International Journal of Civil Engineering and Construction Science
2014; 1(2): 18-25
Published online June 10, 2014 (http://www.aascit.org/journal/ijcecs)

American Association for Science and Technology

On the general evaluation of the maximum allowable drift at the top of shear walls (constant and variable stiffness)

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Citation

Michel Farid Khouri, Wassim Joseph Elias. On the General Evaluation of the Maximum Allowable Drift at the Top of Shear Walls (Constant and Variable Stiffness). *International Journal of Civil Engineering and Construction Science.* Vol. 1, No. 2, 2014, pp. 18-25.

Keywords

Reinforced Concrete Structures,
Shear Walls,
Maximum Allowable Drift,
Constant and Variable Stiffness,
Maximum Strain Limits,
Lateral and Vertical Contribution

Received: April 24, 2014
Revised: May 13, 2014
Accepted: May 14, 2014

Abstract

The latest research on determining maximum allowable displacements at the top of a shear wall (or maximum Drift) has contributed considerably to the improvement of concrete structures. Aiming at limiting maximum displacement, various investigators and seismic codes use values ranging from H/50 to H/2000, (H=building height). Difference shows considerable uncertainty in these limits. This article evaluates maximum allowable drift and provides general formulae for constant and variable stiffness shear walls by using FEM along with structural dynamics and reinforced concrete design considering cracked shear wall sections suggested by UBC and ACI. A comparison is made with various codes.

1. Introduction

Steel structures are allowed to drift more than reinforced concrete structures due to the fact that steel can accommodate tension as well as compression, while concrete tensile strength is generally less than 10% of its compressive strength. Under any tensile movement concrete will crack when subjected to wind and seismic reversals. Many investigators and seismic codes [1-9] suggest values for maximum allowable story drift or maximum allowable lateral displacement but these values differ significantly.

Suggested maximum drift at the top of buildings vary between H/50 and H/2000 where H is the height of the building. In addition to the fact that this difference is large, the question is how the actual structural behavior can be used to determine maximum allowable drift? Some codes such as UBC section 1630.10.2 consider that buildings are allowed to drift H/50 as long as top floors occupants stay confortable without feeling the swing; they also base their estimate on the nonlinear behavior of the structure and materials. Other codes such as the Lebanese code reduce the drift values to H/2000 because land lots are scarce and expensive consequently, the code allows engineers and developers in certain areas to construct without setbacks. This requires two adjacent buildings to be very stiff in order to eliminate/reduce hammering between structures when subjected to seismic or wind loads.

One of the first attempts to evaluate the maximum allowable drift based on real shear wall behavior was done by the author [10], where the maximum allowable drift was determined by assuming a shear building and making use of the finite element analysis along with the structural dynamics and reinforced concrete design. In that initial study, a constant stiffness was assumed from bottom to top of the shear wall. In a later study, drift limitations in a shear wall was done considering a cracked section [11] as suggested by UBC and ACI where shear wall inertia was reduced. Another study was done to evaluate the drift with variable stiffness where the stiffness in a shear wall can vary from bottom to top; in addition, the effect of cracked section reduced inertia was considered [12]. In this article, the previous work done is combined and the effect of the vertical load contribution is added to present complete formulae that can be used to determine the maximum allowable drift for both constant and variable stiffness shear walls and for cracked as well as un-cracked reinforced concrete sections.

2. Shear Building Formulation

The objective is to generate the stiffness matrix for a shear building and afterwards add the contribution of the vertical load. As presented by references [10-12], and after applying boundary conditions, the stiffness matrix of an element on the shear wall is:

$$[K]=\frac{EI}{L^3}\begin{bmatrix}12 & -12\\ -12 & 12\end{bmatrix} \quad (1)$$

Where E is the concrete modulus of elasticity, I is the wall inertia and L is the wall length.

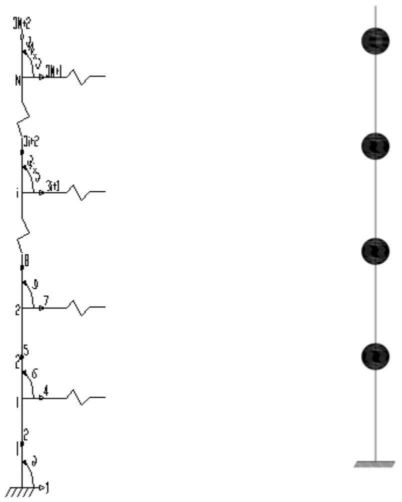

Figure 1. Representation of an N-story building.

