Riccati equation is expressed

$$\dot{P}(t) = A^T(t)P(t) + P(t)A(t) - P(t)BR^{-1}B^TP(t) + Q(t) \quad (5)$$

Expression, with R and Q represent the weighting matrices. Replacing $P(t)$ by its expression (2) in equation (5), and standing up by equalizing the two expressions (4) and (5), one can obtain

$$\dot{\lambda}(t)Z^{-1}(t) - \lambda(t)Z^{-1}(t)\dot{Z}(t)Z^{-1}(t) =$$

$$A^T(t)\lambda(t)Z^{-1}(t) + \lambda(t)Z^{-1}(t)A(t) - \lambda(t)Z^{-1}(t)BR^{-1}B^T\lambda(t)Z^{-1}(t) + Q(t) \quad (6)$$

Thus, we can obtain, multiplying rightly by $Z(t)$, the two sides of the equation (6):

$$\dot{\lambda}(t) - \lambda(t)Z^{-1}(t)\dot{Z}(t) = A^T(t)\lambda(t) + \lambda(t)Z^{-1}(t)A(t)Z(t) - \lambda(t)Z^{-1}(t)BR^{-1}B^T\lambda(t) + Q(t)Z(t)$$

Then, it can rewrite this expression as

$$\dot{\lambda}(t) - \lambda(t)Z^{-1}(t)\{\dot{Z}(t)\} = A^T(t)\lambda(t) + Q(t)Z(t) - \lambda(t)Z^{-1}(t)\{BR^{-1}B^T\lambda(t) - A(t)Z(t)\}$$

And then, it can decompose this expression, in the two expressions

$$\dot{\lambda}(t) = A^T(t)\lambda(t) + Q(t)Z(t);$$

$$\dot{Z}(t) = BR^{-1}B^T\lambda(t) - A(t)Z(t)$$

Finally, these two expressions could have the matrix form

$$\begin{Bmatrix}\dot{\lambda}(t)\\ \dot{Z}(t)\end{Bmatrix} = \begin{bmatrix}A^T(t) & Q(t)\\ BR^{-1}B^T & -A(t)\end{bmatrix}\begin{Bmatrix}\lambda(t)\\ Z(t)\end{Bmatrix} \quad (7)$$

As it is known, from the derivation of the Hamiltonian function, that

$$f_c(t) = -R^{-1}B^T\lambda(t) \quad (8)$$

The expression (7), represent the matrix equation which it has to resolve for $P(t)$ for the closed-loop optimal control.

Suppose that,

$$\begin{bmatrix}A^T(t) & Q(t)\\ BR^{-1}B^T & -A(t)\end{bmatrix} = HM(t) \quad (9)$$

This matrix is called the Hamilton Matrix.

From (2), we have

$$\lambda(t) = P(t)Z(t)$$

Then, from (8), one can obtain

$$f_c(t) = -R^{-1}B^TP(t)Z(t) \quad (10)$$

Integrating the Equation (7) with respect to time, from t_k to t_{k+1}, we obtain

$$Ln\begin{Bmatrix}\lambda(t_{k+1})\\ Z(t_{k+1})\end{Bmatrix} - Ln\begin{Bmatrix}\lambda(t_k)\\ Z(t_k)\end{Bmatrix} = \int_{t_k}^{t_{k+1}} HM(t)dt \quad (11)$$

Suppose that $\Delta T = t_{k+1} - t_k$, a so small step of time for the raison that the Hamiltonian Matrix elements vary linearly.

Suppose that

$$HM(t_i) = HM_i \quad (12)$$

$$HM(t) = \left((HM_{k+1} - HM_k)t + (HM_k t_{k+1} - HM_{k+1}t_k)\right)/\Delta T \quad (13)$$

Integrating the expression (13) with respect to t (from t_k to t_{k+1}), and after simplification, then the equation (11) can be expressed as

$$Ln\begin{Bmatrix}\lambda(t_{k+1})\\ Z(t_{k+1})\end{Bmatrix} - Ln\begin{Bmatrix}\lambda(t_k)\\ Z(t_k)\end{Bmatrix} = (HM_{k+1} + HM_k)\Delta T/2 = X_k$$