From Figure 1, and taking into consideration the boundary conditions presented in reference [11], the structure stiffness matrix can be assembled relative to degrees of freedom 4, 7,…3j+1…3N+1. Using constant and/or variable stiffness shear walls where the lateral stiffness is constant or can vary from the bottom to the top level as desired, the setup of the stiffness matrix for constant stiffness shear wall where $K^{(1)}= K^{(2)}\ldots\ldots=K^{(N)}=K$ and for a variable stiffness shear walls where $K^{(1)} \neq K^{(2)}\ldots \neq K^{(N)}$ are presented in the following section.

3. Base Shear, Displacement and Relative Displacement

To find a relation between the total base shear V, stiffness k and maximum displacement Δ for N stories structure, a triangular distribution of V is assumed. The base shear can be found by any procedure or from any seismic code. This distribution of V gives a formula of the applied force F_i at every level i of the structure as a function of N,

$$F_i = \frac{2iV}{N(N+1)}, \text{ where } \sum_{i=1}^{N} i = \frac{N(N+1)}{2}$$

$F = k.q \Rightarrow q=k^{-1} F$ can be written as:

$$q=\begin{Bmatrix} q_4\\ q_7\\ \vdots\\ \vdots\\ q_{3i+1}\\ \vdots\\ \vdots\\ q_{3N+1=\Delta}\end{Bmatrix}=\frac{2V}{N(N+1)}\frac{1}{k}\cdot\begin{bmatrix}1 & 1 & \ldots & \ldots & 1 & \ldots & \ldots & 1\\ 1 & 2 & \ldots & \ldots & 2 & \ldots & \ldots & 2\\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots\\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots\\ 1 & 2 & \ldots & \ldots & i & & & i\\ \vdots & \vdots & \ddots & & \vdots & \ddots & & \vdots\\ \vdots & \vdots & & \ddots & \vdots & & \ddots & \vdots\\ 1 & 2 & \ldots & \ldots & i & \ldots & \ldots & N\end{bmatrix}\cdot\begin{Bmatrix}1\\ 2\\ \vdots\\ \vdots\\ i\\ \vdots\\ \vdots\\ N\end{Bmatrix}$$

$$q_{3N+1} = \Delta = \frac{2V}{N(N+1)}\frac{1}{k}[1x1 + 2x2 + 3x3 + \ldots + ixi + \ldots + NxN]$$

$$\Delta = \frac{2V}{N(N+1)}\frac{1}{k}\sum_{i=1}^{i=N} i^2, \text{ but } \sum_{i=1}^{i=N} i^2 = \frac{N(N+1)(2N+1)}{6},$$

$$=> \Delta = \frac{(2N + 1)V}{3} \frac{1}{k} \tag{2}$$

Let the vector q represent the displacement in the particular case where the stiffness of all stories is the same as computed and presented in references [10-12]. The objective now is to determine the displacement q' for variable stiffness shear walls.

The stiffness matrix [A] can be written in terms of the lateral stiffness of each story in the following manner:

$$A = \begin{bmatrix} k^1+k^2 & -k^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -k^2 & k^2+k^3 & -k^3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -k^3 & k^3+k^4 & -k^4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -k^4 & \ddots & \ddots & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \ddots & \ddots & -k^i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -k^i & k^i+k^{i+1} & -k^{i+1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -k^{i+1} & k^{i+1}+k^{i+2} & -k^{i+2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -k^{i+2} & \ddots & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & -k^{N-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k^{N-1} & k^{N-1}+k^N & -k^N \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -k^N & k^N \end{bmatrix}$$

Where, $k^{(i)}$ is the stiffness of one story, $k^{(i)} = \sum_{j=1}^{n} 12 \frac{E_j I_j}{L_j^3}$.