Or, by elevate it to a power

$$\begin{Bmatrix}\lambda(t_{k+1})\\ Z(t_{k+1})\end{Bmatrix} = e^{(HM_{k+1}+HM_k)\Delta T/2}\begin{Bmatrix}\lambda(t_k)\\ Z(t_k)\end{Bmatrix} \quad (14)$$

The matrix $e^{(HM_{k+1}+HM_k)\Delta T/2}$ has the same dimensions like HM_i, then we can compute it by the Taylor series, as

$$XP_{k+1} = e^{(HM_{k+1}+HM_k)\Delta T/2} = e^{X_k} = \sum_{k=0}^{\infty}\frac{X_k^k}{k!} \quad (15)$$

We can suppose that $\lambda_k = \lambda(t_k)$ and $Z_k = Z(t_k)$, and we subdivide XP_{k+1} on four equal dimensions matrices, such as

$$XP_{k+1} = \begin{bmatrix}XP_{k+1}^{11} & XP_{k+1}^{12}\\ XP_{k+1}^{21} & XP_{k+1}^{22}\end{bmatrix} \quad (16)$$

As we suppose that $P = \lambda Z^{-1}$ (Eq. 2), then we can obtain for any step of time, using the subdivision of Eq. (19)

$$P_{k+1} = \left(XP_{k+1}^{11}\lambda_k + XP_{k+1}^{12}Z_k\right)\left(XP_{k+1}^{21}\lambda_k + XP_{k+1}^{22}Z_k\right)^{-1} \quad (17)$$

Multiplying rightly the nominator and the denominator of the right side of Eq. (20) by Z_k^{-1}, one would obtain then

$$P_{k+1} = (XP_{k+1}^{11}P_k + XP_{k+1}^{12})(XP_{k+1}^{21}P_k + XP_{k+1}^{22})^{-1} \quad (18)$$

For $k = 0$, we can obtain

$$P_1 = (XP_1^{11}P_0 + XP_1^{12})(XP_1^{21}P_0 + XP_1^{22})^{-1} \quad (19)$$

Such that P_0 represent the given initial condition at $k = 0$ (i.e. at $t = 0$).

The Equations (18) and (19) represent the discrete-time solution for the Nonlinear Matrix Differential Equation of Riccati, such that its discrete solution represents the closed-loop optimal control matrix. Note that, as we take a so small step of time ΔT, as the solution being more exact and accurate.

And such that,

$f_c(t) = -R^{-1}B^T\lambda(t)$ and,

$$\lambda(t) = P(t)Z(t)$$

After that determining of P_{k+1} according to the discrete-time algorithm presented, so it is possible to replacing $f_c(t)$ by its expression (10), putting $P(t) = P_{k+1}$ and resolving the system (1) for $Z_{k+1} = Z(t_{k+1})$ and with the seismic exterior vector force $f_{e_{k+1}} = f_e(t_{k+1})$. Then, one can get to the discrete expression of $f_c(t)$ as follows

$$f_{c,k+1} = -R^{-1}B^TP_{k+1}Z_{k+1} \quad (20)$$

This last expression (20) represents the vector of the optimal control force, applied to the total degrees of freedom for one axis of the two horizontal principal directions of the structure have someone to study, and the total force applied by the actuator posed under the base in this direction, represent the total sum of the elements of the vector f_c for any step of time.

3. The Developed Semi-Active Variable Stiffness and Friction Isolator (SAI)

The structure to be analyzed or constructed is supposed constructed on a platform (base), supported perfectly by its role on slide solid metal balls that allow such structure by means of its base to move horizontally freely reported to the ground, except the Semi-Active Isolator proposed which being the connection between. As shown by Figure 1., the two tubes of the two electromechanical actuators are fixed on a inner end metal vertical bar, which in turn fixed by its upper end on the central joint (Joint x-y) of the variable stiffness and friction isolator. The bodies of the actuators (that are connected to the ground acceleration sensors) are fixed to the ground by means of two rails that allow the actuators to slide freely in the perpendicular directions of their bodies and tubes. The Joint 1, 2, 3 and 4 represent articulations of the first edge of any of these joints allow the spring-friction tubes to turn freely and to get any horizontal directions. These articulations are fixed too, to the base of the structure. The Joint x-y represent an articulation for the last end of all of the four cited spring-friction tubes, and can move horizontally to any place of the X-Y square that contains the four tubes.

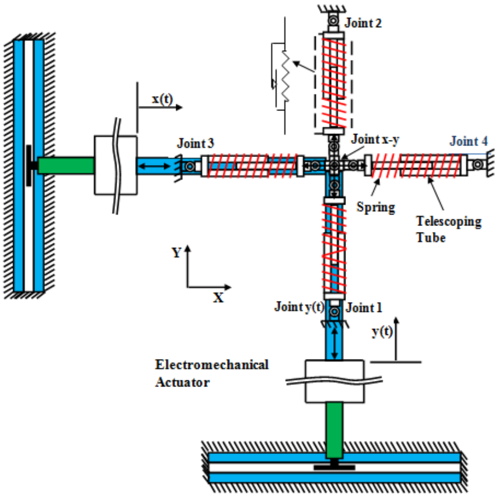

Figure 1. *The Semi-Active Isolator (SAI) connected with the sliding ends actuators.*

Thus, as it can concludes, that this model proposed would get its stability when the tubes are not affected and in this case, the stiffness and friction have lowest values, and these values increase when the displacement and velocity of the base increase, this feature make it a good organ for decreasing the responses of stages when the responses of the base increase.

Firstly, suppose that seismic excitation hits in the X direction only, the displacements of any of the four springs are given by

$$\begin{cases} u_1 = x \\ u_2 = x \\ u_3 = \sqrt{x^2 + L^2} - L \\ u_4 = \sqrt{x^2 + L^2} - L \end{cases}$$

The forces due to the springs and frictions, according to the X-axis are expressed

$$F_{SX} = k_e \left[u_1 + u_2 + u_3 \frac{x}{\sqrt{x^2 + L^2}} + u_4 \frac{x}{\sqrt{x^2 + L^2}} \right]$$

$$= 2k_e \left[1 + \tfrac{\sqrt{x^2+L^2}-L}{\sqrt{x^2+L^2}} \right] x = K_{eqX} x \tag{21}$$

$$F_{FX} = c_e \left[u_1 + u_2 + u_3 \frac{x}{\sqrt{x^2 + L^2}} + u_4 \frac{x}{\sqrt{x^2 + L^2}} \right]$$

$$= 2k_e \left[1 + \tfrac{\sqrt{x^2+L^2}-L}{\sqrt{x^2+L^2}} \right] \dot{x} = \quad C_{eqX} \dot{x} \tag{22}$$

Such that k_e and c_e are the element (one spring-friction tube) stiffness and friction coefficient, x and $\dot{x}$ are the displacement and velocity of the base, L is the length of one element in its rest case (initial case) and K_{eqX} and C_{eqX} are the equivalent stiffness and friction coefficient of the model, according to X axis for an earthquake hitting in the direction X only.

Thus, the equivalent stiffness and friction coefficient are expressing as

$$K_{eqX} = 2k_e \left[1 + \left(\sqrt{x^2 + L^2} - L \right) / \sqrt{x^2 + L^2} \right] \tag{23}$$

$$C_{eqX} = 2c_e \left[1 + \left(\sqrt{x^2 + L^2} - L \right) / \sqrt{x^2 + L^2} \right] \tag{24}$$

Thus, we can remark that

$$\begin{bmatrix} K_{eqX} = 2k_e \\ C_{eqX} = 2c_e \end{bmatrix} \qquad \text{for } x = 0$$

$$\begin{bmatrix} K_{eqX} = 2k_e \left[1 + \left(\sqrt{2} - 1 \right) / \sqrt{2} \right] \\ C_{eqX} = 2c_e \left[1 + \left(\sqrt{2} - 1 \right) / \sqrt{2} \right] \end{bmatrix} \qquad \text{for } x = L$$

The same expressions as (23) and (24) can be deducted for the Y direction, replacing x by y.