The stiffness matrix for a constant stiffness shear wall in the case where $k^{(1)} = k^{(2)} = \ldots = k^{(i)} = \ldots = k^{(n)} = k$, is as demonstrated in a reference [10]:

$$B = k \begin{bmatrix} 2 & -1 & 0 & \cdots & \cdots & 0 & \cdots & \cdots & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & \cdots & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & \ddots & \vdots & \ddots & 0 & 0 & 0 & 0 \\ \vdots & 0 & -1 & \ddots & \ddots & 0 & \cdots & 0 & \cdots & 0 & \vdots \\ \vdots & \vdots & \ddots & \ddots & 2 & -1 & \ddots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & -1 & 2 & -1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & -1 & 2 & -1 & \ddots & \vdots & \vdots \\ \vdots & 0 & 0 & 0 & \cdots & 0 & -1 & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & 0 & \vdots & \ddots & \vdots & \ddots & \ddots & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & \cdots & \cdots & 0 & \cdots & \cdots & 0 & -1 & 1 \end{bmatrix}$$

$$B^{-1} = \left(\frac{1}{k}\right) \begin{bmatrix} 1 & 1 & 1 & \cdots & \cdots & 1 & 1 & 1 & \cdots & \cdots & 1 \\ 1 & 2 & 2 & \cdots & \cdots & 2 & 2 & 2 & \cdots & \cdots & 2 \\ 1 & 2 & 3 & \cdots & \cdots & 3 & 3 & 3 & \cdots & \cdots & 3 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & \cdots & i & i & i & \cdots & \cdots & i \\ 1 & 2 & 3 & \cdots & \cdots & i & i+1 & i+1 & \cdots & \cdots & i+1 \\ 1 & 2 & 3 & \cdots & \cdots & i & i+1 & i+2 & \cdots & \cdots & i+2 \\ \vdots & \vdots & \vdots & \ddots & & \vdots & \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots & \vdots & & \ddots & \vdots \\ 1 & 2 & 3 & \cdots & \cdots & i & i+1 & i+2 & \cdots & \cdots & N \end{bmatrix}$$

At this time matrix C can be found such as: A = C x B => C = A x B^{-1}, =>

$$C = \frac{1}{k} \begin{bmatrix} k^1 & k^1-k^2 & k^1-k^2 & \cdots & \cdots & k^1-k^2 & k^1-k^2 & \cdots & \cdots & \cdots & k^1-k^2 \\ 0 & k^2 & k^2-k^3 & \cdots & \cdots & k^2-k^3 & k^2-k^3 & \cdots & \cdots & \cdots & k^2-k^3 \\ 0 & 0 & k^3 & \ddots & \ddots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \ddots & \vdots & \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \ddots & \ddots & k^{i-1}-k^i & k^{i-1}-k^i & \cdots & \cdots & \cdots & k^{i-1}-k^i \\ \vdots & \vdots & \vdots & \ddots & \ddots & k^i & k^i-k^{i+1} & \cdots & \cdots & \cdots & k^i-k^{i+1} \\ 0 & 0 & 0 & \cdots & \cdots & 0 & k^{i+1} & \cdots & \cdots & \cdots & k^{i+1}-k^{i+2} \\ 0 & 0 & 0 & \cdots & \cdots & \vdots & 0 & \ddots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \ddots & k^{N-1} & k^{N-1}-k^N \\ 0 & 0 & 0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 & k^N \end{bmatrix}$$

The shear force Fi at any level i is computed as: F = A x q', and A = C x B => F = C x (B x q'). Therefore, once the values of the displacements q' are known, the shear forces can be calculated.

Now, find matrix D as follows: A = B x D => D = B^{-1} x A, and D = C^T.

$$D = \frac{1}{k} \begin{bmatrix} k^1 & 0 & \cdots & \cdots & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ k^1-k^2 & k^2 & 0 & \cdots & 0 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ k^1-k^2 & k^2-k^3 & k^3 & \ddots & 0 & 0 & \vdots & 0 & 0 & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & \vdots & 0 & 0 & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ k^1-k^2 & k^2-k^3 & \cdots & \ddots & k^{i-1}-k^i & k^i & 0 & 0 & 0 & 0 & 0 \\ k^1-k^2 & k^2-k^3 & \cdots & \cdots & k^{i-1}-k^i & k^i-k^{i+1} & k^{i+1} & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \ddots & k^{N-1} & 0 \\ k^1-k^2 & k^2-k^3 & \cdots & \cdots & k^{i-1}-k^i & k^i-k^{i+1} & k^{i+1}-k^{i+2} & \cdots & \cdots & k^{N-1}-k^N & k^N \end{bmatrix}$$