The displacements of any of the four springs according to X-axis, due to an earthquake hits on an arbitrary direction in the X-Y plan, are expressed as follows

$$\begin{cases} u_1 = \left| \sqrt{(L + x)^2 + y^2} - L \right| \\ u_2 = \left| \sqrt{(L - x)^2 + y^2} - L \right| \\ u_3 = \left| \sqrt{(L + y)^2 + x^2} - L \right| \\ u_4 = \left| \sqrt{(L - y)^2 + x^2} - L \right| \end{cases} \tag{25}$$

The absolute values implicate that the displacements and by consequence, equivalent stiffness must be positive whatever the signs of x and y.

The resistant stiffness force according to X-axis, due to such indicate earthquake, is obtained by

$$F_{SX} = k_e \left[u_1 \frac{L + x}{\sqrt{(L + x)^2 + y^2}} + u_2 \frac{L - x}{\sqrt{(L - x)^2 + y^2}} + u_3 \frac{x}{\sqrt{(L + y)^2 + x^2}} + u_4 \frac{x}{\sqrt{(L - y)^2 + x^2}} \right]$$

Replacing u_i by their values (25), one can obtained the total stiffness and friction resistant forces, according to the X-axis, by

$$F_{SX} = k_e \left[\frac{\left|\sqrt{(L+x)^2+y^2}-L\right|}{\sqrt{(L+x)^2+y^2}} \left(\frac{L}{x}+1\right) + \frac{\left|\sqrt{(L-x)^2+y^2}-L\right|}{\sqrt{(L-x)^2+y^2}} \left(\frac{L}{x}-1\right) + \frac{\left|\sqrt{(L+y)^2+x^2}-L\right|}{\sqrt{(L+y)^2+x^2}} + \frac{\left|\sqrt{(L-y)^2+x^2}-L\right|}{\sqrt{(L-y)^2+x^2}} \right] x \quad (26\text{-}1)$$

$$F_{FX} = c_e \left[\frac{\left|\sqrt{(L+x)^2+y^2}-L\right|}{\sqrt{(L+x)^2+y^2}} \left(\frac{L}{x}+1\right) + \frac{\left|\sqrt{(L-x)^2+y^2}-L\right|}{\sqrt{(L-x)^2+y^2}} \left(\frac{L}{x}-1\right) + \frac{\left|\sqrt{(L+y)^2+x^2}-L\right|}{\sqrt{(L+y)^2+x^2}} + \frac{\left|\sqrt{(L-y)^2+x^2}-L\right|}{\sqrt{(L-y)^2+x^2}} \right] \dot{x} \quad (26\text{-}2)$$

The equivalent stiffness and friction according to X-axis are then given by

$$K_{eqX} = k_e \left[\frac{\left|\sqrt{(L+x)^2+y^2}-L\right|}{\sqrt{(L+x)^2+y^2}} \left(\frac{L}{x}+1\right) + \frac{\left|\sqrt{(L-x)^2+y^2}-L\right|}{\sqrt{(L-x)^2+y^2}} \left(\frac{L}{x}-1\right) + \frac{\left|\sqrt{(L+y)^2+x^2}-L\right|}{\sqrt{(L+y)^2+x^2}} + \frac{\left|\sqrt{(L-y)^2+x^2}-L\right|}{\sqrt{(L-y)^2+x^2}} \right] \quad (27\text{-}1)$$

$$C_{eqX} = c_e \left[\frac{\left|\sqrt{(L+x)^2+y^2}-L\right|}{\sqrt{(L+x)^2+y^2}} \left(\frac{L}{x}+1\right) + \frac{\left|\sqrt{(L-x)^2+y^2}-L\right|}{\sqrt{(L-x)^2+y^2}} \left(\frac{L}{x}-1\right) + \frac{\left|\sqrt{(L+y)^2+x^2}-L\right|}{\sqrt{(L+y)^2+x^2}} + \frac{\left|\sqrt{(L-y)^2+x^2}-L\right|}{\sqrt{(L-y)^2+x^2}} \right] \quad (27\text{-}2)$$

The total forces due to the stiffness and friction and the equivalent stiffness and friction according to Y-axis are obtained with the same manner, for an earthquake excited on an arbitrary direction, or by replacing x by y and y by x in the equations (26) and (27).

4. Illustration and Demonstrative Numerical Example

As an demonstrative example for the utility and useful, of the algorithm presented in section 2 for the computing of the nonlinear optimal control feedback force, and the Semi-Active variable stiffness and friction Isolator given in section 3, a seven floors prototype structure, with one horizontal X-axis degree of freedom by floor, was uncontrolled, active controlled and semi-active controlled analyzed. For the structure, equal masses are supposed concentrates at the level of the floors and equal stiffness is supposed attaching between any two floors. The damping is deducted according to the percentage of mass and stiffness formula (Rayligh formula). The mass, stiffness and damping matrices are then given by

$$M = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m \end{bmatrix}$$

$$K(t) = \begin{bmatrix} k & -k & 0 & 0 & 0 & 0 & 0 & 0 \\ -k & 2k & -k & 0 & 0 & 0 & 0 & 0 \\ 0 & -k & 2k & -k & 0 & 0 & 0 & 0 \\ 0 & 0 & -k & 2k & -k & 0 & 0 & 0 \\ 0 & 0 & 0 & -k & 2k & -k & 0 & 0 \\ 0 & 0 & 0 & 0 & -k & 2k & -k & 0 \\ 0 & 0 & 0 & 0 & 0 & -k & 2k & -k \\ 0 & 0 & 0 & 0 & 0 & 0 & -k & k + k_b \end{bmatrix}$$

$$C = 0.05M + 0.05K(t)$$

The elastic stiffness between any two floors and the masses at the top of any floor are considered equal, and are take the values $k = 2.5(2\pi)^2$ and $m = 0.25$. The Force-Displacements constitutive model of the material is considered bilinear with the hardening stiffness $k_h = k/5$. For the base stiffness k_b indicated in the $K(t)$ matrix is taken equal k (for the uncontrolled and active controlled structure) and K_{eqX} (for the semi-active structure), in this last case we take for the base damping $C(8,8) = 0.05m + 0.05k + C_{eqX}$ such that, $k_e = k/2$ and $c_e = 12.5$. The maximal elastic displacement is supposed equals to $D_0 = 0.055$, and the free length of any spring and friction tube is taken $L = 0.12$. The weighting matrix R is supposed diagonal with a value 0.1 at each diagonal element, and the second weighting matrix Q is proposed being equals

$$Q = \begin{bmatrix} M^{-1}K(t) & 0 \\ 0 & M^{-1}C \end{bmatrix}$$

The structure is supposed submitted to the Modified El-Centro earthquake shown by Figure 2. The unities taken for data and results entities are the Kilogram, Newton, Second and Meter.

The uncontrolled, active controlled and semi-active (for the first, fourth and seventh floors) displacements versus time are shown by Figures 3.-8. and the floors forces versus time by the Figures 9.-11. The variations, of the base displacement and of the control feedback force for the controlled structure, and of the fraction K_{eqX}/k (for the semi-active controlled structure) versus time are given by Figures 12.-14. The variation of K_{eqX}/k according to the variation of the semi-active base displacement is showing by the Figure 15.