$$D^{-1}=\frac{k}{\prod\limits_{i=1}^{i=N} k^{i}}\begin{bmatrix}
\prod\limits_{\substack{i=1\\(i\neq 1)}}^{i=N} k^{i} & 0 & 0 & \cdots & 0 & 0 & \cdots\ 0 & 0\\
\left(k^{2}-k^{1}\right)\prod\limits_{\substack{i=1\\(i\neq 1;2)}}^{i=N} k^{i} & \prod\limits_{\substack{i=1\\(i\neq 2)}}^{i=N} k^{i} & 0 & \cdots & 0 & 0 & \cdots\ 0 & 0\\
\left(k^{2}-k^{1}\right)\prod\limits_{\substack{i=1\\(i\neq 1;2)}}^{i=N} k^{i} & \left(k^{3}-k^{2}\right)\prod\limits_{\substack{i=1\\(i\neq 2;3)}}^{i=N} k^{i} & \prod\limits_{\substack{i=1\\(i\neq 3)}}^{i=N} k^{i} & \ddots\ \ddots & \vdots & \vdots & \ddots\ \ddots\ \vdots & \vdots\\
\vdots & \vdots & \ddots & \ddots\ \ddots & 0 & 0 & 0\ 0\ 0 & 0\\
\vdots & \vdots & \ddots & \ddots\ \ddots & \prod\limits_{\substack{i=1\\(i\neq j-1)}}^{i=N} k^{i} & \ddots & 0\ 0\ 0 & 0\\
\left(k^{2}-k^{1}\right)\prod\limits_{\substack{i=1\\(i\neq 1;2)}}^{i=N} k^{i} & \left(k^{3}-k^{2}\right)\prod\limits_{\substack{i=1\\(i\neq 2;3)}}^{i=N} k^{i} & \cdots & \cdots\cdots & \left(k^{j}-k^{j-1}\right)\prod\limits_{\substack{i=1\\(i\neq j;\,j-1)}} k^{i} & \prod\limits_{\substack{i=1\\(i\neq j)}}^{i=N} k^{i} & \ddots\ 0\ \ddots & 0\\
\vdots & \vdots & \ddots & \ddots\ \ddots & \vdots & \ddots & \ddots\ \ddots\ 0 & \vdots\\
\left(k^{2}-k^{1}\right)\prod\limits_{\substack{i=1\\(i\neq 1;2)}}^{i=N} k^{i} & \left(k^{3}-k^{2}\right)\prod\limits_{\substack{i=1\\(i\neq 2;3)}}^{i=N} k^{i} & \cdots & \cdots\cdots & \left(k^{j}-k^{j-1}\right)\prod\limits_{\substack{i=1\\(i\neq j;\,j-1)}} k^{i} & \cdots & \cdots\cdots\cdots & \prod\limits_{\substack{i=1\\(i\neq N)}}^{i=N} k^{i}
\end{bmatrix}$$

Where $\prod\limits_{i=1}^{i=N} k^{(i)} = K^{(1)}.\ K^{(2)}\ldots\ldots\ K^{(N)}$.

F = A x q' => q' = A-1 x F, but A-1 = D-1 x B-1 =>
q' = D^{-1} x [B^{-1} x F], and [B^{-1} F] = q and since q is equal to:

$$q=\frac{V}{k}\begin{bmatrix}
1\left[1-\frac{\left(1^{2}-1\right)}{6X}\right]\\
2\left[1-\frac{\left(2^{2}-1\right)}{6X}\right]\\
\vdots\\
(i-1)\left[1-\frac{\left((i-1)^{2}-1\right)}{6X}\right]\\
i\left[1-\frac{\left(i^{2}-1\right)}{6X}\right]\\
\vdots\\
N\left[1-\frac{\left(N^{2}-1\right)}{6X}\right]
\end{bmatrix}=\begin{bmatrix}
q_{4}\\ q_{7}\\ \vdots\\ q_{3(i-1)+1}\\ q_{3i+1}\\ \vdots\\ q_{3N+1}
\end{bmatrix}\equiv\begin{bmatrix}
q_{1}\\ q_{2}\\ \vdots\\ q_{i-1}\\ q_{i}\\ \vdots\\ q_{N}=\Delta
\end{bmatrix}$$

$$\Rightarrow q'_{j}=\frac{V}{\prod\limits_{i=1}^{N} k^{(i)}}\left\{\sum_{i=1}^{i=j-1}\left[\left[i-\frac{\left(i^{2}-1\right)}{6X}\right]\cdot\left[k^{(i+1)}-k^{(i)}\right].\left[\prod_{\substack{l=1\\(l\neq i;i+1)}}^{l=N} k^{(l)}\right]\right]+j\left[1-\frac{\left(j^{2}-1\right)}{6X}\right]\cdot\prod_{\substack{i=1\\(i\neq j)}}^{i=N} k^{(i)}\right\}\qquad(3)$$

The value of the relative displacement $q'_j - q'_{j-1}$ is given by:

$$q'_j - q'_{j-1} = \frac{V}{\prod_{i=1}^{i=N} k^{(i)}} \left\{ (j-1)\left[1 - \frac{(j-1)^2 - 1}{6X}\right] \cdot \left[k^{(j)} - k^{(j-1)}\right] \cdot \left[\prod_{\substack{l=1 \\ (l \neq j-1;\, j)}}^{l=N} k^{(l)}\right] + j\left[1 - \frac{j^2 - 1}{6X}\right] \prod_{\substack{i=1 \\ (i \neq j)}}^{i=N} k^{(i)} - (j-1)\left[1 - \frac{(j-1)^2 - 1}{6X}\right] \prod_{\substack{i=1 \\ (i \neq j-1)}}^{i=N} k^{(i)} \right\}$$

which can be reduced to:

$$q'_j - q'_{j-1} = \frac{V}{k^{(j)}}\left[\frac{2X - j^2 + j}{2X}\right] \qquad (4)$$

4. Computing Maximum Strain

Similar to what was presented in references [10-12] and using references [13] and [14], it was demonstrated that the strain in the x direction along a shear wall between the two levels (j-1) and j is:

$\varepsilon_i = -\frac{y}{L^3}(12x - 6L)(q'_j - q'_{j-1})$, and adding the effect of the vertical load, the strain equation becomes:

$$\varepsilon_i = \{-\frac{y}{L^3}(12x - 6L) - \frac{P_i}{A_i.E}\}(q'_j - q'_{j-1}) \qquad (5)$$

where y is the algebraic distance measured from the neutral axis to the extreme fiber of the shear wall section; y is considered to be positive in the opposite direction of the deflection. The maximum value of ε_i is obtained when maximum value of y is replaced, x is the abscissa of the section along the shear wall between the two levels (j-1) and j, and ε_x is the strain in x direction.

Note that equation (4) includes the effect of the vertical load above the level considered where L is the story height and P_i is the axial load at level i applied at the center of the shear wall with an area of A_i and modulus of elasticity E.

By substituting the values of $(q'_j - q'_{j-1})$ from eq. (3) into eq. (4), the strain formula becomes:

$$\varepsilon_i = \{-\frac{y}{L^3}(12x - 6L) - \frac{P_i}{A_i.E}\}\left(\frac{2X - j^2 + j}{2X}\right)\left(\frac{V}{k^{(j)}}\right) \qquad (6)$$

The maximum value of the strain ε_i between levels (i-1) and i is found as follows:

$$\frac{\partial}{\partial X}[\varepsilon_i] = -12\,\frac{y}{L^3}\left(\frac{2X - j^2 + j}{2X}\right)\left(\frac{V}{k^{(j)}}\right) < 0$$

The function ε_i is decreasing which means that the maximum strain in a shear wall, between levels (j-1) and j, occurs at the bottom of the shear wall (x = 0, j=0 and i=1), where P_1 is the axial load at level 1 applied at the center of the shear wall with an area of A_1 and this maximum strain is equal to:

$$\varepsilon_{(x=0)} = \frac{6y}{L^2}\left(\frac{V}{k^{(j)}}\right) - \frac{P_1}{A_1.E} \qquad (7)$$

5. Maximum Displacement

The maximum drift at the top of the shear wall is reached when the strain of the reinforcement in the tensile zone at the critical section of the shear wall is equal to ε_{st} (maximum allowable strain in steel), and the strain in the extreme fiber of the compression zone in the same section is equal to ε_c = maximum strain limit of concrete in compression = 0.003. So the critical section in the shear wall is considered to have the behavior described in Figure 2.[15]