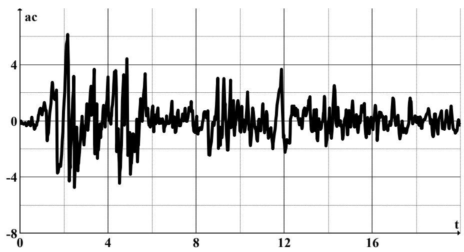

Figure 2. *The Modified El-Centro earthquake.*

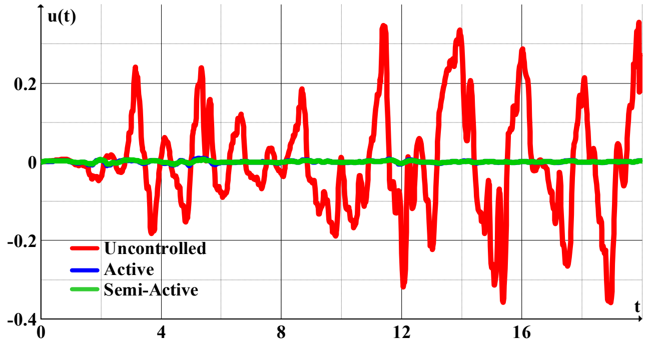

Figure 3. *Controlled and uncontrolled displacements of the first floor.*

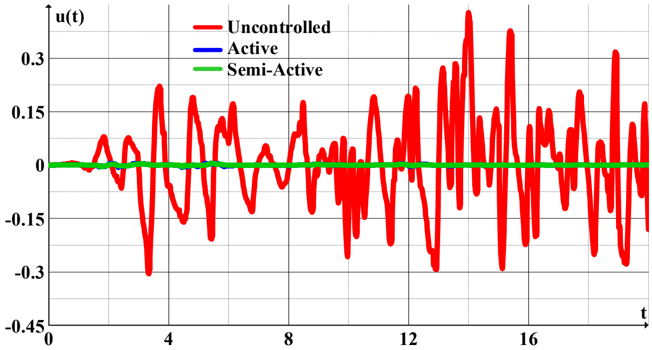

Figure 4. *Controlled and uncontrolled displacements of the fourth floor.*

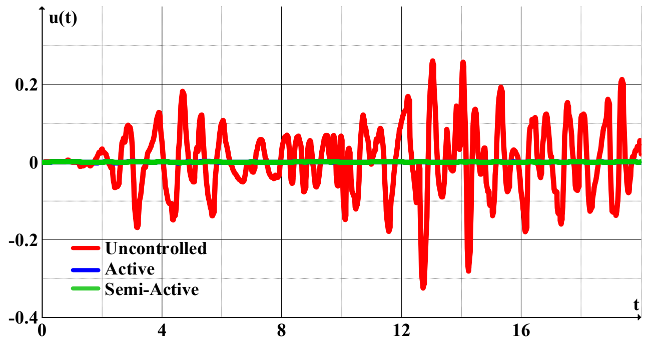

Figure 5. *Controlled and uncontrolled displacements of the seventh floor.*

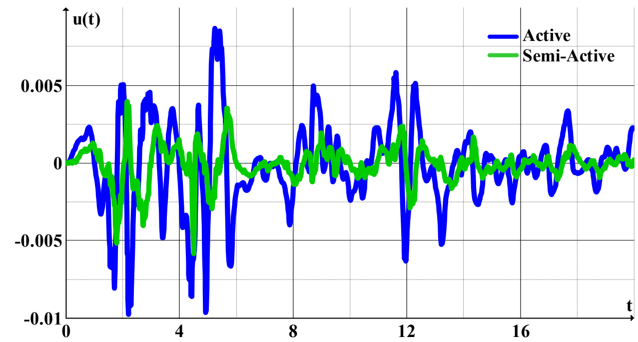

Figure 6. *Active and Semi-Active Controlled displacements of the first floor.*

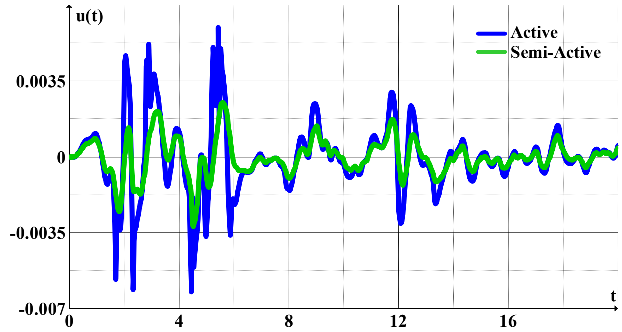

Figure 7. *Active and Semi-Active Controlled displacements of the fourth floor.*

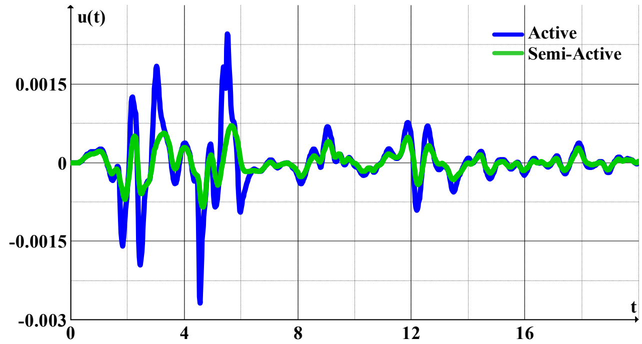

Figure 8. *Active and Semi-Active Controlled displacements of the seventh floor.*