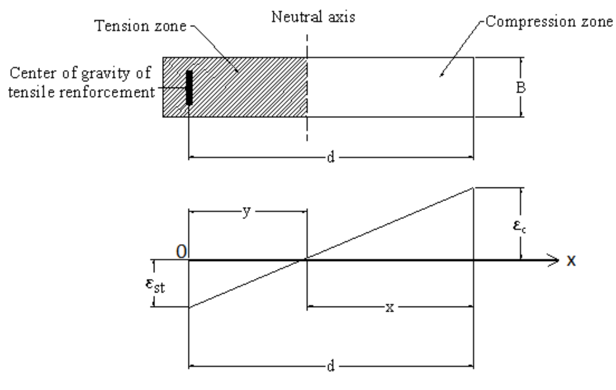

Figure 2. Balanced Reinforced Concrete Section.

From eq. (3) we have the maximum displacement at the top of the shear wall Δ is given as follows:

$$\Delta = q'_N = \frac{V}{\prod_{i=1}^{N} k^{(i)}}\left\{\sum_{i=1}^{i=N-1}\left[\left[i - \frac{(i^2 - 1)}{6X}\right] \cdot \left[k^{(i+1)} - k^{(i)}\right] \cdot \left[\prod_{\substack{l=1 \\ (l \neq i;\, i+1)}}^{l=N} k^{(l)}\right]\right] + N\left[1 - \frac{(N^2 - 1)}{6X}\right] \cdot \prod_{\substack{i=1 \\ (i \neq N)}}^{i=N} k^{(i)}\right\}$$

Let

$$R = \frac{1}{\prod_{i=1}^{N} k^{(i)}} \left\{ \sum_{i=1}^{i=N-1} \left[\left[i - \frac{(i^2-1)}{6X} \right] \cdot \left[k^{(i+1)} - k^{(i)} \right] \cdot \left[\prod_{\substack{l=1 \\ (l \neq i;\, i+1)}}^{l=N} k^{(l)} \right] \right] + N \left[1 - \frac{(N^2-1)}{6X} \right] \cdot \prod_{\substack{i=1 \\ (i \neq N)}}^{i=N} k^{(i)} \right\} \quad (8)$$

But $\Delta = R\,V$, and replacing V in equation 6 =>

$$\varepsilon_{(x=0)} = \frac{6y}{L^2}\left(\frac{\Delta}{R\ k^{(j)}}\right) - \frac{P_1}{A_1.E} \quad (9)$$

From similar triangles of the balanced section of Figure 2, the following can be written:

$$\frac{y}{\varepsilon_{st}} = \frac{x}{\varepsilon_c} \Rightarrow y = \frac{\varepsilon_{st}}{\varepsilon_{st} + \varepsilon_c} d \quad (10)$$

The maximum strain in the shear wall presented in eq. (9) at the level of steel should be smaller than εy (the yield strain of steel):

$$\varepsilon_{(x=0)} = \frac{6y}{L^2}\left(\frac{\Delta}{R\ k^{(j)}}\right) - \frac{P_1}{A_1.E} \le \varepsilon_{st} \quad (11)$$

Replace y from eq. (10) =>

$$\Delta \le \frac{R\ k^{(N)}(\varepsilon_{st} + \varepsilon_c)(\varepsilon_{st} + \frac{P_1}{A_1.E})L^2}{6.\varepsilon_{st}.d} \quad (12)$$

If the strain in the steel is considered to stay within ε_y (the yielding strain of steel), the maximum allowable displacement at the top of a variable stiffness shear wall obeys the following equation:

$$\Delta_v \le \frac{R\ k^{(N)}(\varepsilon_y + \varepsilon_c)(\varepsilon_y + \frac{P_1}{A_1.E})L^2}{6.\,\varepsilon_y.d} \quad (13)$$

For constant stiffness shear walls from bottom to top, eq. (13) becomes:

$$\Delta_c \le \frac{(2N+1)(\varepsilon_y + \varepsilon_c)(\varepsilon_y + \frac{P_1}{A_1.E})L^2}{18.\varepsilon_y.(d)} \quad (14)$$

In this case, the critical section of the shear wall behaves as a balanced section; the limits are reached in the reinforcement in tension and in the concrete in compression at the same time, and at that point, the maximum allowable displacement at the top of the shear wall is reached.

Notice that, in both equations (13) and (14), as the height L of a story increases, the maximum allowable displacement increases, and as d increases the maximum allowable displacement decreases since any small movement tends to cause larger strain at the critical section of the shear wall. On the other hand, and as far as maximum displacement is concerned and disregarding economical and architectural issues, it is better to use larger number of shear walls with small d than to use fewer shear walls with large d; keeping in mind that the inertia of a shear wall is increased cubically as a function of d, and a bigger d will increase the stiffness significantly.

6. Maximum Drift Considering a Cracked Section

According to UBC, Modeling Requirement Section 1630.1.2, "Stiffness Properties of Reinforced Concrete and masonry elements shall consider the effects of cracked sections".[1]

This means that when a designer uses cracked section analysis then according to ACI 318, section 10.10.4.1, the shear wall moment of Inertia would have to be reduced by a factor 0.35 such that the inertia would become 0.35Ig.[12] This will consequently reduce the stiffness of the lateral load resisting elements and will equally increase the drift by the same factor, which means that the drift should be multiplied by a factor of (1/0.35) to get a value taking into consideration the effects of a cracked section suggested by UBC-97 and ACI-318-08. Equation 13 for a cracked section of variable stiffness shear walls becomes:

$$\boxed{\Delta_{vm} \le (\frac{1}{0.35})\Delta_v} \quad (15)$$

and equation 13 for constant stiffness shear walls becomes:

$$\boxed{\Delta_{cm} \le (\frac{1}{0.35})\Delta_c} \quad (16)$$

The formulae of eqns. (15) and (16) suggested by the authors gives the maximum allowable drift as a function of the height of one story, the number of stories, the effective depth of the shear walls used, and the elastic properties of the materials used in the shear wall (steel and concrete).

7. Examples

In order to see how these formulas work, consider an example of a 20 story building with shear walls having variable lateral stiffness as we go up the building as follows:

- from level 1 to 5 the stiffness is k,
- from level 6 to 10 the stiffness is 0.9 k,
- from level 11 to 15 the stiffness is 0.8k,
- and from level 16 to 20 the stiffness is 0.7k.

Yield Strength, f_y = 414 Mpa, Modulus of Elasticity, E_s = 2.105 Mpa, Reinforcement Steel Yield Strain, ε_y = 0.00207, Story Height, L = 3m; Concrete Crushing Strain ε_c = 0.003. For the sake of simplicity of calculations, load P_1 is taken such that the influence area around the shear wall is $(2h)^2$

with a slab thickness of 0.25 m, but in real evaluation it can be calculated exactly, where h is the shear wall length.

Table 1 presents the values of the maximum allowable displacement for constant and variable stiffness shear walls with length h= 3m and effective depth d=0.9h=2.7m, and for h=4 with d=3.6m. The shear walls have a constant width b=0.3m.

Table 1. *Allowable drift for Example 1 using Eqns (14)&(15).*

Story	Height (m)	Dm (Constant Stiffness)	Dm (Variable Stiffness)	Dm (Constant Stiffness)	Dm (Variable Stiffness)
		h=3, d=2.7 m		h=4, d=3.6 m	
1	3	0.0093	0.0093	0.0073	0.0073
2	6	0.0157	0.0157	0.0124	0.0124
3	9	0.0220	0.0220	0.0173	0.0173
4	12	0.0282	0.0282	0.0222	0.0222
5	15	0.0345	0.0345	0.0270	0.0270
6	18	0.0408	0.0400	0.0321	0.0313
7	21	0.0470	0.0460	0.0370	0.0362
8	24	0.0533	0.0522	0.0419	0.0410
9	27	0.0596	0.0584	0.0468	0.0458
10	30	0.0658	0.0645	0.0518	0.0508
11	33	0.0721	0.0629	0.0567	0.0493
12	36	0.0784	0.0684	0.0616	0.0536
13	39	0.0846	0.0737	0.0665	0.0579
14	42	0.0909	0.0792	0.0714	0.0622
15	45	0.0972	0.0848	0.0764	0.0666
16	48	0.1034	0.0789	0.0813	0.0619
17	51	0.1097	0.0837	0.0862	0.0657
18	54	0.1160	0.0885	0.0911	0.0694
19	57	0.1222	0.0932	0.0961	0.0733
20	60	0.1285	0.0980	0.1010	0.0770

Results show that the effect of adding the contribution of the axial load P_1 on the drift limitation value for a 20 story building is around an addition of 10% increase in the story drift depending whether the structure is with constant or variable stiffness. This percentage increases progressively with increasing the number of stories due to the increase in the axial load P_1.