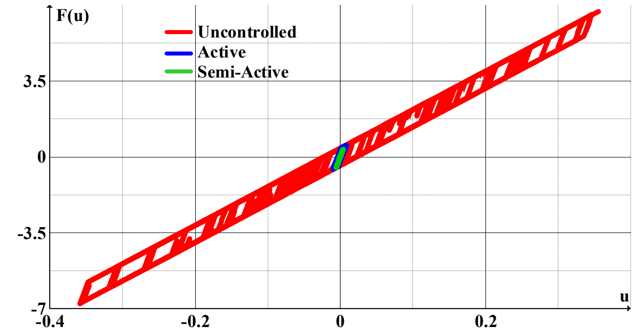

Figure 9. *Controlled and uncontrolled forces of the first floor.*

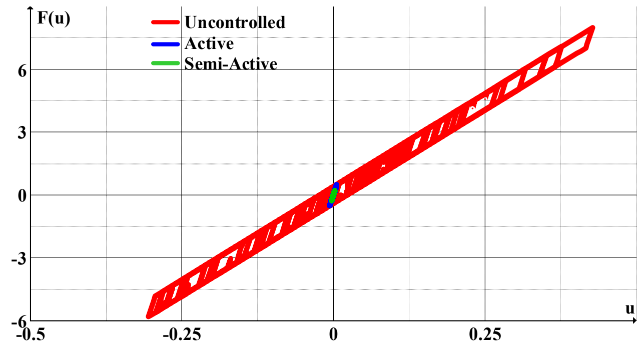

Figure 10. *Controlled and uncontrolled forces of the forth floor.*

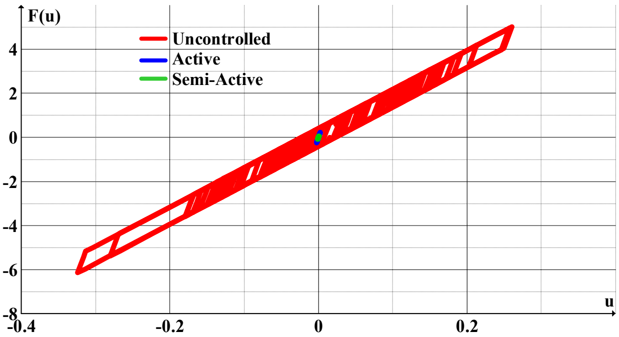

Figure 11. *Controlled and uncontrolled forces of the seventh floor.*

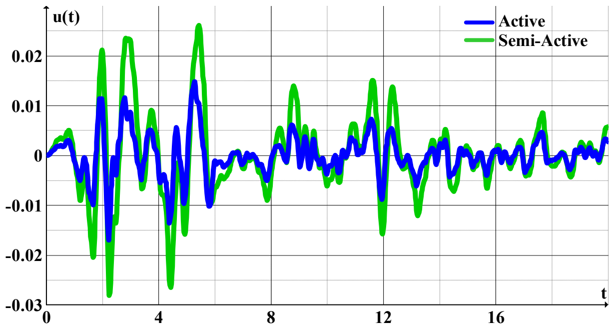

Figure 12. *Active and Semi-Active Controlled base displacement.*

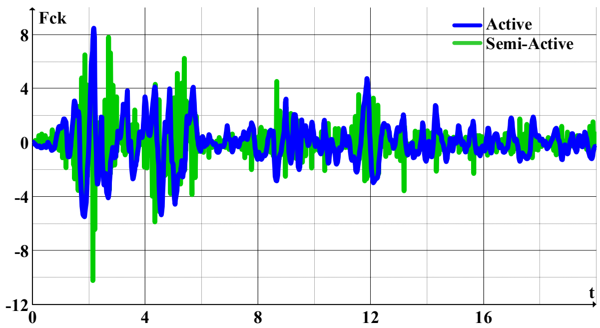

Figure 13. *Active and Semi-Active Control force provoked by the actuator.*

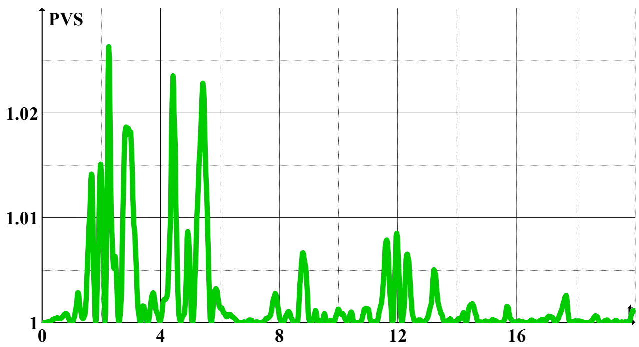

Figure 14. *Passive variable stiffness reported to the structural stiffness vs. time.*

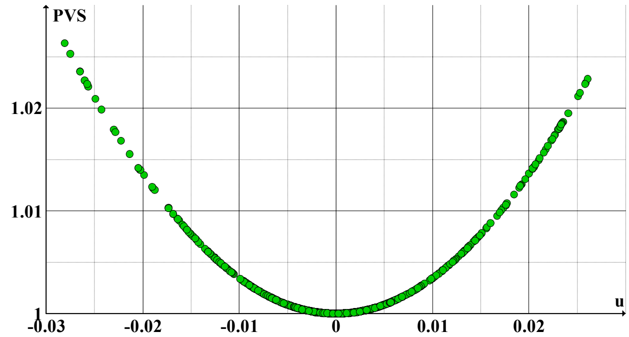

Figure 15. *Passive variable stiffness reported to the structural stiffness vs. base displacements.*

5. Discussion of the Results

As shown by Figure 3. to Figure 8. and as indicated by Table 1. that there is no comparison between the uncontrolled and active controlled displacements, as we talk about 427 mm and 324 mm as greatest relative displacement for the first and the seventh floors for an uncontrolled structure, then we can reduce this displacement by 98.5 % to 99.2 % (9.8 to 2.7 mm) when the same structure was Active Controlled. Although, when the base of this controlled structure is replaced by the passive variable stiffness and friction (SAI), such that the stiffness of this organ can vary from k to $1.29k$ and the friction coefficient from 12.5 to 16.2, the Semi-Active displacements are reducing from 40.0 % to 68.3 %. Though, that the grand values of the control feedback forces for the cases of the Semi-Active based structure and the Active controlled structure are 10.2 and 8.4 respectively; but this difference is negligible compared with the differences of the displacements for the cases and the security which offers this last indicated base.

6. Conclusion

As introduced above, that the nonlinearity is an essential characteristic of the structural materials, then any study or analysis without taking in account this material propriety is insignificant and ineffective; among that, all the literature suppose that the Riccati equation solution is constant and reduced in the algebraic through all the time when the earthquake hitting. Moreover, the Semi-Active organs indicated in the literature are sometimes complex; but, because of the insufficiently understanding of the role of these organs for reducing as possible the drifts of the floors, unfortunately that these proposed organs being able to reduce sufficiently the displacements, velocities and accelerations of structural floors.

Table 1. *The Uncontrolled, Active and Semi-Active Controlled maximal displacements of different floors.*

	Uncontrolled and Controlled maximal Displacements					Controlled maximal Displacements		
	Uncontrolled	Active	S-Active	% A/U	% SA/U	Active	S-Active	% SA/A
1st floor	0.35746	0.00976	0.00586	2.7	1.6	0.00976	0.00586	60.0
4th floor	0.42755	0.00624	0.00321	1.5	0.8	0.00624	0.00321	51.4
7th floor	0.32431	0.00268	0.00085	0.8	0.3	0.00268	0.00085	31.7

In this study, a nonlinear discrete-time algorithm for the resolution of the Riccati equation which indeed, take in one account the nonlinearity of the real structural properties such as the stiffness and damping matrices. Thus, a good passive variable stiffness and friction mechanical organ is proposed to be a base of the analyzed structure, such model is stiffened and fractioned more accordingly of the increasing of the base displacement. This property gives it a grand role for reducing the drifts of the analyzed structure. As mentioned in the previous section that because of the developments in this study, as we talk of tenth of centimeters for the relative displacements of the structural floors for the case of uncontrolled structure, then we talk only of about some millimeters and even less, for the case of Semi-Active controlled structure, and this returns to the role of the real analyzed controlled structure, and the role of the good passive variable stiffness and friction taken.

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