7.1. Comparison of Results with Various Codes

On performing a comparison for the maximum allowable lateral displacement between eqn.(16) and selected seismic codes as presented in Table 2 below, it can be observed that results of this equation lie between the formula suggested by Mark Fintel [16] and the French codes PS92 [3], while UBC97 [1] gives a larger value. The important point that needs to be made is that the suggested eqns. (15) and (16) considers the geometry of the shear wall and the effective depth of the reinforcing steel. They also consider the height of a story, while the code formulas consider only the height of the building disregarding any structural and material properties.

Table 2. *Allowable drift comparison between Eqn. (15) and various codes.*

Story	Height (m)	Dm (Constant Stiffness) Khouri-Elias	Dm (Constant Stiffness) Lebanese Code "H/2000"	Dm (Constant Stiffness) M. Fintel "H/500"	Dm (Constant Stiffness) PS92 "H/250"	Dm (Constant Stiffness) UBC97-IBC2006 "H/50"
		h=3, d=2.7 m				
1	3	0.0093	0.0015	0.006	0.012	0.06
2	6	0.0157	0.003	0.012	0.024	0.12
3	9	0.0220	0.0045	0.018	0.036	0.18
4	12	0.0282	0.006	0.024	0.048	0.24
5	15	0.0345	0.0075	0.03	0.06	0.3
6	18	0.0408	0.009	0.036	0.072	0.36
7	21	0.0470	0.0105	0.042	0.084	0.42
8	24	0.0533	0.012	0.048	0.096	0.48
9	27	0.0596	0.0135	0.054	0.108	0.54
10	30	0.0658	0.015	0.06	0.12	0.6
11	33	0.0721	0.0165	0.066	0.132	0.66
12	36	0.0784	0.018	0.072	0.144	0.72
13	39	0.0846	0.0195	0.078	0.156	0.78
14	42	0.0909	0.021	0.084	0.168	0.84
15	45	0.0972	0.0225	0.09	0.18	0.9
16	48	0.1034	0.024	0.096	0.192	0.96
17	51	0.1097	0.0255	0.102	0.204	1.02
18	54	0.1160	0.027	0.108	0.216	1.08
19	57	0.1222	0.0285	0.114	0.228	1.14
20	60	0.1285	0.03	0.12	0.24	1.2

8. Conclusion

In this study, shear building was analyzed using the finite element method for both constant and variable stiffness shear walls and the contribution of the vertical load to the strain was considered. The shear was obtained as a function of the displacement. A value for the displacement at any story was obtained, and from which a function for the relative displacement between two stories was then determined. Using the above, an equation for the maximum strain was resolved. A limiting value for the maximum displacement within the elastic limits was obtained as a function of the height of a story, the stiffness of a story, number of stories, effective depth d of a shear wall, the yield strain of steel ε_y and the maximum allowable concrete strain ε_c.

On the other hand, the value h/50 suggested by UBC97 [1] and IBC 2006 [2] generates large strains at the bottom of a shear wall; it is important to note that even though UBC and other codes consider the non-linear inelastic behavior of the structure and high drift values correspond to a flexible structure thereby lower lateral forces, such large displacements limits may be dangerous when the structure depends on shear walls for lateral stiffness. With this in mind, it is pertinent to mention that UBC provides a restriction on inelastic inter-story drift which will be evaluated in a future work.

It is now left for the designing engineer to evaluate his structure and decide/choose a maximum allowable strain limit for concrete and steel, and determine the corresponding maximum allowable displacement values. Finally, the formulae suggested by the authors can serve as a starting point after which the designing engineer would know that the shear wall in question has passed the elastic limit.

Acknowledgements

The author thanks Optimal Engineering Consulting and Contracting, (OECC) for sponsoring the major part of this work and special thanks to the Lebanese University, Faculty of Engineering, Branch II for sponsoring part of this on-going research.

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